Theory of $b \to sl^+l^-$

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19th International Workshop on Weak Interactions and Neutrinos, October 6–11, 2003

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- Status of $\bar{B} \to K^{(*)} l^+ l^-$ Decays
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Introduction

Why are we interested in rare decays?

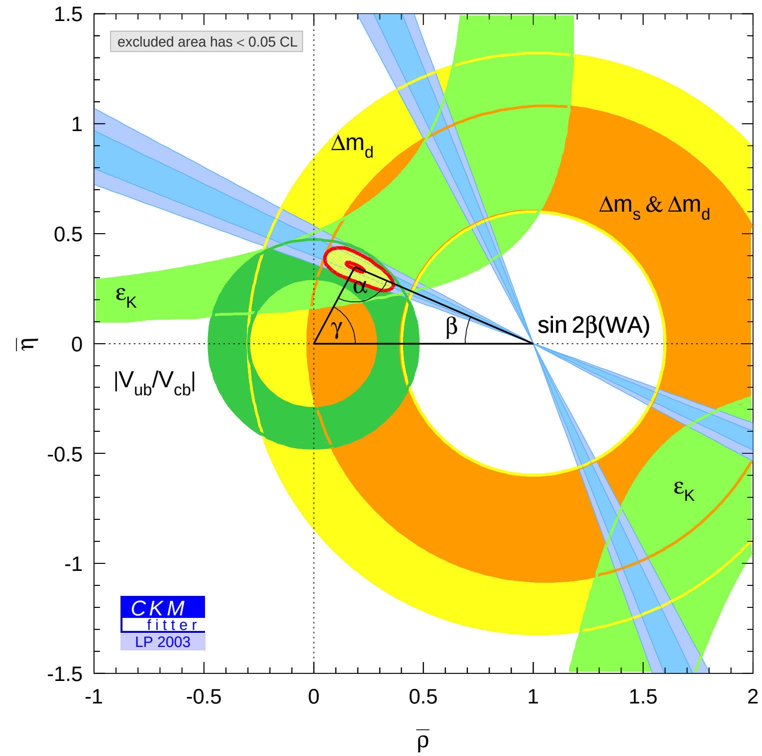
- Processes are particularly interesting when their suppression is associated with some, hopefully broken, conservation law
- Most significant example in this respect are proton decay and $\mu \to e\gamma$: transitions completely forbidden within the SM
- Unique perspectives in rare decays are those opened by precision studies of $|\Delta B| = 1$ FCNCs: $\bar{B} \to X_{s,d}\gamma$, $\bar{B} \to X_{s,d} l^+ l^-$, $\bar{B} \to K^{(*)} l^+ l^-$, $\bar{B}_{s,d} \to l^+ l^-$ and $\bar{B} \to X_{s,d} \nu \bar{\nu}$
  - forbidden at the tree-level within the SM
  - suppressed by the hierarchical structure of the CKM matrix
  - likely to be dominated by short-distance physics

precise determination of the flavor structure of the SM

enhanced sensitivity to physics beyond the SM

Unitarity Triangle

[Höcker et al. '03]



Present indirect and direct information on the CKM matrix already provide serious constraints on possible new sources of quark-flavor mixing

However,

- only tree-level and $|\Delta F| = 2$ amplitudes appear in usual UT fits
- some observables suffer from irreducible theoretical uncertainties at the 10% level

On the other hand,

- NP could in principle affect $|\Delta F| = 1$ and $|\Delta F| = 2$ loop-induced amplitudes in a very different way
- it would be desirable to base the fits only on observables with theoretical errors at the percent level

→ rare B decays are essential to address these two points

Status of $\bar{B} \to X_s l^+ l^-$ Decay

Beside $\bar{B} \to X_s \gamma$, the inclusive $\bar{B} \to X_s l^+ l^-$ transition provides a natural framework to perform high-precision studies of quark-flavor dynamics:

- GIM mechanism in the partonic amplitude introduces sensitivity to CKM factor $V_{ts}^* V_{tb}$
- Precise calculation of the inclusive rate within perturbative QCD in the heavy-quark limit $m_b \gg \Lambda_{\rm QCD}$:

$$\Gamma(\bar{B} \to X_s l^+ l^-) \xrightarrow[\text{HQE, OPE}]{m_b \to \infty} \Gamma(b \to s l^+ l^-)$$

- Systematic control of the suppressed non-perturbative corrections:

power corrections in $\Lambda_{\rm QCD}/m_b$

$\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$ and $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$ under control except from the endpoint region

[Ali et al. '96; Bauer & Burrell '99]

effects from intermediate $c\bar{c}$ pairs

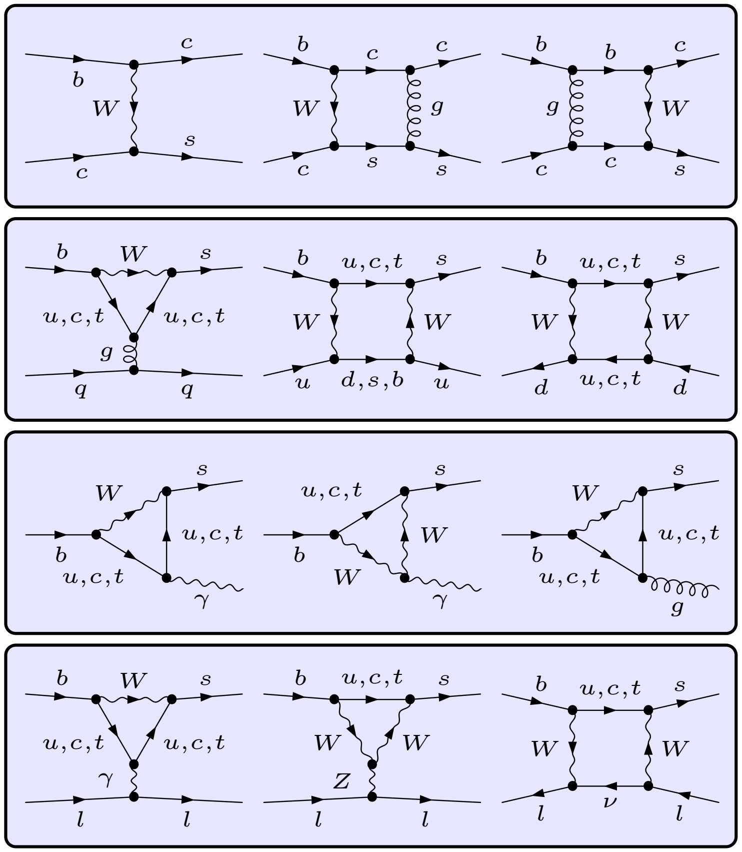
$\mathcal{O}(\Lambda_{\rm QCD}^2/m_c^2)$ well under control away from the charm resonance region

[Chen et al. '97; Buchalla et al. '97]

→ inclusive mode provides theoretical clean decay distributions: R, BR, $A_{\rm FB}$, ...

Low-Energy Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}\times\text{QED}} + \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \, \vec{C}^T(\mu)\vec{Q}$$



- Current-current operators:

$$Q_1 = (\bar{s}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma^\mu T^a b_L)\,,$$

$$Q_2 = (\bar{s}_L \gamma_\mu c_L)(\bar{c}_L \gamma^\mu b_L)\,.$$

- QCD penguins:

$$Q_3 = (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_q (\bar{q}\gamma^\mu q)\,, \ldots\,,$$

$$Q_6 = (\bar{s}_L \gamma_\mu \gamma_\nu \gamma_\rho T^a b_L) \textstyle\sum_q (\bar{q}\gamma^\mu \gamma^\nu \gamma^\rho T^a q)\,.$$

