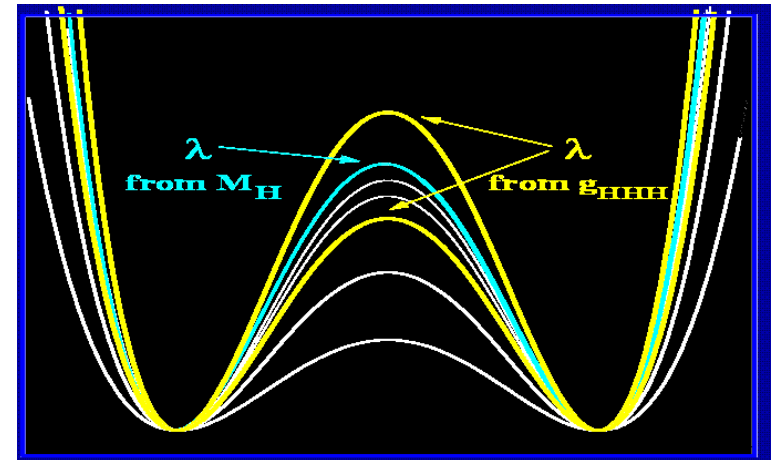


# Testing the Higgs Potential at $e^+e^-$ and Hadron Colliders

Albert De Roeck  
CERN



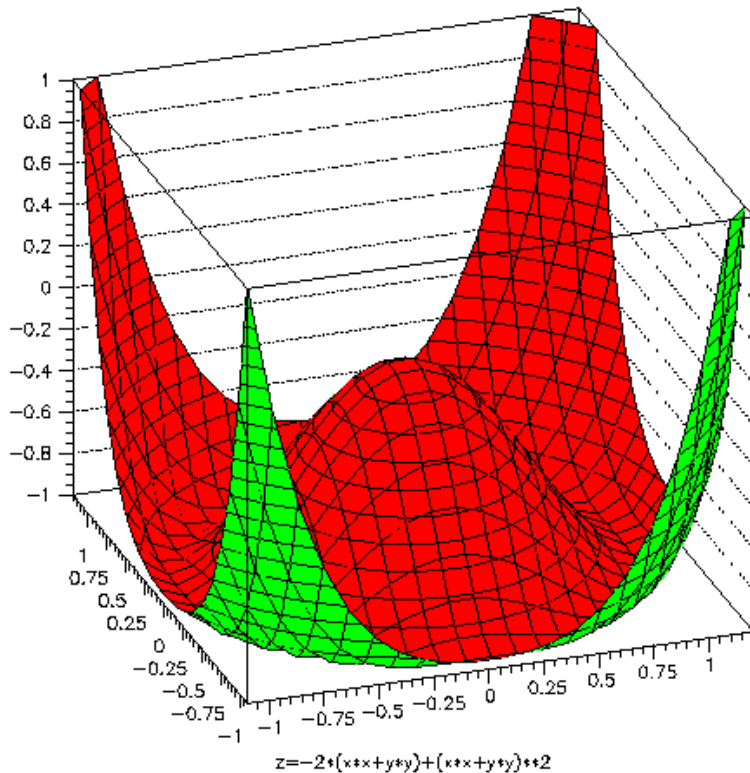
Thanks to M. Battaglia, U. Bauer, E. Boos, D. Rainwater

# Reconstructing the Higgs Potential

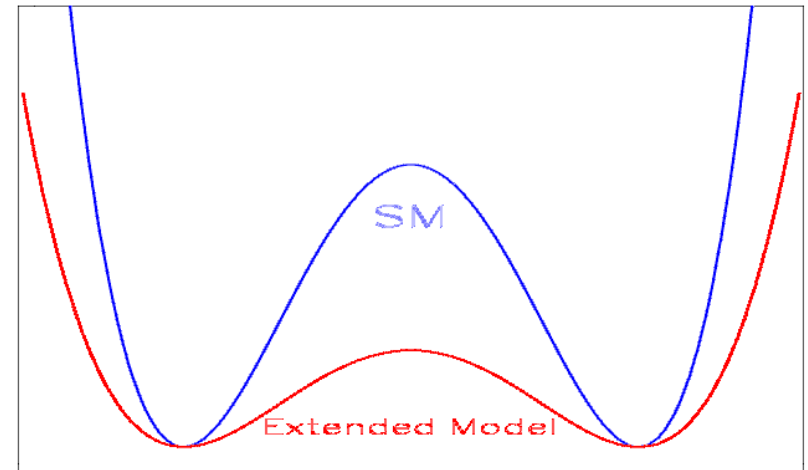
The shape of the Higgs potential

Determination of the shape needed for the complete investigation of the Higgs profile and to obtain a direct proof of the mechanism of EW symmetry breaking

$$V(\Phi^* \Phi) = \lambda \left( \bar{\Phi}^* \Phi - \frac{1}{2} v^2 \right)^2$$



$V\Phi$



$\Phi$

Tests may reveal the extended Nature of the Higgs sector

# Higgs potential

- Higgs potential can be expressed in terms of the Higgs field as:

$$V_H = \frac{m_H^2}{2} H^2 + \frac{m_H^2}{2v} H^3 + \frac{m_H^4}{8v^2} H^4$$

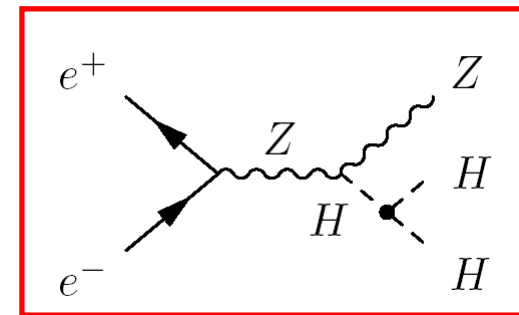
$$\lambda_{HHH} = 3m_H^2/v^*$$

$$\lambda_{HHHH} = 3m_H^2/v^2$$

$$v = (\sqrt{2G_F})^{-1/2} \simeq 246\text{GeV}$$

Access to the potential via Higgs self coupling (triple Higgs coupling), i.e. via double Higgs production

Quartic Higgs self-coupling 2-3 orders of magnitude smaller than triple Higgs coupling → concentrate on  $\lambda_{HHH}$



\* Sometimes other notation used:  $\lambda_{SM} = \frac{m_H^2}{2v^2}$   $V(\eta_H) = \frac{1}{2} m_H^2 \eta_H^2 + \lambda v \eta_H^3 + \frac{1}{4} \tilde{\lambda} \eta_H^4$

# Experimental prospects

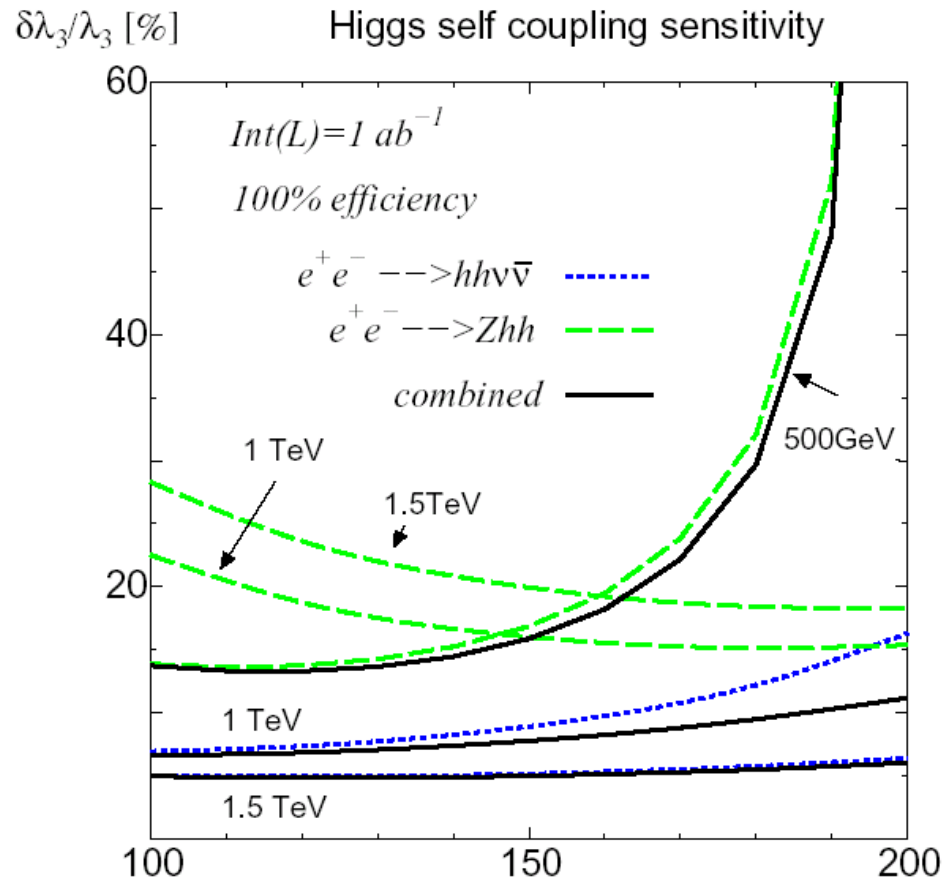
Colliders for which double Higgs production has been studied

- **TeV Class Linear Collider (TESLA, NLC, JLC)**  
 $e^+e^-$ ,  $\sqrt{s} = 500-1000 \text{ GeV}$   $(0.5-2) \text{ ab}^{-1}$   $2-3 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- **Photon collider**  
 $e^+e^-$ ,  $\sqrt{s} = 500-1000 \text{ GeV}$   $(0.5-2) \text{ ab}^{-1}$   $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- **Multi-TeV Linear Collider**  
 $e^+e^-$ ,  $\sqrt{s} = 1-5 \text{ TeV}$   $(3-5) \text{ ab}^{-1}$   $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- **LHC**  
 $pp$ ,  $\sqrt{s} = 14 \text{ TeV}$   $0.3-0.6 \text{ ab}^{-1}$   $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
- **SLHC**  
 $pp$ ,  $\sqrt{s} = 14 \text{ TeV}$   $3-6 \text{ ab}^{-1}$   $10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- **VLHC**  
 $pp$ ,  $\sqrt{s} = 200 \text{ TeV}$   $0.3-1.2 \text{ ab}^{-1}$   $2 \cdot 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$

# $\lambda_{HHH}$ at a LC

Processes:  $e^+e^- \rightarrow hhZ$      $e^+e^- \rightarrow (WW)\nu\bar{\nu} \rightarrow hh\nu\bar{\nu}$ .

Y.Yasui et al LCWS02



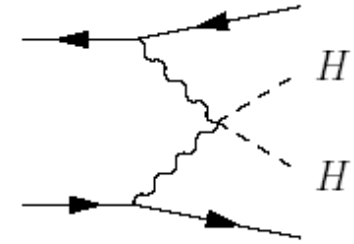
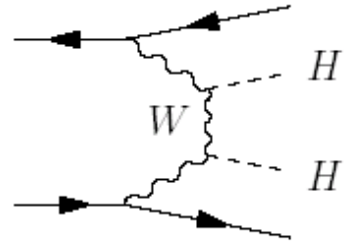
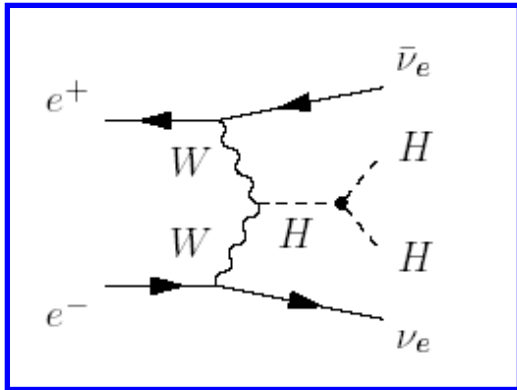
Parton level calculation  
e- beams assumed polarized

