AN SO(10) THEORY FOR NEUTRINO MASS

R. N. MOHAPATR[illegible]
~~TRIESTE~~ '03.

- CHALLENGES OF NEUTRINO PHYSICS

- SEESAW AND SUSY GUT

- MINIMAL SO(10) THEORY FOR ν-MASS AND DARK MATTER

EVIDENCES FOR $m_\nu \neq 0$

- SOLAR + ATMOSPHERIC
  KAMLAND + K2K
  $\Rightarrow$ $m_\nu \neq 0$ & $\theta_{\alpha i} \neq 0$.

- **MIXING MATRIX** (IN THE BASIS WHERE e, μ, τ MASS EIGENSTATES)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\alpha i} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{\alpha i} \approx \begin{pmatrix} c & s & \varepsilon \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{s}{\sqrt{2}} & \frac{c}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

ALLOWED RANGES:

$$\Delta m^2_{\odot} \approx 4.7\times10^{-5}\,eV^2 - 10^{-4}\,eV^2$$

$$\sin^2 2\theta_{\odot} \approx .71 - .94$$

(3σ)

DE HOLANDA, SMIRNOV
hep-ph/03092

$$\Delta m^2_A = 1.4\times10^{-3} - 5.1\times10^{-3}\,eV^2$$

(3σ)

$$\sin^2 2\theta_A > .86$$

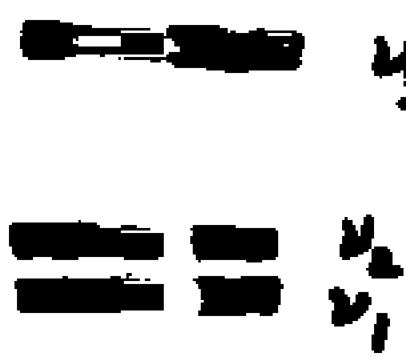
$$U_{e3} \leq .16 - .2$$

ν-MASS PATTERN

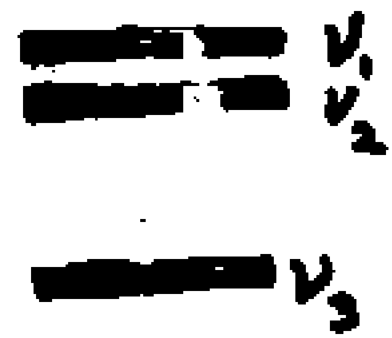
UNKNOWN AND IS AN IMPORTANT SOUGHT AFTER ITEM !!

- ^3H-DECAY: $m_i \lesssim 2.2\,eV$; WMAP: $\Sigma m_i \lesssim .71$ to 2 eV
- PATTERN:

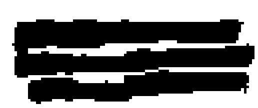
(i) <u>NORMAL</u>:



(ii) <u>INVERTED</u>



(iii) <u>DEGENERATE</u>



THEORETICAL CHALENGES FROM ν- MASS OBSERVATION

✓1. WHY $m_\nu \ll m_{e,u,d}$?

2. WHY θ_{12}, θ_{23} SO LARGE ?

IS IT COMPATIBLE WITH GUTS - Q-L UNIFICATION

3. WHY $\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \simeq \frac{m_2}{m_3} \gg \frac{m_1}{m_2}$?

4. CP-VIOLATION (BORIS KAYSER TALK)

(i) $m_\nu \ll m_{u,d,e}$?

SEESAW MECHANISM

STD MODEL + ν_R

• MODELS WITHOUT PARITY SYM

TYPE I:

$$\mathcal{M}_\nu = - M_{\nu D} M_R^{-1} M_{\nu D}^T$$

(GELLMANN, RAMOND, SLANSKY; YANAGIDA; R.N.M., SENJANOVIĆ; GLASHOW …)

MODELS WITH PARITY SYM:
(LR, SO(10), E_6, …; SU(3)…)

↓

TYPE II:

$$\mathcal{M}_\nu = f v_L - M_{\nu D} \frac{1}{f v_R} M_{\nu D}^T$$

(R.N.M., SENJANOVIĆ '80; LAZARIDES, SHAFI, WETTERICH '81)

$$v_L \simeq \frac{v_{wk}^2}{v_R}$$

WHY IS SEESAW MECHANISM SO APPEALING ?

(A) $\nu_R \Rightarrow$ COMPLETE QUARK-LEPTON SYM.

(B) MAKES PARITY AN ASYMPTOTI[C] SYM. OF WEAK INTERACTION

(C) $\Delta m^2_A \Rightarrow M_R \doteq 10^{15}$ GeV

IMPLYING THAT M_R COULD BE LINKED TO GUT SCALE,

SO(10) AS THE GUT GR[OUP]

(D) N_R-DECAY CAN EXPLAIN THE ORIGIN OF MATTER; FUKUGITA YANAGIDA,..

(ii) LARGE MIXINGS

- DEPENDS ON MASS PATTERN:

- TWO LEADING CANDIDATES:

(A)

$$m_L = \begin{pmatrix} \varepsilon_1 & M_1 & M_2 \\ M_1 & \varepsilon_2 & \varepsilon_3 \\ M_2 & \varepsilon_3 & \varepsilon_4 \end{pmatrix}$$

$\varepsilon_i \ll M_i \Rightarrow$ APPROX. $L_e - L_\mu - L_\tau$ SYM

INVERTED HIERARCHY

B. NORMAL HIERARCHY

$$m_\nu = \begin{pmatrix} \epsilon^{n'} & \epsilon^{n} & b\epsilon \\ \epsilon^{n} & 1+a\epsilon & 1 \\ b\epsilon & 1 & 1+\epsilon \end{pmatrix} \sqrt{\Delta m^2_A}$$

$\epsilon \sim \lambda = .22$; $a, b \sim 1$

$n, n' \geq 1$

SIMILARITY OF MASS PATTERN TO QUARKS & e, μ, ...