- Magnetic penguins:

$$Q_7 = \frac{e^2}{g^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}\,,$$

$$Q_8 = \frac{1}{g} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}\,.$$

- Semileptonic operators:

$$Q_9 = \frac{e^2}{g^2} (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_l (\bar{l}\gamma^\mu l)\,,$$

$$Q_{10} = \frac{e^2}{g^2} (\bar{s}_L \gamma_\mu b_L) \textstyle\sum_l (\bar{l}\gamma^\mu \gamma_5 l)\,.$$

→ $b \to s l^+ l^-$ can probe aspects of flavor physics not accessible with $b \to s\gamma$

Recent Perturbative Standard Model Calculations

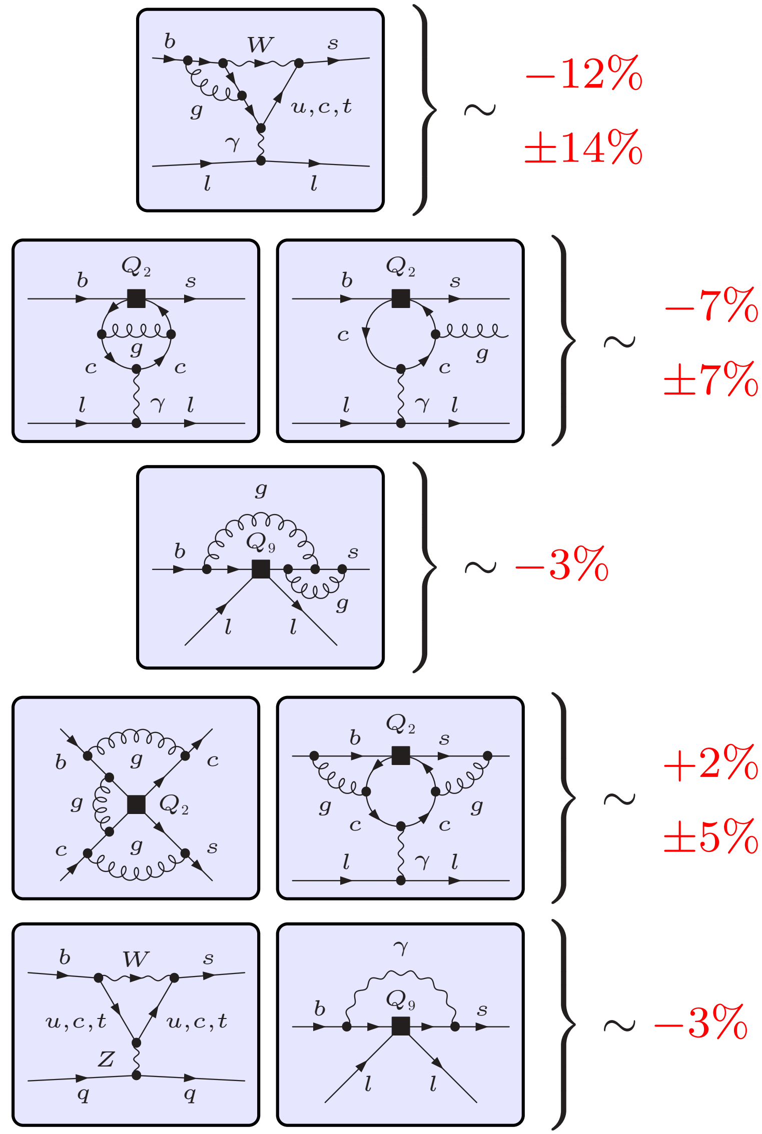
Resummation of NNLO QCD logarithms nearly finished:

- Two-loop $\mathcal{O}(\alpha_s^2)$ matching corrections $\vec{C}(\mu_W)$
  [Bobeth, Misiak & Urban '99]
- Two-loop $\mathcal{O}(\alpha_s^2)$ matrix elements $\langle Q_{1,2}(\mu_b)\rangle$
  [Asatrian et al. '01, '02; Ghinculov et al. '02]
- Two-loop $\mathcal{O}(\alpha_s^2)$ matrix element $\langle Q_9(\mu_b)\rangle$
  [Bobeth, Gambino, Gorbahn & UH '03]
- Three-loop $\mathcal{O}(\alpha_s^3)$ mixing $Q_{1\text{–}6} \to Q_{1\text{–}6,9}$
  [Gambino, Gorbahn & UH '03]
- Two-loop $\mathcal{O}(\alpha_s^2)$ matrix elements $\langle Q_{3\text{–}6}(\mu_b)\rangle$ still missing

