Jet Physics and QCD Simulation

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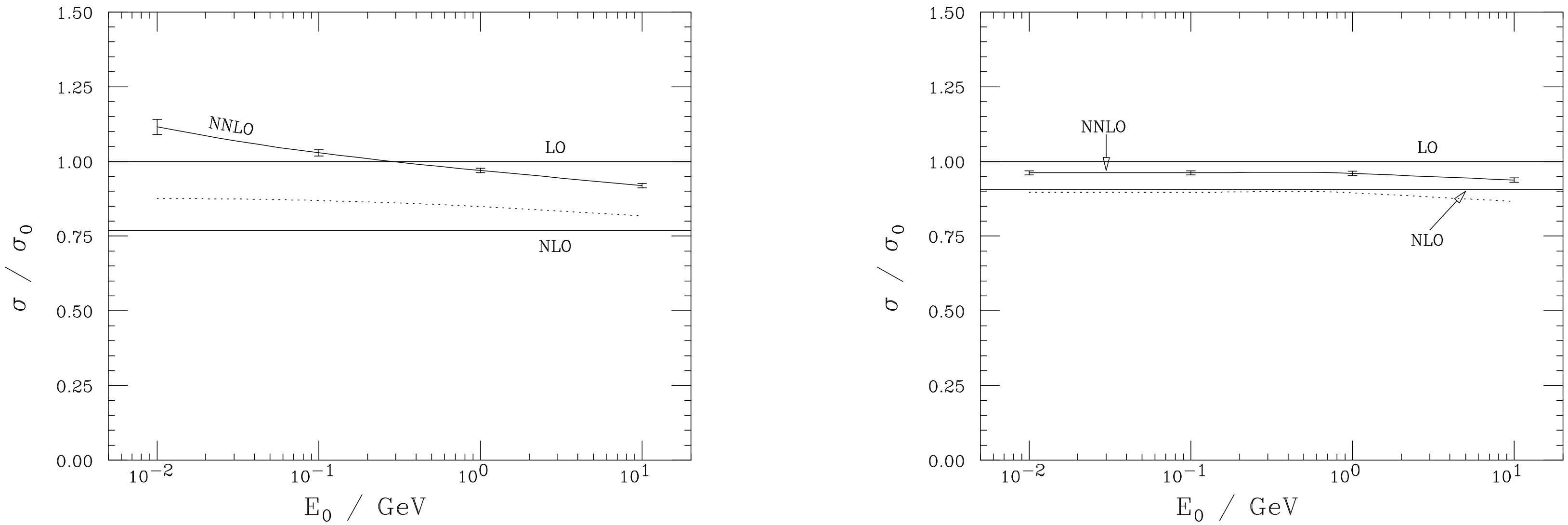
- Issues in Jet Physics
  - Cone jet algorithms
  - $k_\perp$ jet algorithm
- Issues in QCD simulation
  - Improving shower variables
  - Combining matrix elements and showers
  - Multiscale showering
- Herwig++
  - Overview
  - Some results (e^+e^-, Drell-Yan)
  - Outlook

S. Gieseke, P. Stephens and BW, JHEP **0312** (2003) 045 [hep-ph/0310083]

S. Gieseke, A. Ribon, M. H. Seymour, P. Stephens and BW, JHEP **0402** (2003) 005 [hep-ph/0311208]

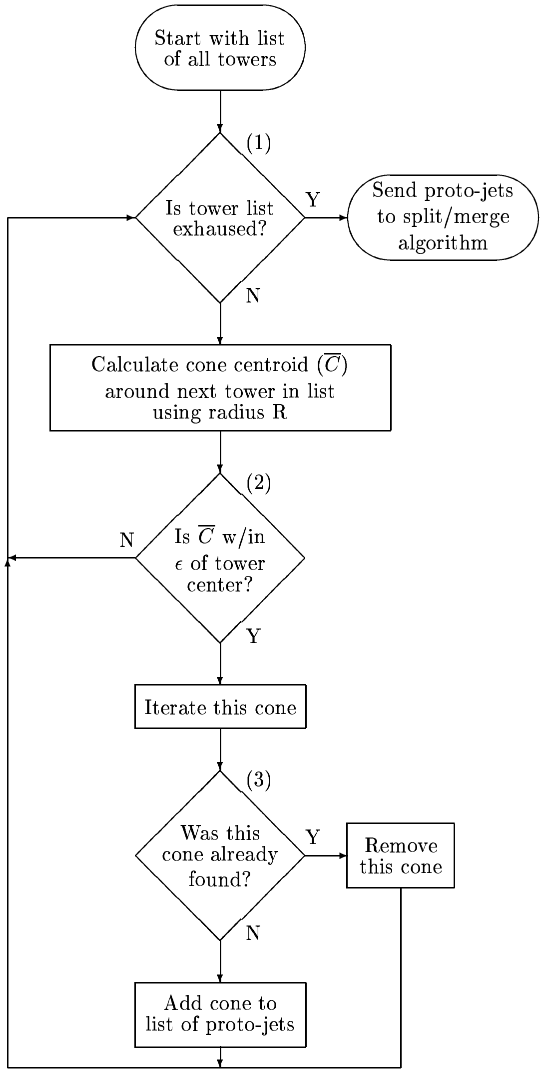
Legacy Cone Algorithm

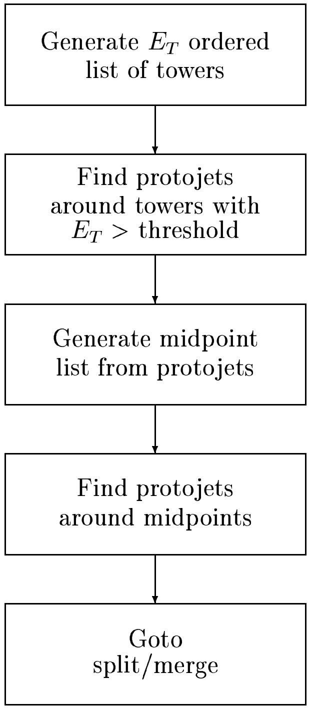
- Use of "seeds" prevented infrared safety ⇒ log divergence (at NNLO)
- Solved by "midpoint" algorithm:

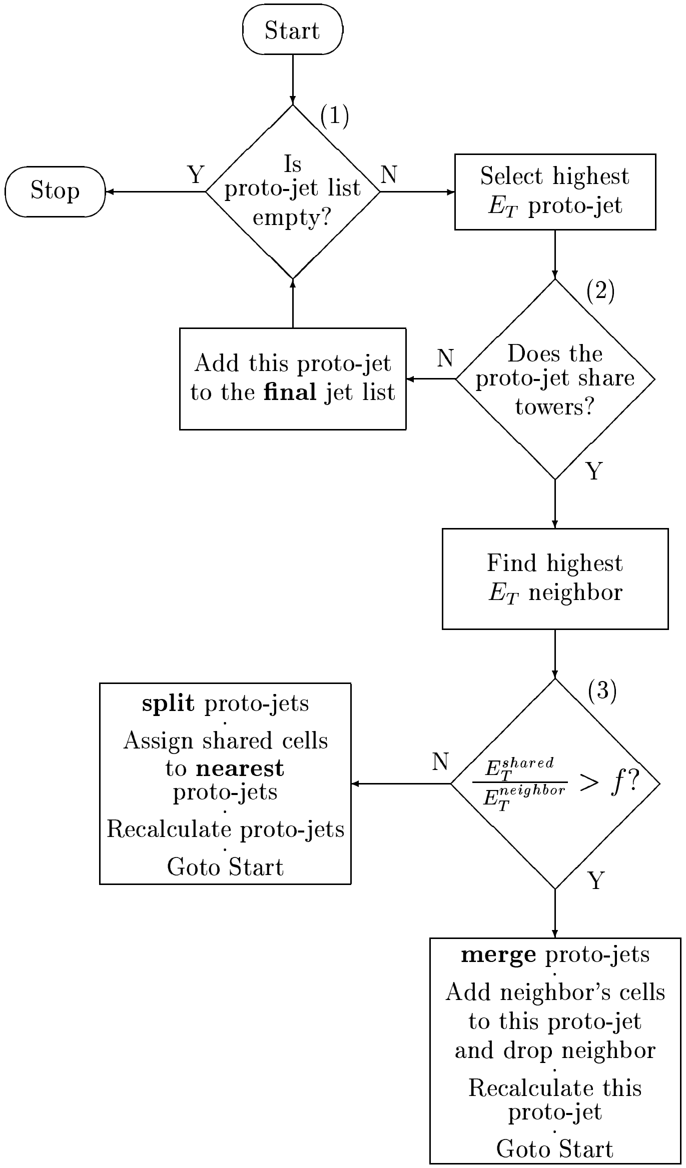


M Seymour, NPB513 (1998) 269 [hep-ph/9707338]

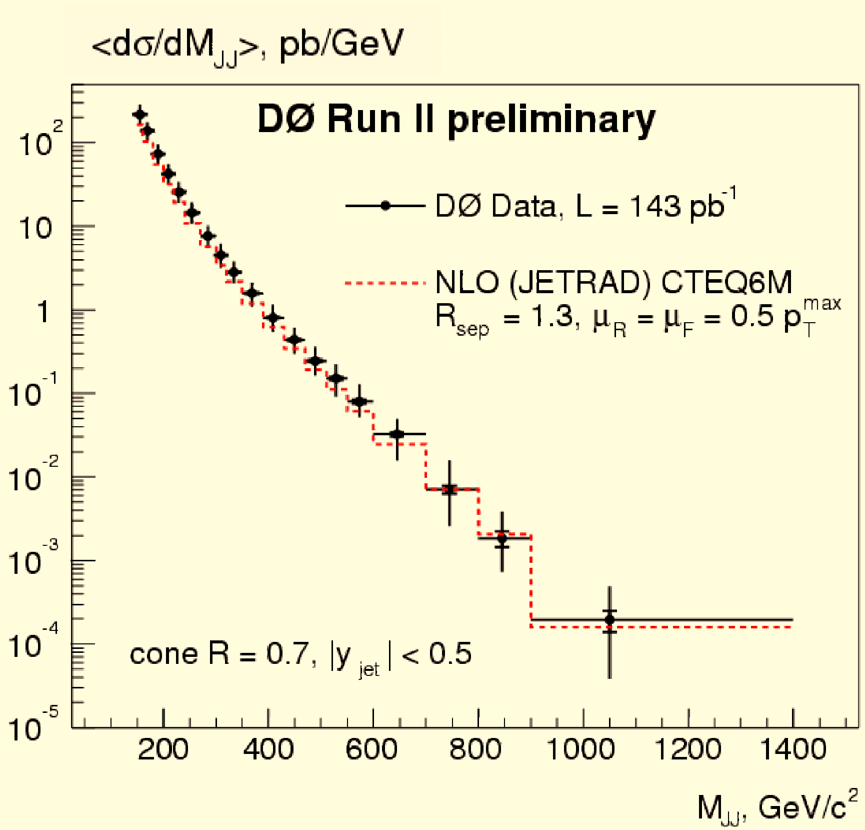
Improved Legacy Cone Algorithm

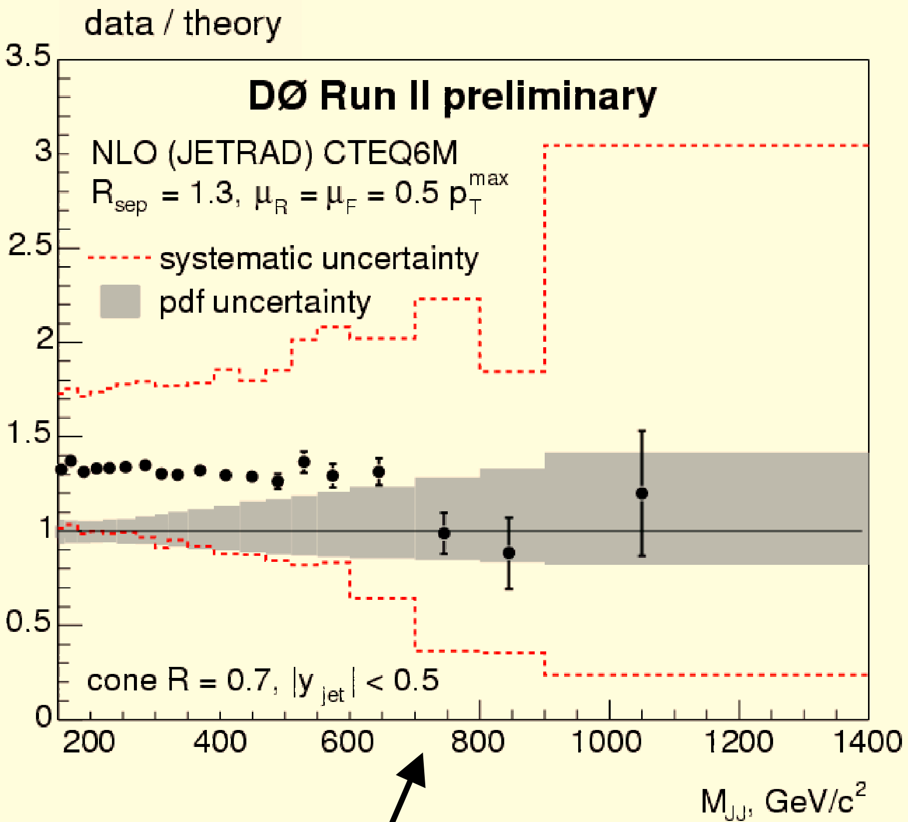






Dijet Mass Cross section



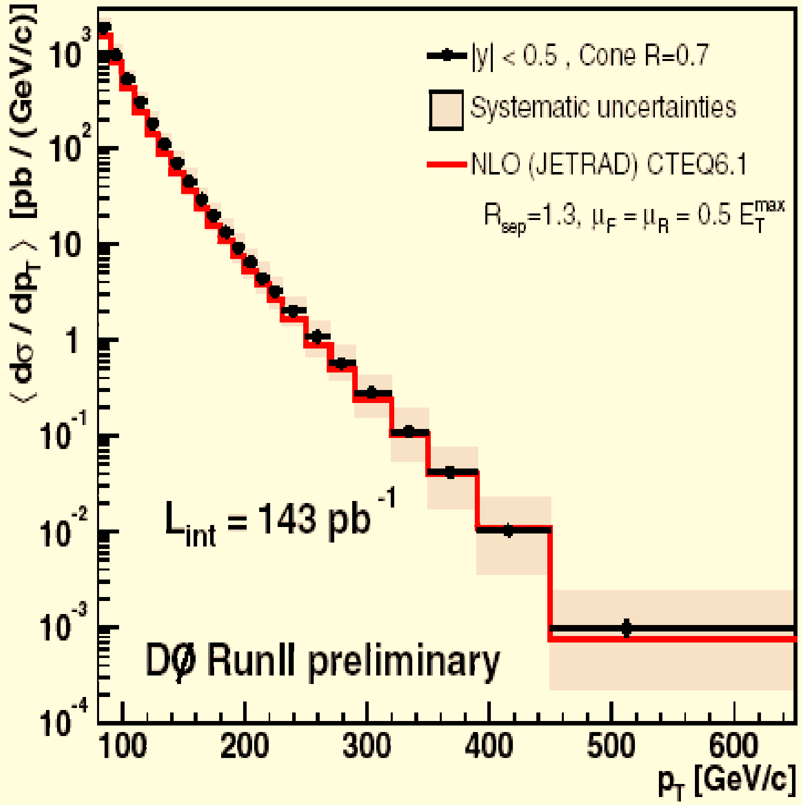


Resonances or excess at high mass:

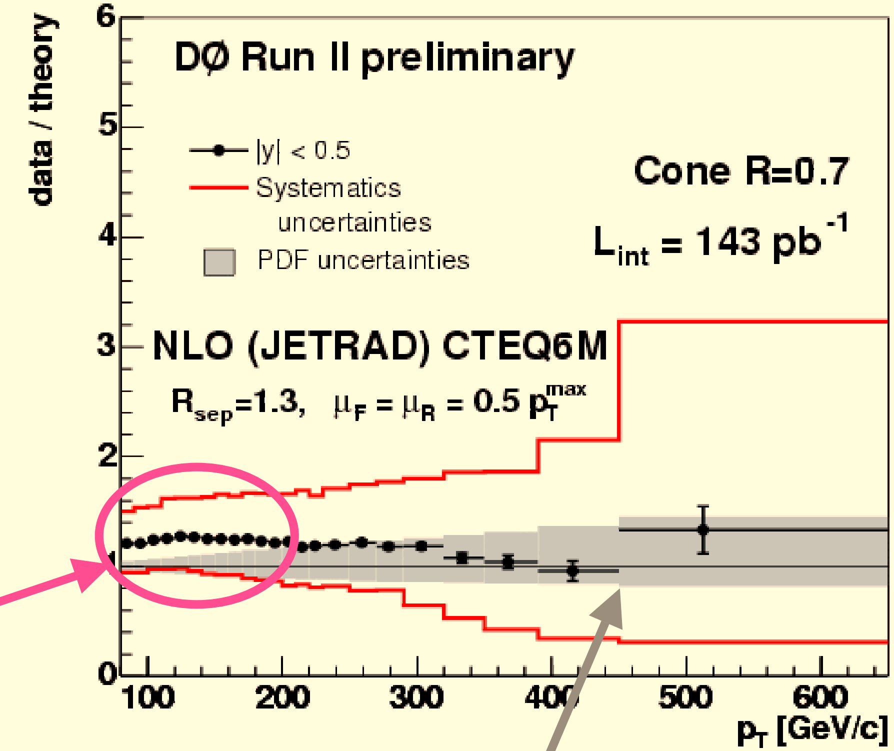
- o new particles
- o quark compositeness

data/theory agree within large systematic errors (mainly jet-energy scale)

Inclusive Jet P_T Cross section



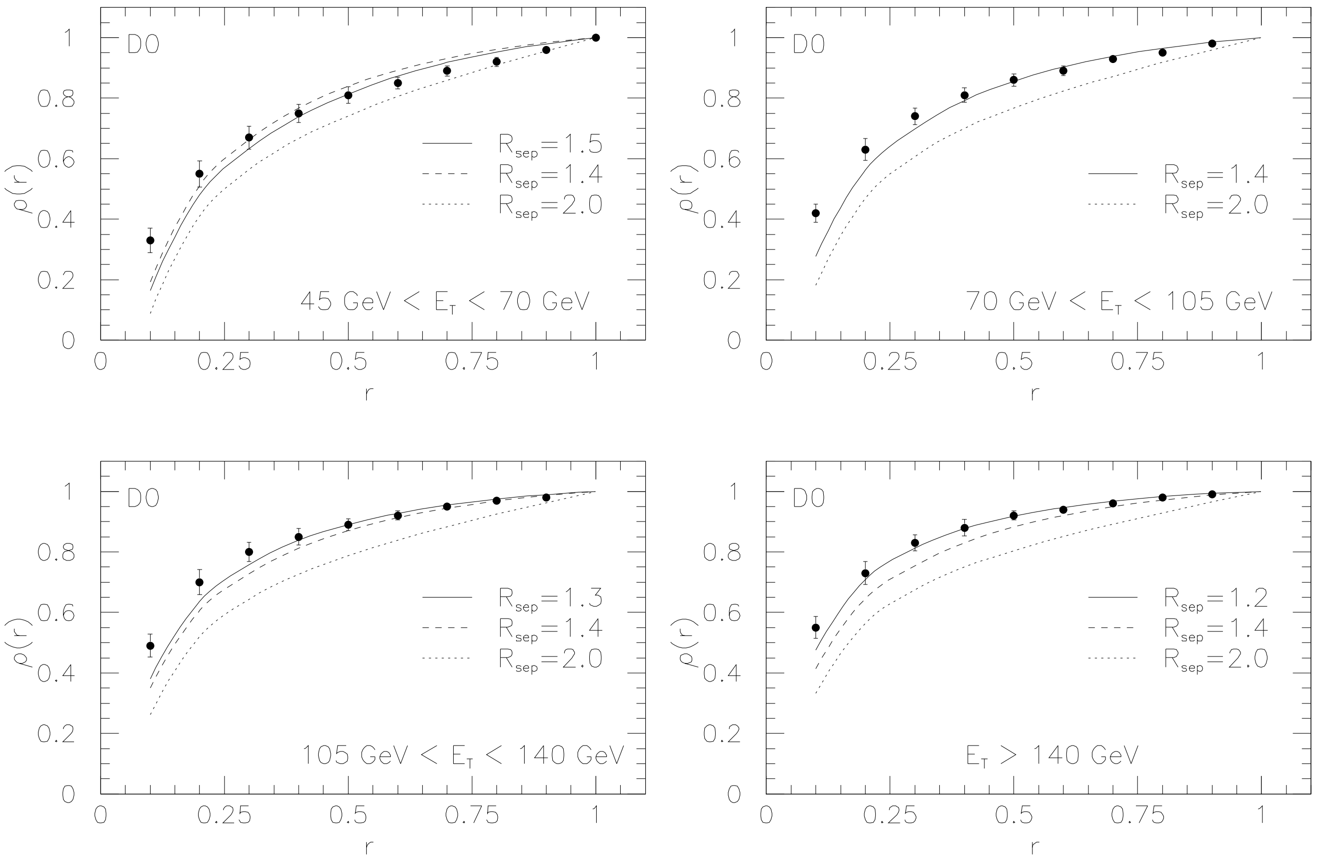
Agreement with theory within systematic uncertainties (dominated by jet-energy scale)



Hadronization correction needed?

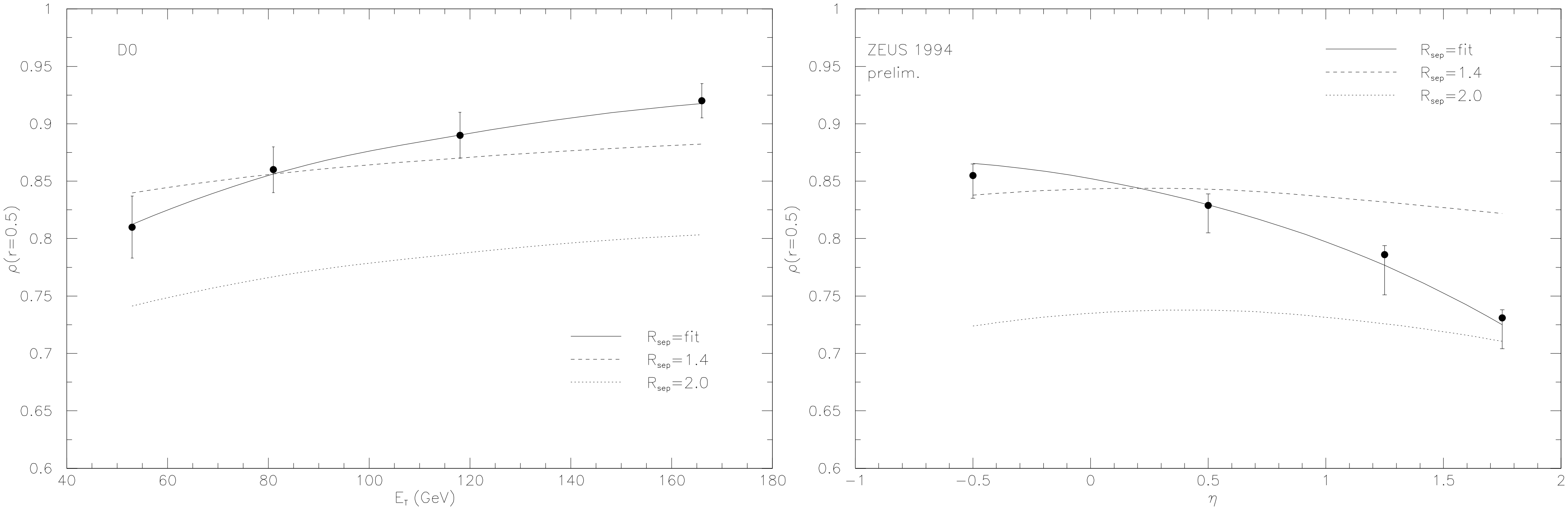
NLO uncertainty due to gluon @ high x

The $R_{\rm sep}$ problem



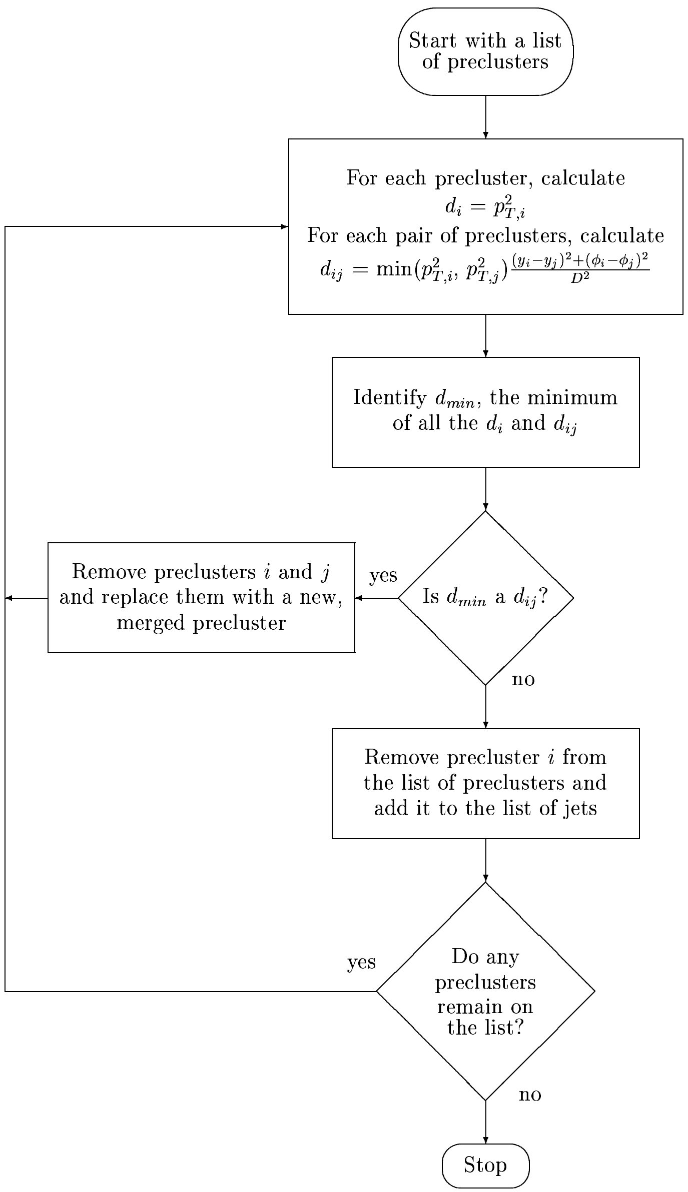
$$\rho(r) = \int_0^r E_T \, d\sigma \Big/ \int_0^R E_T \, d\sigma$$

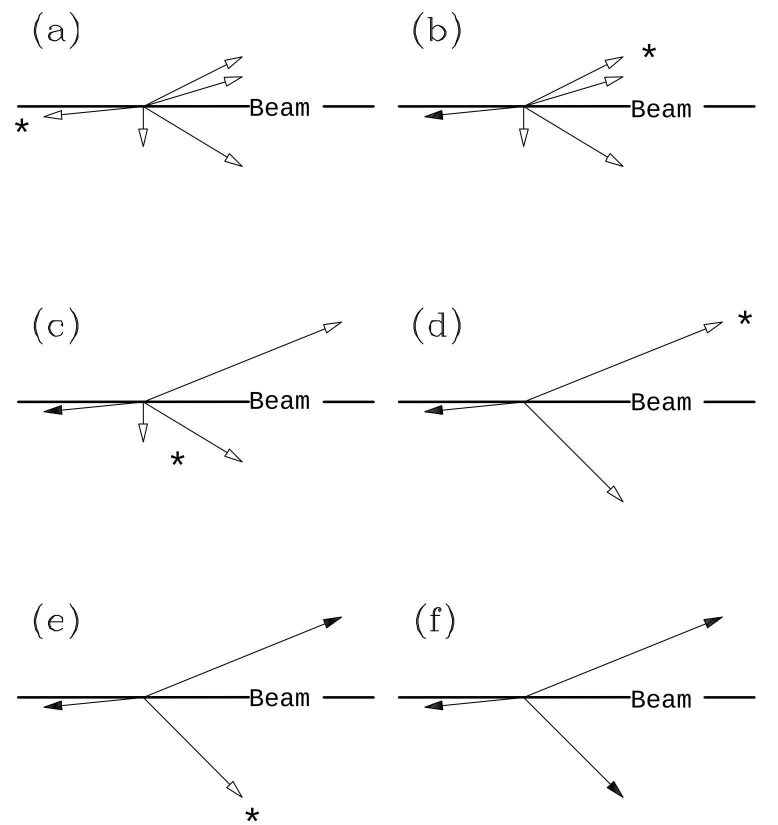
R_{sep} is a free parameter!



