Rare Kaon decays and CKM unitarity tests

Giancarlo D'Ambrosio
INFN-Sezione di Napoli

Lake Geneva, 8th October 03

- Based mainly on CKM workshop for CKM unitarity tests
- on work with G. Buchalla and G. Isidori hep-ph/0308008 and
- G.D., G. Ecker, G. Isidori, and J. Portoles, JHEP 08 (98) 004, hep-ph/9808289

Outline

- Chiral Perturbation theory
- Weinberg scattering lenghts and K_{l4}
- CKM unitarity and V_{us}, V_{ud}
- Direct CP violating contribution to $K_L \to \pi^0 e^+ e^-$
- CP conserving $K_L \to \pi^0 e^+ e^-$
- $K_L = K_2 + \epsilon K_1 \to \pi^0 e^+ e^-$: New results from NA48KS
- Conclusions

Chiral Perturbation Theory

χPT effective field theory based on the two assumptions

- π's are the Goldstone boson of $SU(3)_L \otimes SU(3)_R \to SU(3)_V$
- *(chiral) power counting* i.e. the theory has a small expansion parameter: $p^2/\ \Lambda^2_{\chi SB}$: $\Lambda_{\chi SB} \sim 4\pi F_\pi \sim 1.2$ GeV

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}^2_{\Delta S=0} + \mathcal{L}^4_{\Delta S=0} + \cdots = \frac{{F_\pi}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \to l\nu,\ \pi\pi \to \pi\pi,\ K \to \pi ..} + \overbrace{\sum_i L_i O_i}^{K \to \pi ..} + \cdots$$

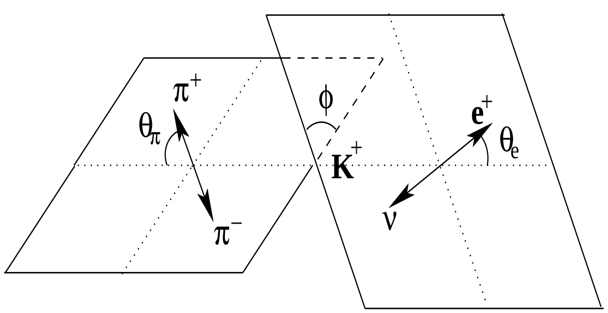
Fantastic chiral prediction $A_{\pi\pi} \sim (s - m^2_\pi)/{F_\pi}^2$ Weinberg, Colangelo *et al*

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+ \gamma\gamma,\, K \to \pi l^+ l^-} + \cdots$$

K_{l4} and $\pi\pi$ strong phases $\delta_I^l(s)$

$$K^+ \to \pi^+\pi^- l\nu \Longrightarrow \text{form factors} \qquad F_i(s) = f_i(s) e^{i\delta_0^0(s)} + ..$$

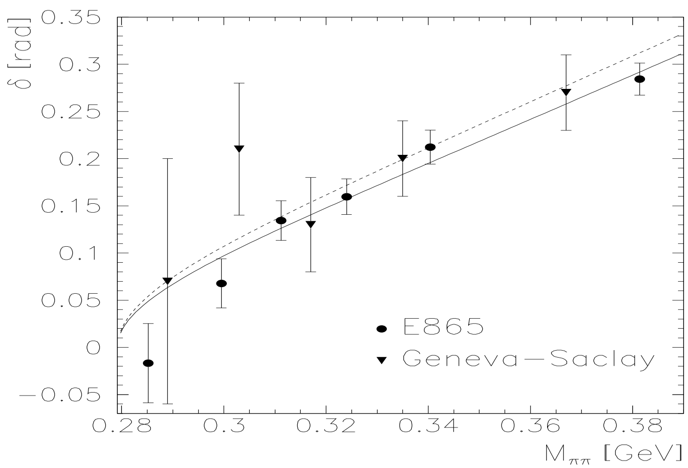
$$A_{\pi\pi(s)} \sim (s - m_\pi^2)/F_\pi^2 + \quad h.o. \qquad \Leftrightarrow \delta_I^l(s)$$



- Look angular plane asymmetry $\pi\pi$ $l\nu$
- $\delta_I^l(s \sim 4\ m_\pi^2)$, $\pi\pi$ phase shifts near threshold $\Longrightarrow a_I^l$
- a_0^0 strongly sensitive to $\langle 0|\bar{q}q|0\rangle$

a_0^0: BNL-E865 and THEORY

$$a_0^0 \begin{cases} 0.16 & O(p^2) & & \text{Weinberg 79} \\ 0.20 \pm 0.01 & O(p^4) & \text{parameter free pre.} & \text{Gasser Leutwyler} \\ 0.220 \pm 0.005 & O(p^6) & \text{+analy.+disper.} & \text{Bijnens et. al} \end{cases}$$