• STD MODEL + ϕ_2, η^+

$$M_{\nu,\phi} = \frac{f_{\alpha\beta}}{16\pi^2 M}(m_\alpha^2 - m_\beta^2)$$

$$= \begin{pmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 0 \end{pmatrix}$$

(Zee, Wolfenstein

REALISTIC BUT PREDICTS:

• $$\sin^2 2\theta_\odot = 1 - \frac{1}{16}\left(\frac{\Delta m_\odot^2}{\Delta m_A^2}\right)^2$$

$$\geq .99 .$$

EXPT. $\leq .94$

(SNO SALT PHASE).

LARGE MIXING FOR QUASI-DEG. ν'S:

$\times$

Q-L UNIFICATION

➡ AT SEESAW SCALE

$$\theta_{quark} \approx \theta_{lepton}$$

- TYPE II SEESAW FOR QUASI-DEGENERACY

$$\mathcal{M}_\nu = f v_L - \frac{1}{v_{BL}} M_{\nu D} f^{-1} M_{\nu D}^T$$

(AT $\mu = v_B$

- RADIATIVE MAGNIFICATION OF SMALL MIXINGS:

$$\mathcal{M}_\nu(M_Z) = \begin{pmatrix} 1+\delta_e & & \\ & 1+\delta_\mu & \\ & & 1+\delta_\tau \end{pmatrix} \mathcal{M}_\nu(v_{BL}) \begin{pmatrix} 1+\delta_e & & \\ & 1+\delta_\mu & \\ & & \end{pmatrix}$$

$\delta_\cdot - h_l^2 \ln v_{BL}$

AT SEESAW SCALE

$$\theta_{12} = \theta_{CABIBBO}, \quad \theta_{23} = V_{cb}; \quad \theta_{13} = V_{ub}$$

PARIDA, RAJASEKARAN, R.N.

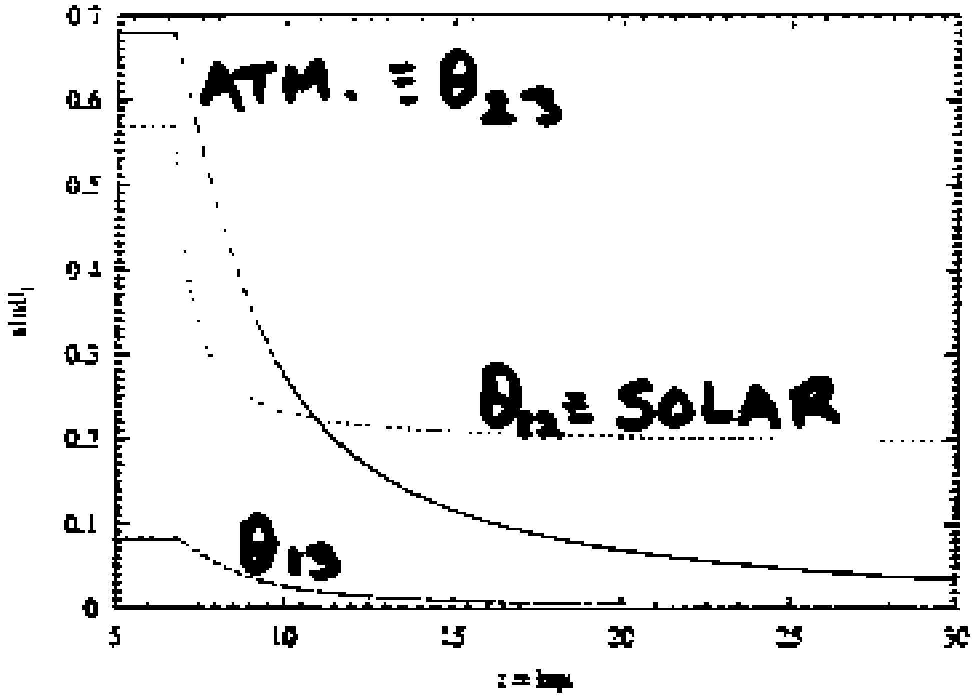


FIG. 1: Evolution of small quark-like mixings at the see-saw scale to bilarge neutrino mixings at low energies. The solid, dashed and dotted lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively, as defined in the text.

WEAK SCALE VALUES:

$$\sin^2 2\theta_A = .84 - .94$$
$$\sin^2 2\theta_\odot = .78 - .86$$
$$U_{e3} \simeq .08$$
$$m_{ee} > .1 \text{ eV}$$

FROM MASS MATRIX

TO

FUNDAMENTAL THEORY

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.

SEESAW APPROACH

$\Rightarrow$ STD MODEL $+ \nu_R$

$\Rightarrow$ $M_R \approx M_U$

- SUSY GUT A NATURAL FRAMEWORK TO STABILIZE GAUGE HIER[illegible]
- OTHER VIRTUES OF SUSY

a) DARK MATTER WITH R-PARITY

b) EW SYM. BREAKING (UNDER STRESS ?)

SO(10): A NATURAL GUT THEORY FOR m_ν

(i) $\nu_R \subset \{16\}_F : \begin{pmatrix} u \\ d \end{pmatrix}_{i, {L \atop R}} \begin{pmatrix} \nu \\ e \end{pmatrix}_{L \atop R} \ldots$

(ii) $U(1)_{B-L} \subset SO(10)$

- ALL INGREDIENTS FOR SEESA[W]

- ALL SUSY SO(10) MODELS[:]
DO NOT HAVE A DARK MATTE[R]

- $16_H \supset \tilde{\nu}_{R,H} \Rightarrow \langle \tilde{\nu}_{R,H} \rangle \neq$
(ALBRIGHT'S TALK) $\quad$ $B-L=1$

- $(126)_H \supset \Delta_{\nu_R \nu_R}$ $\quad$ $B-L=2$

ADVANTAGE OF 126_H

- $\langle 16_H \rangle$ BREAKS R-PARITY WHEREAS $\{126\}_H$ GUARANTEES EXACT R-PARITY !!