Higher order EW effects under control:

- LO $\mathcal{O}(\alpha/\alpha_s)$ and NLO $\mathcal{O}(\alpha)$ QED effects
  [Bobeth, Gambino, Gorbahn & UH '03]

→ affect inclusive and exclusive modes



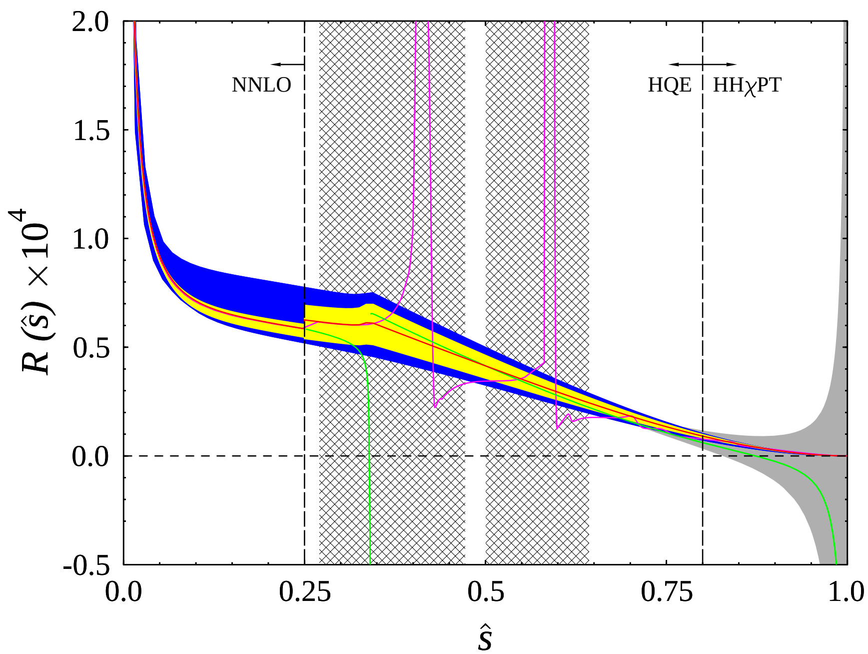
Dilepton Invariant Mass Spectrum

The normalized dilepton invariant mass spectrum can be written as:

$$R(\hat{s}) = \frac{1}{\Gamma(b \to X_c e \bar{\nu}_e)} \frac{d\Gamma(b \to X_s l^+ l^-)}{d\hat{s}} = \left(\frac{\alpha_{\rm em}}{4\pi}\right)^2 \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{(1-\hat{s})^2}{f(z)\kappa(z)}$$
$$\times \left[4\left(1+\frac{2}{\hat{s}}\right)\left|\widetilde{C}_{7,R}^{\rm eff}\right|^2 + (1+2\hat{s})\left(\left|\widetilde{C}_{9,R}^{\rm eff}\right|^2 + \left|\widetilde{C}_{10,R}^{\rm eff}\right|^2\right) + 12\,{\rm Re}\left(\widetilde{C}_{7,R}^{\rm eff}\widetilde{C}_{9,R}^{\rm eff\,*}\right) + \delta_R\right]$$

where $\hat{s} = q^2/m_b^2$ is the invariant dilepton mass

- Leading power corrections are smaller than 5% in both perturbative domains
- Low-$\hat{s}$: NNLO QCD and EW corrections lower SM prediction by 20% and reduce scale uncertainties from $\pm 20\%$ to $\pm 5\%$
- High-$\hat{s}$: scale uncertainties remain at the 10% level, as two-loop $\mathcal{O}(\alpha_s^2)$ matrix elements of $Q_{1,2}$ are unknown in this region



