WIN 03
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# The Majorana Questions

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# The Majorana Questions

Why do we think neutrinos are Majorana particles ($\bar{\nu} = \nu$) ?

How can we test whether $\bar{\nu} = \nu$? Is neutrinoless double beta decay the only way?

If $\bar{\nu} = \nu$, what are the implications for $\cancel{CP}$?

What are Majorana $\cancel{CP}$ phases? What $\cancel{CP}$ effects can they produce? Can we observe these phases?

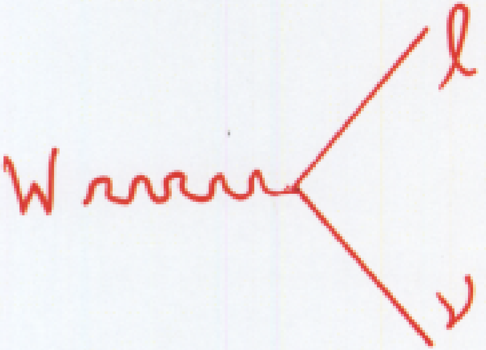
We will assume CPT invariance.

$$\cancel{CPT} \Longrightarrow \left\{ \begin{array}{l} \text{Neutrinos are not} \\ \text{Majorana particles} \end{array} \right\}$$

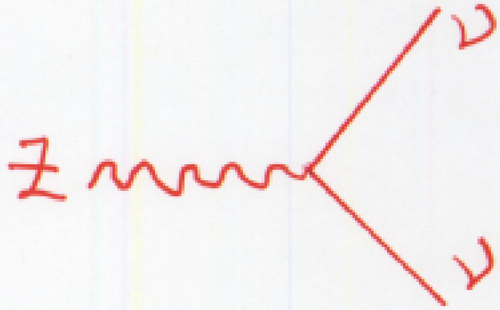
(Barenboim, Beacom, Borissov, B.K.)

# MAJORANA NEUTRINOS
— or —
# DIRAC NEUTRINOS?

The S(tandard) M(odel)




and

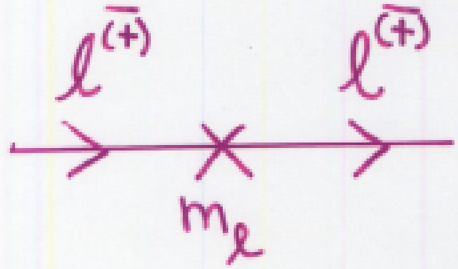



couplings conserve the Lepton Number L defined by—

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

So do the Dirac charged-Lepton mass terms

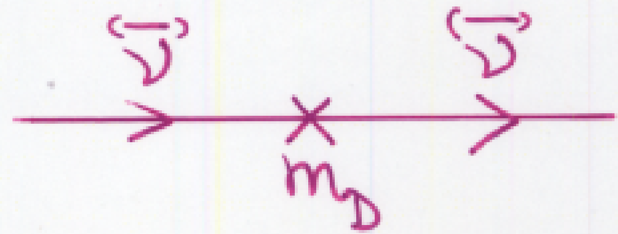
$$m_\ell \bar{\ell}_L \ell_R$$



Original SM: $m_\nu = 0$.

Why not add a Dirac mass term,

$m_D \bar{\nu}_L \nu_R$




Then everything conserves L, so for each mass eigenstate $\nu_i$,

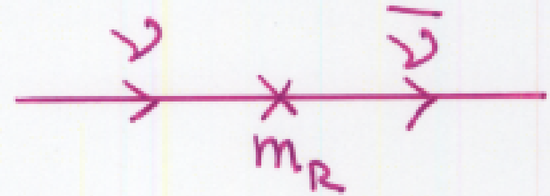
$\bar{\nu}_i \neq \nu_i$ (Dirac neutrinos)

$[L(\bar{\nu}_i) = -L(\nu_i)]$

The Dirac mass term required $\nu_R$.