BNL-E865 $a_0^0 = 0.216 \pm 0.013$
+ChPT+Th [hep-ph/0301040]

$K(k) \to \pi(p) l\nu$ and CKM unitarity

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ V_{ub} negligible

$$M(K_{l3}) = \frac{G_F}{\sqrt{2}} C V_{us} \left[f_+(t)(p+k)_\mu + f_-(t)(k-p)_\mu \right] L^\mu \qquad t = (k-p)^2$$

- CVC $\Longrightarrow f_+(0) = 1 + ..,$ $f_-(0) = 0$ .. Ademollo-Gatto
- $f_-(t) \Leftrightarrow f_0(t), f_+(t),$ $f_{+,0}(t) = f_+(0)(1 + \lambda_{+,0} t/m_\pi^2)$
- $\lambda_{+,0}$ measured, TH $f_+(0) = 1 + f_2 + f_4$

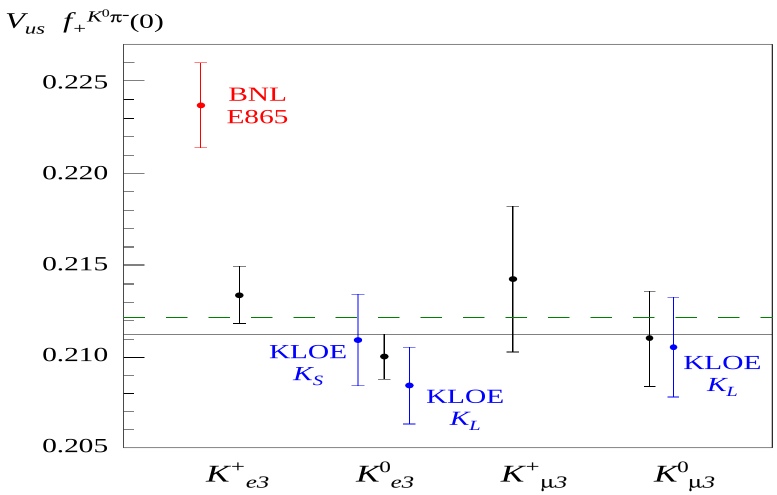
  CHPT $f_2 = -0.023 \pm$ "0" Gasser Leutwyler

  Guess $f_4 = -0.016 \pm 0.008$ Leutwyler Roos

$K(k) \to \pi(p) l\nu$ and CKM unitarity

$$\Gamma(K^i_{l3}) = \mathcal{N}_i |V_{us}|^2 |f_+(0)|^2 I(\lambda_+, \lambda_0)$$

$I(\lambda_+, \lambda_0)$ stable



e.m./Isospin corrections+TH. $f_+(0)$

$$|V_{us}| = 0.2196 \pm 0.0019_{\rm exp} \pm 0.0018_{\rm th}$$

E865 $\quad 0.2272 \pm 0.0023_{\rm exp} \pm 0.0018_{\rm th}$

CKM, Isidori

V_{us} and V_{ud}

$$|V_{us}| = 0.2196 \pm 0.0019_{\rm exp} \pm 0.0018_{\rm th}$$ CKM, Isidori et al.

$$|V_{us}|_{\rm E865} = 0.2272 \pm 0.0023_{\rm exp} \pm 0.0018_{\rm th}$$

$$|V_{us}|_{\rm Hyperons} = 0.2250 \pm 0.0027_{\rm exp} \pm 0.0018_{\rm th}$$ Cabibbo,Swallow, Winston

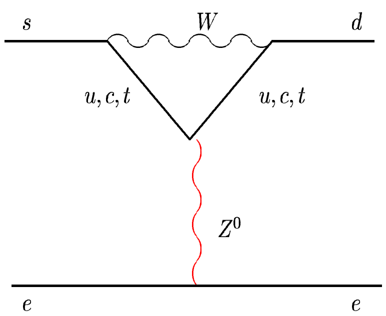
V_{ud}

- Superallowed $\Longrightarrow |V_{ud}| = 0.9740 \pm 0.0005 \stackrel{\rm Unit.}{\Longrightarrow} |V_{us}| = 0.2269 \pm 0.0021$
- Beta decay, g_A, g_V, $|V_{ud}| = 0.9731 \pm 0.0015$
  Grenoble $|V_{ud}| = 0.9718 \pm 0.0013$?
- News expected soon from KLOE,NA48