[illegible]EE, R.N.M. '95
AULAKH, MELFO, [illegible], [illegible] ..

$$\because \quad R = (-1)^{3(B-L)+2S}$$

$\langle 126_H \rangle \neq 0$ BREAKS B-L BY 2 UNITS
$\Rightarrow$ R-PARITY IS A GOOD SYM. OF LOW ENERGY THEORY.

- MAKES MSSM NATURAL (NO QLd^c, $u^c d^c d^c$ TERMS)

$\Rightarrow$ GUARANTEES STABLE DARK MATTER.

MINIMAL SO(10)

COMMON DEFINITION

"MY MODEL IS MORE MINIMAL THAN YOURS."

BETTER DEFINITION

- "ALLOW ALL COUPLINGS POSSIBLE IN YOUR SUPERPOT."
- "USE NO SYMMETRIES OTHER THAN THE GAUGE SYM."

A MINIMAL SO(10) AND m_ν

ABU, R.N.M./92

<u>MINIMAL HIGGS:</u> $\{10\}$ $\{\overline{126}\}$

$$h\,\psi\psi\{10\} + f\,\psi\psi\{\overline{126}\}$$

$$\Rightarrow M_u = h\langle 10\rangle_u + f\langle\overline{126}\rangle_u$$

$$M_d = h\langle 10\rangle_d + f\langle\overline{126}\rangle_d$$

$$M_\ell = h\langle 10\rangle_d - 3f\langle\overline{126}\rangle_d$$

$$M_{\nu D} = h\langle 10\rangle_u - 3f\langle\overline{126}\rangle_u$$

$$M_\nu = f\langle 126\rangle_{LL} - M_{\nu D}(f v_R)^{-1} M_{\nu D}^T$$

I.E. $v_L \simeq M_W$

$$\Rightarrow M_\nu \simeq f\,\frac{v_{\ldots}^2}{2 M_{\ldots}}$$

(… … …)

PARAMETER COUNTING

x

h :	3
f :	6
4 VEVS :	4

$13 - 1\,(M_Z) = 12$

ALL DERMINED BY

$m_{u,c,t}$; $m_{d,s,b}$; $m_{e,\mu,\tau}$; U_{CKM}.

- FLAVOR STRUCTURE FOR NEUTRINOS COMPLETELY DETERMINED !!

(ALL ~~OF~~ IN SQUARK, SLEPTON SECTOR)

TWO INGREDIENTS OF THE MODEL THAT EXPLAIN LARGE NEUTRINO MIXINGS:

— x —

(A) SUMRULE FOR M_ℓ:

$$k M_\ell = r M_d + M_u$$

⇒ CHARGED LEPTON MIXINGS $\approx \lambda$
$\approx .2$

$$U_{PMNS} = U_\ell^\dagger U_\nu \simeq U_\nu$$

3) TYPE II SEESAW AND A SUMRULE FOR M_ν:

IF $M_\nu = f\langle 126\rangle_{LL} + \text{SMA}$

$$\Rightarrow \boxed{M_\nu \simeq c(M_d - M_\ell)}$$

<u>TWO GEN.</u> (BAJC, SENJANOVIĆ, VISSANI, 2002)

$$M_d \simeq m_b \begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix} ; \quad M_\ell \simeq m_\tau \begin{pmatrix} \lambda^2 & \lambda \\ \lambda^2 & 1 \end{pmatrix}$$

PHENOMENOLOGICALLY WE KNOW:

MSSM

$$m_b(M_U) = m_\tau(M_U)(1+\varepsilon)$$

$$\varepsilon \approx -.09 \text{ to } .2$$

($\tan\beta = 10$)

$$\Rightarrow \quad M_\nu = c(M_d - M_\ell)$$

$$= c \begin{pmatrix} \lambda^2 + \lambda & \lambda^2 \\ \lambda^2 & \lambda^2 \end{pmatrix} m_\tau$$

- $\theta_{PMNS,23} \approx \theta^\nu_{23} \approx$ LARGE;
- $m_2 \ll m_3$; $\Delta m^2_\odot \ll \Delta m^2_A$

DOES IT WORK FOR 3-GENERATIONS?

Goh, R.N.M., Ng
hep-ph/0303055

$$M_{d,\ell} = m_{b,\tau} \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\Rightarrow \quad \theta'_{12} \sim \lambda; \quad \theta'_{23} \sim \lambda^2; \quad \theta'_{13} \sim \lambda^3.$$

$$M_\nu = c\,(M_d - M_\ell)$$

$$M_\nu \simeq c \begin{pmatrix} \lambda^4 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}$$

$\Rightarrow \theta_{23}, \theta_{12}$ LARGE; $\theta_{13} \simeq \lambda$; $\frac{m_2}{m_3} \sim \lambda \simeq 1/5$

$$\frac{m_t^2}{M_U} \approx \frac{4\times10^4}{2\times10^{16}} = .02$$

DETAILED COMPUTATION.

GUT SCALE VALUES:

TABLE [illegible]
DAS, PARIDA
hep-ph/0010004

INPUT.

input observable	$tan\beta = 10$
m_u (MeV)	$0.7238^{+0.1365}_{-0.1467}$
m_c (MeV)	$210.3273^{+19.0036}_{-21.2264}$
m_t (GeV)	$82.4333^{+30.2676}_{-14.7686}$
m_d (MeV)	$1.5036^{+0.4235}_{-0.2304}$
m_s (MeV)	$29.9454^{+4.3001}_{-4.5444}$
m_b (GeV)	$1.0636^{+0.1414}_{-0.0865}$
m_e (MeV)	0.3585
m_μ (MeV)	$75.6715^{+0.0578}_{-0.0501}$
m_τ (GeV)	$1.2922^{+0.0013}_{-0.0012}$

INPUT

$$V_{CKM} = \begin{pmatrix} 0.974836 & 0.222899 & -0.00319129 \\ -0.222638 & 0.974217 & 0.0365224 \\ 0.0112498 & -0.0348928 & 0.999328 \end{pmatrix}$$

FITS TO $m_{e,\mu,\tau} \Rightarrow r, k$

$m_{s,d,c} < 0$

$-.78 \leq r \leq -.74$
$.23 \leq k \leq .26$

⇐ 2nd & 3rd Gen.