With $\nu_R$ introduced, no SM principle prevents the occurrence of the Majorana mass term

$$m_R \overline{\nu_R^c} \nu_R$$




This does not conserve L, and now

$$\bar{\nu}_i = \nu_i \quad \text{(Majorana neutrinos)}$$

[No conserved L to distinguish $\bar{\nu}_i$ from $\nu_i$]

We note that $\bar{\nu}_i = \nu_i$ means —

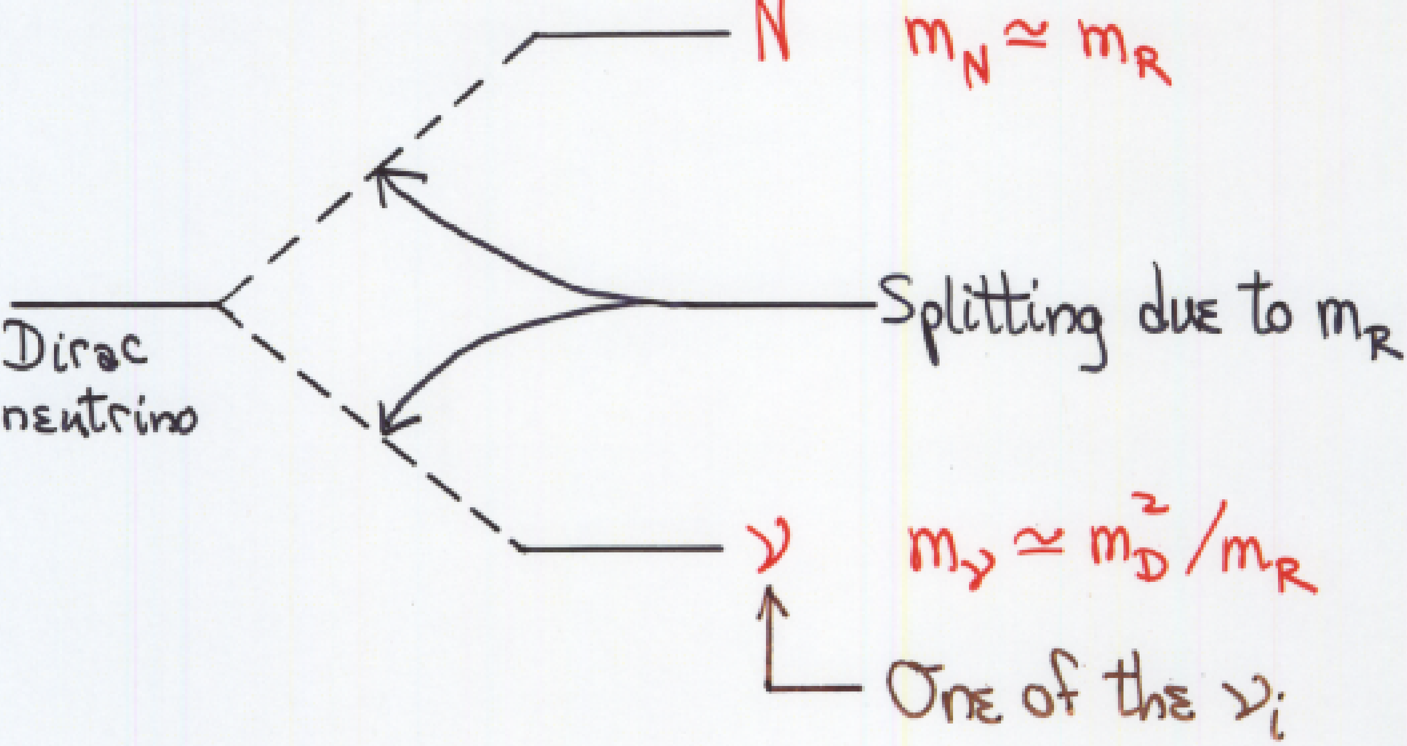
$$\bar{\nu}_i(h) = \nu_i(h).$$

helicity

In the See-Saw Mechanism,

$$\mathcal{L}_{mass} \sim \left[\overline{\nu_L}, \overline{\nu_R^c}\right] \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix}$$

with $m_R \gg m_D \sim m_{q\,or\,l}$.



# Predictions

- Each $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)
- The light neutrinos have heavy partners N

How heavy ??

$$m_N \sim \frac{m_{top}^2}{m_\nu} \sim \frac{m_{top}^2}{0.05\,eV} \sim 10^{15}\,GeV$$

Near the GUT scale.