HHZ mode dominant at  
500 GeV

HHνν mode dominant  
at high beam energies

Promising sensitivity

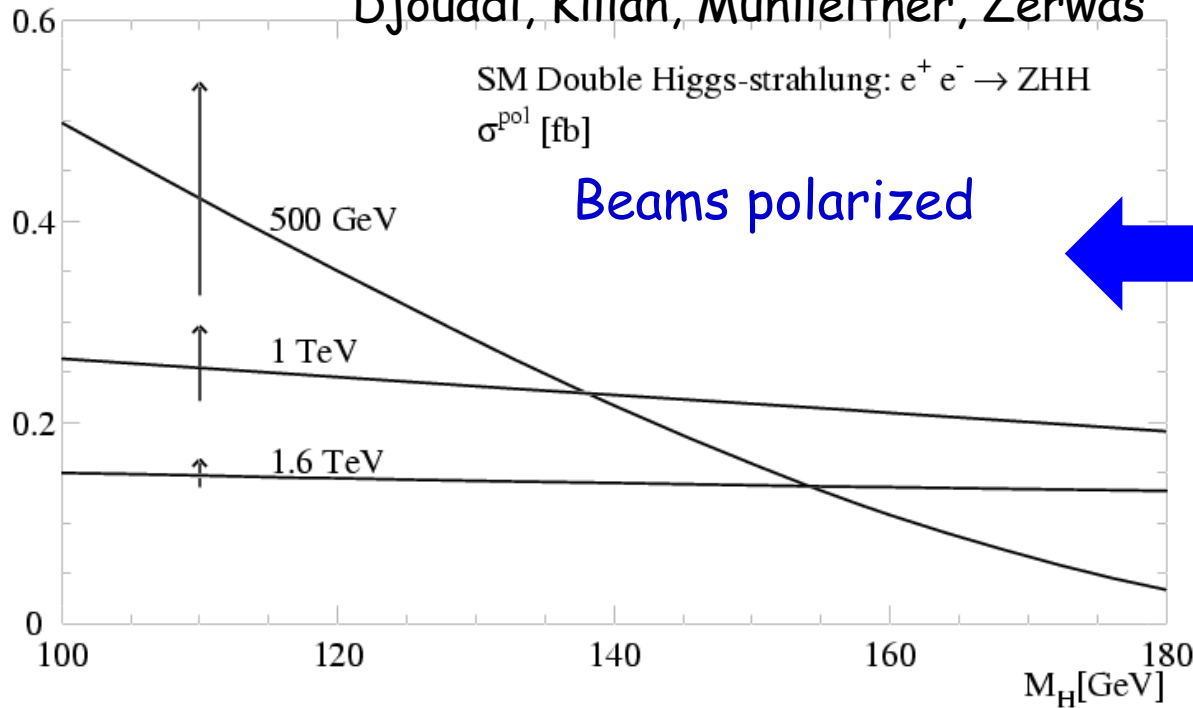
# $e^+e^- \rightarrow ZHH$



Only first diagram sensitive to  $\lambda_{HHH}$

Other diagrams dilute the sensitivity

Djouadi, Kilian, Muhlleitner, Zerwas



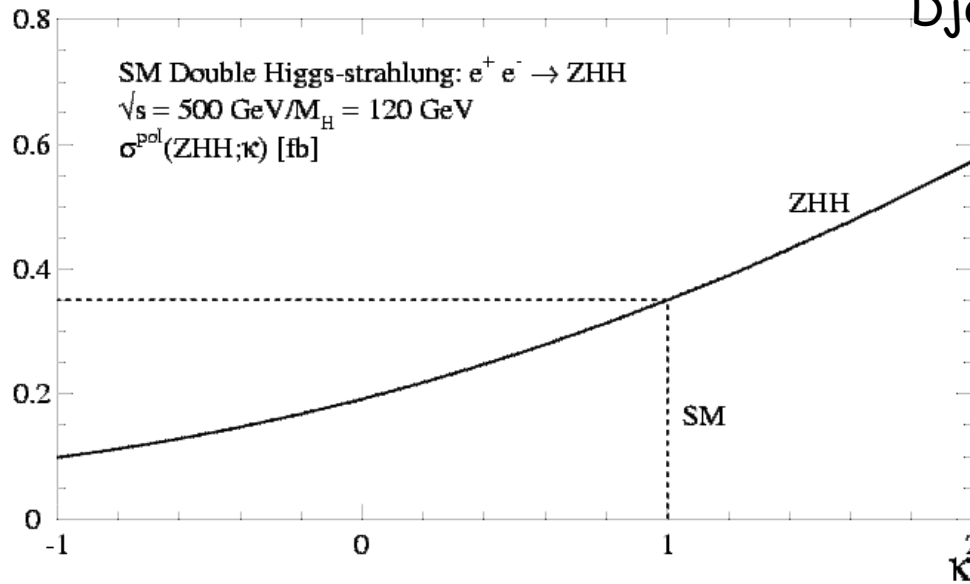
Cross sections for three collider energies. Small!

Arrows show variation of  $\lambda_{HHH}$  from 1/2 to 3/2 the SM value

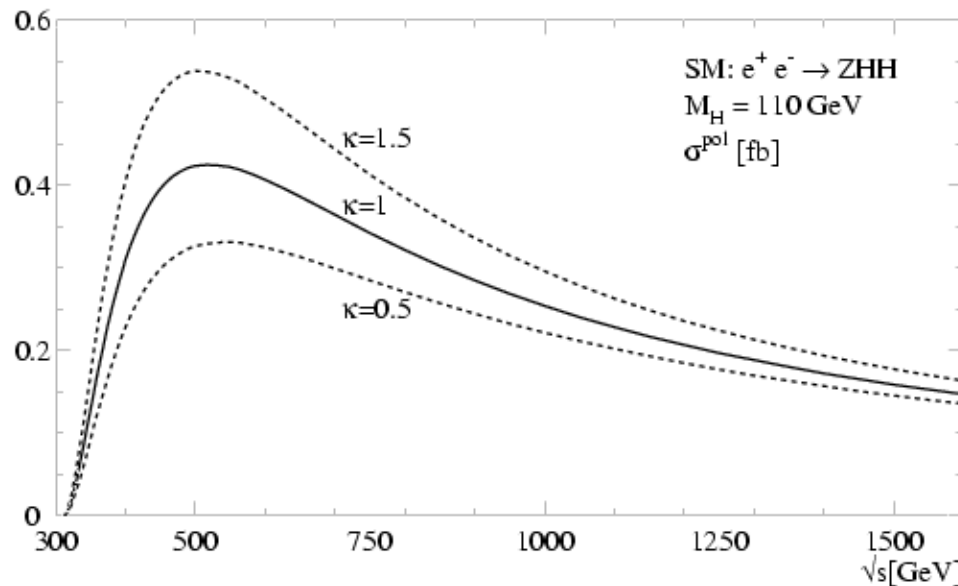


# $e^+e^- \rightarrow ZHH$

Djouadi, Kilian, Muhlleitner, Zerwas



Variation of the cross section with the modified trilinear coupling  $\kappa\lambda_{HHH}$



Energy dependence for a fixed Higgs mass with a modified trilinear coupling  $\kappa\lambda_{HHH}$

# Results: $e^+e^- \rightarrow ZHH$ @ 500 GeV

Castanier, Gay, Lutz, Oloff

$hhZ \rightarrow b\bar{b}b\bar{b}l^+l^-$      $hhZ \rightarrow b\bar{b}b\bar{b}q\bar{q}$     Final states

- Detector simulation (SIMDET fast simulation/ TESLA Detector)
- 2 b-quarks identified/ Neural Network
- Backgrounds ( $WW, Z\gamma, ZZ, WWZ, ZZZ, ttH, hZ...$ )

$m_h(\text{GeV}/c^2)$	$\sigma_{hhZ}(\text{fb})$	$N_{hhZ}^{500}$	$\epsilon_{hhZ}$	$\Delta\sigma/\sigma$		
				$\mathcal{L} = 500\text{fb}^{-1}$	$1000\text{fb}^{-1}$	$2000\text{fb}^{-1}$
120	0.186	93.	43%	24.1%	17.3%	11.6%
130	0.149	74.	43%	26.6%	19%	17.7%
140	0.115	57.	39%	32%	23%	17%



selection	variable	$\Delta\lambda/\lambda$		
		$\mathcal{L} = 500\text{fb}^{-1}$	$1000\text{fb}^{-1}$	$2000\text{fb}^{-1}$
B	$\mathcal{B}^{\text{recoil}}$	42.2%	30.3%	20.3 %
D	NN output	35.7%	22.6%	18.0%

$S/\sqrt{B}=4.2$

23% (40%)  
Precision for  
1  $\text{ab}^{-1}$  at 500GeV  
for  
 $m_H=120(140)$  GeV

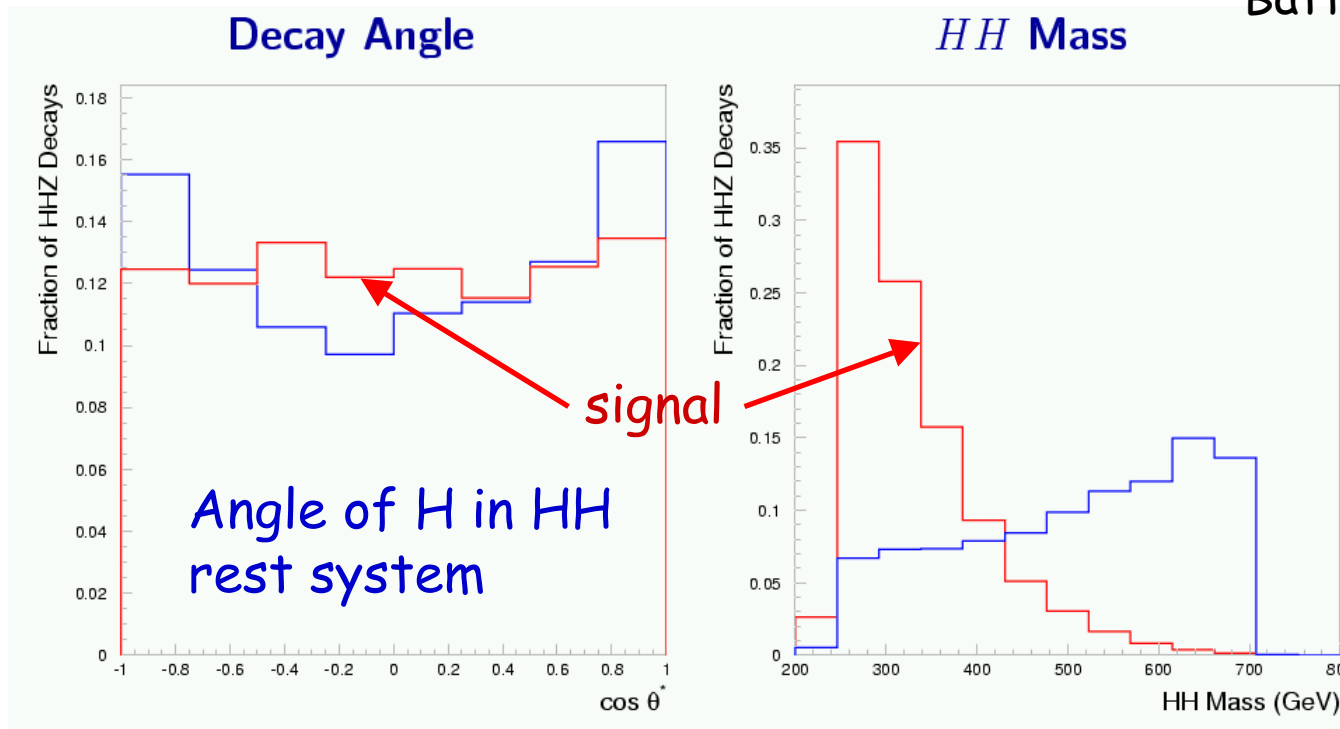


# Results: $e^+e^- \rightarrow ZHH$ @ 500 GeV

Additional diagrams dilute the  $\lambda_{HHH}$  sensitivity of the  $\sigma(e^+e^- \rightarrow HHZ)$  measurement:

Try to exploit the scalar decay and phase space properties to enhance sensitivity to the HHH vertex contribution

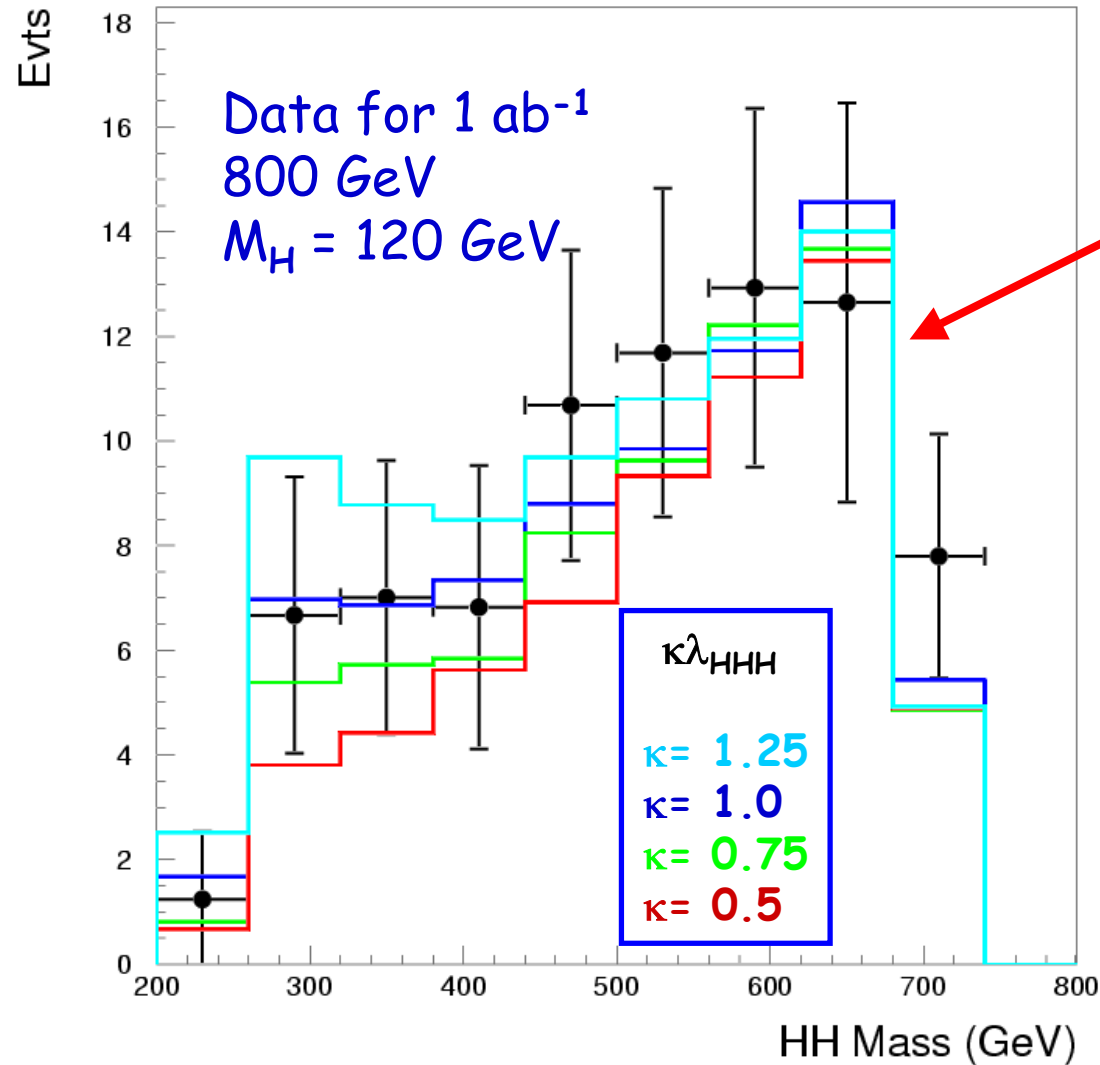
Battaglia, Boos, Yao



Angle of H in HH rest system

Use in likelihood fit...