SOME DETAILS:

SUM RULES:

- $\tilde{M}_\ell = -3\tilde{M}_d + 4\, U_{CKM}^T \tilde{M}_u U_{CKM}$
- $M_\nu = \left(\frac{m_b}{m_\tau} + 3\right)\tilde{M}_d - 4 U_{CKM}^T \tilde{M}_u U_{C\ldots}$

$$\Rightarrow \tilde{M}_\ell \simeq \begin{pmatrix} -3\tilde{m}_d & 4\lambda\tilde{m}_c + 4A^2\lambda^5(\rho - i\eta) & 4A\lambda^3 \\ & & |1-\rho\ldots \\ - & -3\tilde{m}_s & 4A\lambda^2 \ldots \\ - & - & 1 \ldots \end{pmatrix}$$

$\tilde{m}_d \sim 2\lambda^4;\ \tilde{m}_c \sim \lambda^4;\ \tilde{m}_s \sim .7\lambda^2;\ \tilde{m}_e \sim \lambda^5$

$$\Rightarrow \quad \tilde{m}_e \simeq -3\tilde{m}_d - 16A^2|1-\rho-i\eta|^2\lambda^6$$

$\tilde{m}_d < 0$, FINE TUNING TO GET $\tilde{m}_e$!!

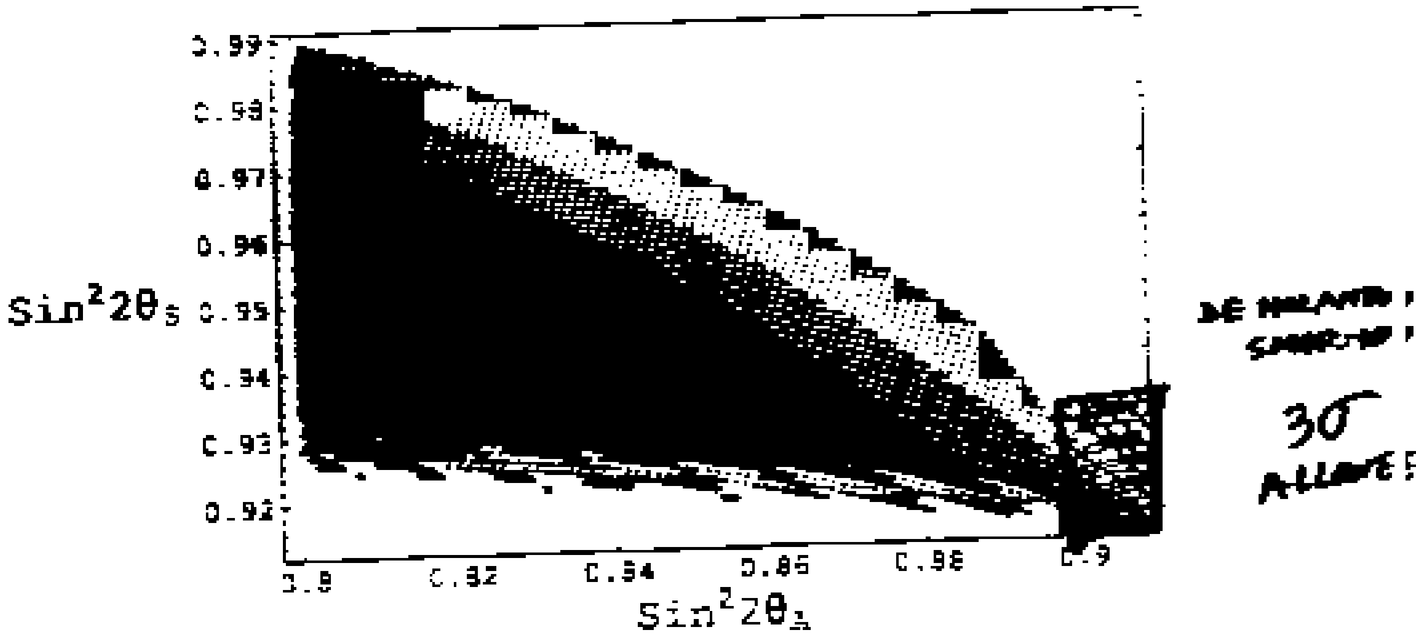


FIG. 1. The figure shows the predictions for $\sin^2 2\theta_S$ and $\sin^2 2\theta_A$ for the range of quark masses in table I. Note that $\sin^2 2\theta_S \geq 0.9$ and $\sin^2 2\theta_A \leq 0.9$

ALLOWED $Sin^2 2\theta_A$ = .85 – 1   99% C

.9 – 1   90% C

K2K, SUPER-K, COMBINED.