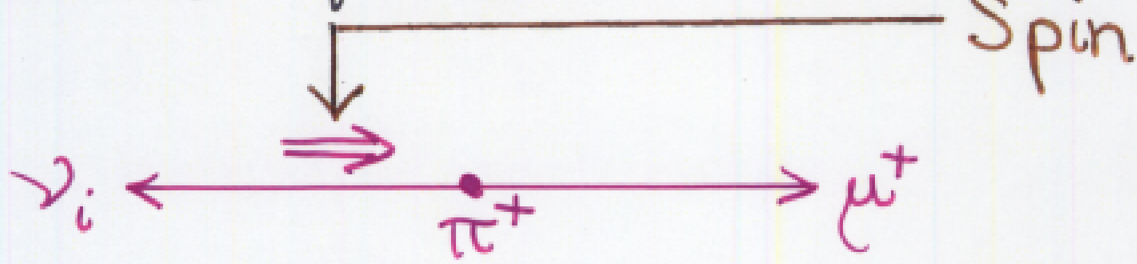
Review of see-saw: hep-ph/0211134

How can we confirm that $\overline{\nu_i} = \nu_i$ ?

# Ideas That Do Not Work

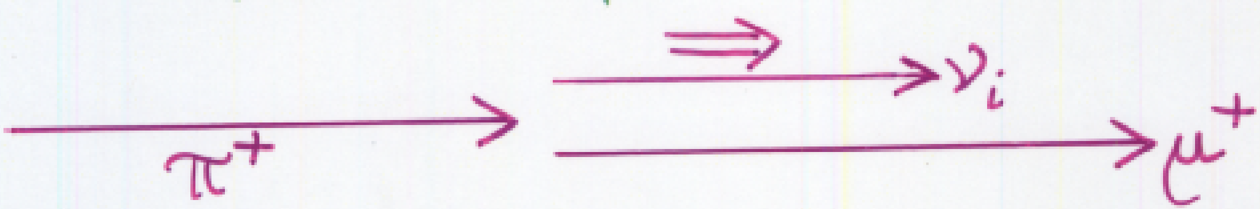
## 1) Give the neutrino a Boost

Produce a $\nu_i$ via—



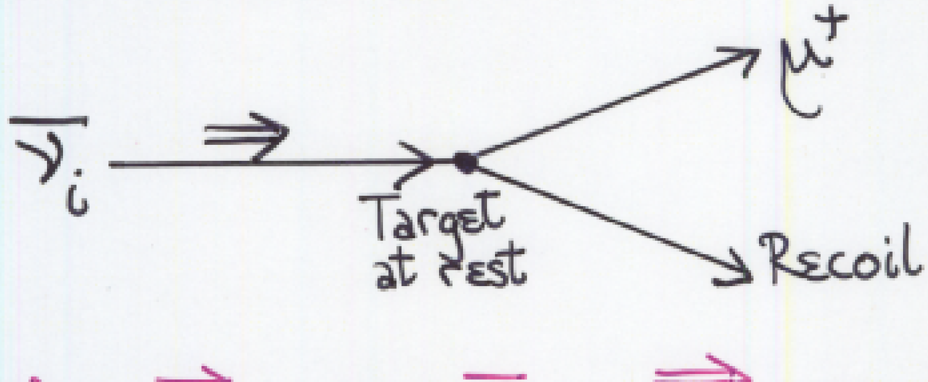

Pion Rest Frame

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$




Lab. frame

The SM weak interaction causes—




If $\nu_i \xrightarrow{\Rightarrow} = \bar{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make $\mu^+$ too.

# Minor technical difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_{\nu_i}}$$

$$\Rightarrow E_\pi(\text{Lab}) \gtrsim 10^5\,\text{TeV} \text{ if } m_{\nu_i} \sim 0.05\,\text{eV}$$

Fraction of all $\pi$-decay $\nu_i$ that get helicity flipped

$$\simeq \left(\frac{m_{\nu_i}}{E_\nu(\pi \text{ Rest Frame})}\right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05\,\text{eV}$$

Since L-violation comes only from Majorana neutrino masses, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & L Stodolsky)