# Results: $e^+e^- \rightarrow ZHH$ @ 500 GeV



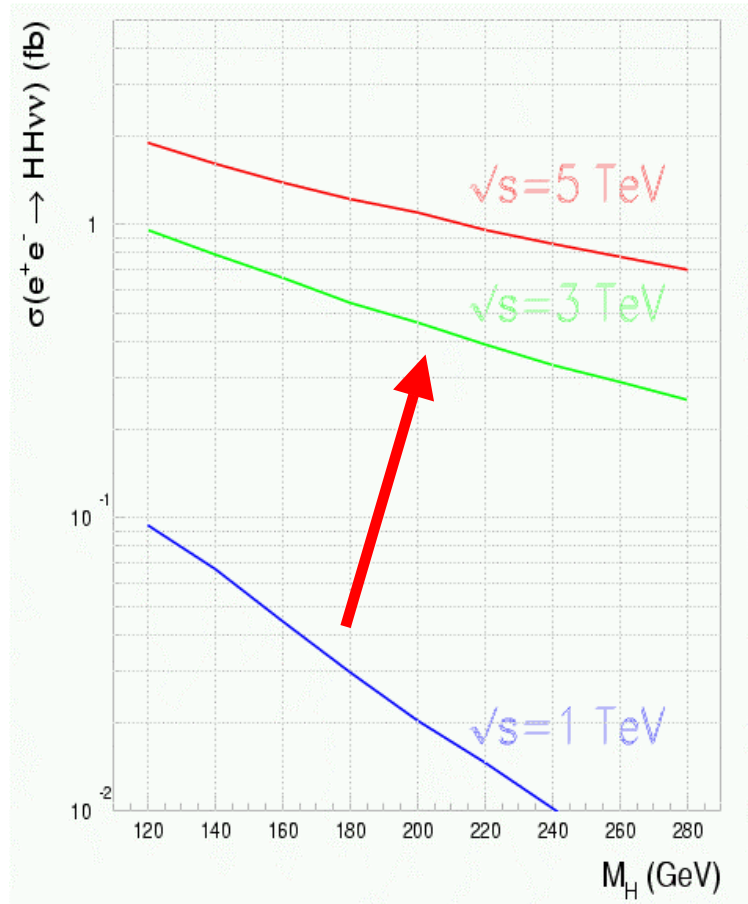
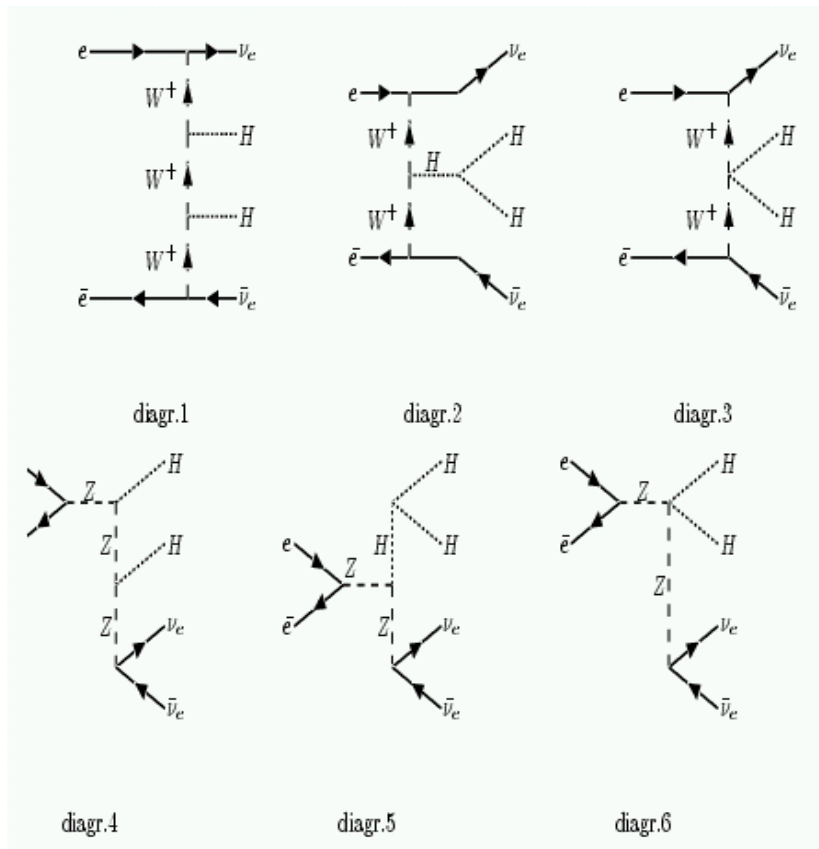
Results including  
a  $\chi^2$  fit to the  $M_{HH}$   
distribution

$\sqrt{s}$ (TeV)	$\sigma_{HHZ}$ Only	$M_{HH}$ Fit
0.5	$\pm 0.23$ (stat)	$\pm 0.20$ (stat)
0.8	$\pm 0.35$ (stat)	$\pm 0.29$ (stat)

20% precision for 1 ab<sup>-1</sup>  
at 500 GeV  
⇒ Improvement O(15%)

# $e^+e^- \rightarrow HH\nu\nu$

Higher energies:  $HH\nu\nu$  significantly larger cross section than  $HHZ$



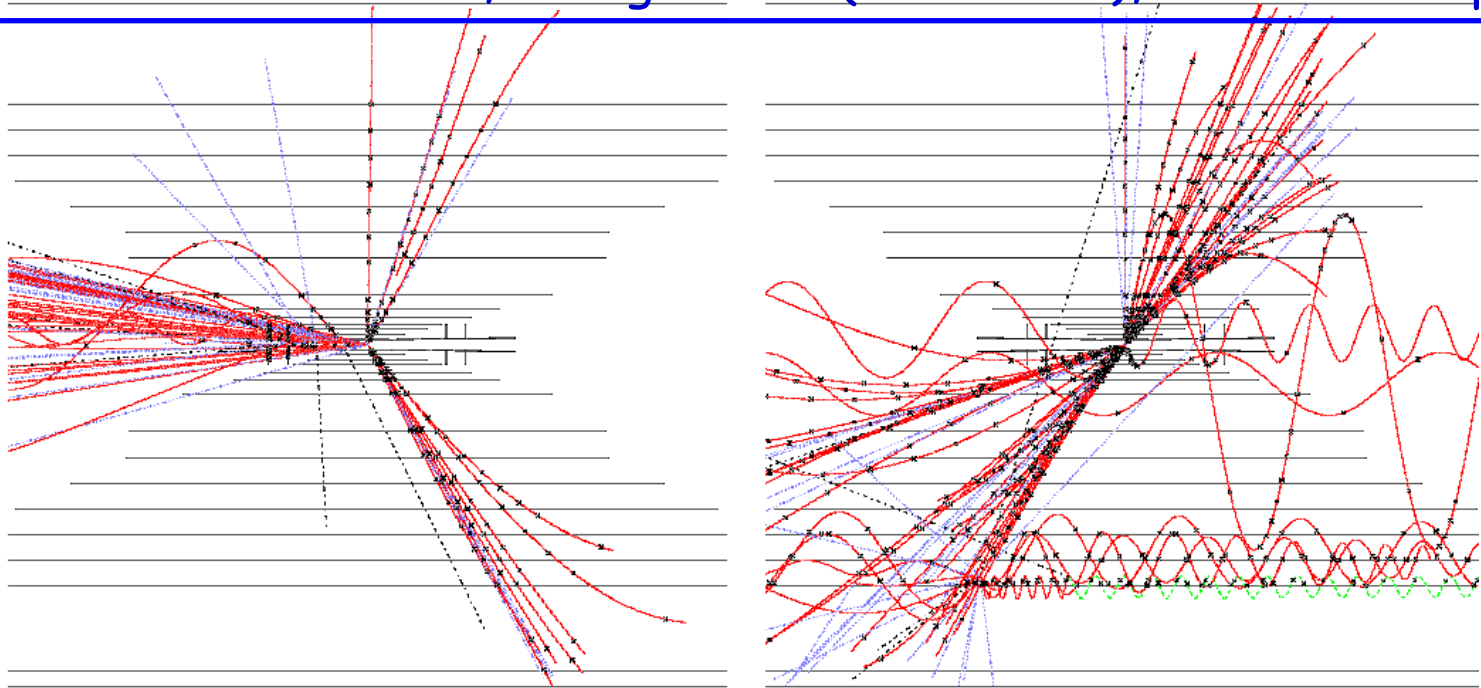
Study:  $HH \rightarrow b\bar{b}b\bar{b}, W^+W^-W^+W^-$

# $e^+e^- \rightarrow HH\nu\nu$ (3 TeV)

Battaglia, Boos, Yao, ADR

$$e^+e^- \rightarrow HH\nu\bar{\nu} \rightarrow b\bar{b} b\bar{b} + E_{\text{missing}}$$

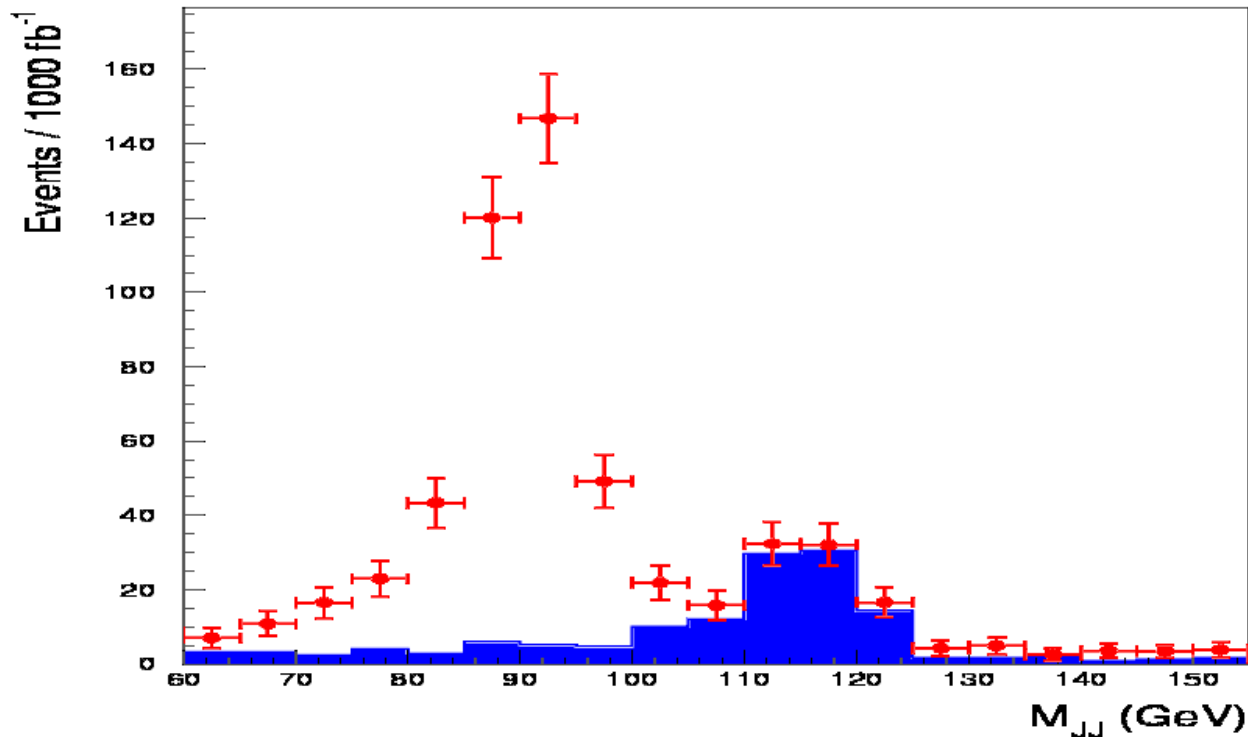
Include detector simulation, backgrounds (COMPHEP), CLIC beam spectra...



Experimental challenge: Higgses often forward!  
Require 4B's or 4W's

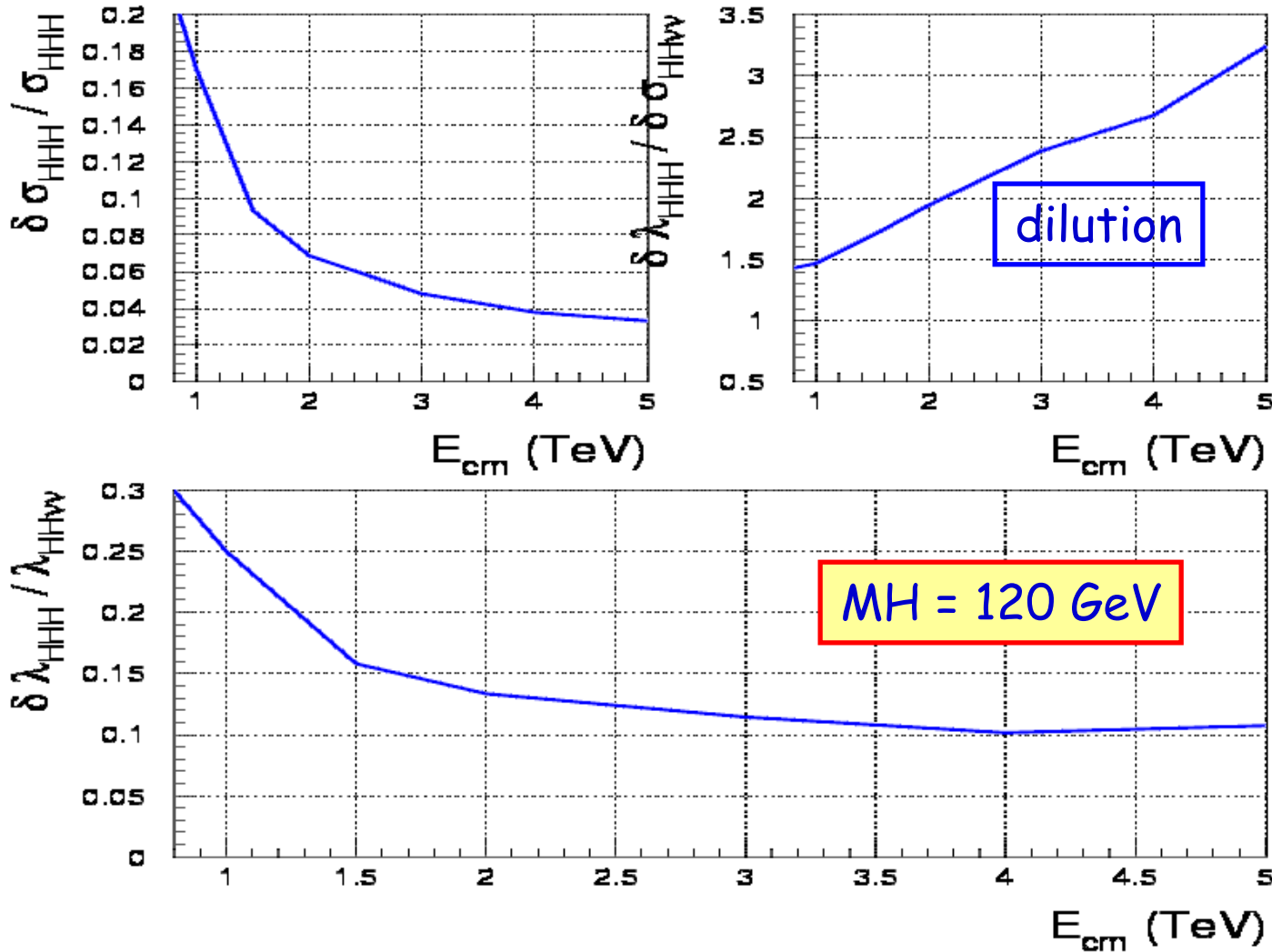
# $e^+e^- \rightarrow HH\nu\nu \rightarrow bbbb\nu\nu$

