Fermion Masses and Mixings in the Little Flavon Model

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The Experiments

Fermion Masses

m_t/m_c	248 ± 70
m_c/m_u	325 ± 200
m_b/m_s	40 ± 10
m_s/m_d	430 ± 300
m_τ/m_μ	17
m_μ/m_e	207
m_τ/m_e	3484
$\Delta m^2_\odot/\Delta m^2_\oplus$	$0.014 - 0.076$

Mixing Matrices

$\|V_{us}\|$	$0.219 - 0.226$
$\|V_{ub}\|$	$0.002 - 0.005$
$\|V_{cb}\|$	$0.037 - 0.043$
$\|V_{td}\|$	$0.004 - 0.014$
$\|V_{ts}\|$	$0.035 - 0.043$
$\sin 2\beta$	$0.705^{+0.042}_{-0.032}$
$\tan^2\theta_\odot$	$0.23 - 0.69$
$\sin^2 2\theta_\oplus$	$0.8 - 1.0$
$\sin^2\theta_{13}$	< 0.09

hope

$\Downarrow$

a model to explain
some of these features

quarks

$$\mathcal{M}_u^{RL}/\mathrm{Gev} = 109 \begin{pmatrix} 0.000020 & 0.00020 & 0.0061\, e^{i\,0.7\,\pi} \\ 0.00034\, e^{i\,1.3\,\pi} & 0.0041\, e^{i\,0.7\,\pi} & 0.18\, e^{i\,1.4\,\pi} \\ 0.0061\, e^{i\,0.2\,\pi} & 0.044\, e^{i\,0.8\,\pi} & 1.6\, e^{i\,0.2\,\pi} \end{pmatrix}$$

only
a numerical example

$$\mathcal{M}_d^{RL}/\mathrm{Gev} = 2.8 \begin{pmatrix} 0.00016\, e^{i\,1.4\,\pi} & 0.00057\, e^{i\,1.2\,\pi} & 0.034\, e^{i\,0.4\,\pi} \\ 0.0061\, e^{i\,1.2\,\pi} & 0.032\, e^{i\,0.4\,\pi} & 0.89\, e^{i\,0.3\,\pi} \\ 0.0051\, e^{-i\,0.3\,\pi} & 0.036\, e^{i\,\pi} & 1.6\, e^{-i\,0.5\,\pi} \end{pmatrix}$$

leptons

FINE TUNING

$$\mathcal{M}_e^{RL}/\mathrm{Gev} = 2.3 \begin{pmatrix} 0.00079 & 0.035 & 0.045 \\ -0.0061 & 0.39 & 0.37 \\ 0.0074 & 0.32 & 0.39 \end{pmatrix}$$

...can they come from
well-defined textures
+
parameters O(1)?

?

$$\mathcal{M}_\nu^{LL}/\mathrm{Gev} = 10^{-9} \begin{pmatrix} 0.0043 & -1.3 & 0.093 \\ -1.3 & 0.76 & 0.46 \\ 0.093 & 0.46 & -7.0 \end{pmatrix}$$

quarks

$$\mathcal{M}_u^{RL}/\mathrm{Gev} = 247 \begin{pmatrix} \lambda_{11}(\varepsilon_1\varepsilon_2)^6 & \lambda_{12}(\varepsilon_1\varepsilon_2)^5 & \lambda_{13}(\varepsilon_1\varepsilon_2)^3 \\ \lambda_{21}(\varepsilon_1\varepsilon_2)^4 & \lambda_{22}(\varepsilon_1\varepsilon_2)^3 & \lambda_{23}(\varepsilon_1\varepsilon_2) \\ \lambda_{31}(\varepsilon_1\varepsilon_2)^3 & \lambda_{32}(\varepsilon_1\varepsilon_2)^2 & \lambda_{33}1 \end{pmatrix}$$