# The Idea That Can Work — Neutrinoless Double Beta Decay

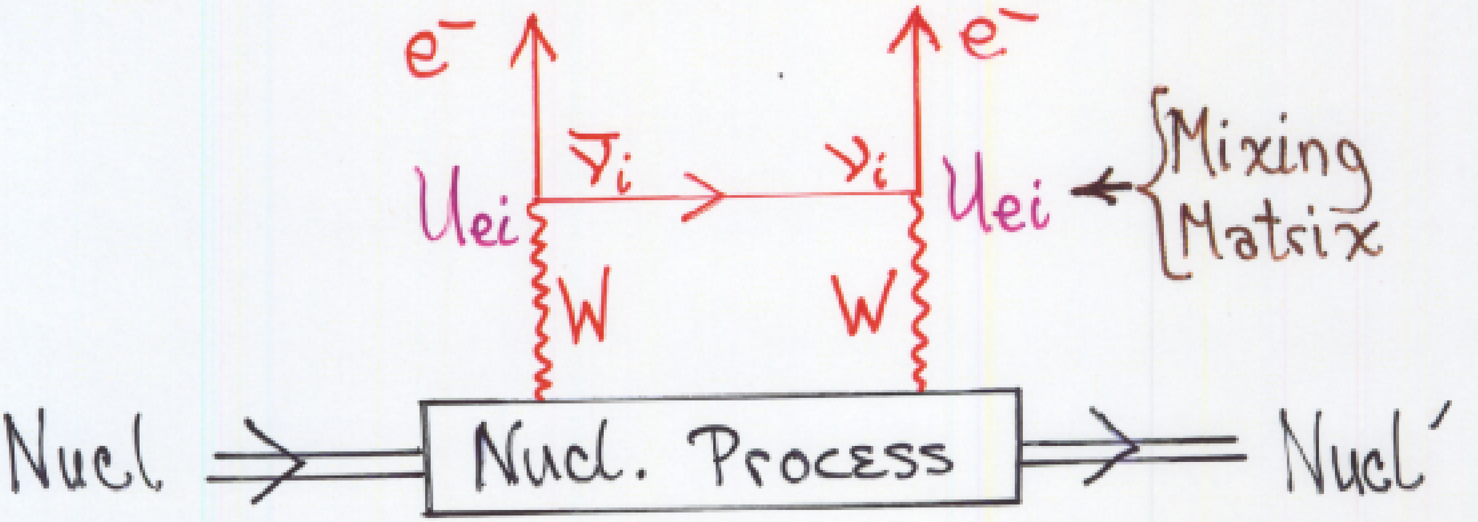
This process is —

$$\mathrm{Nucl} \to \mathrm{Nucl}' + 2e^- \qquad [0\nu\beta\beta].$$

$$\mathrm{Nucl} \to \mathrm{Nucl}' + 2e^-$$

$\Rightarrow \not{L}$; $\bar{\nu}_i = \nu_i$; a Majorana mass term

The dominant mechanism is expected to be—




Mass($\nu_i$)

$\bar{\nu}_i$ is emitted $\left[RH + O\left(\frac{m_i}{E}\right) LH\right]$

$\therefore$ Amp[$\nu_i$ contribution] $\propto m_i$

$$\text{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

$0\nu\beta\beta$ violates L. Standard Model interactions conserve L. The $\cancel{L}$ in $0\nu\beta\beta$ comes from underlying Majorana mass terms.

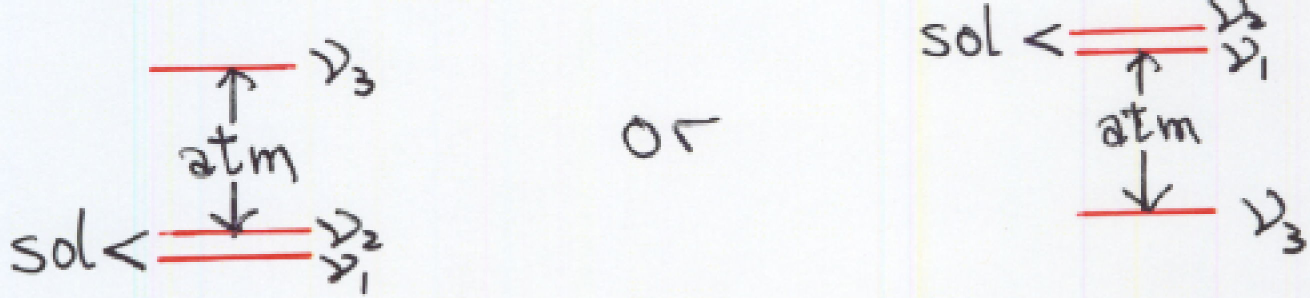
$\therefore$ Amp[$0\nu\beta\beta$] $\propto$ $\nu$ mass

# How Large Is $m_{\beta\beta}$?

## How sensitive need an experiment be?

Suppose there are only 3 neutrino mass eigenstates. (More might help.)