Motivations

SM at short distance predicts the current⊗current structure for $K \to \pi e^+ e^-$

$$\mathcal{H} \sim \frac{G_F \alpha}{\sqrt{2} M_W^2} \quad \overline{s}_L \gamma_\mu d_L \quad \overline{e}_L \gamma^\mu e_L \quad \left[\sum_q V_{qs}^* V_{qd} \, m_q^2 \right] + h.c$$



$$\left[A^2 \lambda^5 \, (1 - \rho - i\eta) m_t^2 + \lambda m_c^2 \right]$$

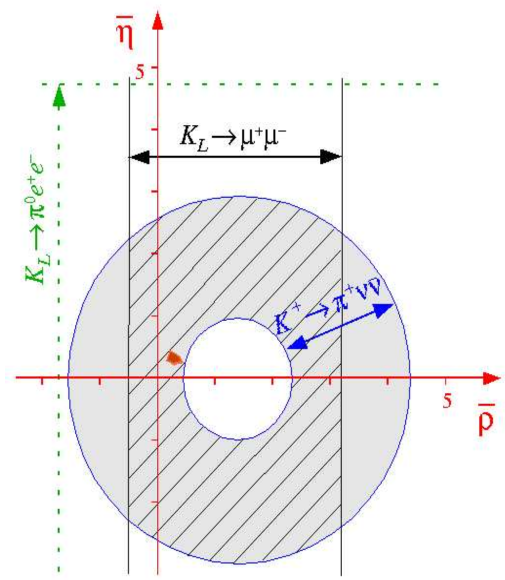
Gilman,Wise; Buchalla,Buras, Lautenbacher

$$K_L \to \pi^0 e^+ e^- \quad \begin{cases} \text{CP violating} \\ \text{sensitivity to new physics} \\ Im\lambda_t = \Im(V_{ts}^* V_{td}) \end{cases}$$

Impact on New Physics from $K_L \to \pi^0 e^+ e^-$

- CKM-fit: B- and K- $\Rightarrow$ $Im\lambda_t = \Im(V_{ts}^* V_{td}) = (1.3 \pm 0.11) \cdot 10^{-4}$

- But limits from K-physics only very weak Colangelo-Isidori,Buras-Silvestrini



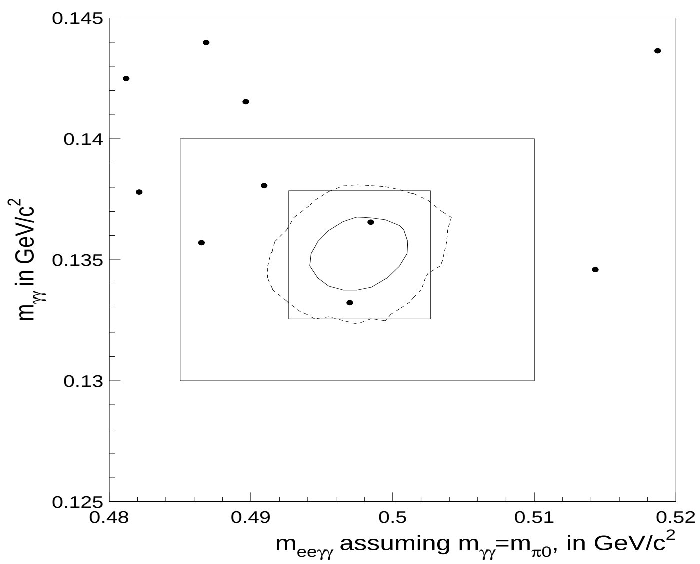
Isidori

$K_L \to \pi^0 e^+ e^-$ vs. $K_L \to \pi^0 \nu \overline{\nu}$

- $K_L \to \pi^0 \nu\overline{\nu} \overbrace{(2.8 \pm 1.0)}^{\text{SM}} \cdot 10^{-11}$ $\overbrace{< 5.9 \cdot 10^{-7}}^{\text{KTeV}}$ no e.m. bck.
- $$K_L \to \pi^0 e^+ e^- : \overbrace{\sim 1 \cdot 10^{-11}}^{\text{SM}} \quad \text{But} \begin{cases} K_L \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^- \\ K_L = K_2 + \epsilon K_1 \to \pi^0 e^+ e^- \end{cases}$$
- – Greenlee bck.
  $$Br(K_L \to e^+ e^- \gamma\gamma) \quad \begin{cases} (5.8 \pm 0.3) \cdot 10^{-7} & \text{No kin. cut} \\ 1 \cdot 10^{-10} & \text{kin. cut} \end{cases}$$
- $\frac{\text{signal}}{\text{bck.}} \sim 0.1$ But bck. can be known accurately (QED)$\Longrightarrow$ statistics