M Klasen & M Kramer, PRD 56 (1997) 2702 [hep-ph/9701247]

The $k_\perp$ cluster algorithm





Inclusive Jet Cross section: K_T algorithm

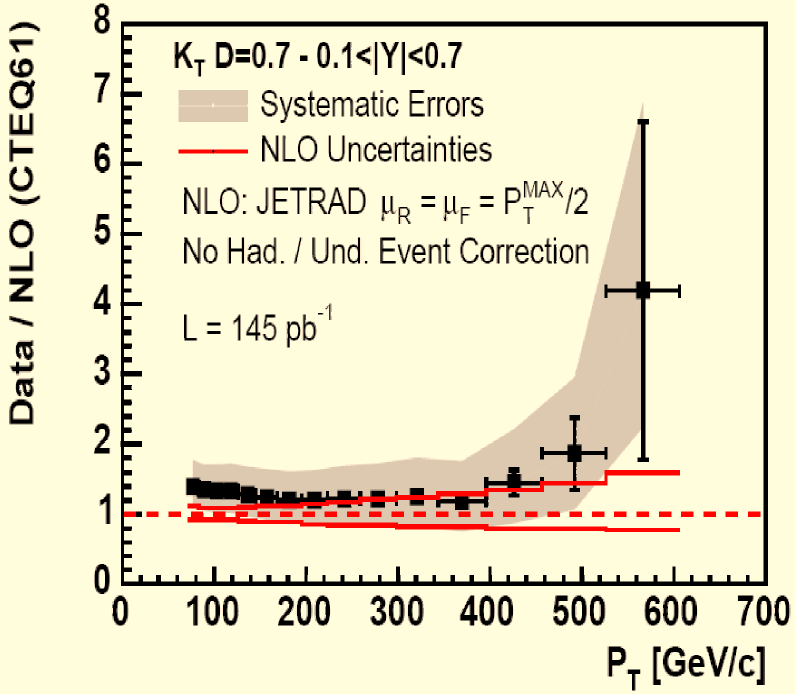
- Inclusive K_T algorithm

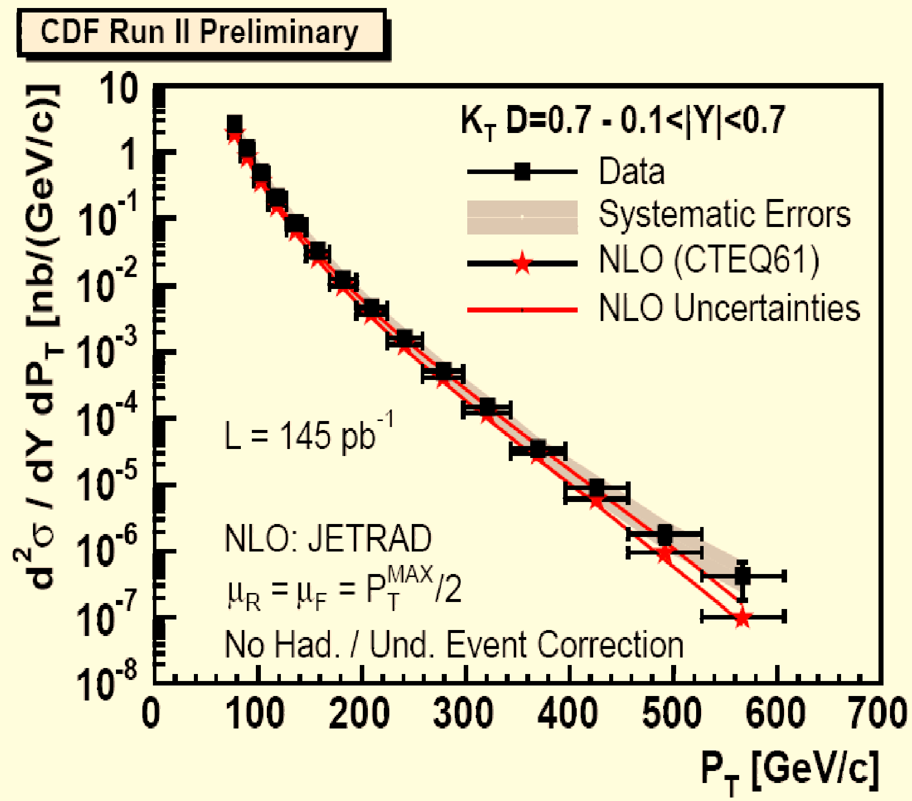
$$d_{ij} = \min(P^2_{T,i}, P^2_{T,j}) \frac{\Delta R^2}{D^2} \quad \Delta R^2 \rightarrow (y_i - y_j)^2 + (x_i + x_j)^2$$

$D^2 \rightarrow$ jet size

$$d_i = (P_{T,i})^2$$

- Infrared and collinear safe
- No merging / splitting





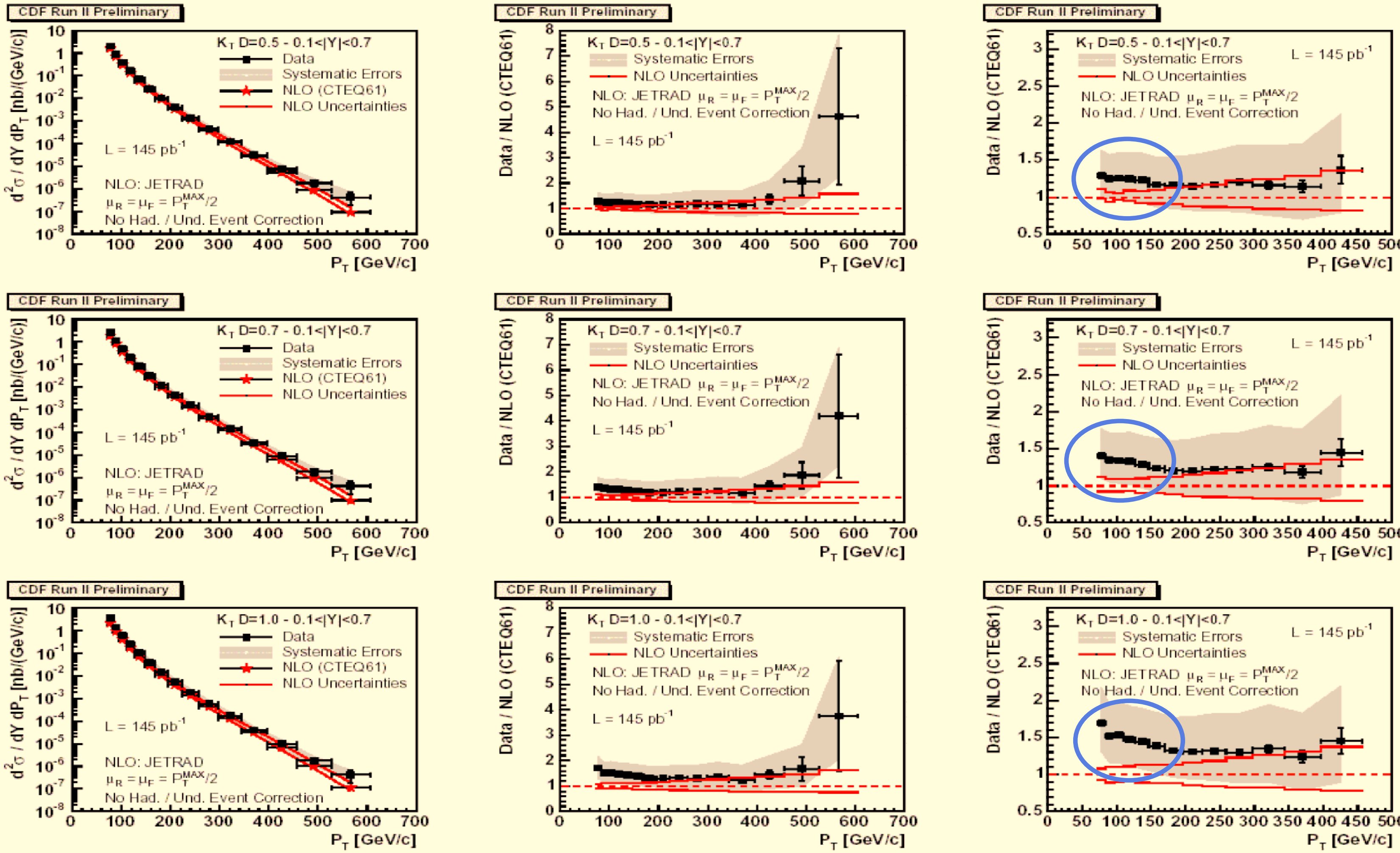
- Reasonable data-theory agreement
- NLO still needs to be corrected for Hadronization /Underlying Event

Inclusive Jet Cross Section K_T vs D

D=0.5

D=0.7

D=1



Frank Chlebana and Andrey Korytov talks for details

Increasing D data departs from pQCD NLO

× more soft contributions

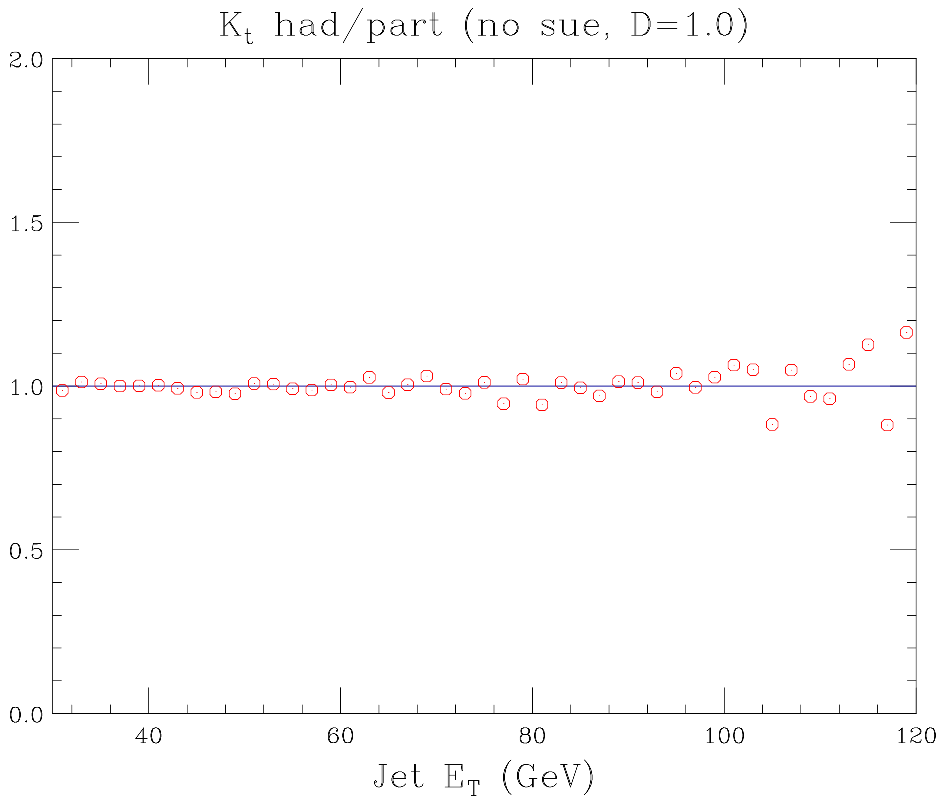
$k_\perp$-algorithm: hadron/parton-level comparisons

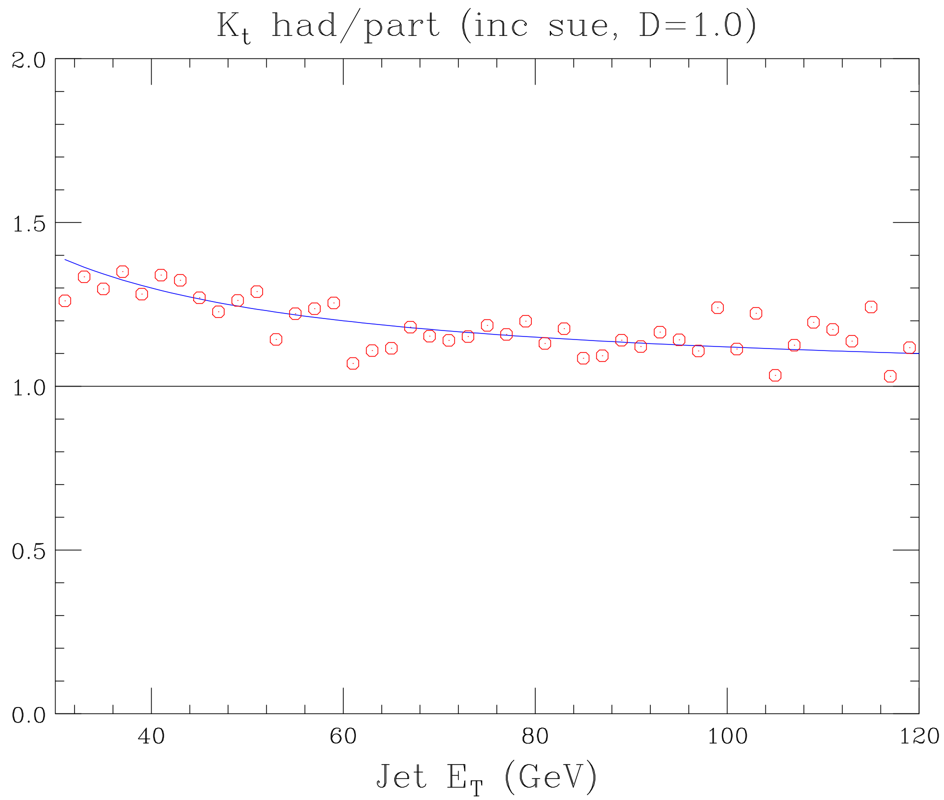
- Underlying event
  - $(dE_T/d\eta\, d\phi)_{u.l.} \sim 0.6$ GeV $\Rightarrow$ $0.6\,\pi D^2 \sim 2D^2$ GeV
  - $d\sigma/dE_T \propto E_T^{-n} \to (E_T - 2D^2)^{-n} \sim E_T^{-n}\left(1 + \frac{2nD^2}{E_T}\right)$
- Hadronization
  - Offset ($\sim$ constant?) linear in D ('splash-in/out' ?)

$$\text{HERWIG} \quad \Rightarrow \quad \frac{(d\sigma/dE_T)_{\rm hadr}}{(d\sigma/dE_T)_{\rm part}} \sim 0.8 + 0.2\,D + \frac{12D^2}{E_T}$$

$$\text{CDF data} \quad \Rightarrow \quad \frac{(d\sigma/dE_T)_{\rm hadr}}{(d\sigma/dE_T)_{\rm NLO}} \sim 1.1 + 0.2\,D + \frac{16D^2}{E_T}$$

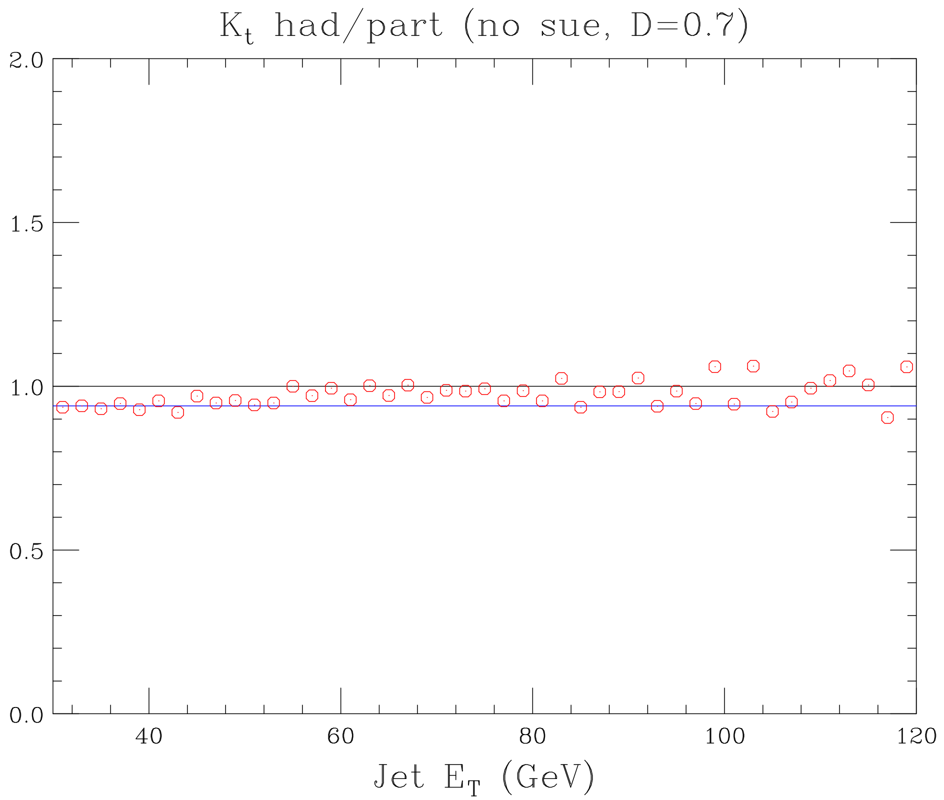
$k_\perp$-algorithm: HERWIG studies (D=1.0)