Then the spectrum looks like




For the mixing matrix $U$, we assume the atmospheric mixing is maximal, and take the solar mixing to be in the Large Mixing Angle MSW region.

W5 Then —

$\cancel{CP}$

Majorana Phases    Dirac Phase

$$\mathcal{U} \simeq \begin{array}{cc} \begin{bmatrix} c\, e^{i\frac{\alpha_1}{2}} & s\, e^{i\frac{\alpha_2}{2}} & s_{13} e^{-i\delta} \\ -\frac{s}{\sqrt{2}} e^{i\frac{\alpha_1}{2}} & \frac{c}{\sqrt{2}} e^{i\frac{\alpha_2}{2}} & \frac{1}{\sqrt{2}} \\ \frac{s}{\sqrt{2}} e^{i\frac{\alpha_1}{2}} & -\frac{c}{\sqrt{2}} e^{i\frac{\alpha_2}{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} & \begin{matrix} e \\ \mu \\ \tau \end{matrix} \\ \begin{matrix} \nu_1 & \quad\quad \nu_2 & \quad\quad \nu_3 \end{matrix} & \end{array}$$

$c \equiv \cos\theta_\odot$, $s \equiv \sin\theta_\odot$, $s_{13} \equiv \sin\theta_{13}$

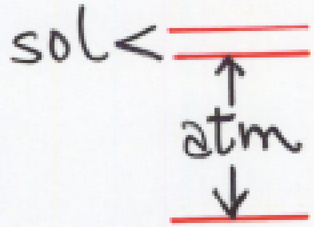
From solar and KamLAND data,

$$0.24 \lesssim \sin^2\theta_\odot \lesssim 0.36 \quad \text{(SNO; 90\% CL)}$$

From CHOOZ bound on $\bar{\nu}_e$ oscillation,

$$\sin^2\theta_{13} \lesssim 0.05 \quad \text{(CHOOZ; 90\% CL)}$$

If the spectrum looks like —




$\leftarrow m_0 \gtrsim \sqrt{\Delta m^2_{atm}} \simeq 40 \text{ meV}$

then —

$$m_{\beta\beta} \cong m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right)}$$

$$m_0 \cos 2\theta_\odot \leq m_{\beta\beta} \leq m_0$$

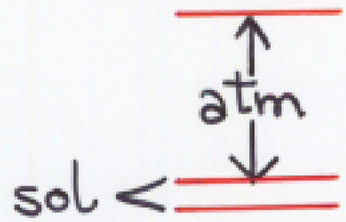
At 90% CL,

$m_0 > 36$ meV (SuperK); $\cos 2\theta_\odot > 0.28$ (SNO),

so

$$\underline{m_{\beta\beta} > 10 \text{ meV}}\,.$$

If the spectrum looks like—




then

$0 < m_{\beta\beta} <$ Present Bound [(0.3–1.0) eV].

(Petcov et al.)

## Analyses of $m_{\beta\beta}$ vs. Neutrino Parameters

Barger, Bilenky, Farzan, Giunti, Glashow, Grimus, BK, Kim, Klapdor-Kleingrothaus, Langacker, Marfatia, Monteno, Pascoli, Päs, Peres, Petcov, Rodejohann, Smirnov, Vissani, Whisnant, Wolfenstein, Murayama, Peña-Garay

Review of $\beta\beta$ Decay: Elliott & Vogel

# Majorana CP-Violating Phases

The 3x3 quark mixing matrix: 1 $\cancel{CP}$ phase

When $\overline{\nu_i} = \nu_i$ —

The 3x3 lepton mixing matrix: 3 $\cancel{CP}$ phases

The 2 extra phases, $\alpha_1$ and $\alpha_2$, are called Majorana phases.

Each Majorana phase is associated with a particular $\nu$ mass eigenstate $\nu_i$:

$$U_{\alpha i} = U^0_{\alpha i} e^{i\frac{\alpha_i}{2}} \; ; \; \text{all } \alpha . \qquad [U]$$

Majorana phases have physical consequences only in physical processes that involve violation of L.

They do not affect $\nu$ flavor oscillation, but they do affect $0\nu\beta\beta$.