SK, SNO, KAMLAND; $Sin^2 2\theta_S$ = .66 – .99   99%

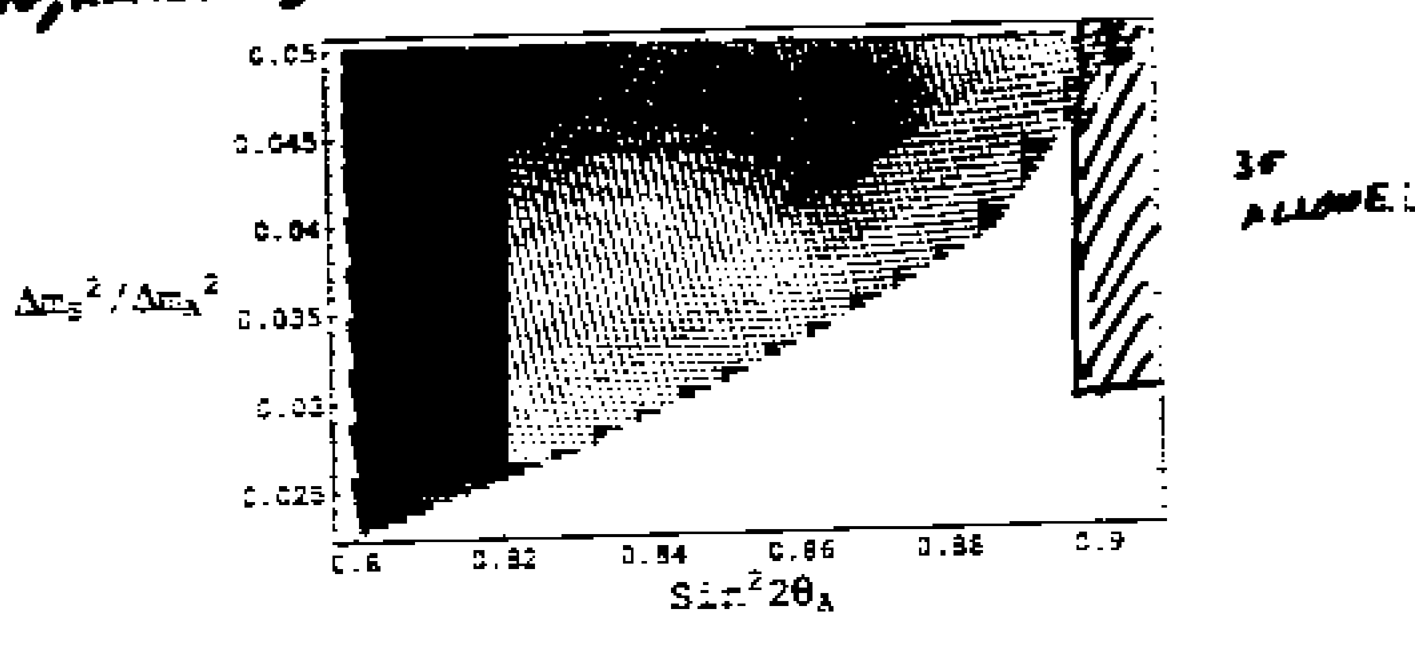


FIG. 2. The figure shows the predictions for $\sin^2 2\theta_A$ and $\Delta m_S^2 / \Delta m_A^2$ for the range of quark masses and mixings that fit charged lepton masses.

$$U_{e3} \simeq \frac{V_{ub}}{\frac{m_b}{m_\tau} - 1} \approx \frac{\lambda^3}{\lambda^2} \approx \lambda$$

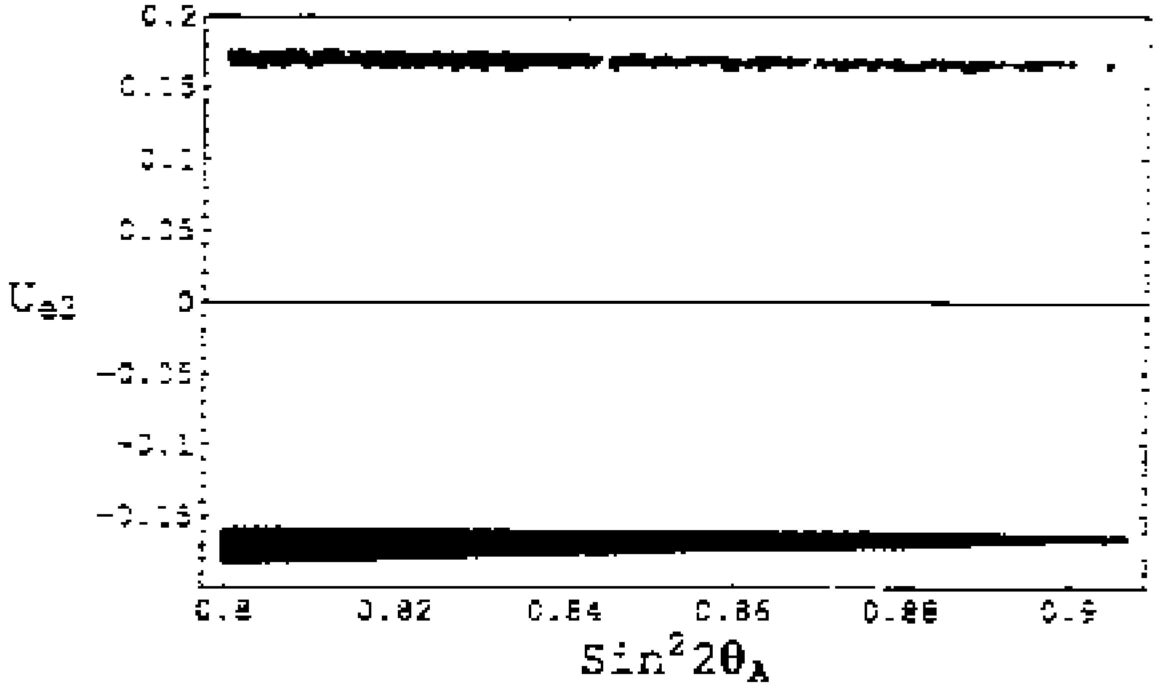
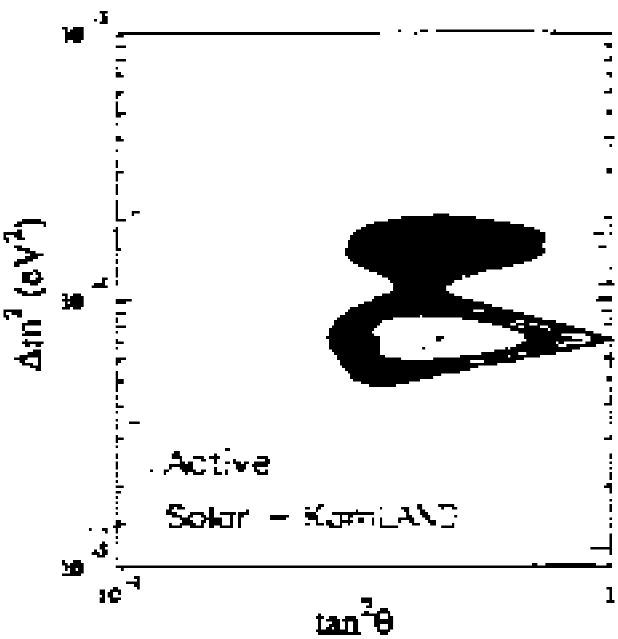