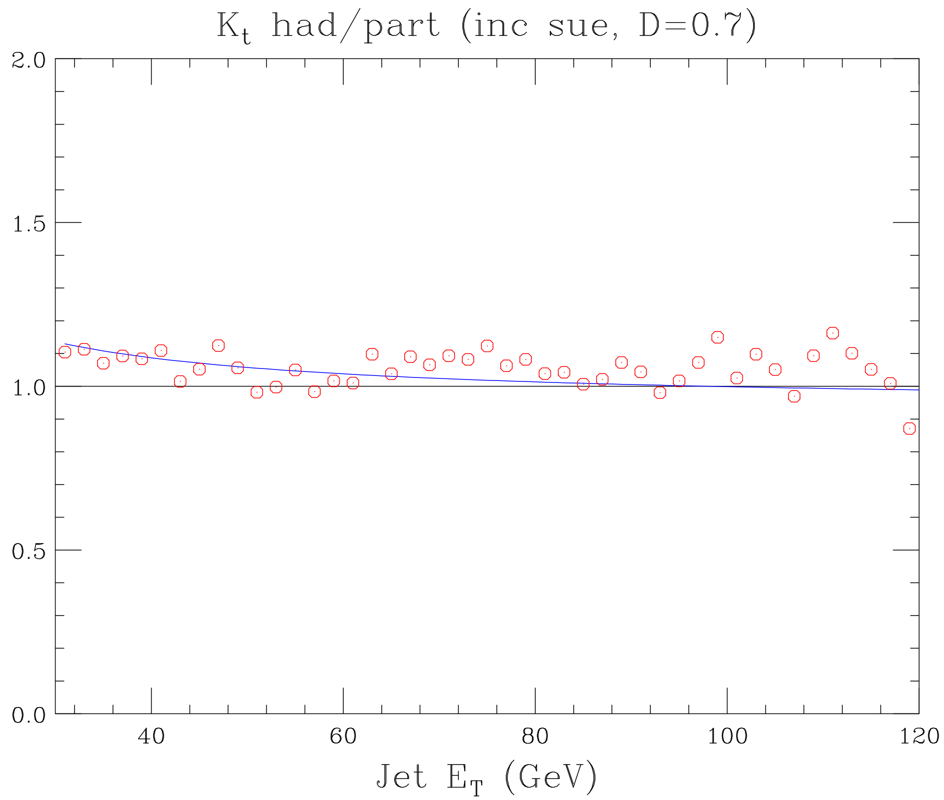




- Splash-in/out cancel, but large underlying event contribution

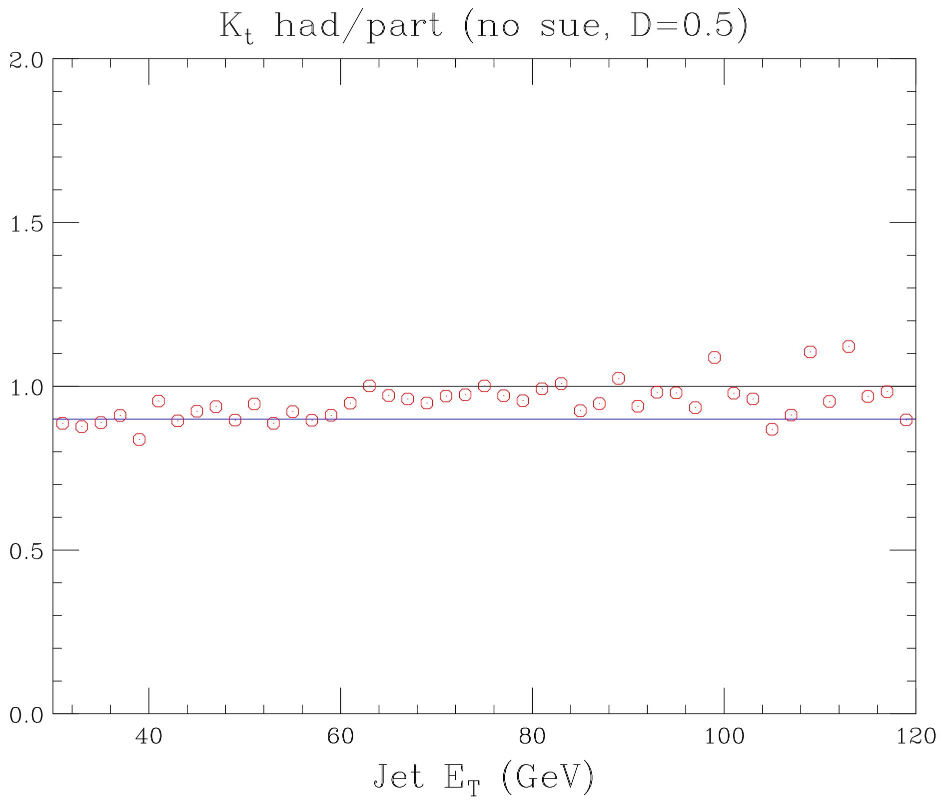
$k_\perp$-algorithm: HERWIG studies (D=0.7)

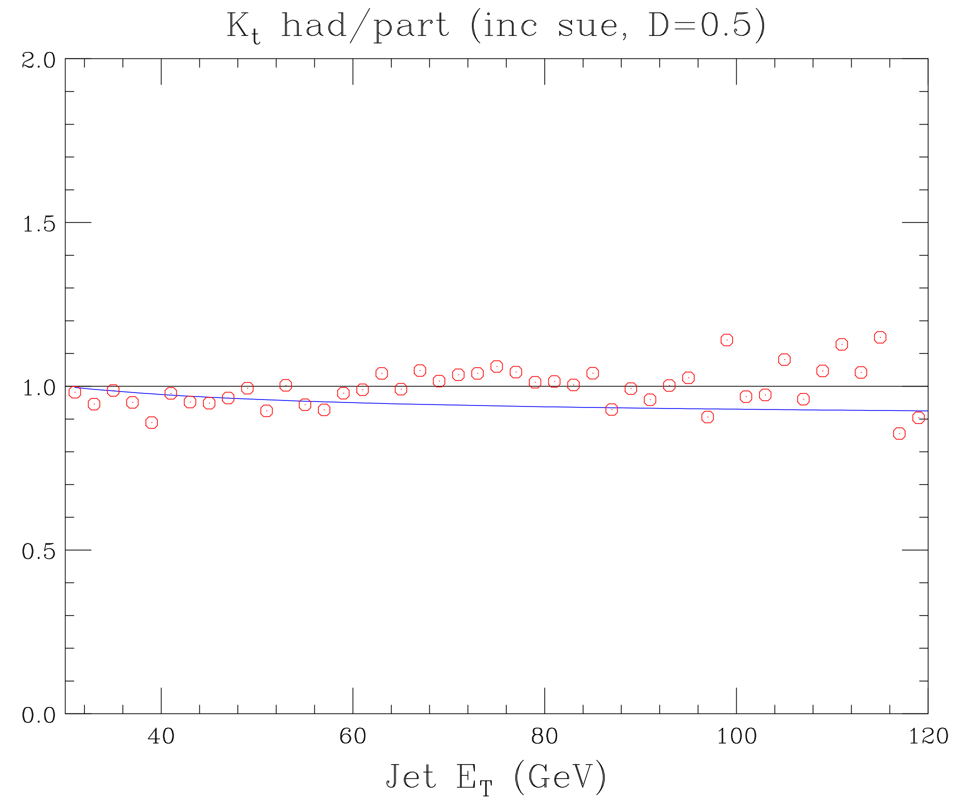




- Small net splash-out, less underlying event contribution

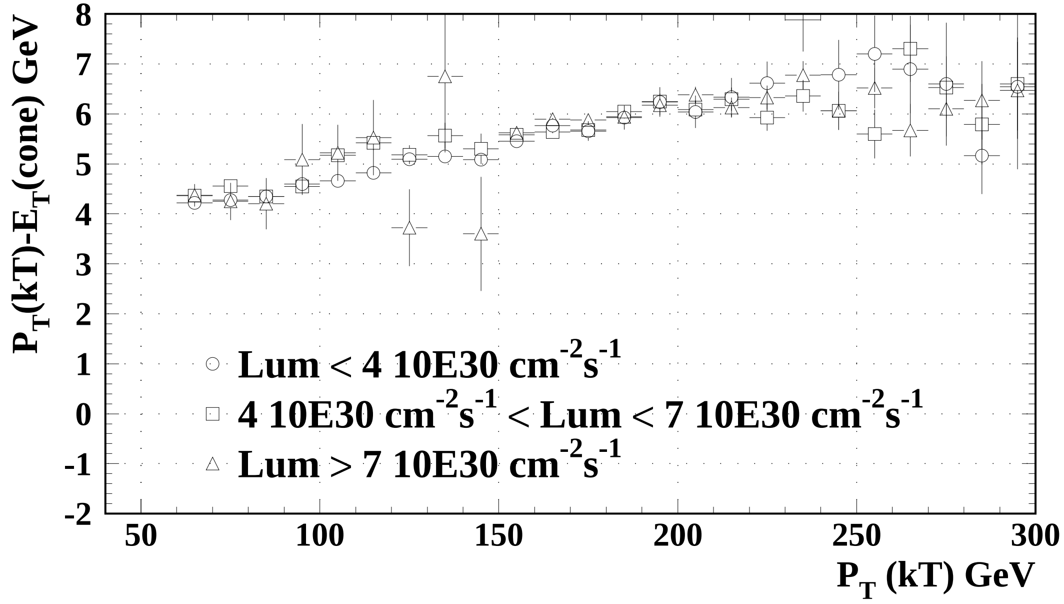
$k_\perp$-algorithm: HERWIG studies (D=0.5)