$Br(K_L \to \pi^0 e^+ e^-)$ KTeV

- '97('99) $2.6 \cdot 10^{11}\ K_L$
- expected bck. 1 evt. '97 ('99) 2 evt. (1)
- $Br < 5.1 \cdot 10^{-10}$ (3.5) combined $< 2.8 \cdot 10^{-10}$



KTeV

Foreseen statistics to measure the Direct- CP-violating part in the SM $\Im\lambda_t$ at 30% : 1.000 more K_L

Control over three contributions

- Direct CP violation in $K_L \to \pi^0 e^+ e^-$
- CP conserving $K_L \to \pi^0 e^+ e^-$
- $K_L = K_2 + \epsilon K_1 \to \pi^0 e^+ e^-$

Short distance contribution to $K_L \to \pi^0 e^+ e^-$

$K_2 \to \pi^0 (e^+ e^-)_{J=1}$ dominated by the s.d.

$$Q_{7V} = \bar{s}\gamma^\mu (1-\gamma_5) d\, \bar{\ell}\gamma_\mu \ell \,, \qquad Q_{7A} = \bar{s}\gamma^\mu (1-\gamma_5) d\, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

$$\begin{aligned} B(K_L \to \pi^0 e^+ e^-)_{\mathrm{CPV-dir}} &= \frac{\tau(K_L)}{\tau(K^+)} \frac{B(K^+_{e3})}{|V_{us}|^2} (y_{7A}^2 + y_{7V}^2) \left[\Im(V_{ts}^* V_{td})\right]^2 \,, \\ &= (2.45 \pm 0.22) \times 10^{-12} \left[\frac{\Im\lambda_t}{10^{-4}}\right]^2 \end{aligned}$$

where

$$\Im\lambda_t = \Im(V_{ts}^* V_{td}) \xrightarrow{\mathrm{SM}} (1.33 \pm 0.11) \times 10^{-4} \qquad \text{Buchalla et al,CKM}$$

$$K_L(p) \to \pi^0(p_3)\gamma(q_1)\gamma(q_2)$$

Lorentz + gauge invariance $\Rightarrow$ $M \sim$ $A(y,z)$ $B(y,z)$

	$A(y,z)$	$B(y,z)$
	$\gamma\gamma$	$\gamma\gamma$
$y = p\cdot(q_1-q_2)/m_K^2$, $z = (q_1+q_2)^2/m_K^2$	$J=0$	D $-$ wave too
$r_\pi = m_\pi/m_K$	$F^{\mu\nu}F_{\mu\nu}$	$F^{\mu\nu}F_{\mu\lambda}\partial_\nu K_L \partial^\lambda \pi^0$

- $$\frac{d^2\Gamma}{dydz} \sim z^2|A+B|^2 + \left(y^2 - \left(\frac{(1+r_\pi^2-z)^2}{4} - r_\pi^2\right)\right)^2 |B|^2 \qquad S, B$$
- Different gauge structure $\Rightarrow$ B$\neq$0 at $z \to 0$ (collinear photons).

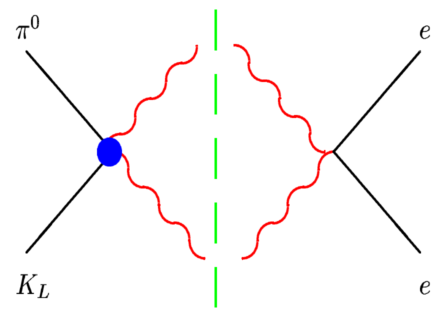
Crucial role in $K_L \to \pi^0 e^+ e^-$

A suppressed by m_e/m_K

B is not

Morozumi et al, Flynn Randall

Sehgal Heiliger, Ecker et al., Donoghue et al.



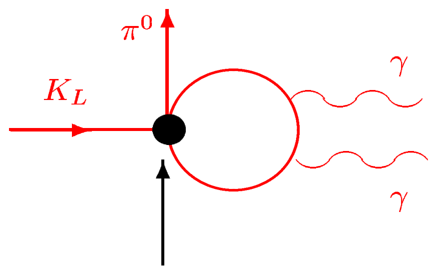
$K_L \to \pi^0 \gamma\gamma$

- $O(p^4)$

Ecker, Pich, de Rafael; Cappiello,G.D

CT | Loop

0



only A

But $\frac{\Gamma(K_L \to \pi^0\gamma\gamma)_{p_4}}{\Gamma(K_L \to \pi^0\gamma\gamma)_{\exp}} \sim \frac{1}{2.5}$

G_8 from $K_S \to 2\pi$

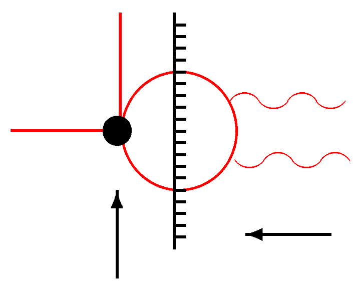
- $O(p^6)$ A, B from:

3 CT′s

$F_{\mu\nu} F^{\mu\alpha} \partial_\alpha K_L \partial^\nu \pi^0$

$F^2 \; \partial K_L \partial \pi^0$

$F^2 \; m_K^2 K_L \pi^0$



Cappiello, G.D., Miragliulo

Cohen, Ecker, Pich

Full description of unitarity cut

$$A(K \to 3\pi) = a + b\; Y + c\; Y^2 + d\; X^2$$

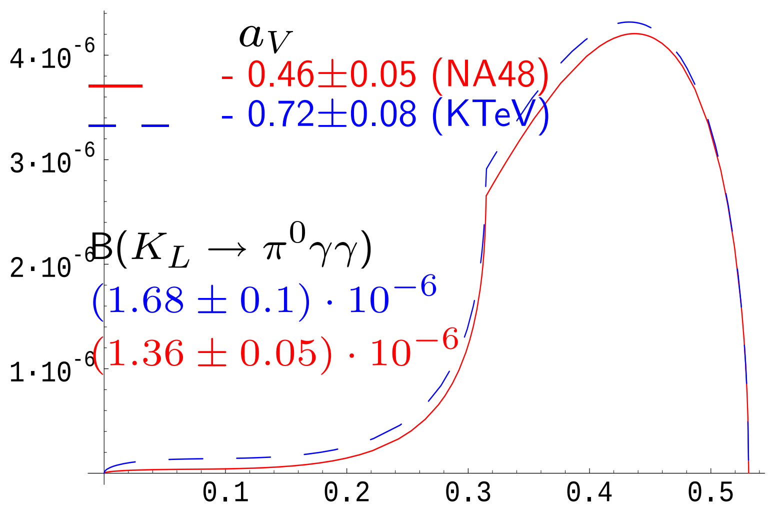
$$A_{\rm CT} = \alpha_1(z - r_\pi^2) + \alpha_2$$
$$B_{\rm CT} = \beta$$

VMD $\Rightarrow$ 1 coupling a_V $\quad(\sim -0.6$ G.D., Portoles)

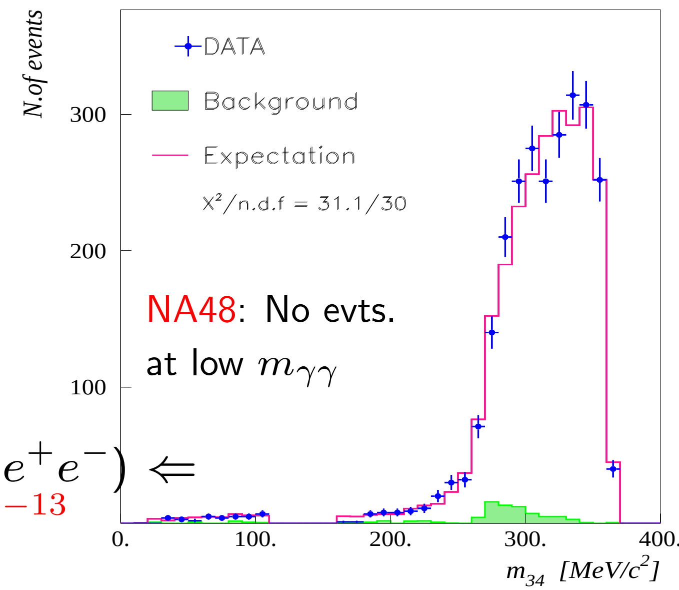
(Ecker, Pich, de Rafael;Sehgal et al.)

$$\alpha_1 = \frac{\beta}{2} = -\frac{\alpha_2}{3} = -4a_V \sim 2 \qquad \text{n.d.a.}\sim 0.2$$

- KTeV and NA48: 1 parameter fit (a_V) with all the unitarity corrections



B($K_L \to \pi^0 e^+ e^-$) $\Leftarrow$
$< 5 \cdot 10^{-13}$

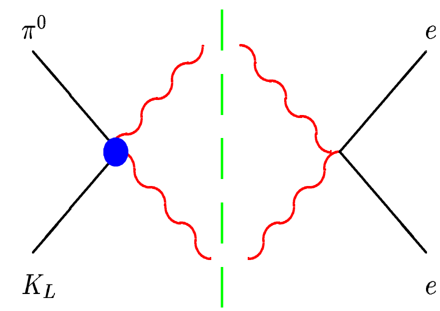


$K_L \to \pi^0 \gamma^*(q_1)\gamma^*(q_2) \to \pi^0 e^+ e^-$: need for a form factor

$B(z) \sim B(0)$ over all the interesting physical range (Dalitz plot analysis, explicit model dependence)

$$B(K_L \to \pi^0 \gamma\gamma)_{M_{\gamma\gamma}<110 \text{ MeV}} = 2.0 \times 10^{-9} \times |B(0)|^2$$

Generally one computes the model independent imaginary part for $K_L \to \pi^0 \to \pi^0 e^+ e^-$. Also the dispersive part has to be computed



$\sim \ln \Lambda \Rightarrow$ form factor $f(q_1^2, q_2^2)$.