FIG. 3. The figure shows the predictions of the model for $\sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

SOLAR



ATMOSPHERIC

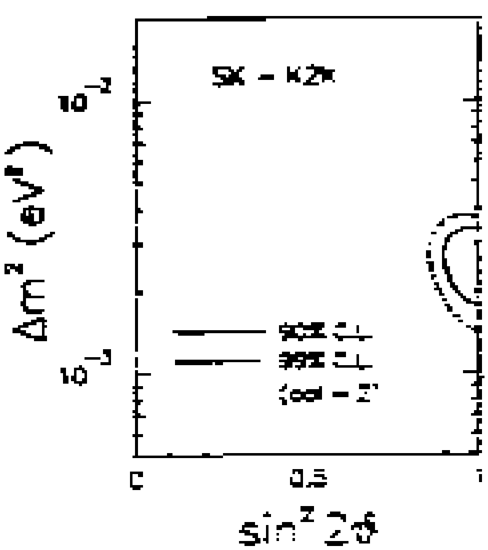


Fig. 3. Allowed region obtained from the analysis of Super-Kamiokande atmospheric and K2K data in terms of $\nu_\mu \to \nu_\tau$ oscillations. Figure from Ref. [12].

INCLUDE CP-PHASES:

7-PHASES - α_u NOT RELEVANT, $\alpha_{c,t,d,s,b}$ DETERMINED BY m_e FIT & MAXIMAL MIXING CONDITION

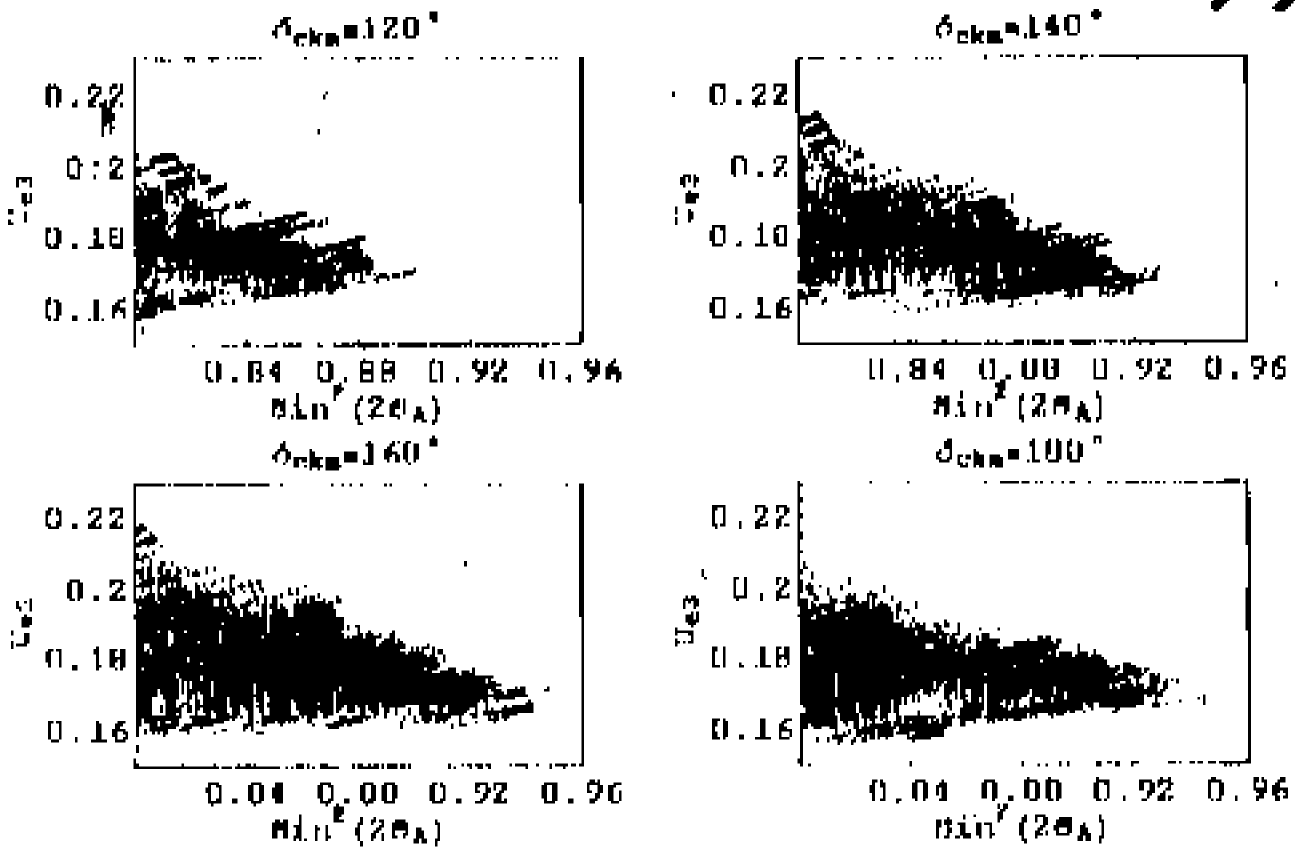


FIG. 6. The figure shows the predictions for $\sin^2 2\theta_A$ and U_{e3} for the range of quark masses and mixings that fit charged lepton masses and where all CP phases in the fermion masses are included, for four different values of δ_{CKM}.