Select b-tagged events with significant missing ET, force 4-jets  
Studies include detector simulation, backgrounds...



$$\frac{\delta\sigma_{HH\nu\nu}}{\sigma_{HH\nu\nu}} \simeq 0.12 \text{ ab}^{-1}$$

# Results: $e^+e^- \rightarrow HH\nu\nu$

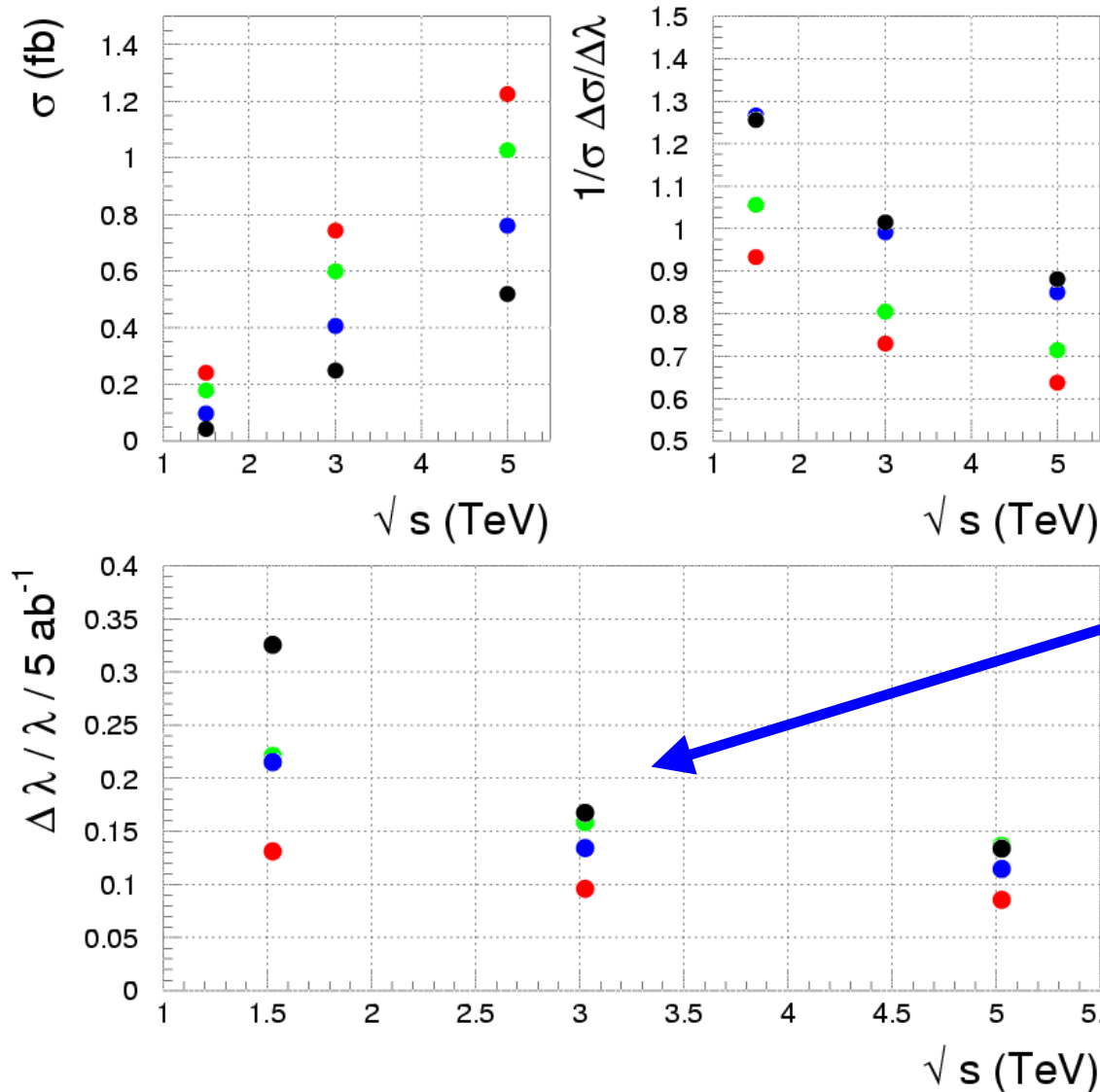


12% precision  
for  $3 \text{ ab}^{-1}$  at  
3 TeV



# Results: $e^+e^- \rightarrow HH\nu\nu$

New



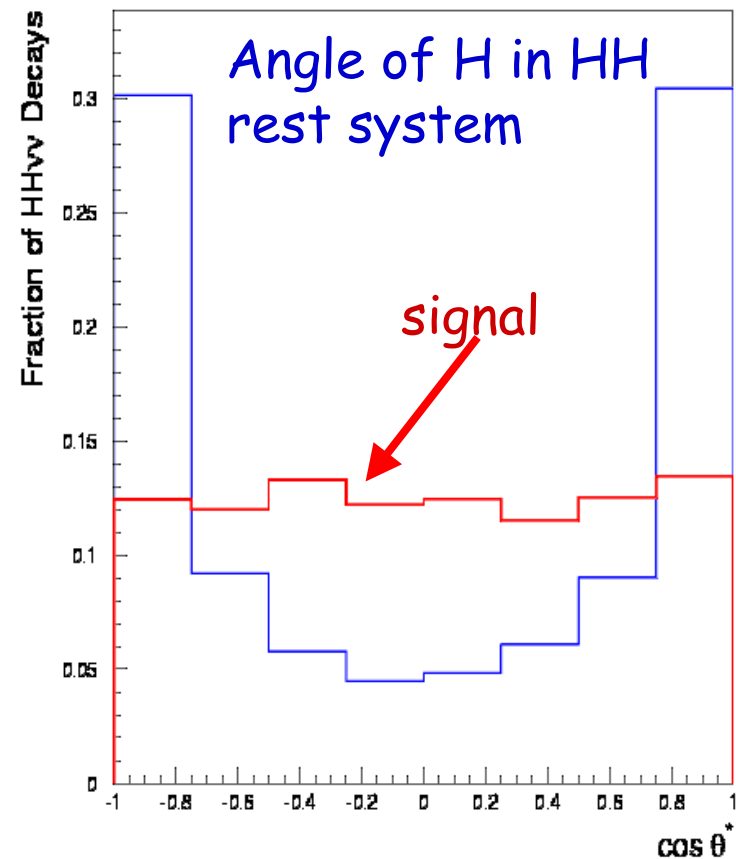
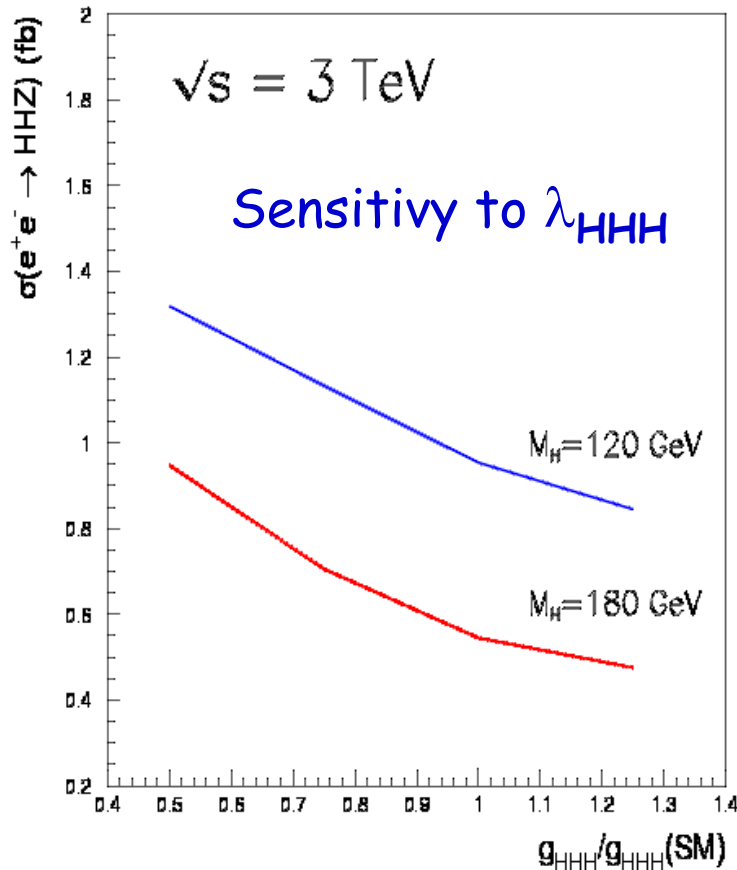
Precision on  $\lambda_{HHH}$  for Higgs masses in the range

- $m_H = 120 \text{ GeV}$
- $m_H = 140 \text{ GeV}$
- $m_H = 180 \text{ GeV}$
- $m_H = 240 \text{ GeV}$

For  $5 \text{ ab}^{-1}$  of data

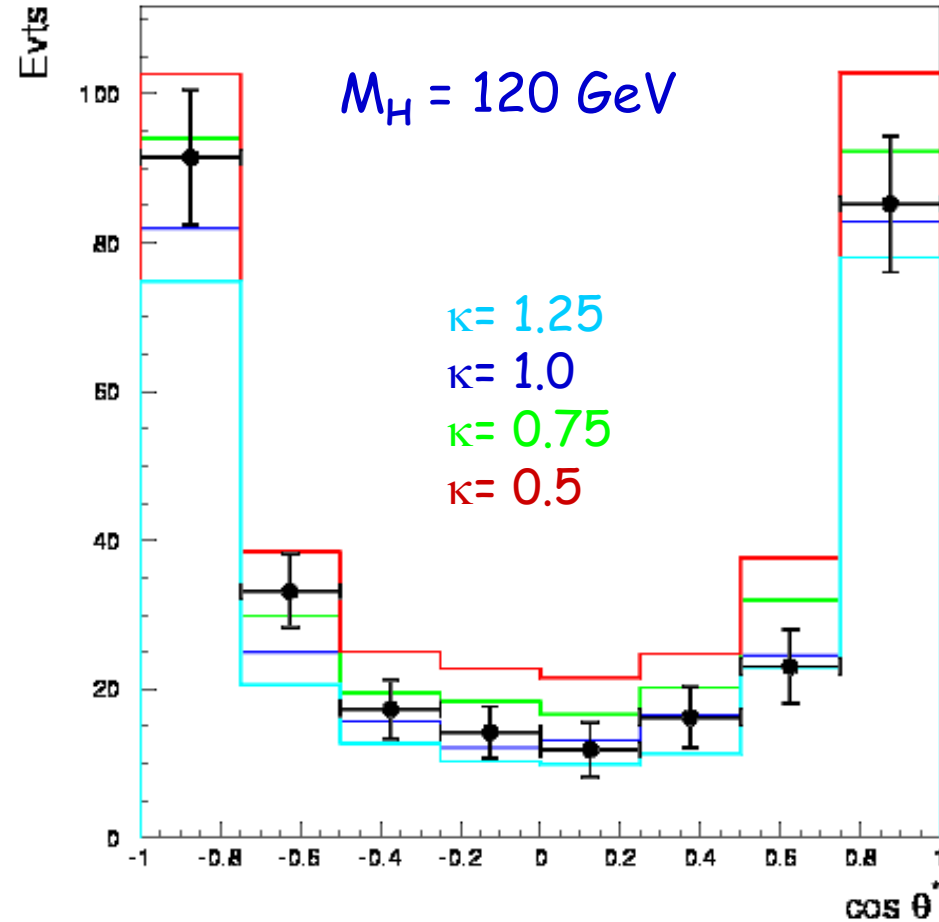
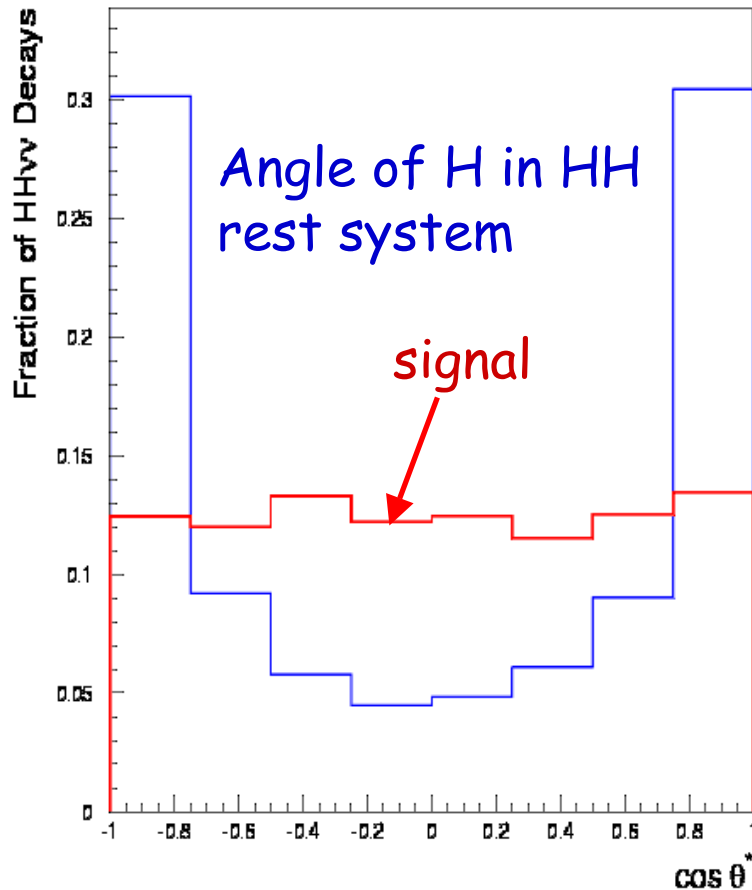
# Results: $e^+e^- \rightarrow HH\nu\nu$