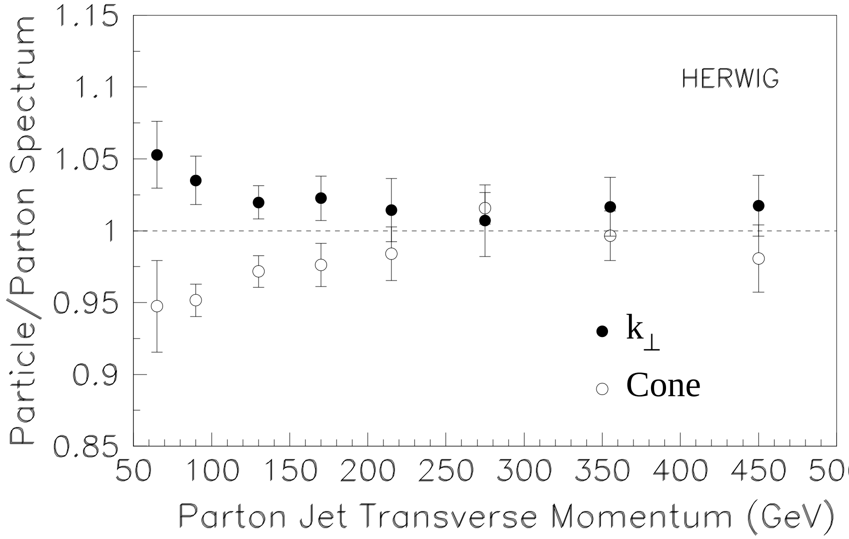




- Splash-out dominant, little underlying event contribution

$k_\perp$–cone comparisons

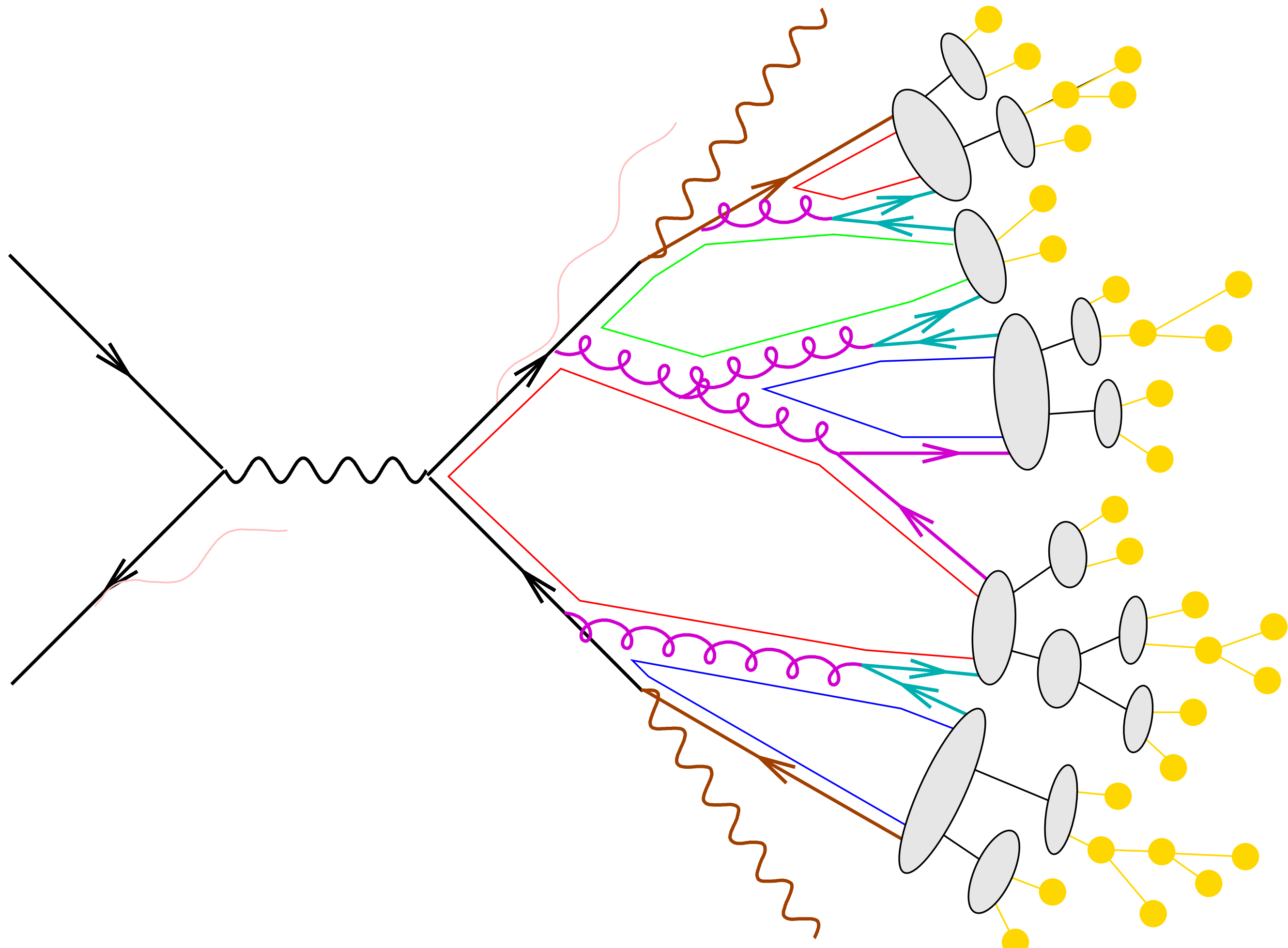




Conclusions

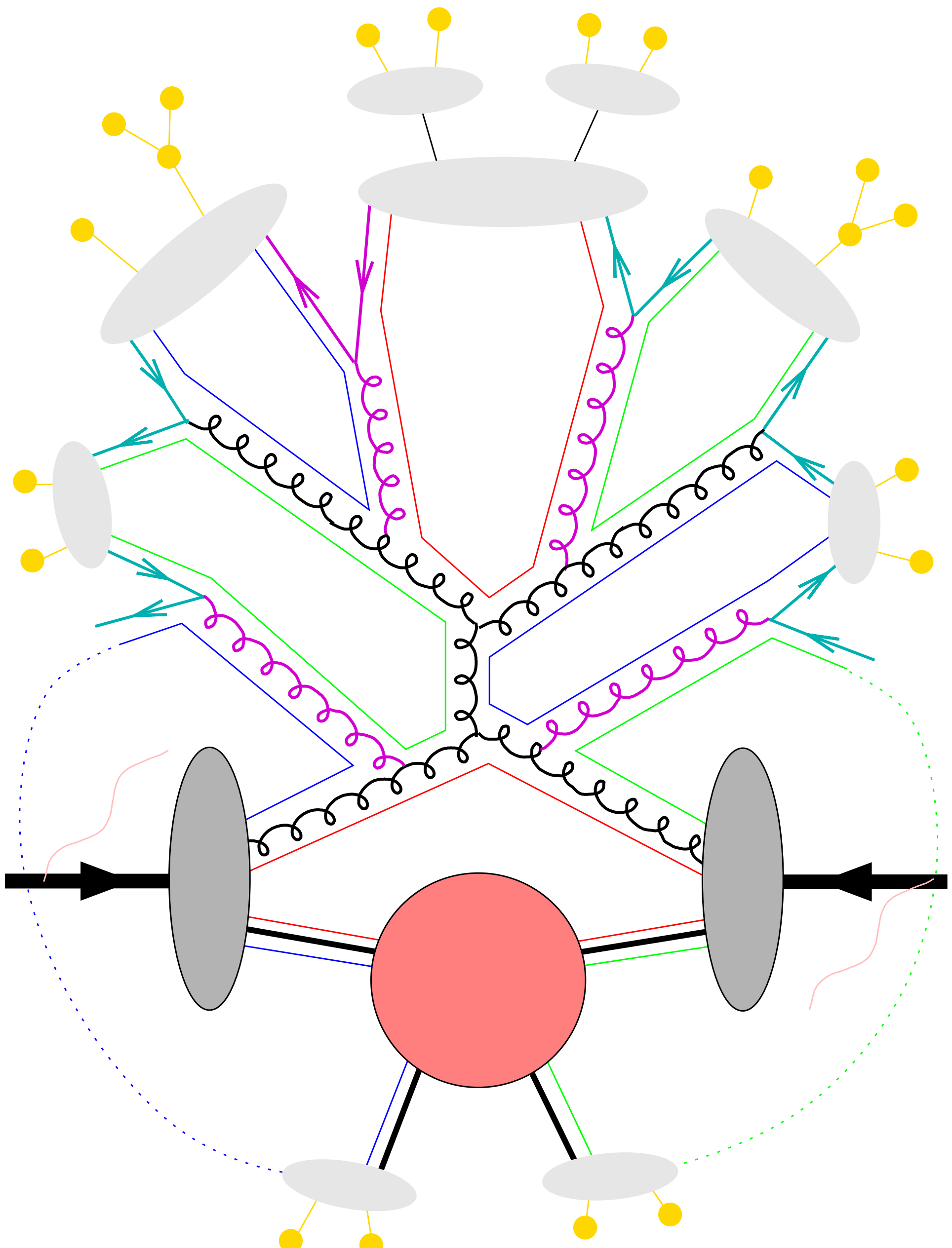
- Cone jets lose energy outside cone, $k_\perp$ jets drag in energy
- Need more studies of dependence on E_T and D
- MC@NLO for jet production needed!

Event Generators: e^+e^-



- hard scattering
- (QED) initial/final state radiation
- partonic decays, e.g. $t \to bW$
- parton shower evolution
- nonperturbative gluon splitting
- colour singlets
- colourless clusters
- cluster fission
- cluster $\to$ hadrons
- hadronic decays

Additional Complications in pp



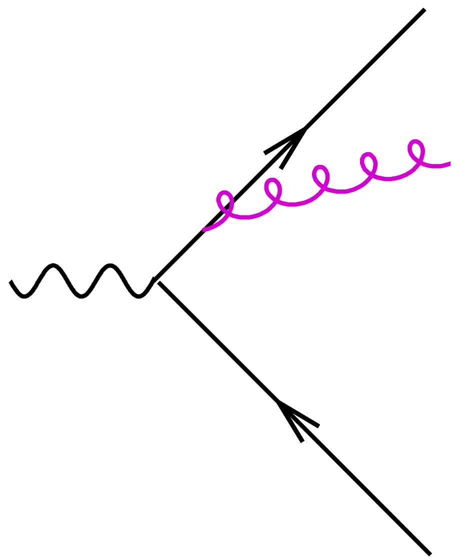
- ★ backward parton evolution
- ★ underlying event

Collinear Enhancement (Light Partons)

- ME involving $q \to qg$ (or $g \to gg$) strongly enhanced whenever emitted gluon is almost collinear. Propagator factor

$$\frac{1}{(p_q + p_g)^2} \approx \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

  - soft+collinear divergences.
  - dominant contribution to the ME.



- Collinear factorization

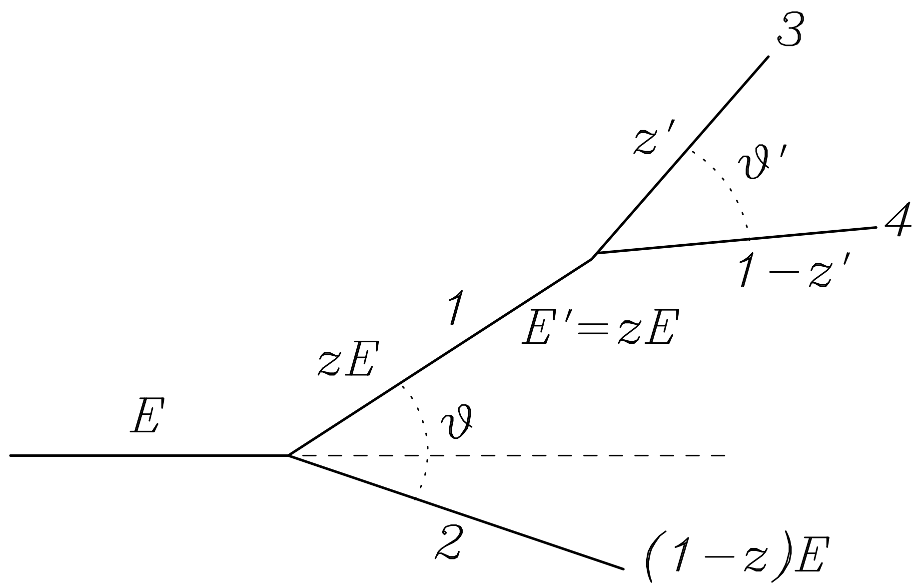
$$|M_{p+1}|^2 d\Phi_{p+1} \approx |M_p|^2 d\Phi_p \frac{dt}{t} \frac{\alpha_s}{2\pi} P(z) dz d\phi$$

$$P(z) = C_F \frac{1+z^2}{1-z}$$

$\Rightarrow$ Parton shower MC.

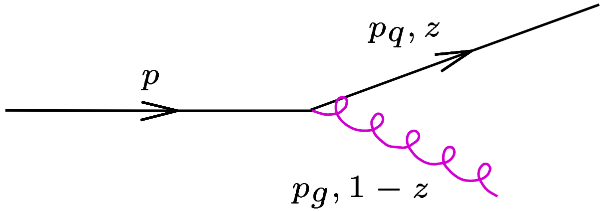
- Shower resums leading collinear logarithmic contributions.

Soft Gluon Coherence



- Soft gluon 3 ($z' \ll 1$) at angle $\theta' > \theta$ cannot resolve individual colour charges of partons 1 & 2
  - emitted coherently by partons 1 & 2
  - effectively emitted by parent of 1 & 2 $\Rightarrow$ angular ordering
  - not equivalent to a veto on $\theta' > \theta$
- Angular ordered shower resums leading soft logarithmic contributions.
  - but angular evolution variables $\Rightarrow$ dead regions

Quasi–Collinear Limit (Heavy Quarks)



- Sudakov basis p, n with $p^2 = m^2$ ('forward'), $n^2 = 0$ ('backward'), $p_\perp^2 = -\boldsymbol{p}_\perp^2$

$$
\begin{aligned}
p_q &= zp + \beta_q n - p_\perp \\
p_g &= (1-z)p + \beta_g n + p_\perp
\end{aligned}
$$

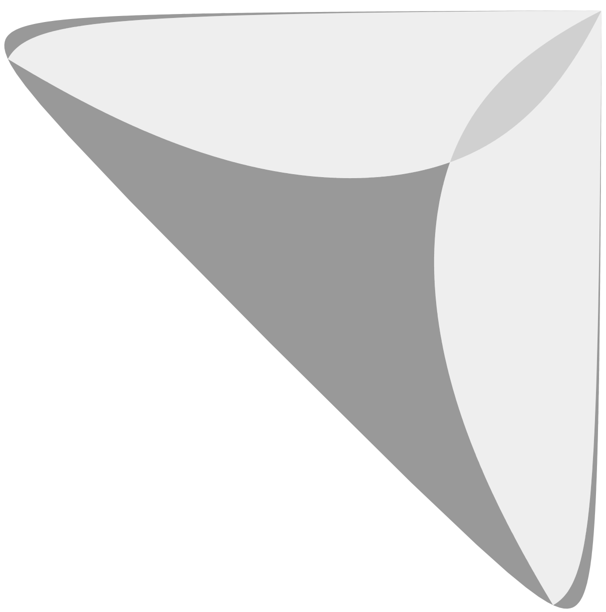
- Quasi-collinear limit (Catani et al.): for $|\boldsymbol{p}_\perp| \sim m \ll p_+$

$$
\begin{aligned}
P_{qq}(z, \boldsymbol{p}_\perp^2, m^2) &= C_F \left[\frac{1+z^2}{1-z} - \frac{2z(1-z)m^2}{\boldsymbol{p}_\perp^2 + (1-z)^2 m^2} \right] \\
&\equiv \frac{C_F}{1-z} \left[1 + z^2 - \frac{2m^2}{z\tilde{q}^2} \right]
\end{aligned}
$$

- Generalised angular variable: for $m \to 0\,,\quad \tilde{q} \sim |\boldsymbol{p}_\perp|/z(1-z) \sim E\theta$
- Collinear limit: for $p_\perp \to 0\,,\quad \tilde{q} \sim m/z\,,\quad P_{qq} \sim C_F(1-z)$

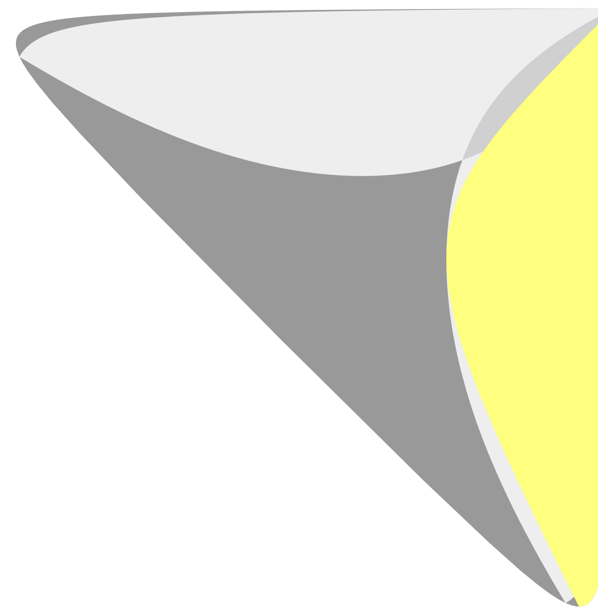
Dead regions in $q\bar{q}g$ phase space

- Consider $(x, \bar{x})$ phase space for $e^+e^- \to q\bar{q}g$



Fortran HERWIG

Herwig++



Comparison

✓ No overlapping regions in phase space.

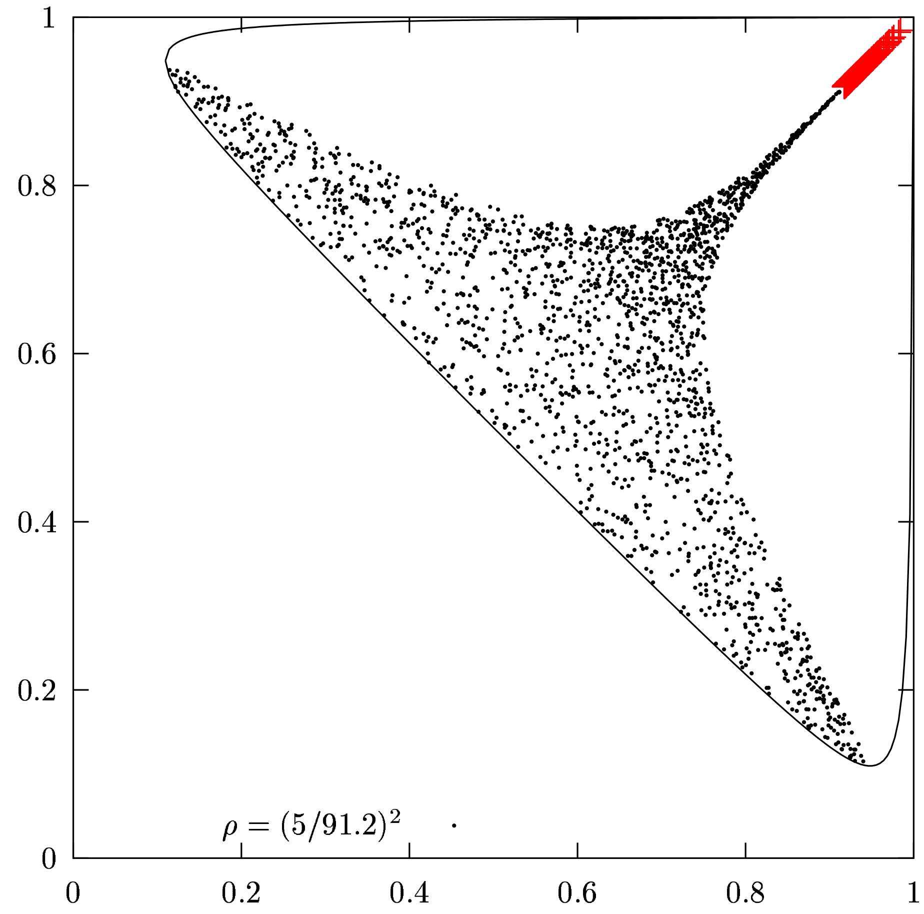
✓ Smooth coverage of soft gluon region.

✓ No collinear dead cones.

✗ Larger non-collinear dead region.