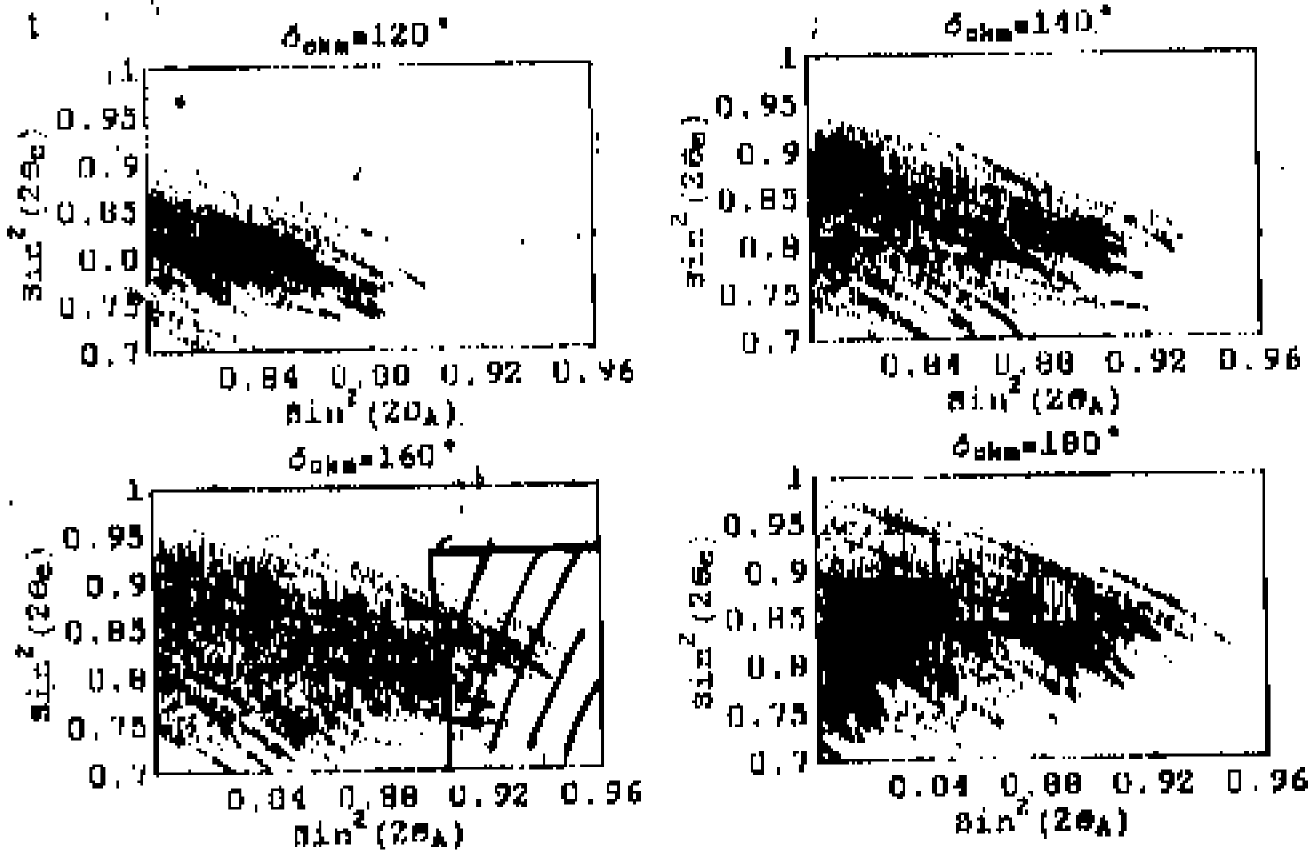


FIG. 4. The figure shows the predictions for $\sin^2 2\theta_A$ and $\sin^2 2\theta_\odot$ for the range of quark masses and mixings that fit charged lepton masses in the presence of all CP phases in the fermion masses. The four different panels give the predictions for different values of the CKM phase ($\tan \delta_{CKM} = \frac{\eta}{\rho}$). Note that all these values are outside the one sigma region of the present standard model fit to all CP violating data. Note that the case $\delta_{CKM} = 180$ includes the CP conserving case discussed in sec. IV.