Improve further the results by using the scalar nature of the Higgs



# Results: $e^+e^- \rightarrow HH\nu\nu$

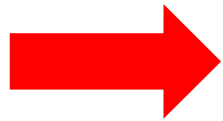
Improve the results further by using the scalar nature of the Higgs



Fit to the cross sections and the normalized shape of the  $|\cos\theta|$  distribution

# Results: $e^+e^- \rightarrow HH\nu\nu$

Results for 3 TeV and 5  $\text{ab}^{-1}$



$M_H$ (GeV)	$\sigma_{HH\nu\nu}$ Only	$ \cos\theta^* $ Fit
120	$\pm 0.094$ (stat)	$\pm 0.070$ (stat)
180	$\pm 0.140$ (stat)	$\pm 0.080$ (stat)

Including a fit to the  $\cos\theta$  distribution improves the results with 25-30%

Further improvement using polarized beams is possible ( $\times \sim 1.5$ )

The quartic couplings will remain elusive:

The table below gives the cross section in atobarns

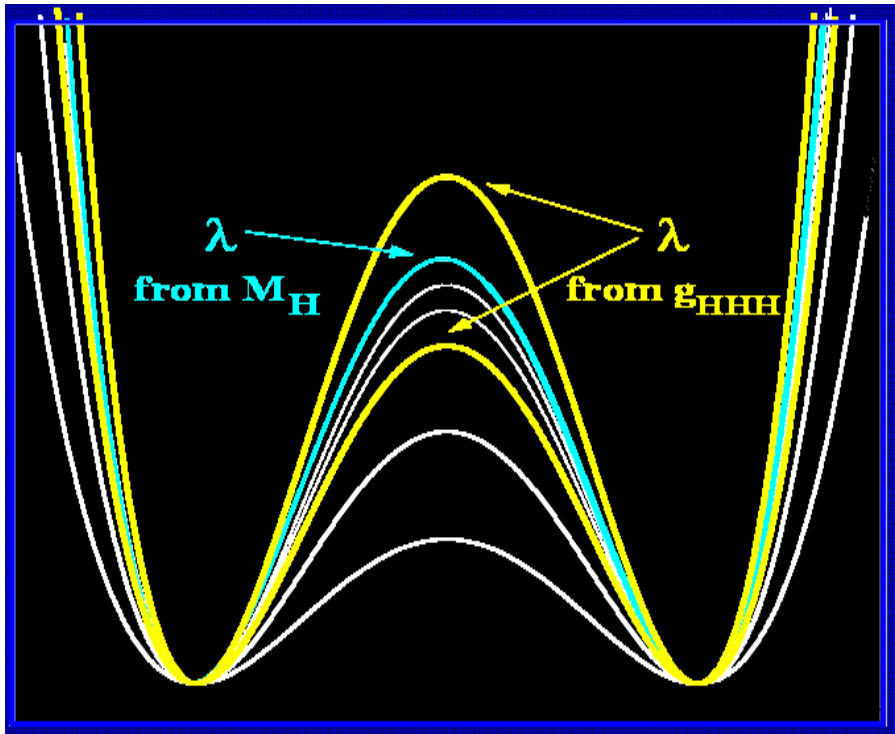
I.e. one expects 1-2 (5) events/year for CLIC at 5 (10) TeV

$\sqrt{s}$	$g_{HHHH}/g_{HHHH}^{SM} = 0.9$	$g_{HHHH}/g_{HHHH}^{SM} = 1.0$	$g_{HHHH}/g_{HHHH}^{SM} = 1.1$
3 TeV	0.400	0.390	0.383
5 TeV	1.385	1.357	1.321
10 TeV	4.999	4.972	4.970

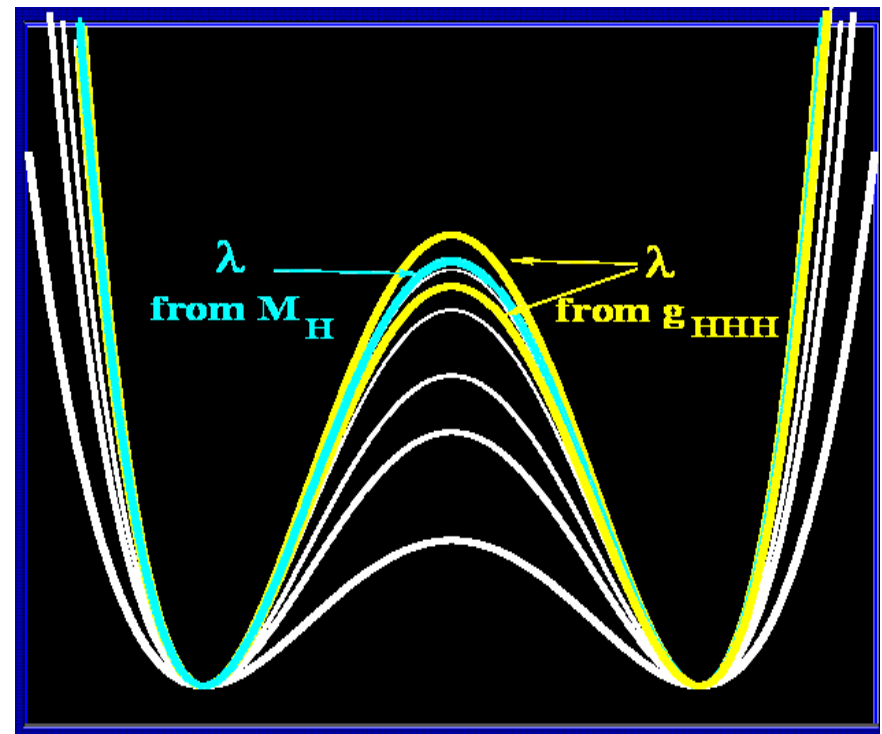
# The Higgs potential

"Visual example": compare measurement from Higgs mass with results from  $\lambda_{HHH}$  measurements

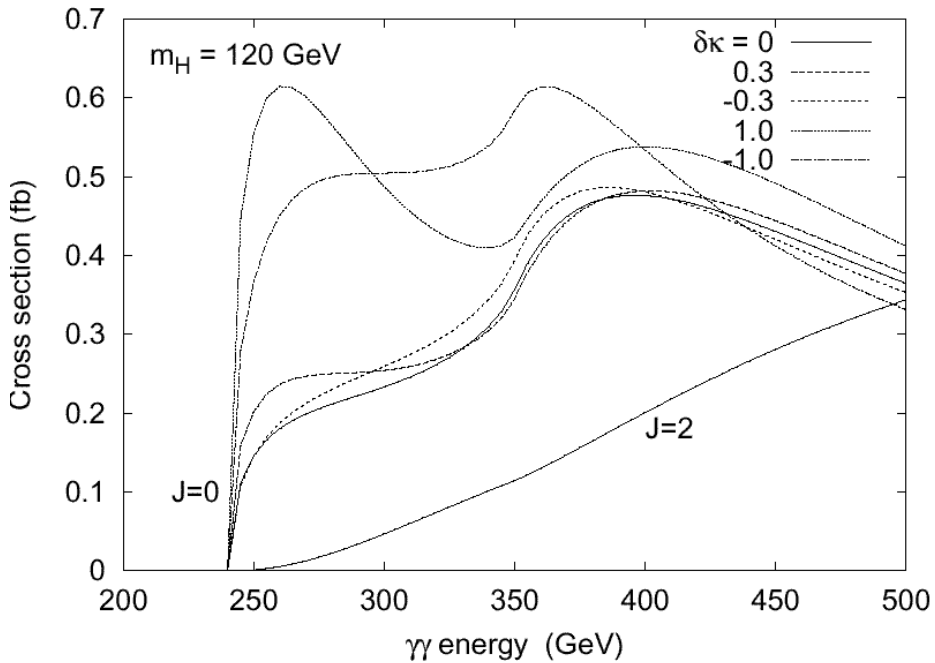
For 500 GeV  $e+e^-$



For 3 TeV  $e+e^-$



# Photon Collider



D. Asner et al.

$$\delta\mathcal{L}_{\text{Higgs}} = -\frac{\delta\kappa}{2} \frac{m_H^2}{v} \left[ H^3 + \frac{3}{v} G^+ G^- H^2 \right] + \dots,$$

$$e^+e^- \rightarrow HHZ \Leftrightarrow \gamma\gamma \rightarrow HH$$

Event rates similar

Final state cleaner (higher efficiency)?

Parton level cross sections

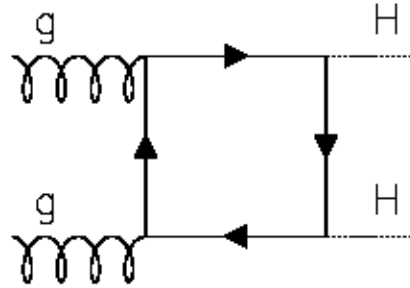
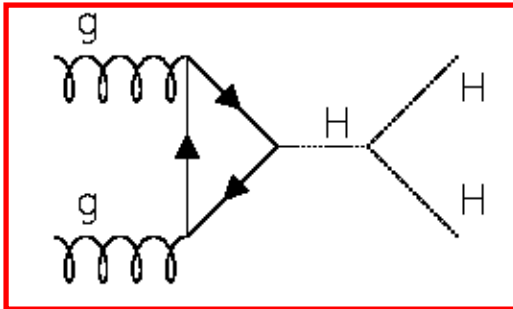
	$\sqrt{s_{ee}} = 500 \text{ GeV}$			$\sqrt{s_{ee}} = 800 \text{ GeV}$		
	$\int \mathcal{L}_{th} \text{ (fb}^{-1}/10^7 \text{ s)}$	$\sigma \text{ (fb)}$	Event yield	$\int \mathcal{L}_{th} \text{ (fb}^{-1}/10^7 \text{ s)}$	$\sigma \text{ (fb)}$	Event yield
Spin-0	40	0.3	13	120	0.3	39
Spin-2	20	0.1	1-2	60	0.2	1-2
$e^+e^-$	160	0.2	32	250	0.15	38

⇒ needs a study of the dilution, backgrounds, detector effects

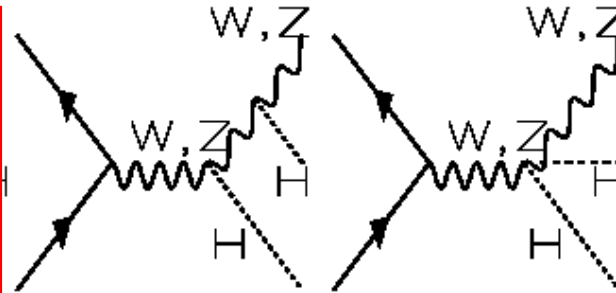
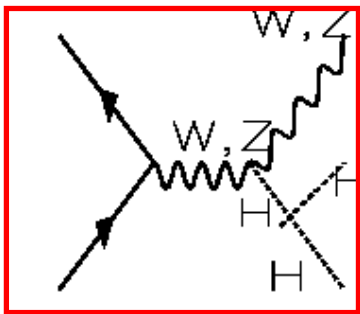


# Hadron Colliders: $PP \rightarrow HHX$

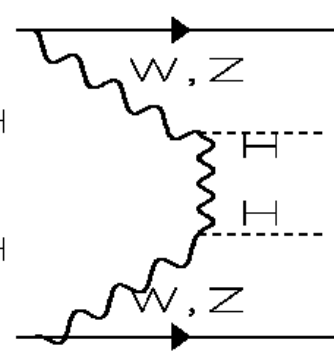
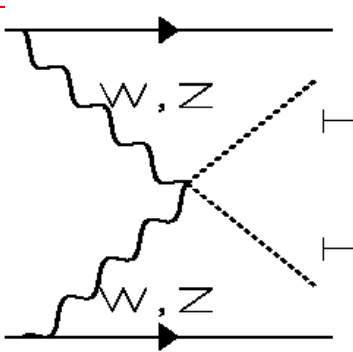
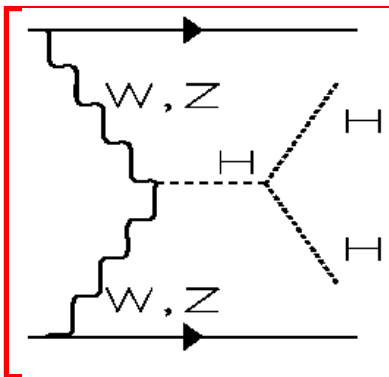
## Contributing diagrams



gluon-gluon fusion



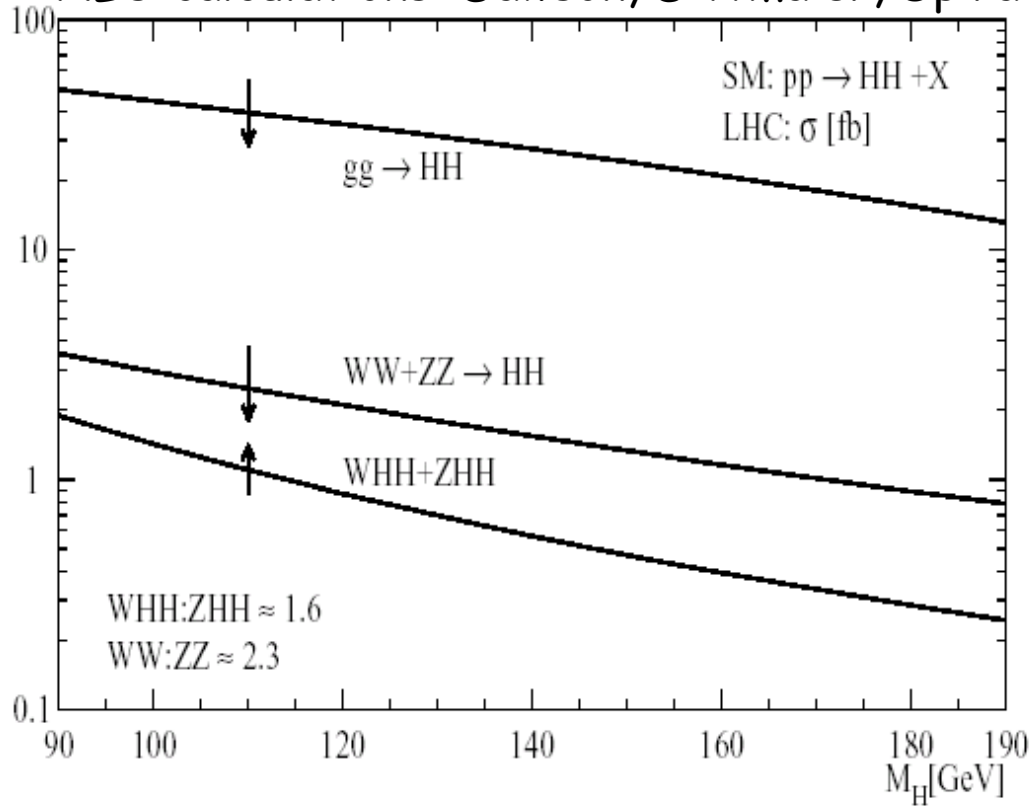
W or Z strahlung



WW and ZZ fusion

# PP $\rightarrow$ HHX

NLO calculations: Dawson, Dittmaier, Spira



Cross sections  
for three processes  
Small!