Short distance forbidden at leading order in α_s by Furry theorem

Matching with short distance is believed to be achieved with Vectors.

$$B(z, y; q_1^2, q_2^2) = B(z) \times f(q_1^2, q_2^2)$$

$$f(q_1^2, q_2^2) = 1 + \alpha \left(\frac{q_1^2}{q_1^2 - m_V^2} + \frac{q_2^2}{q_2^2 - m_V^2} \right) + \beta \frac{q_1^2 q_2^2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)}$$

$m_V = m_\rho$. Since we require $f(q_1^2, q_2^2) \to 0$ for large q^2

$$1 + 2\alpha + \beta = 0$$ Buchalla,G.D.,Isidori

Donoghue,Gabbiani use a stronger fall-off form factor ($f \sim 1/q^4$; extra condition $\beta = -\alpha = 1$)

$$K_L \to \pi^0 \gamma^*(q_1)\gamma^*(q_2) \to \pi^0 e^+(k_1) e^-(k_2)$$

$$M\,(\,K_L \to \pi^0\,e^+\,e^-)_{\text{CPC}} = \frac{G_8 \alpha^2 B(z) G(z)}{16\pi^2 m_K^2}\, p\cdot(k_1 - k_2)(p + p_\pi)_\mu \overline{u}(k_2)\gamma^\mu v(k_1)$$

$G(z)$ encodes the form factor $f(q_1^2, q_2^2)$-dependence:

$$G(z) = \frac{2}{3}\ln\left(\frac{m_\rho^2}{-s}\right) - \frac{1}{9} + \frac{4}{3}(1+\alpha) \qquad s = (k_1 + k_2)^2$$

$$Br_{\text{CPC}} = 7.0 \times 10^{-14} \times |B(0)|^2 \times \left\{1 + \left[1.4 + 1.4(1+\alpha) + 0.4(1+\alpha)^2\right]\right\}$$

$$3.5 \times 10^{-4} \times B(K_L \to \pi^0\gamma\gamma)_{M_{\gamma\gamma}<110\ \text{MeV}} \overset{\text{NA48}}{<} 3 \cdot 10^{-12} \qquad \{< 10\}$$

$$G(z) = \frac{2}{3}\ln\left(\frac{m_\rho^2}{-s}\right) - \frac{1}{9} + \frac{4}{3}(1+\alpha)$$

Buchalla,G.D.,Isidori

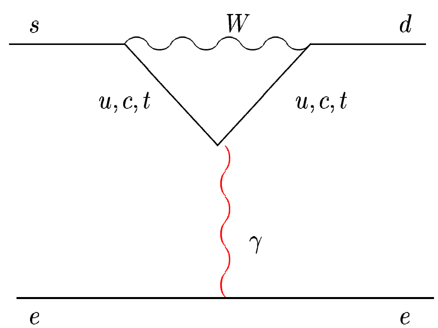
- Two-photon real $\Longrightarrow$ Model-independent; Agreement with Flynn-Randall, Ecker, Pich,de Rafael
- No singularity for $m_e \to 0$
- We disagree with Donoghue-Gabbiani

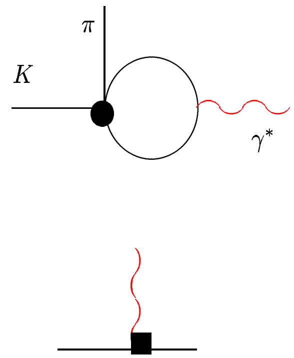
$$G(z) = \frac{2}{3}\ln\left(\frac{m_\rho^2}{-s}\right) - \frac{1}{4}\ln\left(\frac{-s}{m_e^2}\right) + \frac{7}{18}$$

$K^{\pm}(K_S) \to \pi^{\pm}(\pi^0)\ell^+\ell^-$

- short distance $<<$ long distance

LD described by form factor W





$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu\overline{\mu})$, slopes
- $a_i \quad O(p^4)$ Ecker, Pich, de Rafael
- $b_i \quad O(p^6)$ G.D., Ecker, Isidori, Portoles
- $a_+, \; b_+$ in general not related to $a_S, \; b_S$