the textures

$$\mathcal{M}_d^{RL}/\mathrm{Gev} = 247\,(\varepsilon_1\varepsilon_2)^2 \begin{pmatrix} \tilde\lambda_{11}(\varepsilon_1\varepsilon_2)^5 & \tilde\lambda_{12}(\varepsilon_1\varepsilon_2)^4 & \tilde\lambda_{13}(\varepsilon_1\varepsilon_2)^2 \\ \tilde\lambda_{21}(\varepsilon_1\varepsilon_2)^3 & \tilde\lambda_{22}(\varepsilon_1\varepsilon_2)^2 & \tilde\lambda_{23}1 \\ \tilde\lambda_{31}(\varepsilon_1\varepsilon_2)^3 & \tilde\lambda_{32}(\varepsilon_1\varepsilon_2)^2 & \tilde\lambda_{33}1 \end{pmatrix}$$

leptons

$$\mathcal{M}_e^{RL}/\mathrm{Gev} = 0.0034 \begin{pmatrix} \tilde\gamma_{11}\,(\varepsilon_1\varepsilon_2)^4 & \tilde\gamma_{12}\,\varepsilon_1^2\varepsilon_2 & \tilde\gamma_{13}\,\varepsilon_1\varepsilon_2^2 \\ \tilde\gamma_{21}\,\varepsilon_1^2\varepsilon_2^3 & \tilde\gamma_{22}1 & \tilde\gamma_{23}\,(\varepsilon_1\varepsilon_2) \\ \tilde\gamma_{31}\,\varepsilon_1^3\varepsilon_2^2 & \tilde\gamma_{32}\,(\varepsilon_1\varepsilon_2) & \tilde\gamma_{33}1 \end{pmatrix}$$

$$\mathcal{M}_\nu^{LL}/\mathrm{Gev} = 0.0000001 \begin{pmatrix} \gamma_{11}(\varepsilon_1\varepsilon_2)^4 & \gamma_{12}\varepsilon_1^2\varepsilon_2 & \gamma_{13}\varepsilon_1\varepsilon_2^2 \\ \gamma_{21}\varepsilon_1^2\varepsilon_2 & \gamma_{22}\varepsilon_2^2 & \gamma_{23}\varepsilon_1\varepsilon_2 \\ \gamma_{31}\varepsilon_1\varepsilon_2^2 & \gamma_{32}\varepsilon_1\varepsilon_2 & \gamma_{33}\varepsilon_1^2 \end{pmatrix}$$

Low energy scale model

- **horizontal symmetry** $\Rightarrow SU(2)_F \times U(1)_F$ gauged
- **flavons** $\Rightarrow \phi_1, \phi_2$ **doublets of opposite F-hypercharge**

$$\phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \qquad \phi_2 = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$\varepsilon_{1,2}\, f$

vevs of our flavons

$SU(2)_F \times U(1)_F \rightarrow$ **nothing**

- $f \Rightarrow$ **high energy scale** $\Rightarrow$ **flavons arise as pseudo-Goldstone bosons**

Requests

- potential with the desired vacuum
- 3 stable scales $\rightarrow$ f, v_1, v_2
- only small radiative corrections $\rightarrow$ NO FINE TUNING !

$$\Downarrow$$

an inspired Little Higgs scenario

$$\Downarrow$$

Little Flavons

$SU(6) \longrightarrow Sp(6)$

$\Leftarrow \Lambda = 4\pi f$

spontaneously

14 d.o.f.

$\Sigma = e^{i\frac{\Pi}{f}}\Sigma_0$

$\underbrace{U(1)_P}_{global} \times \underbrace{[SU(2) \times U(1)]^2_F}_{gauged} \longrightarrow U(1)_P \times [SU(2) \times U(1)]_F$