Arrows show  
variation of  $\lambda_{HHH}$   
from  $1/2$  to  $3/2$   
the SM value

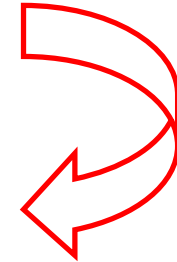
Blondel, Clark, Mazzucato (ATLAS)

ATLAS Study: study decay modes:  $H \rightarrow WW$  and  $H \rightarrow ZZ$   $150 < M_H < 200$  GeV  
LO (!) cross sections. K factor for signal in range 1.5-2  
Detector simulation, backgrounds

# Results: $PP \rightarrow HHX$ (LHC)

$pp \rightarrow HH \rightarrow WWWW \rightarrow lvqqlvqq$  ( $\sigma * BR \simeq 0.24 fb^{-1}$  for  $m_H = 200 GeV$ )  
 $pp \rightarrow HH \rightarrow WWWW \rightarrow lvlvqqqq$  ( $\sigma * BR \simeq 0.12 fb^{-1}$  for  $m_H = 200 GeV$ )  
 $pp \rightarrow HH \rightarrow WWWW \rightarrow lvlvlvqq$  ( $\sigma * BR \simeq 0.072 fb^{-1}$  for  $m_H = 200 GeV$ )  
 $pp \rightarrow HH \rightarrow WWWW \rightarrow lvlvlvlv$  ( $\sigma * BR \simeq 0.006 fb^{-1}$  for  $m_H = 200 GeV$ )  
 $pp \rightarrow HH \rightarrow WWZZ \rightarrow qqqlqq$  ( $\sigma * BR \simeq 0.09 fb^{-1}$  for  $m_H = 200 GeV$ )  
 $pp \rightarrow HH \rightarrow WWZZ \rightarrow lvqqlqq$  ( $\sigma * BR \simeq 0.055 fb^{-1}$  for  $m_H = 200 GeV$ )

Channels studied



600 fb<sup>-1</sup>

	<i>lvqqlvqq</i>	<i>lvqqlvqq</i>	<i>lvlvlvqq</i>	<i>lvlvlvlv</i>	<i>llqqqqqq</i>	<i>lvqqlqq</i>
HH $m_H = 200 GeV$	22	58	5.8	1.	16	6.0
$t\bar{t}$	9	33678	79.	15.	665	17
WW	0	122	0.	0.	0	0
ZZ	0	1240	0.3	0	288	1.8
WZ	6	10	16.0	0	0	33.7
WWW all processes	150	-	-	-	-	-
WH only	37	183	7.1	1.9	50	12.4
ZH	6	322	14.1	11.	98	33.2
Wtt	159	0	0.	0	0	0
Zbb	0	104	0.	0	28	0
$t\bar{t}t$	3	-	-	-	-	-
Total B	333	35659	117.	27.9	1129	98.4
<i>S/B</i>	0.07	$2 * 10^{-3}$	0.05	0.04	0.01	0.06
<i>S/√B</i>	1.2	0.3	0.53	0.19	0.5	0.61

Pretty hopeless for LHC...

# Results: $PP \rightarrow HHX$ (SLHC)

For SLHC ( $6000 \text{ fb}^{-1}$ )  $\lambda_{HHH}$  becomes accessible!

	$lvqqlvqq$	$lvqqlvqq$	$lvlvlvqq$	$lvlvlvlv$	$llqqqqqq$	$lvqqllqq$
HH $m_H = 200 \text{ GeV}$	220	580	58	11.	160	60
$t\bar{t}$	90	336780	790.	150.	6650	170
$WW$	0	1225	0.	0.	0	0
$ZZ$	0	12404	3.5	0	2883	17
$WZ$	60	971	160	0	0	337
$WH$	374	1831	71	19	505	124
$WWW$	1500	-	-	-	-	-
$ZH$	59	3223	141	111.	977	332
$Wtt$	1594	0	3	0	0	0
$Zbb$	0	1045	0.	0	285	0
$t\bar{t}t\bar{t}$	30	-	-	-	-	-
Total B	3333	356595	1169	279	11300	984
$S/B$	0.07	$2 * 10^{-3}$	0.05	0.04	0.01	0.06
$S/\sqrt{B}$	3.1	1.	1.7	0.7	1.5	1.9



Mass (GeV)	200	200 (fit)	170
$\frac{d\sigma}{\sigma}$	27%	17%	20%
$\frac{d\lambda}{\lambda}$	25%	15%	19%

# Results: PP $\rightarrow$ HHX

Baur, Plehn, Rainwater

- Studied signals at LHC, SLHC and VLHC (and LC),
- No detector simulation, but acceptance cuts, efficiencies
- NLO through K-factors
- Use minimum jet distance and  $M_{\text{vis}}$  quantities to improve S/B

For  $M_H > 150$ , examine  $4W \rightarrow$  multipleptons.

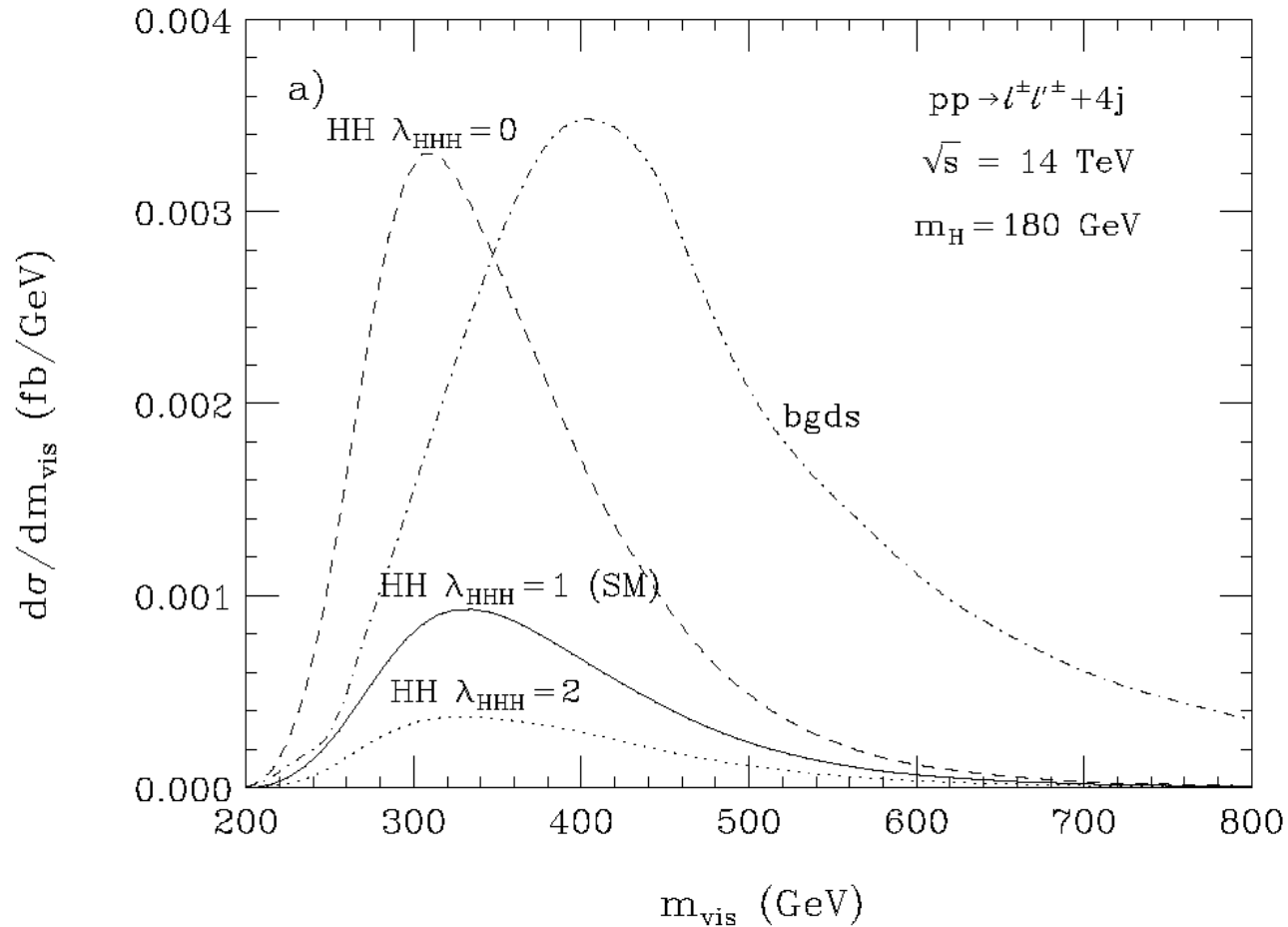
$$l^\pm l^\pm + 4j, \quad l^\pm l^\pm l^\mp + 2j$$

$\rightarrow$  LHC can confirm  $\lambda \neq 0$  (3-pt. Higgs coupling)  
for  $150 < M_H < 200$  GeV.

For  $M_H < 150$ , look for  $b\bar{b}b\bar{b}, b\bar{b}\tau^+\tau^-$ .

$\rightarrow$  totally hopeless at LHC, SLHC,  
1st measurement at VLHC if no TESLA.

# Results: $PP \rightarrow HHX$



$$m_{vis}^2 = \left[ \sum_{i=\ell, \ell', \text{jets}} E_i \right]^2 - \left[ \sum_{i=\ell, \ell', \text{jets}} \mathbf{p}_i \right]^2$$

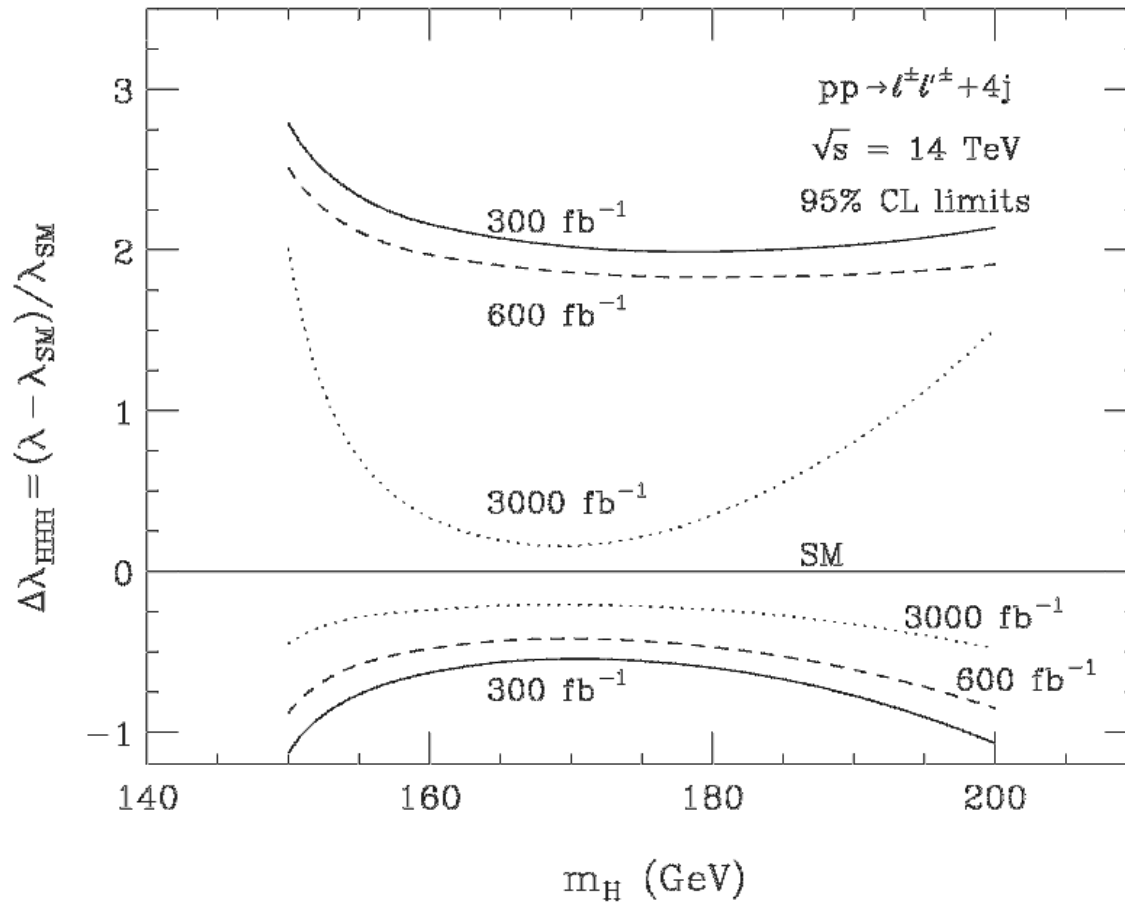
Shape of the  $M_{vis}$  distribution used to derive sensitivity bounds