- Expt. E865

  $K^+ \to \pi^+ e^+ e^- : \quad a_+ = -0.586 \pm 0.010 \quad b_+ = -0.655 \pm 0.044$

  confirmed in $K^+ \to \pi^+ \mu \overline{\mu}$

Problems: a_i b_i same phenomenological size
p^4 p^6 different theoretical order

Probably explained by large VMD. Then we can just parameterize

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 4.6 \times 10^{-9} a_S^2$$

not predicted but dynamically interesting: $a_S \sim \mathcal{O}(1)$ (?): NA48

$K_S \to \pi^0 e^+ e^-$ at NA48/1 Collaboration at CERN

- 7 events observed (with 0.15 expected background events)

$$B(K_S \to \pi^0 e^+ e^-)_{m_{ee} > 165 \text{ MeV}} = \left(3.0^{+1.5}_{-1.2} \pm 0.2\right) \times 10^{-9}$$

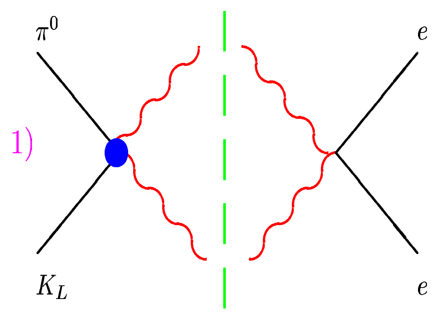
$$|a_S| = 1.08^{+0.26}_{-0.21}$$

Using Vector matrix element and form factor equal to 1

$$B(K_S \to \pi^0 e^+ e^-) = \left(5.8^{+2.9}_{-2.4}\right) \times 10^{-9}$$

$K_L \to \pi^0 e^+ e^-$: summary

$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) \leq 5 \cdot 10^{-10}$ KTeV

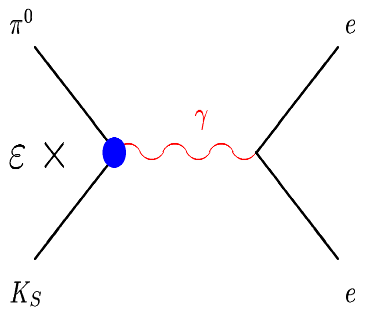


CP conserving NA48

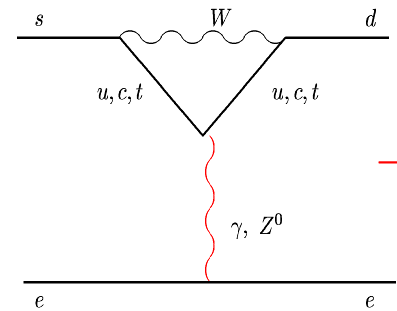
Br$(K_L \to \pi^0 e^+ e^-) < 3 \cdot 10^{-12}$

V-A$\otimes$ V-A $\Rightarrow \langle \pi^0 e^+ e^- | (\bar{s} d)_{V-A} (\bar{e} e)_{V-A} | K_L \rangle$ violates CP

2) $\varepsilon \times$



$+$ 3)



$\longrightarrow$ $\mathrm{Im}\ \lambda_t$

$\lambda_t = V_{td} V^*_{ts}$

$\uparrow$ $\mathrm{B}(K_S \to \pi^0 e^+ e^-) = 4.6 a_S^2 \times 10^{-9}$

Possible large interference: $a_S < -0.5$ or $a_S > 1$; short distance probe even for a_S large

$$|2) + 3)|^2 = \left[15.3\ a_S^2 - 6.8 \frac{Im\lambda_t}{10^{-4}}\ a_S + 2.8 \left(\frac{Im\lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12}$$

$$[17.2 \pm \qquad 9.4 + \qquad 4.7] \quad \cdot 10^{-12}$$

- The large slope for $K^+ \to \pi^+ e^+ e^-$ calls for large VMD
- $K^+ \to \pi^+ e^+ e^-$ receives substantial $\pi\pi$-loop, contrary to $K_S \to \pi^0 e^+ e^-$ (~ 0),
- if we split

$$\left(\frac{a_i^{\rm VMD}}{1 - z m_K^2 / m_V^2} + a_i^{\rm nVMD} \right) \approx \left[\left(a_i^{\rm VMD} + a_i^{\rm nVMD} \right) + a_i^{\rm VMD} \frac{m_K^2}{m_V^2} z \right]$$

Then we can determine both terms from expt.

$$a_+^{\rm VMD} = \frac{m_V^2}{m_K^2} b_+^{\rm exp} = -1.6 \pm 0.1 \ , \qquad a_+^{\rm nVMD} = a_+^{\rm exp} - a_+^{\rm VMD} = 1.0 \pm 0.1$$