# Why do Majorana phases influence processes with $\not{L}$?

Example

$l_e \equiv e, l_\mu \equiv \mu, l_\tau \equiv \tau$

$$l_\beta^- W^+ \longleftarrow \nu \longleftarrow l_\alpha^+ W^-$$

$$\text{Amp} = \sum_i \underbrace{\langle l_\beta^- W^+ | H | \nu_i \rangle}_{\sim U_{\beta i}} \langle \nu_i | H | l_\alpha^+ W^- \rangle$$

$$\langle l_\beta^- W^+ | H | \nu_i \rangle \underset{\text{CPT}}{=} \langle \overline{\nu}_i | H | l_\beta^+ W^- \rangle$$

$$= \langle \nu_i | H | l_\beta^+ W^- \rangle$$

When $\overline{\nu}_i = \nu_i$

Then

$$\mathrm{Amp} \sim \sum_i U_{\beta i} U_{\alpha i}.$$

Suppose the CP phase $\delta = 0$, so $U$ is real apart from the Majorana phases:

$$U_{\alpha i} = |U_{\alpha i}| e^{i\frac{\alpha_i}{2}}.$$

Then

$$\mathrm{Amp} \sim \sum_i |U_{\beta i}||U_{\alpha i}| e^{i\alpha_i}.$$

The relative values of the $\alpha_i$ will clearly affect the interference terms in $|\mathrm{Amp}|^2$.

# Can Majorana Phases Lead to Manifest $\cancel{CP}$?

(de Gouvêa, BK, Mohapatra)

Manifest $\cancel{CP}$:

$$\text{Rate}[\text{Process}] \neq \text{Rate}[\overline{\text{Process}}]$$

Does this happen in $0\nu\beta\beta$?

$0\nu\beta\beta$: $\text{Nucl} \to \text{Nucl}' + 2e^-$

$$\text{Amp} = (\text{Nucl Factor}) \times \left(\sum_i m_i U_{ei}^2\right)$$

$\overline{0\nu\beta\beta}$: $\overline{\text{Nucl}} \to \overline{\text{Nucl}'} + 2e^+$

$$\text{Amp} = (\text{Nucl Factor}) \times \left(\sum_i m_i U_{ei}^{*2}\right)$$

$$\Gamma[0\nu\beta\beta] = \Gamma[\overline{0\nu\beta\beta}]$$

# What does it take to have manifest $\not{CP}$?

$$\text{Amp}[\text{Process}] = \sum_i a_i \underbrace{e^{i\varphi_i}}_{\text{CP-even phase}} \underbrace{e^{i\alpha_i}}_{\text{CP-odd phase}}$$

$$\text{Amp}[\overline{\text{Process}}] = \sum_i a_i e^{i\varphi_i} e^{-i\alpha_i}$$

$$\neq \left\{ \text{Amp}[\text{Process}] \ \underline{\text{or}} \ \text{Amp}^*[\text{Process}] \right\}$$

$0\nu\beta\beta$ lacks CP-even phases.

But

$$\ell_\alpha^+ W^- \longrightarrow \nu \longrightarrow \ell_\beta^- W^+$$

has them.

$$\text{Amp}[\ell_\alpha^+ W^- \to \nu \to \ell_\beta^- W^+] =$$

$$= \underset{\uparrow}{S} \sum_i \underbrace{U_{\alpha i} U_{\beta i}}_{\substack{\text{Has Maj.}\\ \text{phases}}} \frac{m_i}{E} \underbrace{e^{-i m_i^2 \frac{L}{2E}}}_{\substack{\nu \text{ propagator;}\\ \text{CP-even}}}$$

Kinematics

$L$ — Distance

$E$ — helicity suppression

$$\text{Amp}[\ell_\alpha^- W^+ \to \nu \to \ell_\beta^+ W^-] =$$

$$= S \sum_i U_{\alpha i}^* U_{\beta i}^* \frac{m_i}{E} e^{-i m_i^2 \frac{L}{2E}}$$

Suppose only 2 neutrinos matter:

$$U = \begin{array}{c c} & \begin{array}{cc} \nu_1 & \nu_2 \end{array} \\ \begin{array}{c} \nu_e \\ \nu_\mu \end{array} & \begin{bmatrix} c e^{i\frac{\alpha}{2}} & s \\ -s e^{i\frac{\alpha}{2}} & c \end{bmatrix} \end{array}$$