→ in the low-$\hat{s}$ region differential rate allows high-precision test of SM

Branching Ratio: Theory vs. Experiment

- Using $\Gamma(\bar{B} \to X_u e \bar{\nu}_e)$ to normalize the differential rate reduces uncertainty due to m_c/m_b [Chankowski & Slawianowska '03]

- Integrating the differential rate over the low- and high-$\hat{s}$ region gives:

$$\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{0.05<\hat{s}<0.25} = (1.36 \pm 0.06 \pm 0.14) \times 10^{-6}$$

$$\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{0.64<\hat{s}<0.78} = (2.57 \pm 0.23 \pm 0.38) \times 10^{-7}$$

- Integrating the non-resonant differential rate over the entire domain gives:

$$\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{\mathrm{SM}} = (4.33 \pm 0.33 \pm 0.22) \times 10^{-6}$$

- Within errors, SM prediction agrees reasonable with the experimental WA [Nakao '03]:

$$\mathrm{BR}(\bar{B} \to X_s l^+ l^-)_{\mathrm{exp}} = \left(6.2 \pm 1.1 \,{}^{+1.6}_{-1.3}\right) \times 10^{-6}$$

mission accomplished: all $b \to s l^+ l^-$ modes measured

next goal: measurements of the differential decay rates

Forward-Backward Asymmetry

The FB asymmetry can be written as:

$$A_{\rm FB}(\hat s) = \frac{1}{\Gamma(b \to X_c e \bar\nu_e)} \int_{-1}^{1} d\cos\theta \, \frac{d^2\Gamma(b \to X_s l^+ l^-)}{d\hat s \, d\cos\theta} \, {\rm sgn}\,(\cos\theta) =$$

$$\left(\frac{\alpha_{\rm em}}{4\pi}\right)^2 \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{(1-\hat s)^2}{f(z)\kappa(z)} \left[-6\,{\rm Re}\left(\widetilde C_{7,{\rm FB}}^{\rm eff} \widetilde C_{10,{\rm FB}}^{\rm eff\,*}\right) - 3\hat s\,{\rm Re}\left(\widetilde C_{9,{\rm FB}}^{\rm eff} \widetilde C_{10,{\rm FB}}^{\rm eff\,*}\right) + \delta_{\rm FB}\right]$$

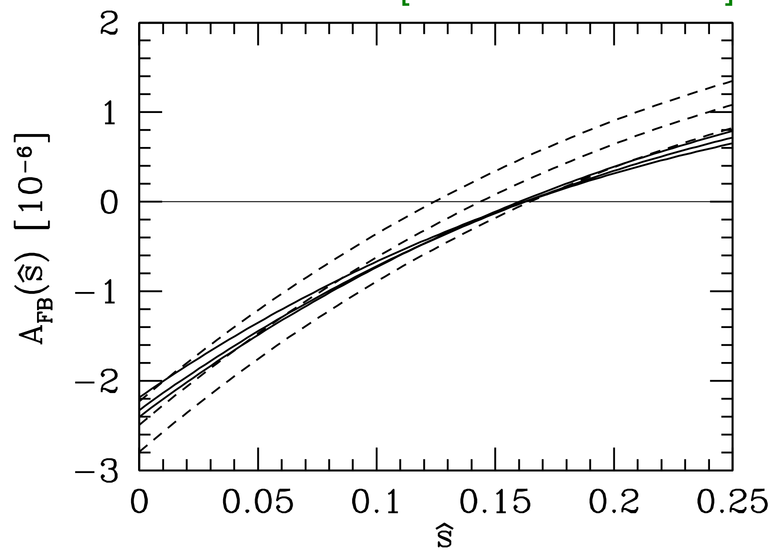
where θ is the angle between the l^+ and the $\bar B$ momenta in the dilepton CM frame