# Results: PP $\rightarrow$ HHX

Baur, Plehn, Rainwater

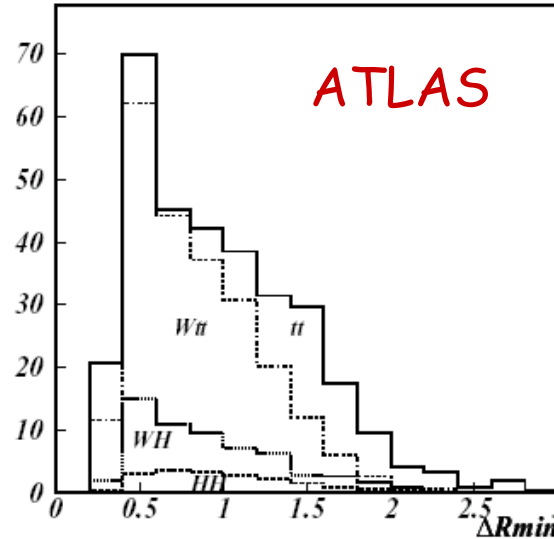
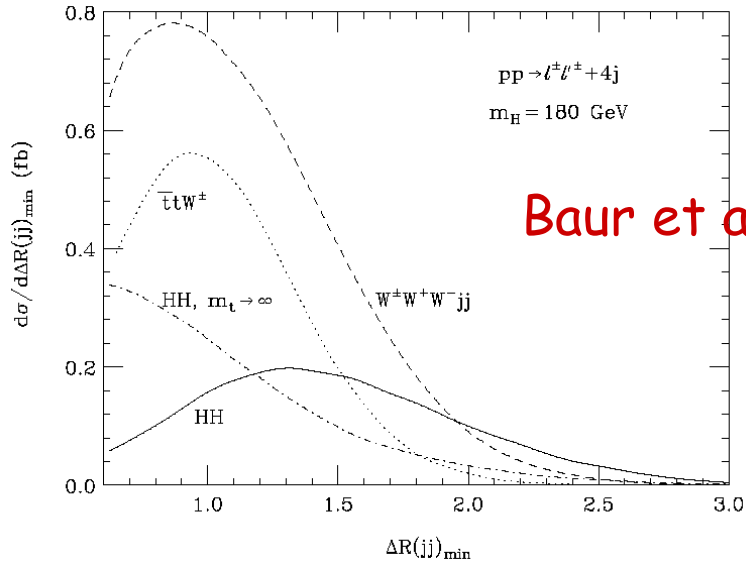
Limits achievable at the 95% CL. for  $\Delta\lambda = (\lambda - \lambda_{SM}) / \lambda_{SM}$



$\lambda = 0$  can be excluded  
at 95% CL.

$\lambda$  can be determined  
to 20-30% with SLHC

# PP $\rightarrow$ HHX



Possible reasons for differences

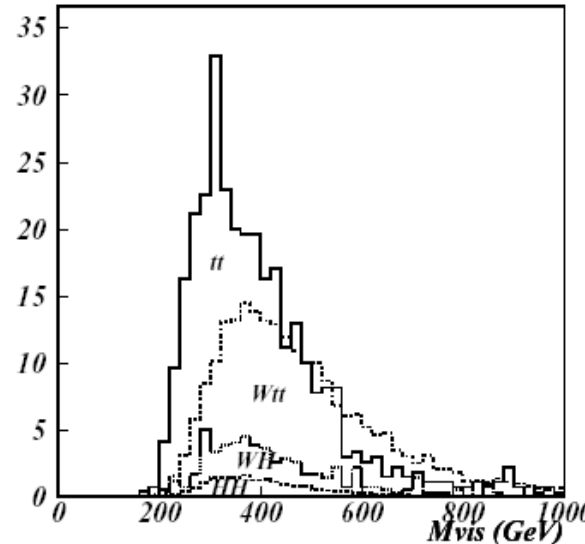
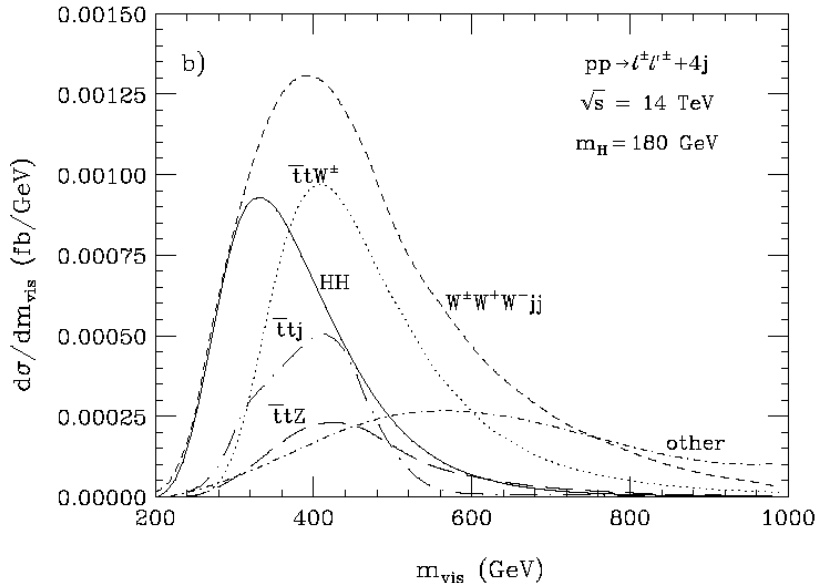
Parton level  $\leftrightarrow$  Detector Level

ATLAS includes parton showers

HO effects

Incorrect simulation ttj of background (ATLAS)? (D.R.)

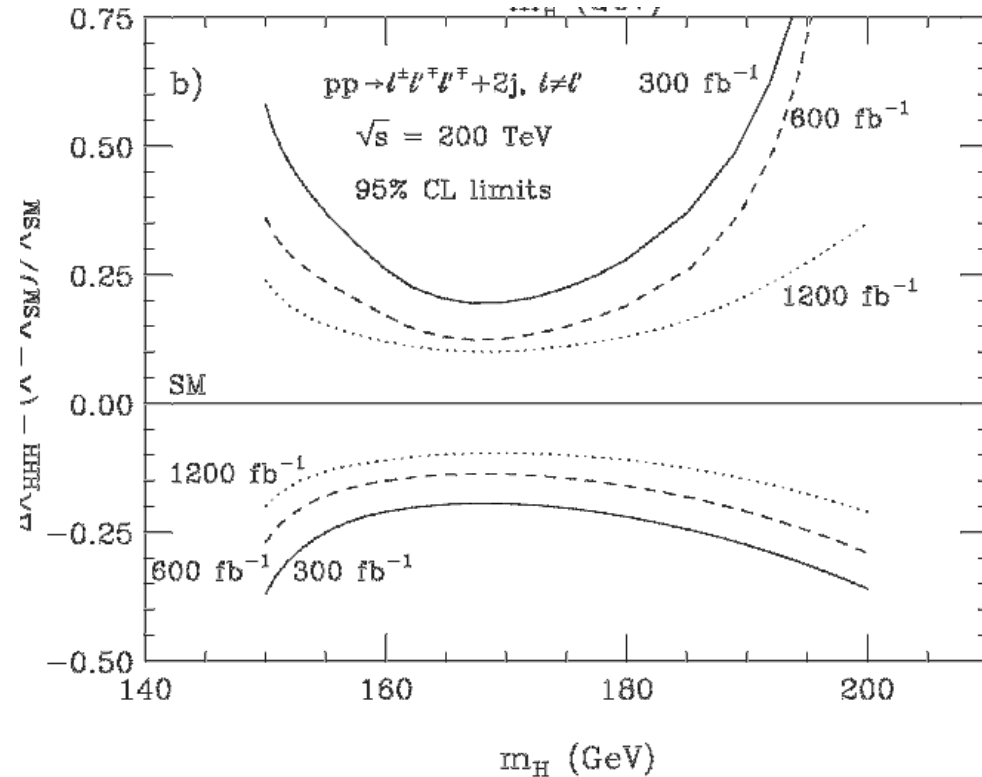
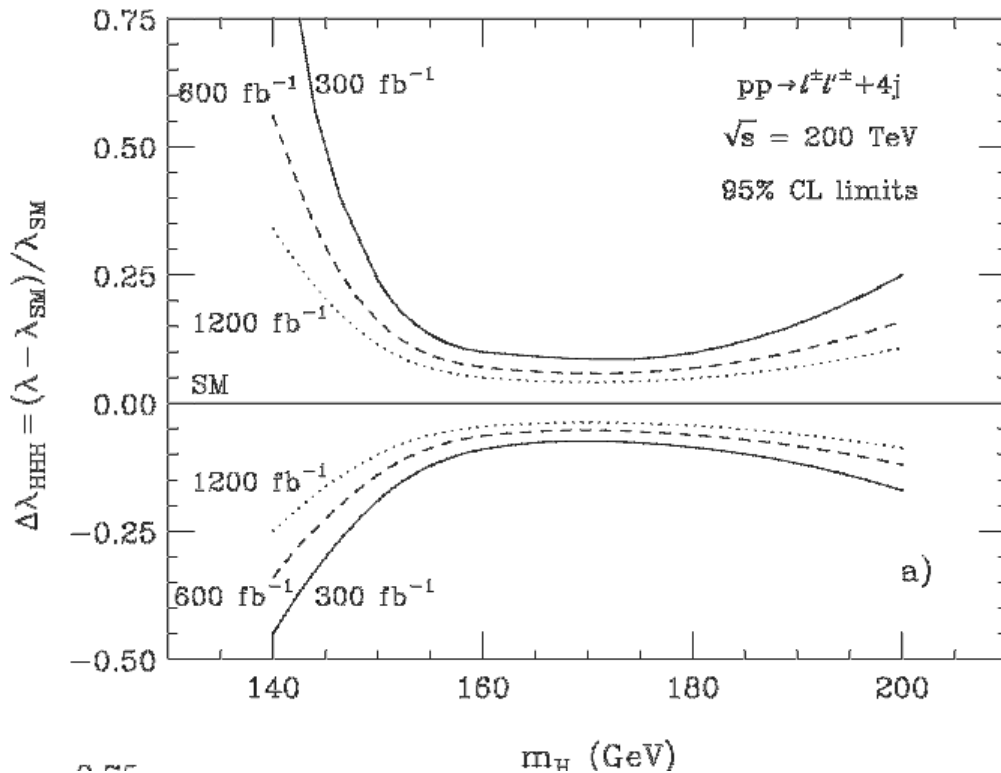
Jury still out...  
CMS starting



# Results: $PP \rightarrow HHX$ (VLHC)

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VLHC  $\sqrt{s}=200$  GeV



Can reach an accuracy of 8-30% (95% CL) for  $M_H$  140-200 GeV

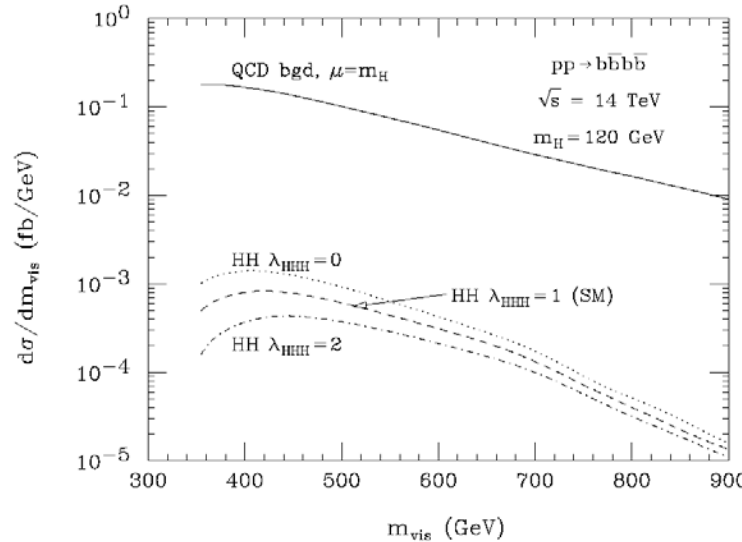
# Lower $M_H$ at pp colliders?

$HH \rightarrow 4b$  AT LHC

$HH \rightarrow b\bar{b}\tau^+\tau^-$  AT LHC

What about  $HH \rightarrow b\bar{b}b\bar{b}$  for lower  $M_H$  @ LHC?

Unfortunately, QCD  $b\bar{b}b\bar{b}$  is factor 1000 larger...

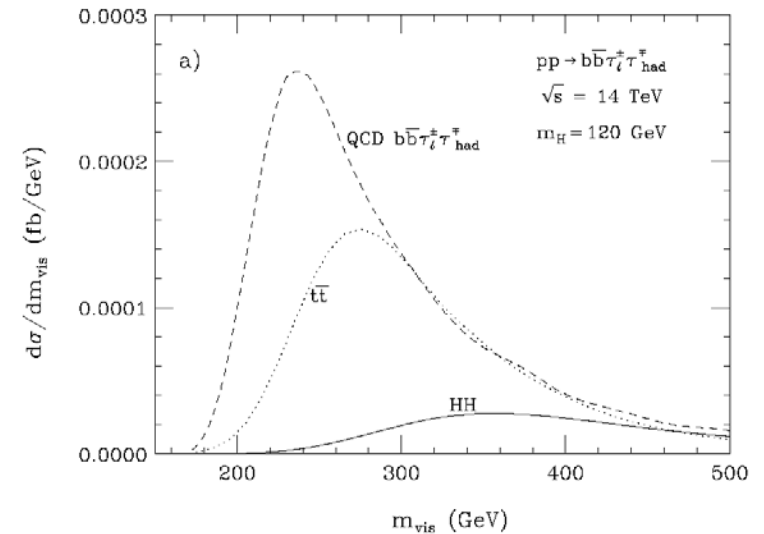


$M_H = 120$  GeV,  $\epsilon_{ID}$  and QCD bkg  
(in terms of  $\lambda_{SM}$ ):

lumi	$1\sigma$ upper	$1\sigma$ lower
LHC, $300 \text{ fb}^{-1}$	+10	-7
SLHC, $3000 \text{ fb}^{-1}$	+6	-3

Difficult...