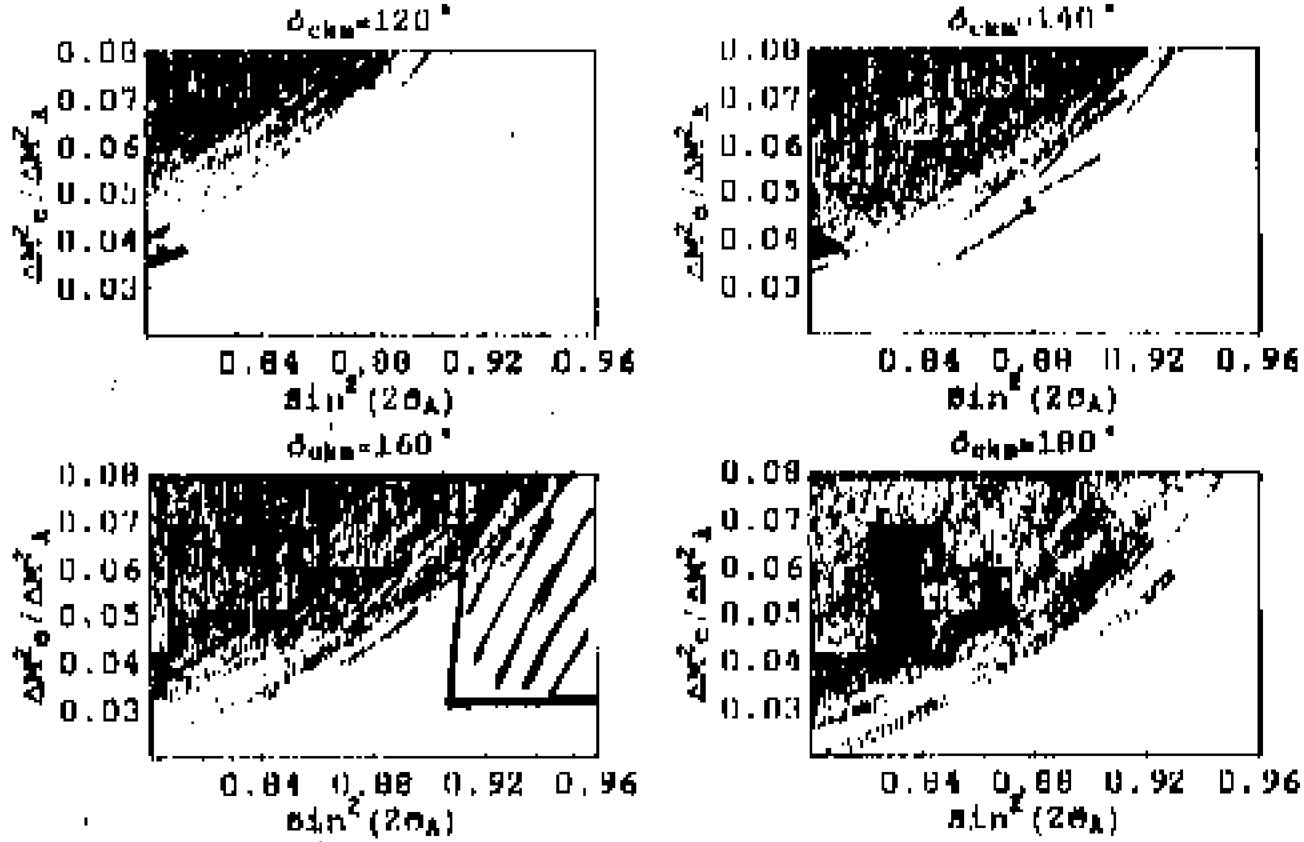


FIG. 5. The figure shows the predictions for $\sin^2 2\theta_A$ and $\Delta m^2_{\odot}/\Delta m^2_A$ for the range of quark masses and mixings that fit charged lepton masses and where all CP phases in the fermion masses are kept subject to the condition that b-tau mass convergence is responsible for large neutrino mixings, for four different values of the δ_{CKM}.

PREDICTION FOR DIRAC PHASE:

—— × ——

7 PHASES: $\alpha_{1,2,3,4,5,6}$
$+ 1$

- α_6 CONTRIBUTES NEGLIGIBLY
- $\alpha_b = \alpha_4 = \alpha_5$ FOR MAXIMAL MIXING
- $\alpha_{a,c}$ FROM m_e FIT

$$\Rightarrow \quad U_{PMNS} = U \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & e^{i\alpha_3} \end{pmatrix}$$

↓ REAL TO λ^0

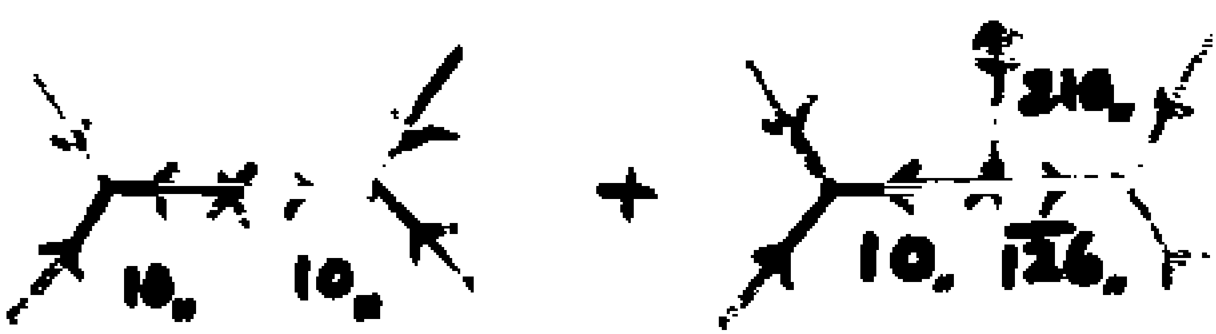
$\Rightarrow$ DIRAC PHASE $\sim \lambda$

PROTON DECAY

(w/ NASRI)

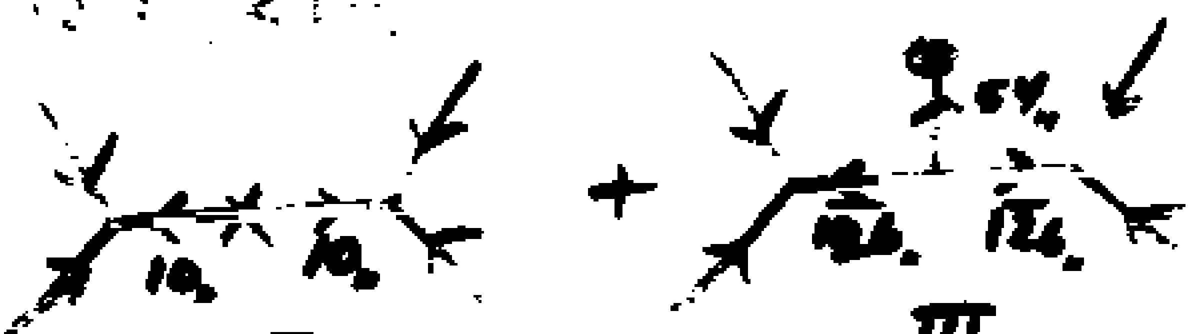
DEPENDS ON HOW SO(10) IS BROKEN:

(i) {210}



I II

(ii) {54} + 210



I III

+ II

$p \to K\nu$ SUPPRESSED BUT NOT $p \to e^+\pi^0$

OTHER ISSUES:

A) SO(10) BREAKING

TYPE II SEE SAW

+ τ_p

$\Rightarrow$ $SO(10) \rightarrow SU(5) \rightarrow MSSM$

$M_U > 2 \times 10^{16}$ GeV

B) POST-GUT GAUGE COUPLING UNIF.

CONCLUSION

(i) $m_\nu \longrightarrow$ SEESAW + COUPLING UNI[FICATION] / DM

$\Rightarrow$ STRONG CASE FOR SO(10) WITH 126_H

(ii) MINIMAL MODEL TESTABLE VIA U_{e3} ;

(iii) CAN SOLVE μ- PROBLEM WITH AN EXTRA R- SYM.