- Position of FB asymmetry zero particularly interesting to determine sign and magnitude of C_7/C_9:

$$\hat s_{0,\rm SM} = 0.162 \pm 0.002 \pm 0.005$$

- NNLO QCD corrections enhance result by 15% and reduce theoretical uncertainties from $\pm 15\%$ to $\pm 5\%$ [Asatrian et al. '02; Ghinculov et al. '02]

[Asatrian et al. '02]



→ FB asymmetry zero provides one of the most sensitive tests of NP

Status of $\bar{B} \to K^{(*)} l^+ l^-$ Decays

Within $b \to s l^+ l^-$ transitions, one can perform interesting test of flavor dynamics also by means of exclusive decays:

- $\bar{B} \to K^{(*)} l^+ l^-$ modes easier accessible in experiment than their inclusive counterpart
- Theoretically, substantial improvement is achieved through QCD-improved factorization:

$$\langle K^{(*)} l^+ l^- | \vec{Q} | \bar{B} \rangle = \vec{C} \, \xi + \Phi_{\bar{B}} \otimes \vec{T} \otimes \Phi_{K^{(*)}} + \mathcal{O}(\Lambda_{\rm QCD}/m_b)$$

  - LEL allows to express the ten independent QCD by three universal soft form factors ξ [Charles et al. '98; Beneke & Feldmann '00]
  - Wilson coefficients $\vec{C}$ and hard scattering kernels $\vec{T}$ are calculable in perturbation theory [Beneke et al. '01]
  - light-cone wave functions $\Phi_{K^{(*)}}$ have been deeply studied using LCSR [Braun & Fylianov '89, '90; Ball et al. '98; Ball & Braun '99]
  - in contrast $\Phi_{\bar{B}}$ and ξ are poorly known, but unquenched LQCD may provide better results in the near future

→ limited understanding of soft physics in general restrains power of exclusive modes

Phenomenology of $\bar{B} \to K^* l^+ l^-$

- Impact of NNLO QCD corrections is sizeable only in low-q^2 region, where they enhance differential rate by 10% [Beneke et al. '01]

- Predictions of decay distributions affected by large uncertainty of $\pm 35\%$ due to form-factors [Ali et al. '00; Beneke et al. '01; Zhong et al. '02]

- To first order FB asymmetry zero free of hadronic uncertainties [Burdman '98; Ali et al. '00; Beneke et al. '01]:

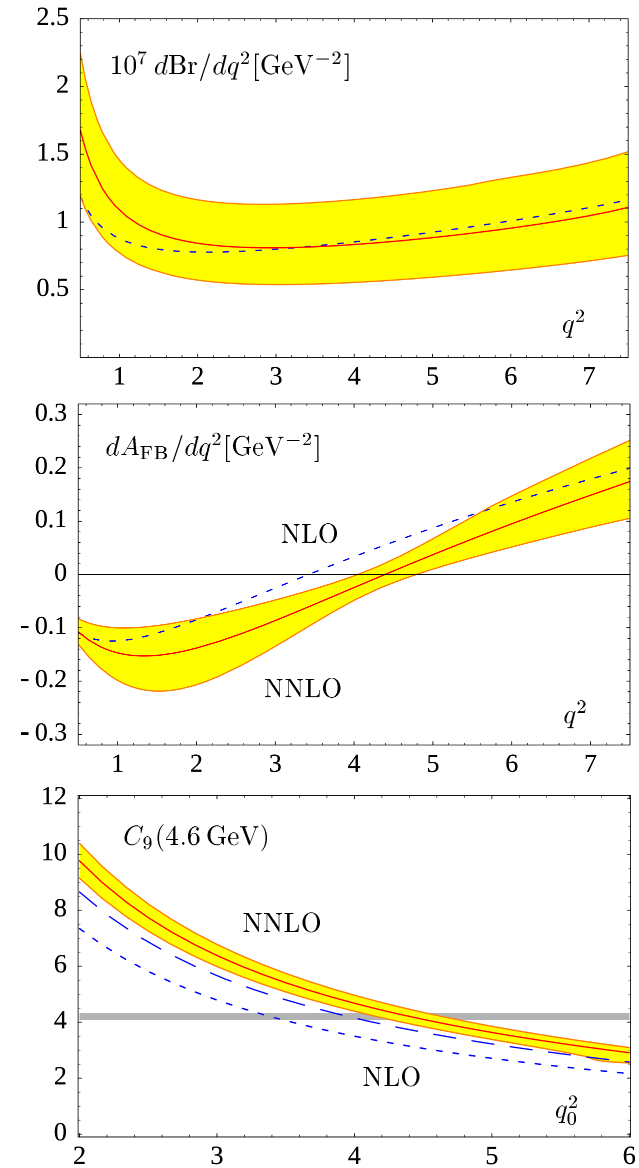
$$q^2_{0,\mathrm{SM}} = (4.17 \pm 0.25 \pm 0.55)\,\mathrm{GeV}^2$$