What about  $HH \rightarrow b\bar{b}\tau^+\tau^-$  @ LHC instead?



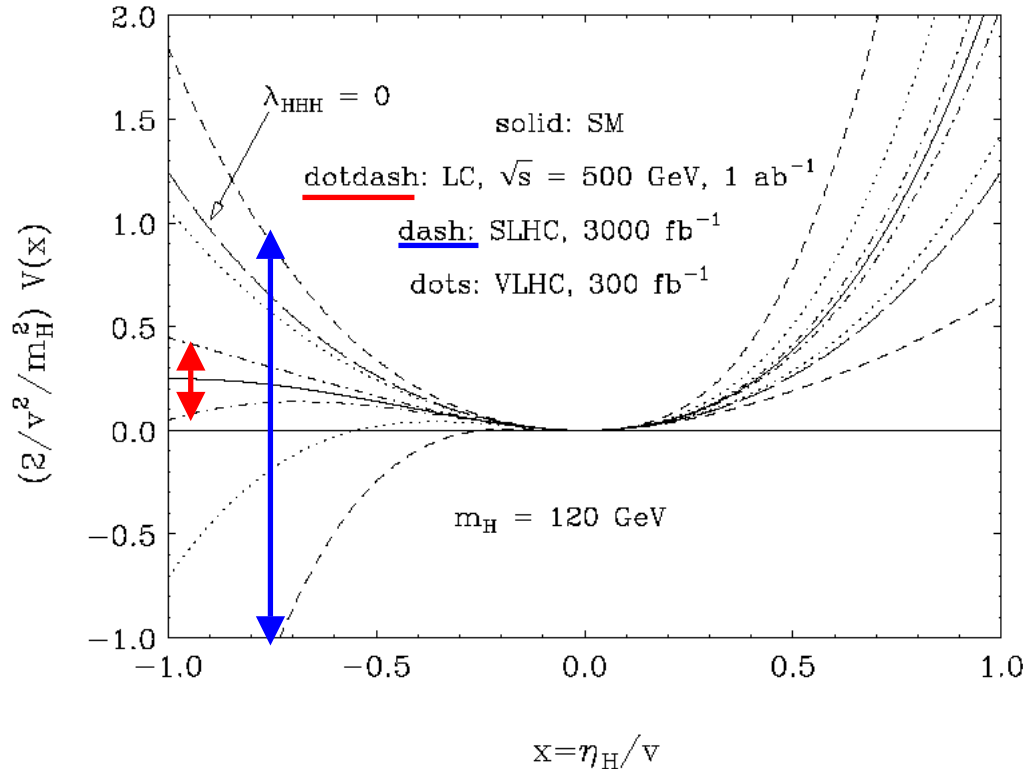
$M_H = 120$  GeV,  $\epsilon_{ID}$  and QCD  $Zb\bar{b}$  bkg  
(in terms of  $\lambda_{SM}$ ):

lumi	$1\sigma$ upper	$1\sigma$ lower
SLHC, $3000 \text{ fb}^{-1}$	+3.1	-1.6
VLHC, $300 \text{ fb}^{-1}$	+1.0	-0.86

Pretty grim, unless no TESLA...

# Comparison for $M_H=120$ GeV

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Precise information  
from a Linear Collider

Hadron colliders  
can not contribute  
much (VLHC?)

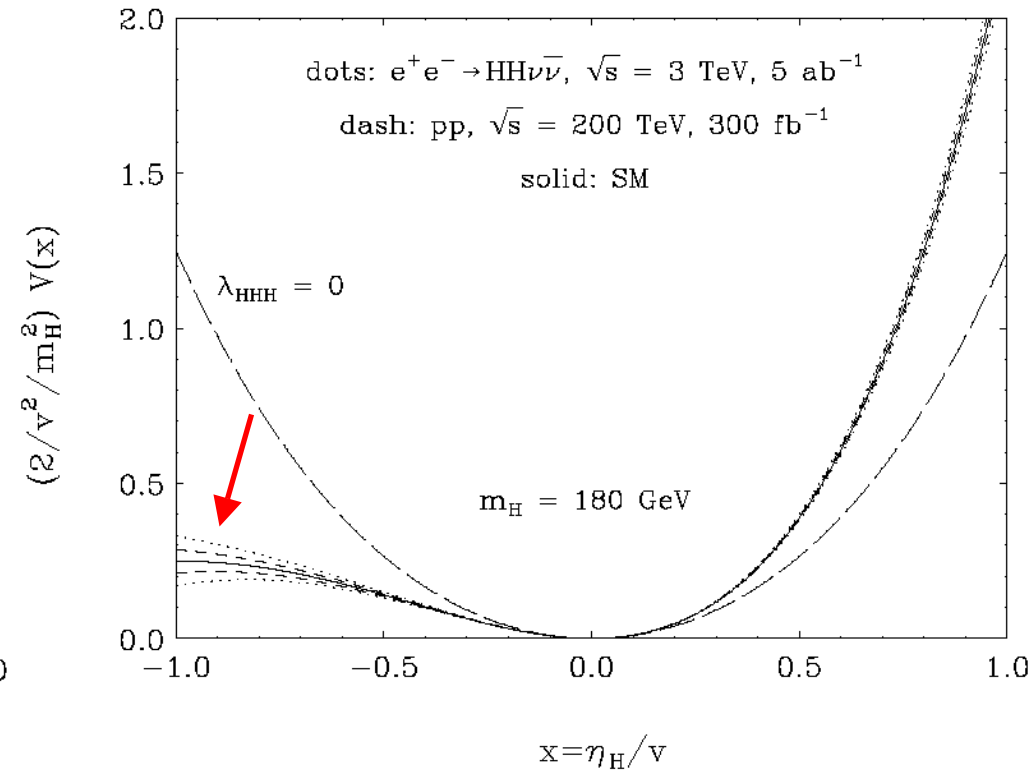
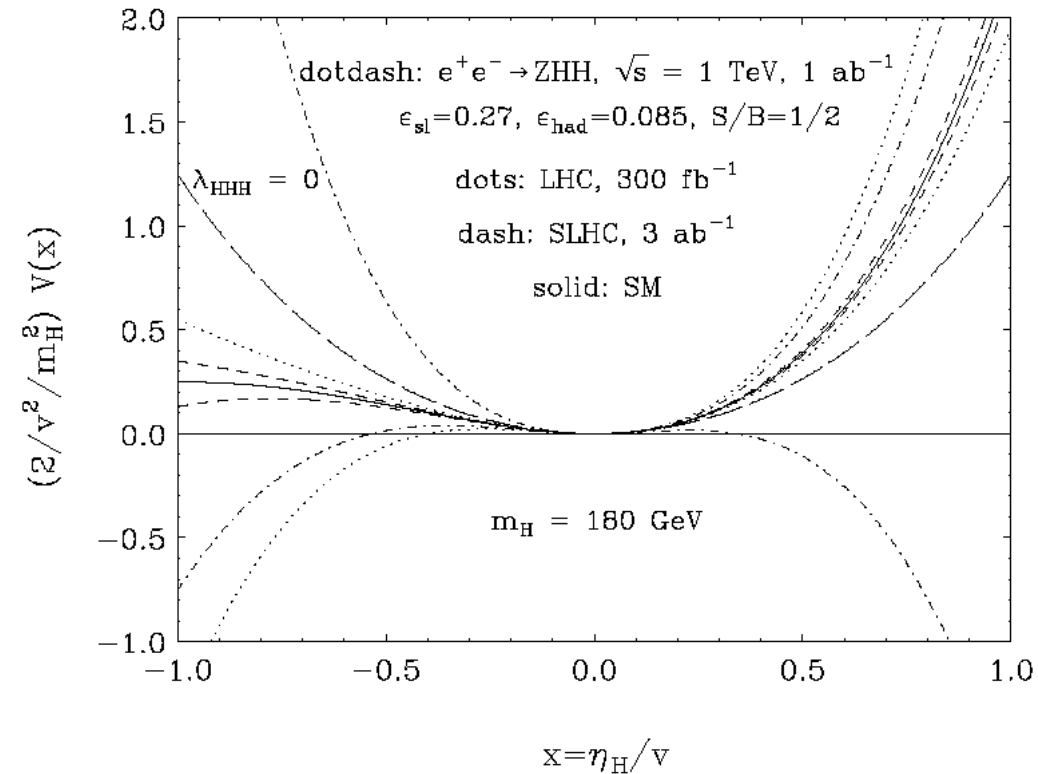
Scaled Higgs potential: 
$$\frac{2}{v^2 m_H^2} V(x) = x^2 + \lambda_{HHH} x^3 + \frac{1}{4} \tilde{\lambda}_{4H} x^4$$

$$x = \frac{\eta_H}{v} \quad \eta_H = \text{Higgs field}$$

$$\tilde{\lambda}_{4H} = \tilde{\lambda} / \lambda_{SM}$$

assume  $\tilde{\lambda}_{4H} = 1$

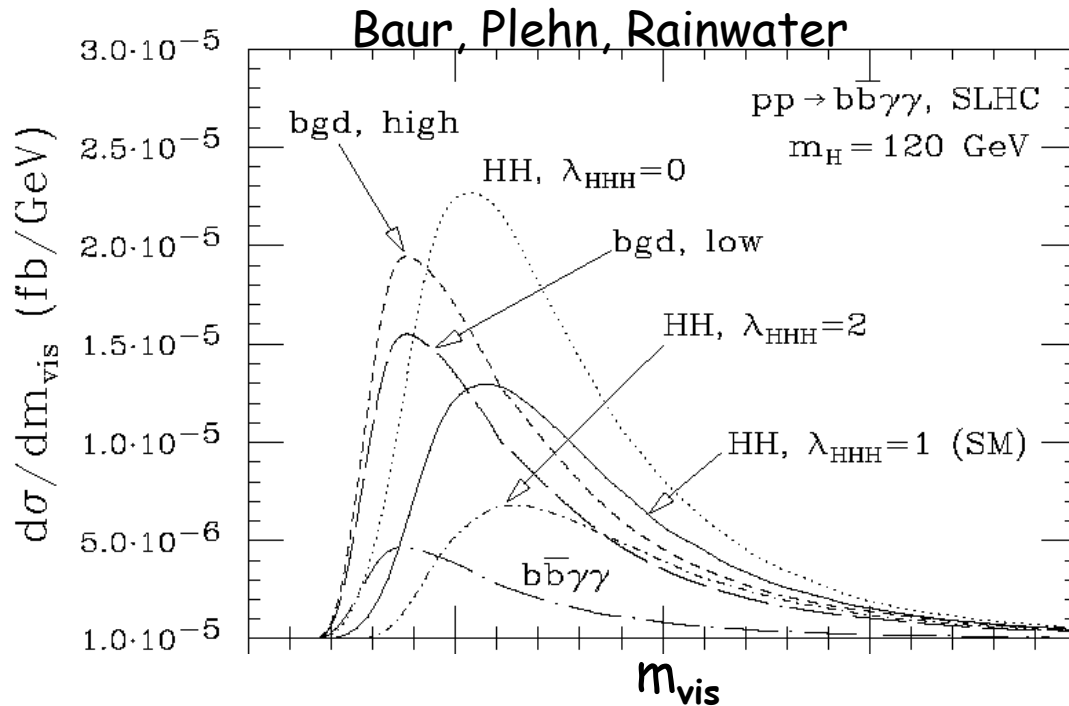
# Comparison for $M_H=180$ GeV



- First hint possible from LHC
- Measurement at SLHC
- LC ( $< \sim 1$  TeV) no information

Both Multi-TeV collider and VLHC can provide very precise measurements

# pp → bbγγ and pp → bbμμ



pp → bbμμ not useable

pp → bbγγ promising

- Apply photon-photon and photon-b separation cuts
- For  $m_H=120$  GeV and  $600 \text{ fb}^{-1}$  expect 6 events at the LHC with  $S/B \sim 2$  (single b tag)
- Interesting measurement at the SLHC (double b tag)

machine	$m_H = 120$ GeV			$m_H = 140$ GeV		
	"hi"	"lo"	bkg. sub.	"hi"	"lo"	bkg. sub.
LHC, $600 \text{ fb}^{-1}$	+1.9 -1.1	+1.6 -1.1	+0.94 -0.74	- -	- -	- -
SLHC, $6000 \text{ fb}^{-1}$	+0.82 -0.66	+0.74 -0.62	+0.52 -0.46	+1.7 -0.9	+1.4 -0.8	+0.76 -0.58
VLHC, $600 \text{ fb}^{-1}$	+0.44 -0.42	+0.42 -0.40	+0.32 -0.30	+0.82 -0.62	+0.66 -0.54	+0.38 -0.34
VLHC, $1200 \text{ fb}^{-1}$	+0.32 -0.30	+0.30 -0.28	+0.26 -0.22	+0.76 -0.58	+0.62 -0.50	+0.36 -0.32

New

Needs accurate prediction of the bbγγ background rate



# Summary

- **LHC**
  - Will have a hard time to measure say something on  $\lambda_{HHH}$
  - Can possibly establish that  $\lambda_{HHH} \neq 0$  if  $150 < M_H < 200$  GeV
- **SLHC**
  - Can measure  $\lambda_{HHH}$  to 20-30% if  $150 < M_H < 200$  GeV
  - Can measure  $\lambda_{HHH}$  to 50-80% if  $120 < M_H < 140$  GeV
- **LC (0.5-1 TeV)**
  - Can measure  $\lambda_{HHH}$  to 20-35% if  $M_H < 140$  GeV
- **CLIC (1-5 TeV)**
  - Can measure  $\lambda_{HHH}$  to 7-15% for  $M_H$  up to 240 GeV
- **VLHC**
  - Can measure  $\lambda_{HHH}$  to 4-15% for  $140 < M_H < 200$  GeV
  - Can measure  $\lambda_{HHH}$  to 20-40% for  $120 < M_H < 140$  GeV

LC results can improve by factor up to 1.3-1.7 when polarised beams are used  
VLHC (SLHC) may need LC to get precise (top Yukawa coupling, HWW...)