Hard Matrix Element Corrections

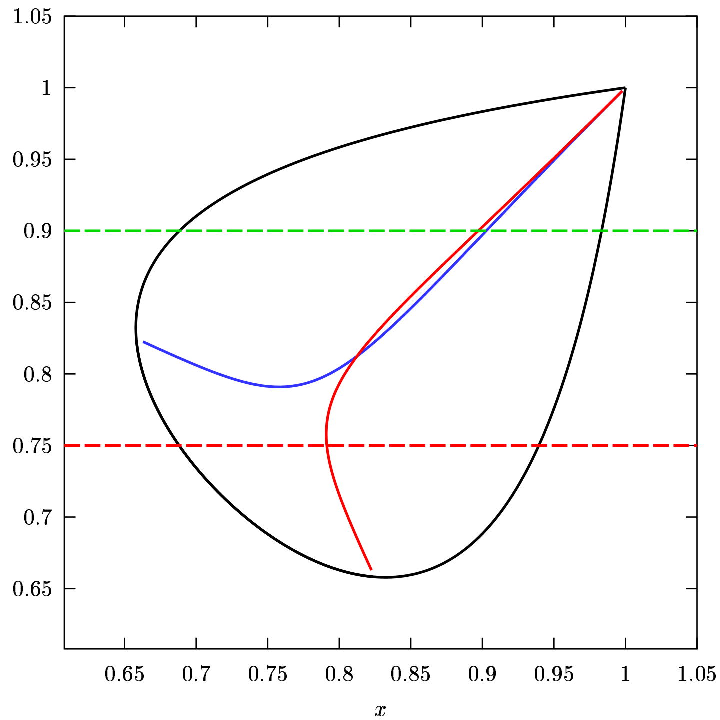
- Points $(x_q, x_{\bar{q}})$ in dead region chosen according to LO $q\bar{q}g$ matrix element and accepted according to ME weight.
- About 3% of all events are actually hard $q\bar{q}g$ events.
- Red points have weight > 1, practically no error by setting weight to one.
- Event oriented according to given $q\bar{q}$ geometry (Kleiss). Quark direction is kept with weight $x_q^2/(x_q^2 + x_{\bar{q}}^2)$.

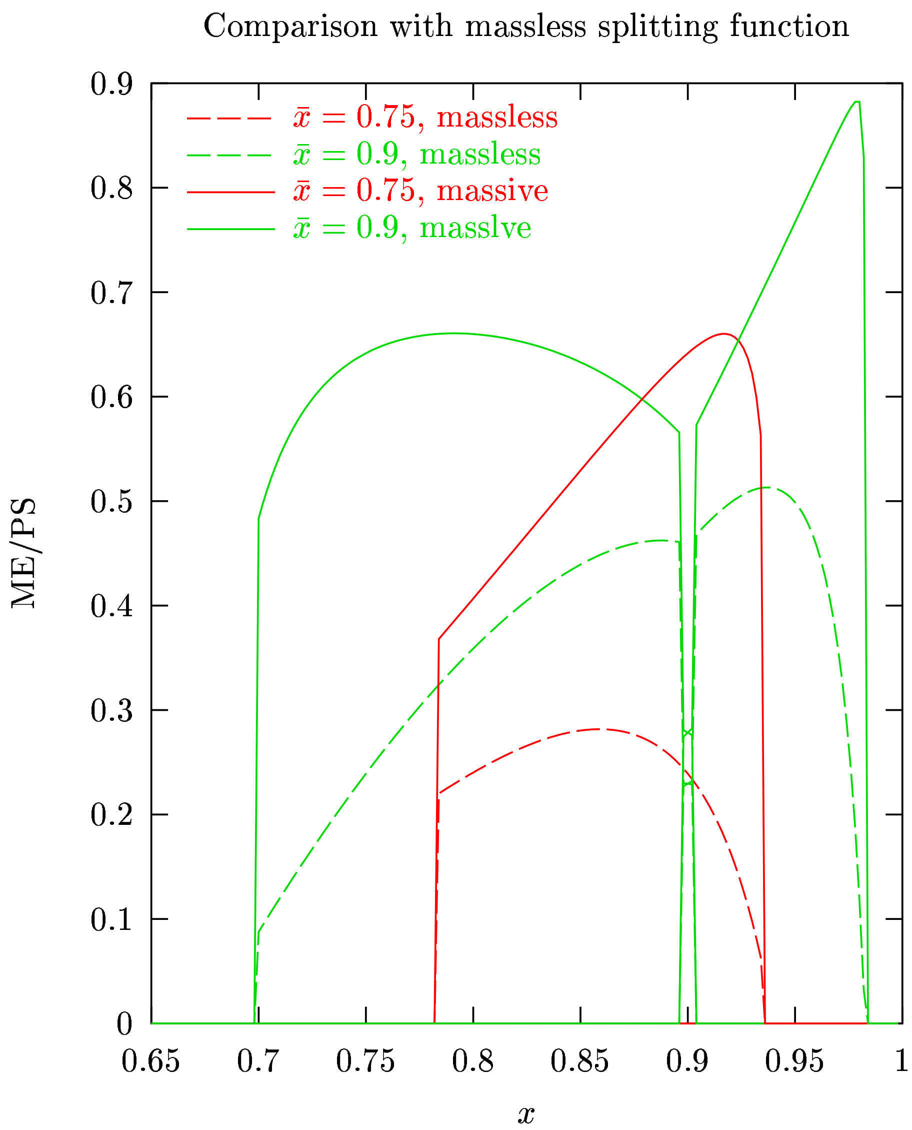


Soft Matrix Element Corrections

- Ratio ME/PS compares emission with result from true ME if slightly away from soft/collinear region.
- Veto on 'hardest emission so far' in $p_\perp$.
- Massive splitting function *very important*!

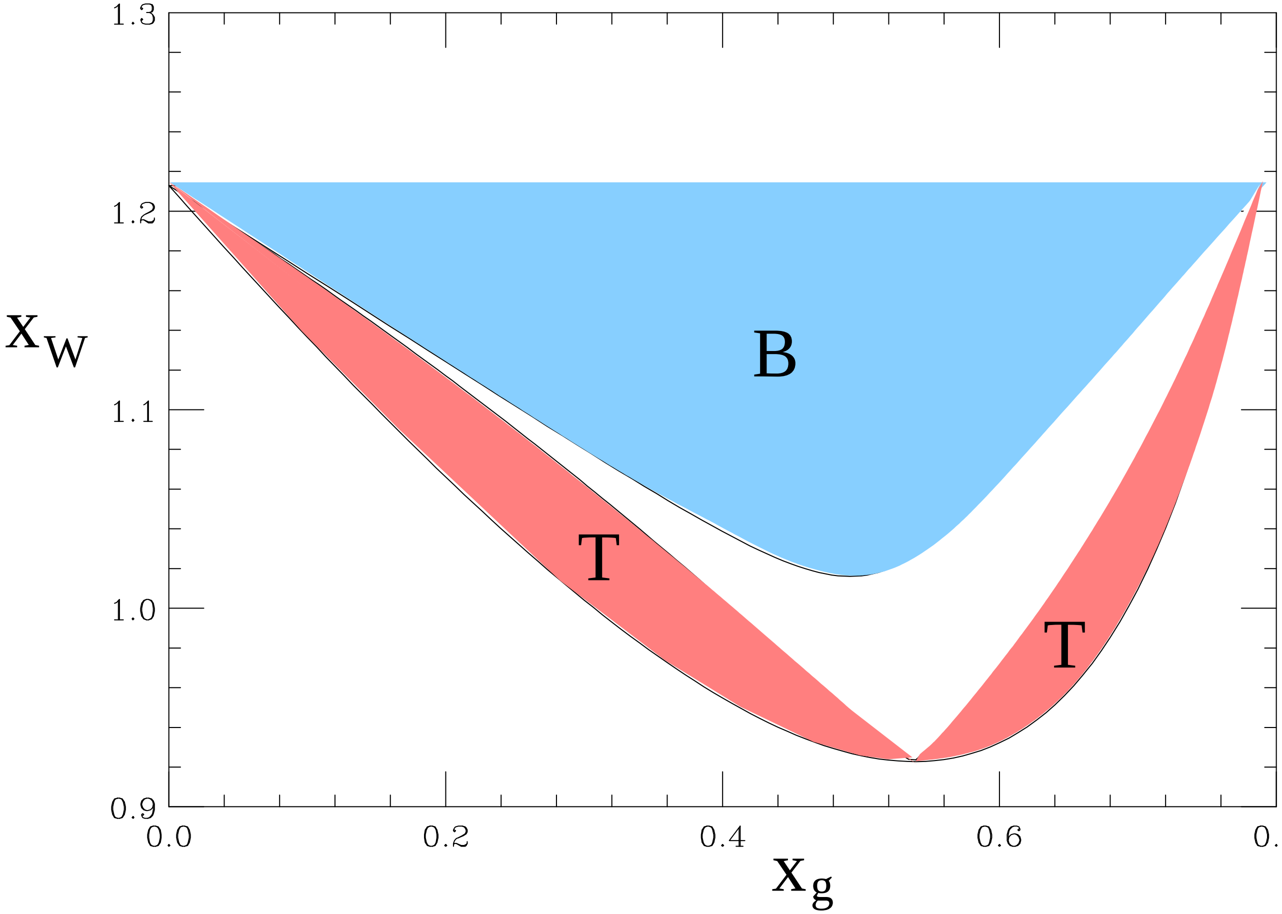
Example with heavy quark, $m^2/Q^2 = 0.1$ ($\approx t\bar{t}$ at 500 GeV)





Heavy Quark Decay

- In $t \to Wb$ ISR from t fills soft and collinear regions $\Rightarrow$ ME correction is finite.



- In Fortran HERWIG, ISR was missing $\Rightarrow$ infrared divergence in ME correction.

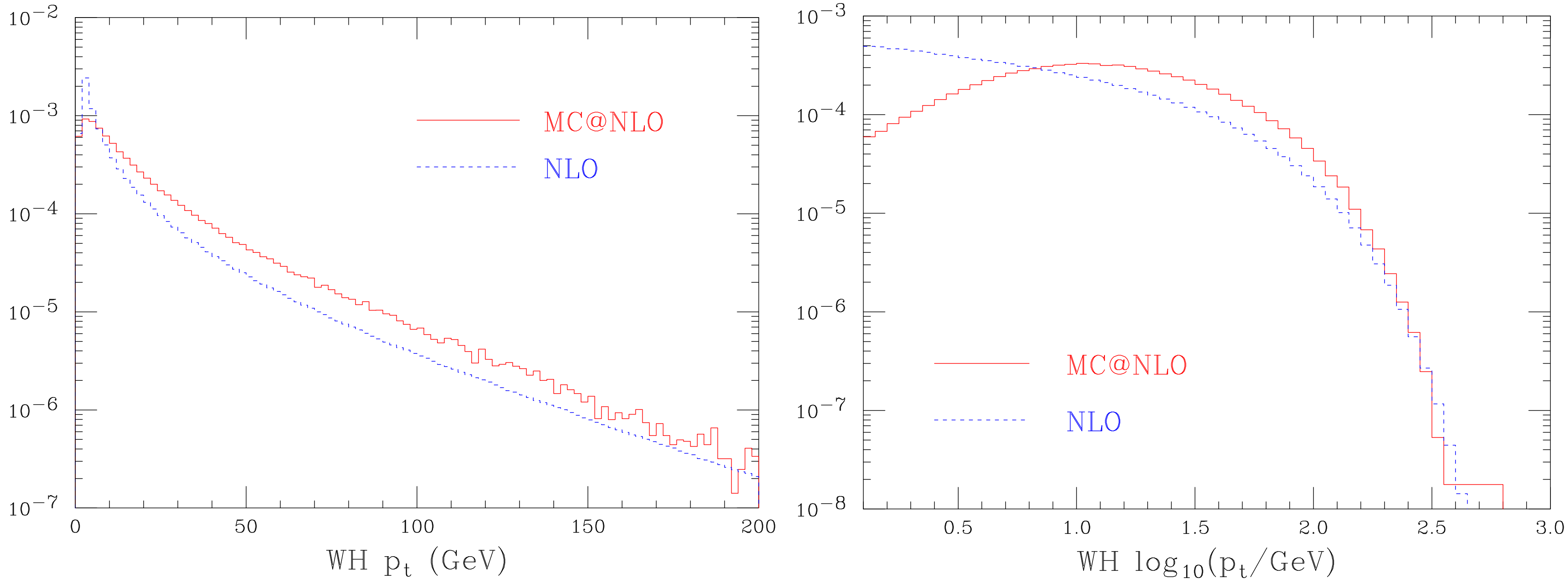
Combining Matrix Elements and Showers

Method of hard+soft matrix element corrections, implemented in HERWIG and PYTHIA, is difficult to extend to NLO or to more complicated processes.

- MC@NLO (+Mrenna, Nason): subtract approximate NLO contributions generated by showers from exact NLO matrix elements.
  - Regularizes divergences of NLO ME
  - All NLO results formally reproduced
  - Shower resums soft & collinear divergences to all orders
  - Existing schemes generate negative weights (in general)
  - Available for V, H, VV, VH, $Q\bar{Q}$ hadroproduction.
- CKKW (+ Krauss, Lönnblad, Mrenna & Richardson): generate ME with k_T-cutoff Q_1, apply corresponding Sudakov form factors, veto $k_T > Q_1$ in showers.
  - Q_1 dependence cancels to NLL
  - Can combine different multiplicity ME's without double counting jet rates (to NLL)
  - No fixed-order results reproduced
  - Difficult to ensure smoothness
  - Studied with HERWIG & PYTHIA, implemented in SHERPA.
- Other schemes: Phase-space slicing (Baer-Reno, Dobbs, Pötter), numerical (Krämer-Soper), ALPGEN+HERWIG (Mangano),. . .

Associated Higgs + Vector Boson Production in MC@NLO

- Transverse momentum of WH pair in $\bar{p}p \to W^+H^0X$ at Tev II

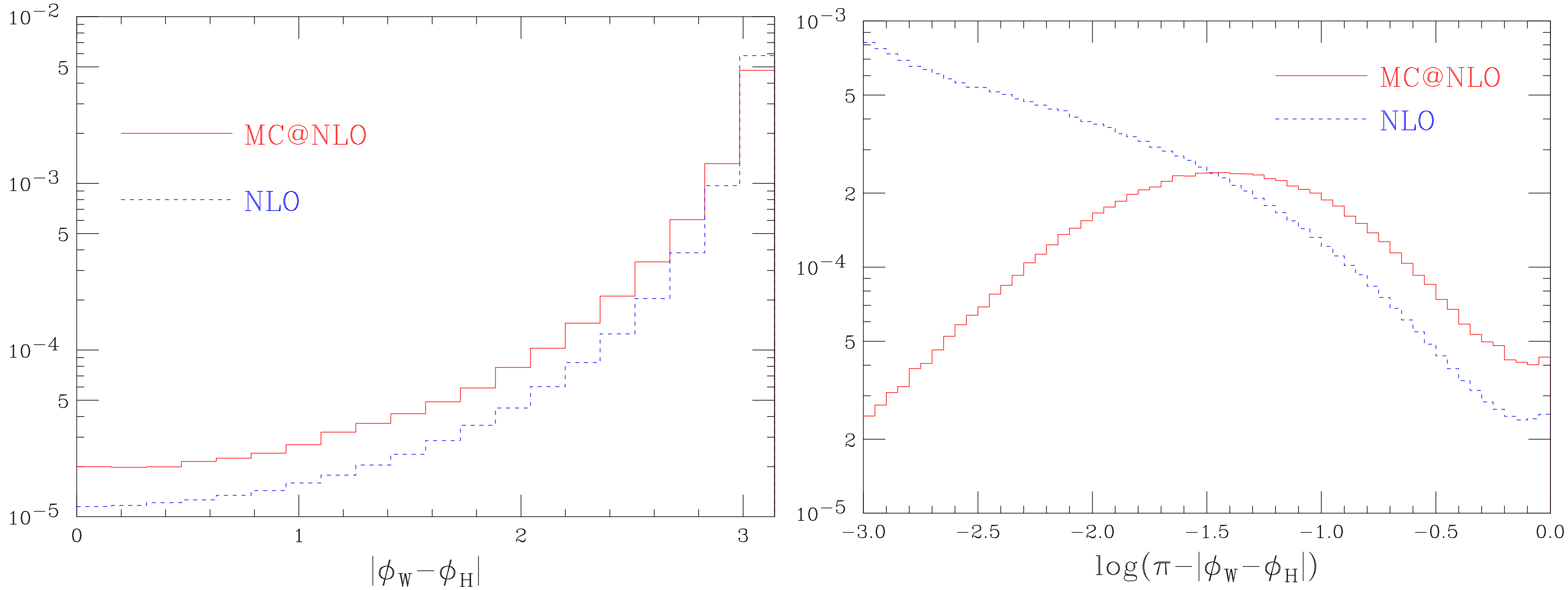


❖ Qualitatively similar to WW: regularizes behaviour at $p_t \to 0$, coincides with NLO at high p_t.