$c \equiv \cos\theta$

$s \equiv \sin\theta$

$\alpha \equiv$ a Majorana phase

M.6]

$$\Gamma[e^+W^- \to \nu \to \mu^- W^+]$$

$$= K\,\frac{\sin^2 2\theta}{E^2}\left[m_1^2 + m_2^2 - 2m_1m_2\cos\left(\Delta m^2\frac{L}{2E} - \alpha\right)\right]$$

(Schechter & Valle)

$$\Gamma[e^-W^+ \to \nu \to \mu^+ W^-]$$

$$= K\,\frac{\sin^2 2\theta}{E^2}\left[m_1^2 + m_2^2 - 2m_1m_2\cos\left(\Delta m^2\frac{L}{2E} + \alpha\right)\right]$$

Here,

$K$ = irrelevant constant $= |S|^2$

$m_{1,2}$ = masses of $\nu_{1,2}$

$\Delta m^2 = m_2^2 - m_1^2$

Note the two rates are not the same.

In the quark sector, the mixing matrix loses its meaning when all quarks of a given charge are degenerate.

What happens here when $m_1 = m_2 \equiv m$?

$$\Gamma[e^+ W^- \to \nu \to \mu^- W^+]$$

$$= \Gamma[e^- W^+ \to \nu \to \mu^+ W^-]$$

$$= K \sin^2 2\theta \, \frac{4m^2}{E^2} \sin^2 \frac{\alpha}{2}$$

When Majorana phases are present, the mixing matrix is still meaningful even when the neutrino masses are of equal size.

# Why?

The Majorana phase $\alpha$ associated with neutrino $\nu_1$ may be viewed as the phase of its mass:

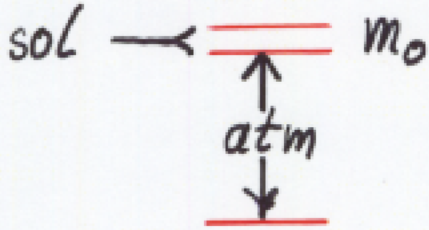
$$\text{mass}(\nu_1) = m_1 e^{i\alpha}$$

Real

Even when $m_1 = m_2$, $\alpha$ distinguishes $\nu_1$ from $\nu_2$.

# Can $\Gamma[0\nu\beta\beta]$ Reveal Majorana Phases?

If the spectrum looks like —




then —

$$m_{\beta\beta} \cong m_0 \sqrt{1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right)}$$

With $\alpha_2 - \alpha_1 \equiv \Delta\alpha$ ,

$$\sin^2\left(\frac{\Delta\alpha}{2}\right) = \frac{1}{\sin^2 2\theta_\odot}\left[1 - \left(\frac{m_{\beta\beta}}{m_0}\right)^2\right] .$$

$\not{CP}$ : $\Delta\alpha \neq 0, \pi$. $\sin^2\left(\frac{\Delta\alpha}{2}\right) \neq 0, 1$.

Experimentally, $1/\sin^2 2\theta_\odot \simeq 1.2$.

Thus,

$$\sin^2\left(\frac{\Delta\alpha}{2}\right) \simeq 1.2\left[1 - \left(\frac{m_{\beta\beta}}{m_0}\right)^2\right].$$

Establishing that $\sin^2\left(\frac{\Delta\alpha}{2}\right) \neq 0, 1$ requires —

- A knowledge of $m_0$ [Tritium?]
- Shrinking the present (factor of three)$^2$ theoretical uncertainty in $\Gamma[0\nu\beta\beta]/m_{\beta\beta}^2$

Studies of Observability of $\Delta\alpha \neq 0, \pi$

Barger, Glashow, Langacker, Marfatia;
Pascoli, Petcov, Rodejohann;
Pascoli, Petcov

# Why Are There 3 Generations?

If baryogenesis arose from $\cancel{CP}$ in quark mixing, we could argue that—

It takes $\geq 3$ generations to have $\cancel{CP}$ in quark mixing.

It takes $\cancel{CP}$ in quark mixing to have baryogenesis.

It takes baryogenesis to have us.

But $\cancel{CP}$ in quark mixing is completely inadequate for baryogenesis.

Majorana phases can produce the manifest $\cancel{CP}$

$$\Gamma[N \to \ell^+ + \text{Higgs}^-] > \Gamma[N \to \ell^- + \text{Higgs}^+]$$

in the early universe. This may be the origin of baryogenesis.

It takes only 2 generations to have manifest $\cancel{CP}$ from Majorana phases.

So why are there 3 ??

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