4 massive
gauge bosons
$m^2 \simeq f^2$
massive singlet
massless $\phi_{1,2}$

potential *á la*
Coleman
Weinberg

BROKEN TO

$\Leftarrow f$

$\Leftarrow v_1$

4 massive
gauge bosons
$m^2 \simeq 10^{-2} f^2$
$+$
4 flavons

$\longleftarrow \underbrace{U(1)_{P'}}_{global}$

$\Downarrow$

textures

$\Leftarrow v_2$

General potential for 2 doublets

$$\begin{aligned} V_4(\phi_1,\phi_2) &= \mu_1^2\phi_1^\dagger\phi_1 + \mu_2^2\phi_2^\dagger\phi_2 + \cancel{(\mu_3^2\tilde{\phi}_1^\dagger\phi_2 + H.c.)} \\ &+ \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) \\ &+ \lambda_4|\tilde{\phi}_1^\dagger\phi_2|^2 + \lambda_5\cancel{[(\tilde{\phi}_1{}^\dagger\phi_2)^2 + H.c.]} \end{aligned}$$

from the quartic term in C.W. and $1-$ loop contributions of RH neutrinos

<u>forbidden</u> by $U(1)_P$

from the Λ^2 term in C.W.

- **Quarks** $\Rightarrow$ **small mixing angles** $\Rightarrow$ **all singlets under** $SU(2)_F$

	Q_{1L}	Q_{2L}	Q_{3L}	u_R	c_R	t_R	d_R	s_R	b_R
$U(1)_F$	3	2	0	-3	-1	0	-4	-2	-2

- **Leptons** $\Rightarrow$ 2 **large angles** $\Rightarrow$ **doublets and singlets**

	l_{eL}	$L_L = (l_\mu, l_\tau)_L$	e_R	$E_R = (\mu, \tau)_R$
$SU(2)_F$	1	2	1	2
$U(1)_F$	-2	$1/2$	2	$1/2$

$\Downarrow$

$\boxed{\text{Yukawa}}$

$\Downarrow$

coupling to Σ in a gauge invariant way

The Numerical Analysis

Fermion Masses

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$\lvert V_{ts}\rvert$	$0.035 - 0.043$	0.039
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$\tan^2\theta_\odot$	$0.23 - 0.69$	0.32
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$\sin^2\theta_{13}$	< 0.09	0.08

$\cdots \Rightarrow \cdots$ **Back to the Textures** $\cdots \Rightarrow \cdots$

$\Rightarrow$ main features determined by vacuum structure alone

…consider the limit $\varepsilon_1 \to 1\,, \varepsilon_2 \to k << 1$

$$\mathcal{M}_d^{(u,d,e)} = R^{(u,d,e)\,\dagger} \mathcal{M}^{(u,d,e)} L^{(u,d,e)} \qquad \mathcal{M}_d^{(\nu)} = L^{(\nu)\,T} \mathcal{M}^{(\nu)} L^{(\nu)}$$

$$\mathrm{CKM} = L^{(u)\,\dagger} L^{(d)} \qquad \mathrm{PMNS} = L^{(e)\,\dagger} L^{(\nu)}$$

$$\mathcal{M}^{u,d\,\dagger}\mathcal{M}^{u,d} = \begin{pmatrix} 0 & O(k^5) & O(k^3) \\ O(k^5) & O(k^4) & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix} \qquad \mathrm{CKM} = \begin{pmatrix} 1 & O(k) & O(k^3) \\ O(k) & 1 & O(k^2) \\ O(k^3) & O(k^2) & 1 \end{pmatrix}$$

$$\mathcal{M}^{\nu} = \begin{pmatrix} 0 & O(k) & O(k^2) \\ O(k) & O(k^2) & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix} \qquad \mathcal{M}^{e\,\dagger}\mathcal{M}^{e} = \begin{pmatrix} 0 & 0 & O(k^2) \\ 0 & 1 & O(k) \\ O(k^2) & O(k) & 1 \end{pmatrix}$$

why we consider $\boxed{\text{the little flavon}}$ a progress with respect to just textures?

- more restrictive $\Rightarrow \begin{cases} \text{we have a lagrangian} \\ \text{all textures in terms of 2 vevs} \end{cases}$
- stable
- defined predictions for flavons (and gauge flavons)
- even though impressive it is only an example $\Rightarrow$ a class of models to study (different groups, potentials, textures)