- Also we can hope $a_i^{\rm VMD}$ obey a short distance relation since i) VMD is a larger scale and ii) NOT affected by $\pi\pi$-loop
- The only operator at short distances is $Q_7 = \overline{s}\gamma^\mu(1-\gamma_5)d\,\overline{\ell}\gamma_\mu\ell$,

$$\mathcal{H}_{eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \Big[\sum_{i=1}^{6,7V} (z_i(\mu) + \tau y_i(\mu)) Q_i(\mu) \ + \ \tau y_{7A}(M_W) Q_{7A}(M_W) \Big]$$

$\tau = -V_{ts}^* V_{td}/V_{us}^* V_{ud}$. The Wilson coefficients $z_7(\mu)$ and $\tau y_7(\mu)$ determine the CPC CPV amplitudes and their relative sign. The isospin structure of Q_{7V} leads

$$(a_S)_{\langle Q_{7V}\rangle} = -(a_+)_{\langle Q_{7V}\rangle}$$

- If this relation is obeyed by the full VMD amplitude

$$(a_S^{\rm VMD})_{\langle Q_{7V}\rangle} = -a_+^{\rm VMD} = 1.6 \pm 0.1$$

in good agreement with NA48 $(|a_S| = 1.08^{+0.26}_{-0.21})$

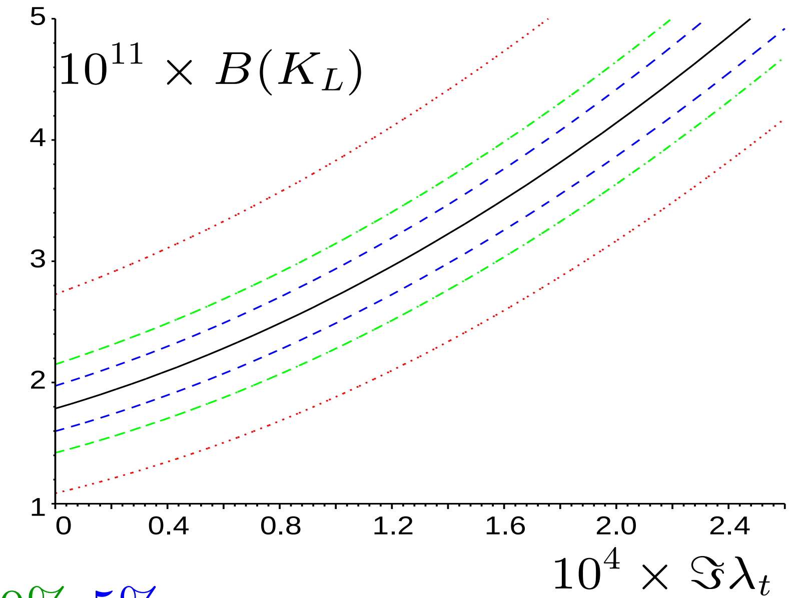
- Having i) separated the contribution better suited to comparison with s.d. (VMD) and ii) realized that this dominates Theoret. and Phenom.(NA48) a_S

- we believe the positive interference of s.d.

$$B(K_L \to \pi^0 e^+ e^-)_{\rm SM} = (3.1^{+1.2}_{-0.9}) \times 10^{-11} \quad a_S$$

KTeV $B(K_L \to \pi^0 e^+ e^-)$

$< 2.8\times 10^{-10}$ at 90 %C.L.



Present error on $a_S = 1.08$, 10% 5%, no error

K-physics bound: $-1.2 \times 10^{-3} < \Im\lambda_t < 1.0 \times 10^{-3}$ at 90 %C.L.

Conclusions

- CHPT Test and SM tests, K_{l3}: $|V_{us}| = 0.2196 \pm 0.0019_{\rm exp} \pm 0.0018_{\rm th}$, $|V_{us}|_{\rm E865} = 0.2272 \pm 0.0023_{\rm exp} \pm 0.0018_{\rm th}$, $|V_{us}|_{\rm unit.} = 0.2269 \pm 0.0021$

- NA48, important info on $K_L \to \pi^0\gamma\gamma$ (now all 3 unknown $\sim$ fixed, chiral-VMD test also $K_S \to \gamma\gamma$ involved)

- Direct CP violation in $K_L \to \pi^0 e^+e^-$ (SM) well known, NP bounds from K-physics important

- CP conserving $K_L \to \pi^0 e^+e^-$ New form factor with explicit model dependence: Still this contribution small

- Dynamical model for $a_S \Rightarrow$ Positive interference expected