- NNLO QCD corrections shift asymmetry zero by 30% toward higher values and reduce scale uncertainties from $\pm 15\%$ to $\pm 5\%$ [Beneke et al. '01]

- FB asymmetry zero allows to test Wilson coefficient of Q_9 with 10% accuracy [Beneke et al. '01]

→ lot to learn from exclusive decays: QCD, NP, ...

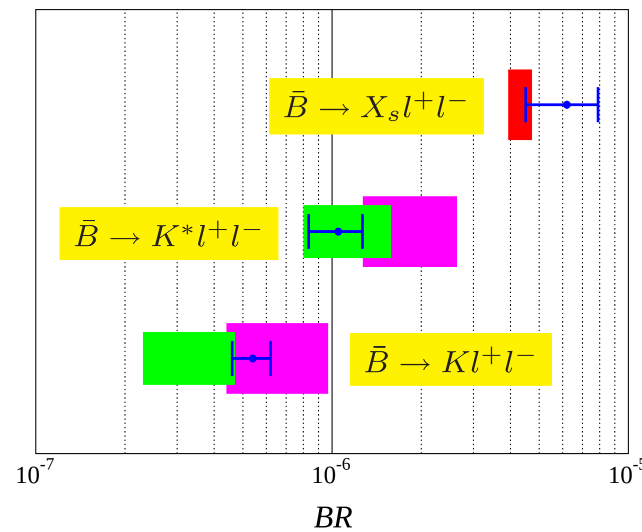
[Beneke et al. '01]



Outlook

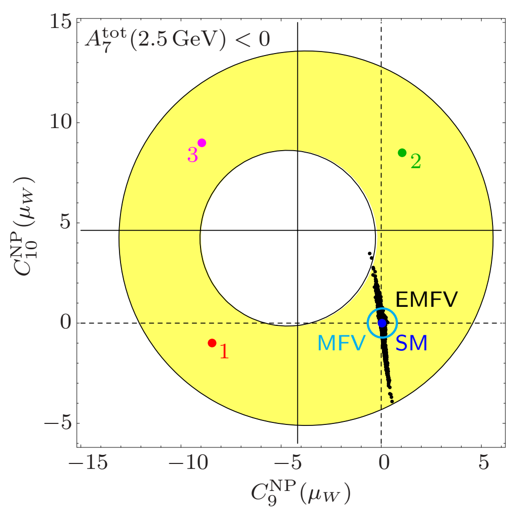
- Within errors, reasonable agreement between experiment and SM predictions for all $b \to s l^+ l^-$ decays
- B factories and upcoming flavor physic experiments at Tevatron and LHC are able to study differential rates and FB asymmetries of $b \to s l^+ l^-$ modes
- In NP with MFV effects on BR are rather small, but in general FB asymmetry can change dramatically

→ unique probes of flavor structure of SM and NP



[Eigen '01]

year	2005	2008
$\mathcal{L}[\text{fb}^{-1}/\text{y}]$	500	1000
yield	365 (280)	728 (565)
$\sigma(\text{stat})[\%]$	7 (9)	5 (6)
$\sigma(\text{sys})[\%]$	7 (12)	6 (10)



[Ali et al. '01]