V Del Duca, S Frixione, C Oleari & BRW, in preparation

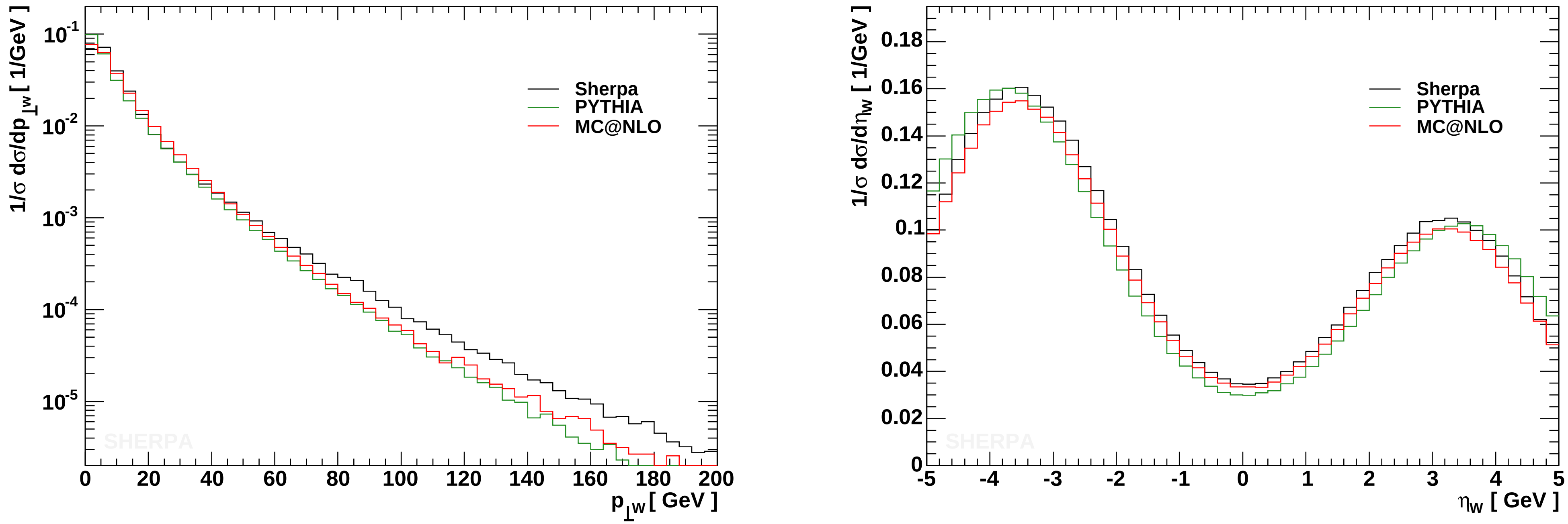
Associated Higgs + Vector Boson Production (cont'd)

- WH azimuthal separation in $\bar{p}p \to W^+H^0X$ at Tev II



  - MC@NLO includes multiple emission at low $\Delta\phi$ and regularizes behaviour at $\Delta\phi \to \pi$.

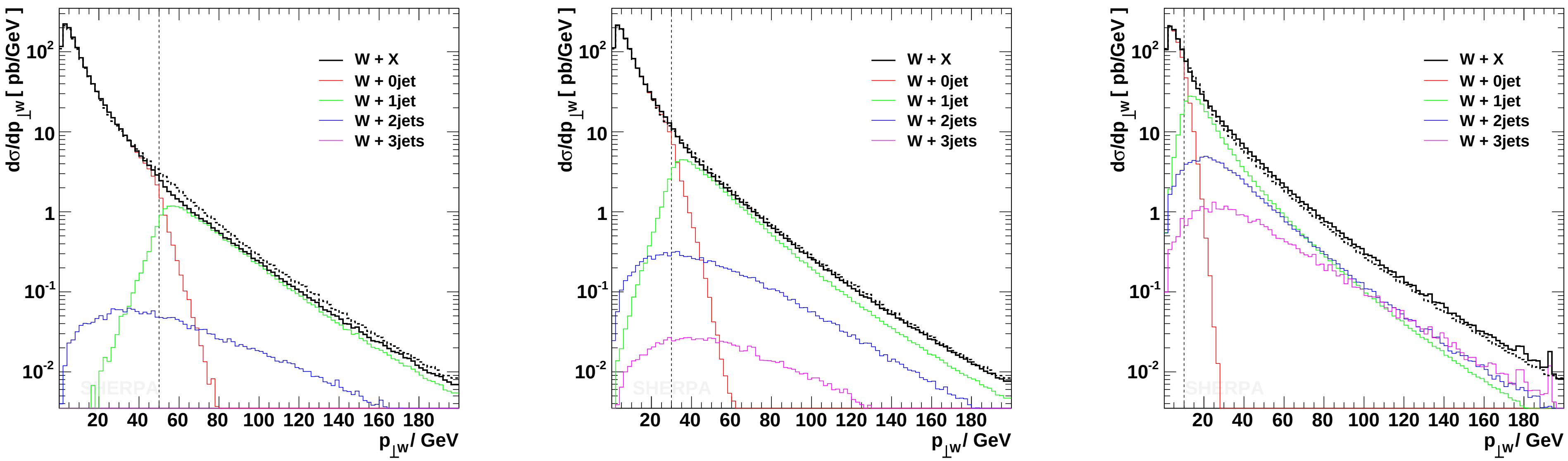
PYTHIA/MC@NLO/SHERPA Comparisons



$p_\perp$ and η distributions of W^- bosons for inclusive production at Tevatron Run II.

F. Krauss, A. Schälicke, S. Schumann & G. Soff, hep-ph/0409106

(In)sensitivity to Matching Scale in SHERPA

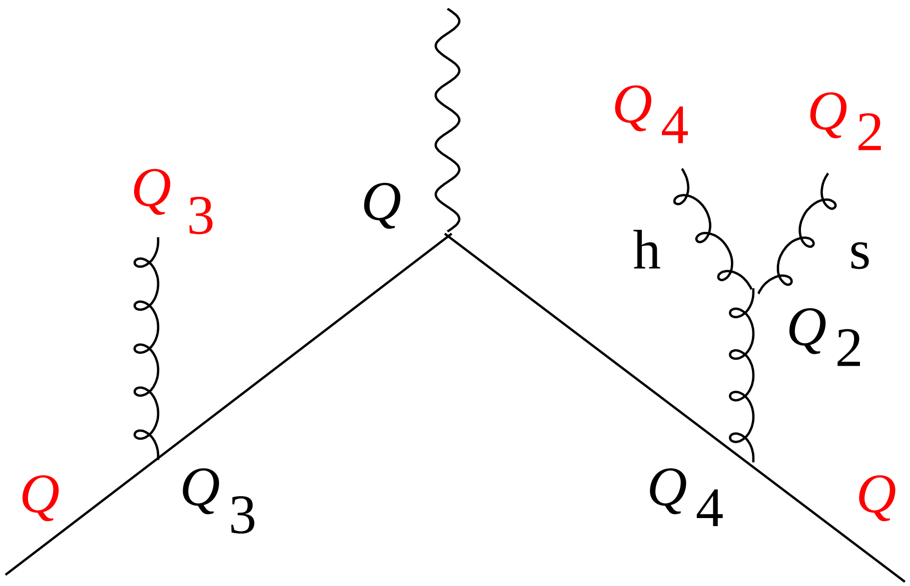


$p_\perp(W^-)$ for $Q_1 = 10$ GeV, 30 GeV and 50 GeV in comparison with $Q_1 = 20$ GeV.

F. Krauss, A. Schälicke, S. Schumann & G. Soff, hep-ph/0409106

Combining ME & PS: Scales

- Coherent branching $\Rightarrow$ evolution in angle, not k_T
- k_T-cutoff Q_1 on ME $\Rightarrow$ veto $k_T > Q_1$ in showers
- However, starting scale for showers is not $\tilde{q} = Q_1$
  - Showers must "fill in" radiation at larger angles, with $\tilde{q} > Q_1$ but $k_T < Q_1$
- Construct parton "histories" (gauge invariant) from clustering sequence
  - Each parton evolves from the $\tilde{q}$ scale at which it was "created" (shown in red)
  - More correctly, generate truncated showers on internal lines (Nason)



Combining ME & PS: Kinematics

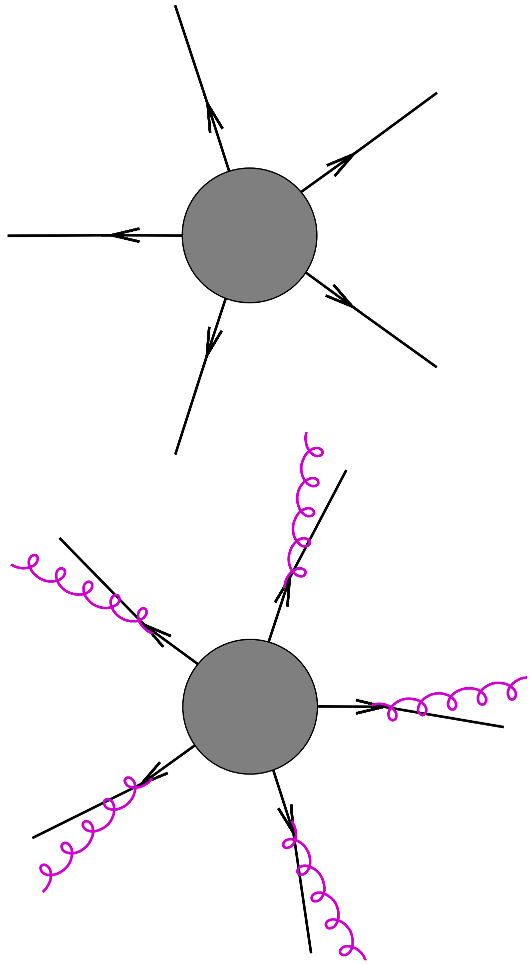
Formally subleading ⇒ important for MC@NLO.
After showering, hard partons have virtualities $q_i^2 \neq m_i^2$
⇒ boost/rescale jets.
Started with

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{m_i^2 + \boldsymbol{p}_i^2}$$

We can rescale 3-momenta with common factor K,

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{q_i^2 + K\boldsymbol{p}_i^2}$$

to preserve overall energy/momentum.
⇒ resulting jets are then boosted accordingly.



New Proposal for NLO + PS Matching

P. Nason, hep-ph/0409146, 14 Sept 04

- Consider n–body LO process: NLO has n– and $(n+1)$–body contributions
  - Factorize $(n+1)$–body phase space: $\Phi(v,r) \to \Phi_v\,\Phi_r$
  - Generate n–body configurations according to NLO rate:

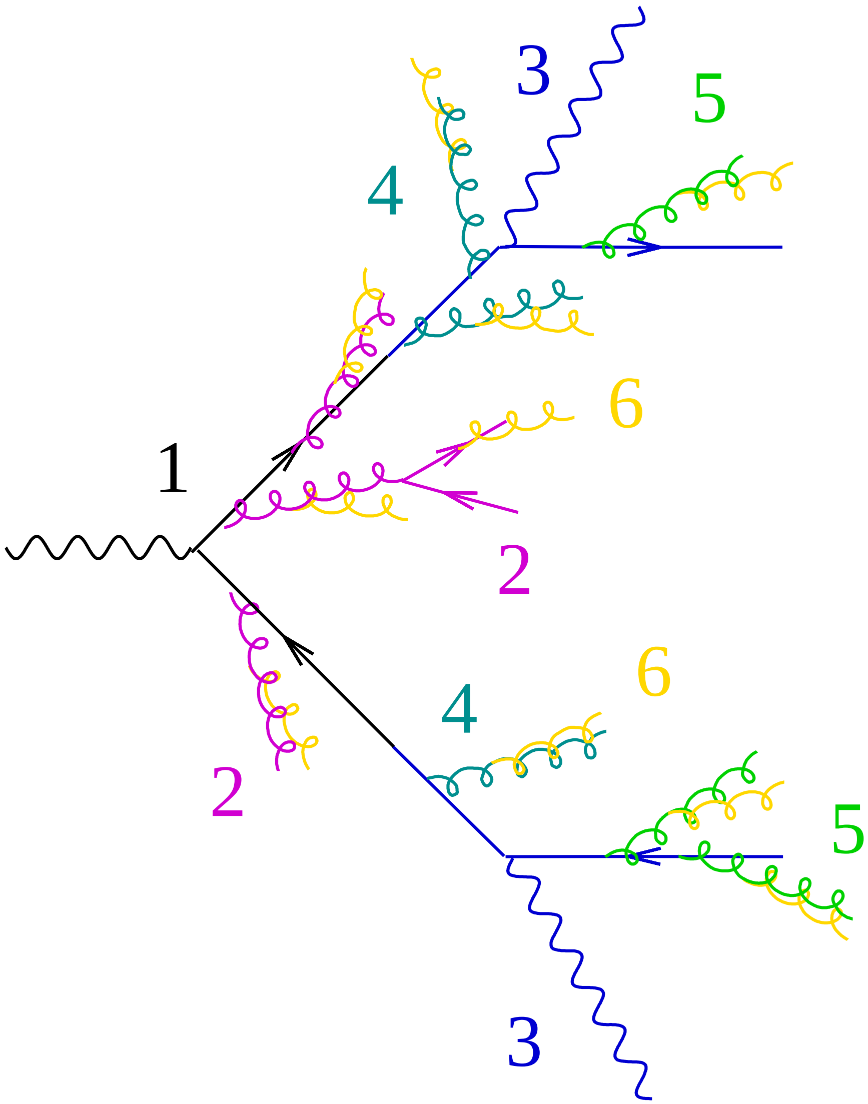
$$\bar{B}(v) = B(v) + V(v) + \int d\Phi_r \left\{R(v,r) - C(v,r)\right\}$$

    where B = Born, V = virtual, R = real emission, C = counterterms
  - Generate first real emission with full ME by using modified Sudakov form factor

$$\Delta_{\rm mod}(p_\perp) = \exp\left\{-\int d\Phi_r \frac{R(v,r)}{B(v)}\theta(k_\perp(v,r) - p_\perp)\right\}$$

  - Assign emission to nearest (in $k_\perp$) leg
  - Generate ‘truncated’ shower on ‘parent’, plus vetoed showers on all $n+1$ legs (all with $k_\perp < p_\perp$) using normal Sudakov factors.
- This generates MC at NLO with no negative weights (as long as $\bar{B} > 0$)
  - Builds in big (spurious?) NNLO contributions by exponentiating whole real emission ME
  - Needs to be tried out.

Multiscale Showering



Example: $t\bar{t}$ production & decay

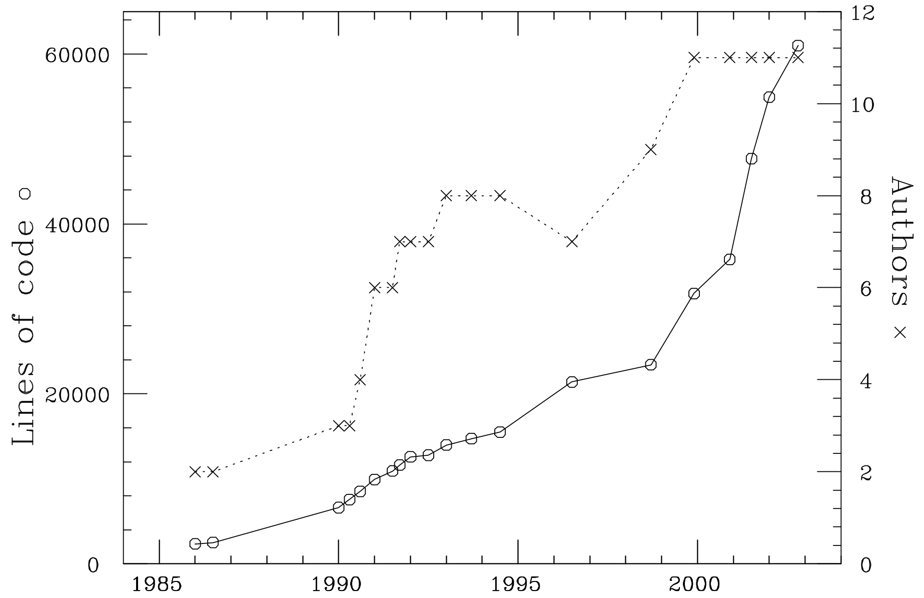
1. Hard process (scale $\sim \hat{s}$)
2. Showers from $t, \bar{t}$ ($\hat{s} \to \Gamma_t$)
3. Decays $t \to Wb$, $\bar{t} \to W\bar{b}$
4. ISR from $t, \bar{t}$ ($m_t \to \Gamma_t$)
5. FSR from $b, \bar{b}$ ($m_t \to \Gamma_t$)
6. Global showering ($\Gamma_t \to \Gamma_b$)

etc.

The Herwig++ Event Generator

A completely new event generator in C++

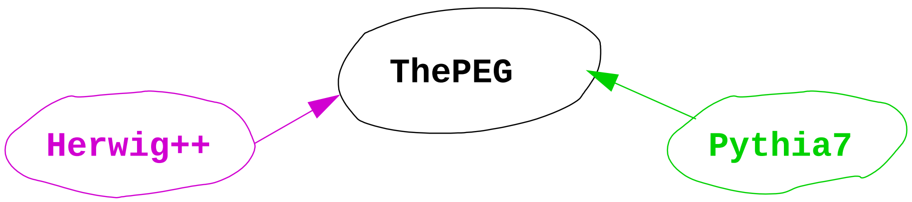
- Aiming at full multi–purpose generator for LHC and future colliders.
- Preserving main features of Fortran HERWIG such as
  - angular ordered parton shower
  - cluster hadronization
- New features and improvements
  - covariant shower formulation
  - improved parton shower evolution for heavy quarks
  - consistent radiation from unstable particles (multiscale evolution)



Growth of Fortran HERWIG

Use of ThePEG in Herwig++

ThePEG = Toolkit for high energy Physics Event Generation
Leif Lönnblad, http://www.thep.lu.se/ThePEG/



Share administrative overhead, common to event generators with Pythia7

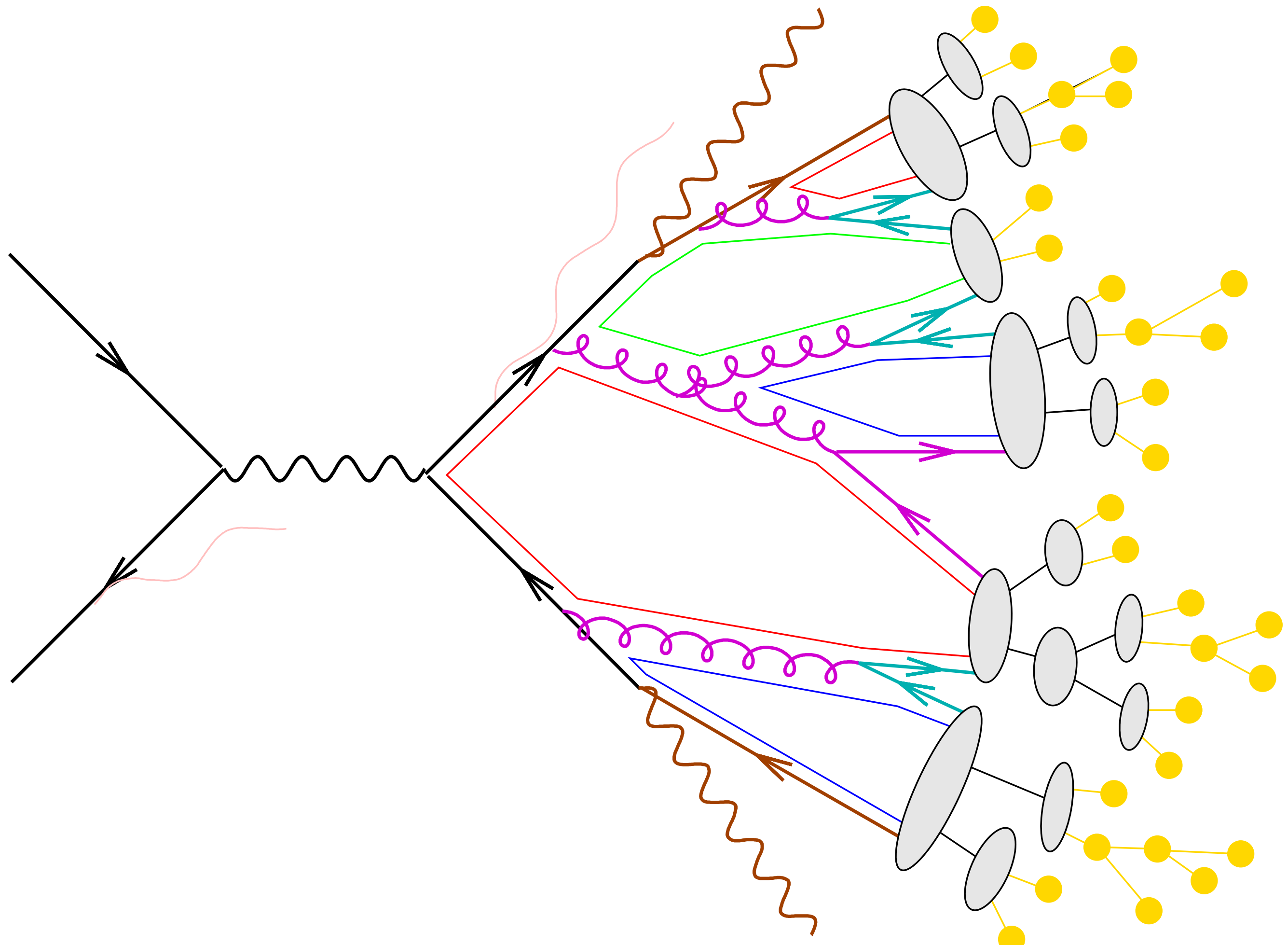
Independent *physics* implementation

Large but very flexible implementation

Common basis for Pythia7/Herwig++

- ✗ Lack of independence.
- ✗ Less possiblity to test codes against each other.
- ✓ Physics is still independent.
- ✓ Beneficial for the user to have the same framework.
- ✓ Running Herwig++ with Lund String Fragmentation from Pythia7 is very simple!

Cluster Hadronization Model



- ★ hard scattering
- ★ (QED) initial/final state radiation
- ★ partonic decays, e.g. $t \to bW$
- ★ parton shower evolution
- ★ nonperturbative gluon splitting
- ★ colour singlets
- ★ colourless clusters
- ★ cluster fission
- ★ cluster $\to$ hadrons
- ★ hadronic decays

Cluster hadronization in a nutshell

- Nonperturbative $g \to q\bar{q}$ splitting ($q = uds$) isotropically. Here, $m_g \approx 750\,\text{MeV} > 2m_q$.
- Cluster formation, universal spectrum (see right)
- Cluster fission until

$$M^p < M^p_{\max} + (m_1 + m_2)^p$$

where masses are chosen from

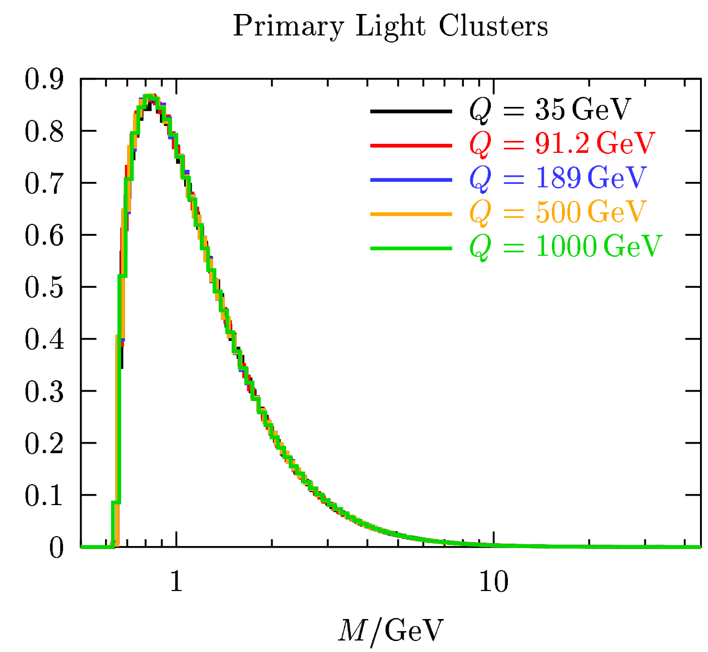
$$M_i = \left[\left(M^P - (m_i + m_3)^P\right) r_i + (m_i + m_3)^P\right]^{1/P},$$

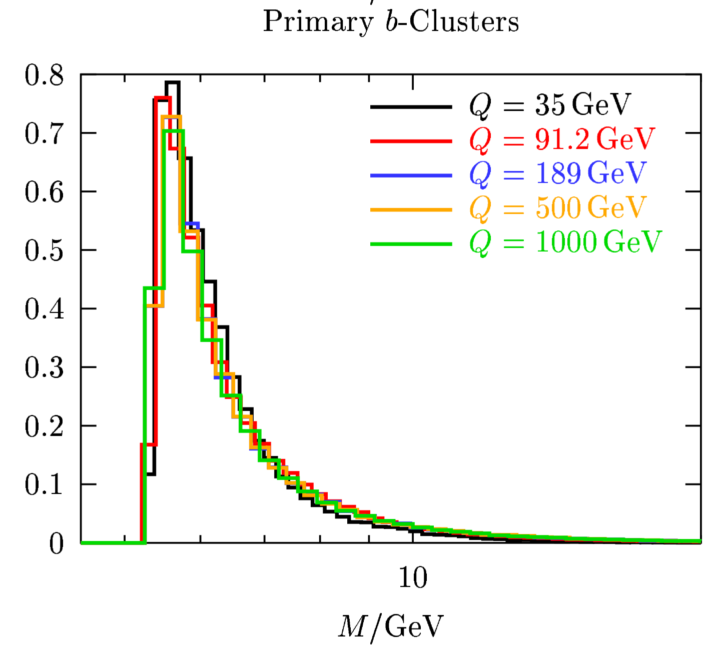
with additional phase space constraints. Constituents keep moving in their original directions.

- Cluster decay

$$P(a_{i,q}, b_{q,j}|i,j) = \frac{W(a_{i,q}, b_{q,j}|i,j)}{\sum_{M/B} W(c_{i,q'}, d_{q',j}|i,j)}.$$

New! Meson/Baryon ratio is parametrized in terms of diquark weight. In Fortran HERWIG the sum ran over all possible hadrons.





$Z^0 \to$ Hadron Multiplicities

Particle	Experiment	Measured	Old Model	Herwig++	Fortran
All Charged	M,A,D,L,O	20.924 ± 0.117	20.22*	20.814	20.532*
γ	A,O	21.27 ± 0.6	23.032	22.67	20.74
π^0	A,D,L,O	9.59 ± 0.33	10.27	10.08	9.88
$\rho(770)^0$	A,D	1.295 ± 0.125	1.235	1.316	1.07
$\pi^\pm$	A,O	17.04 ± 0.25	16.30	16.95	16.74
$\rho(770)^\pm$	O	2.4 ± 0.43	1.99	2.14	2.06
η	A,L,O	0.956 ± 0.049	0.886	0.893	0.669*
$\omega(782)$	A,L,O	1.083 ± 0.088	0.859	0.916	1.044
$\eta'(958)$	A,L,O	0.152 ± 0.03	0.13	0.136	0.106
K^0	S,A,D,L,O	2.027 ± 0.025	2.121*	2.062	2.026
$K^*(892)^0$	A,D,O	0.761 ± 0.032	0.667	0.681	0.583*
$K^*(1430)^0$	D,O	0.106 ± 0.06	0.065	0.079	0.072
$K^\pm$	A,D,O	2.319 ± 0.079	2.335	2.286	2.250
$K^*(892)^\pm$	A,D,O	0.731 ± 0.058	0.637	0.657	0.578
$\phi(1020)$	A,D,O	0.097 ± 0.007	0.107	0.114	0.134*
p	A,D,O	0.991 ± 0.054	0.981	0.947	1.027
Δ^{++}	D,O	0.088 ± 0.034	0.185	0.092	0.209*
Σ^-	O	0.083 ± 0.011	0.063	0.071	0.071
Λ	A,D,L,O	0.373 ± 0.008	0.325*	0.384	0.347*
Σ^0	A,D,O	0.074 ± 0.009	0.078	0.091	0.063
Σ^+	O	0.099 ± 0.015	0.067	0.077	0.088
$\Sigma(1385)^\pm$	A,D,O	0.0471 ± 0.0046	0.057	0.0312*	0.061*
Ξ^-	A,D,O	0.0262 ± 0.001	0.024	0.0286	0.029
$\Xi(1530)^0$	A,D,O	0.0058 ± 0.001	0.026*	0.0288*	0.009*
Ω^-	A,D,O	0.00125 ± 0.00024	0.001	0.00144	0.0009

$Z^0 \to$ Hadron Multiplicities (ctd')

Particle	Experiment	Measured	Old Model	Herwig++	Fortran
$f_2(1270)$	D,L,O	0.168 $\pm$ 0.021	0.113	0.150	0.173
$f_2'(1525)$	D	0.02 $\pm$ 0.008	0.003	0.012	0.012
$D^\pm$	A,D,O	0.184 $\pm$ 0.018	0.322*	0.319*	0.283*
$D^*(2010)^\pm$	A,D,O	0.182 $\pm$ 0.009	0.168	0.180	0.151*
D^0	A,D,O	0.473 $\pm$ 0.026	0.625*	0.570*	0.501
$D_s^\pm$	A,O	0.129 $\pm$ 0.013	0.218*	0.195*	0.127
$D_s^{*\pm}$	O	0.096 $\pm$ 0.046	0.082	0.066	0.043
J/Ψ	A,D,L,O	0.00544 $\pm$ 0.00029	0.006	0.00361*	0.002*
Λ_c^+	D,O	0.077 $\pm$ 0.016	0.006*	0.023*	0.001*
$\Psi'(3685)$	D,L,O	0.00229 $\pm$ 0.00041	0.001*	0.00178	0.0008*

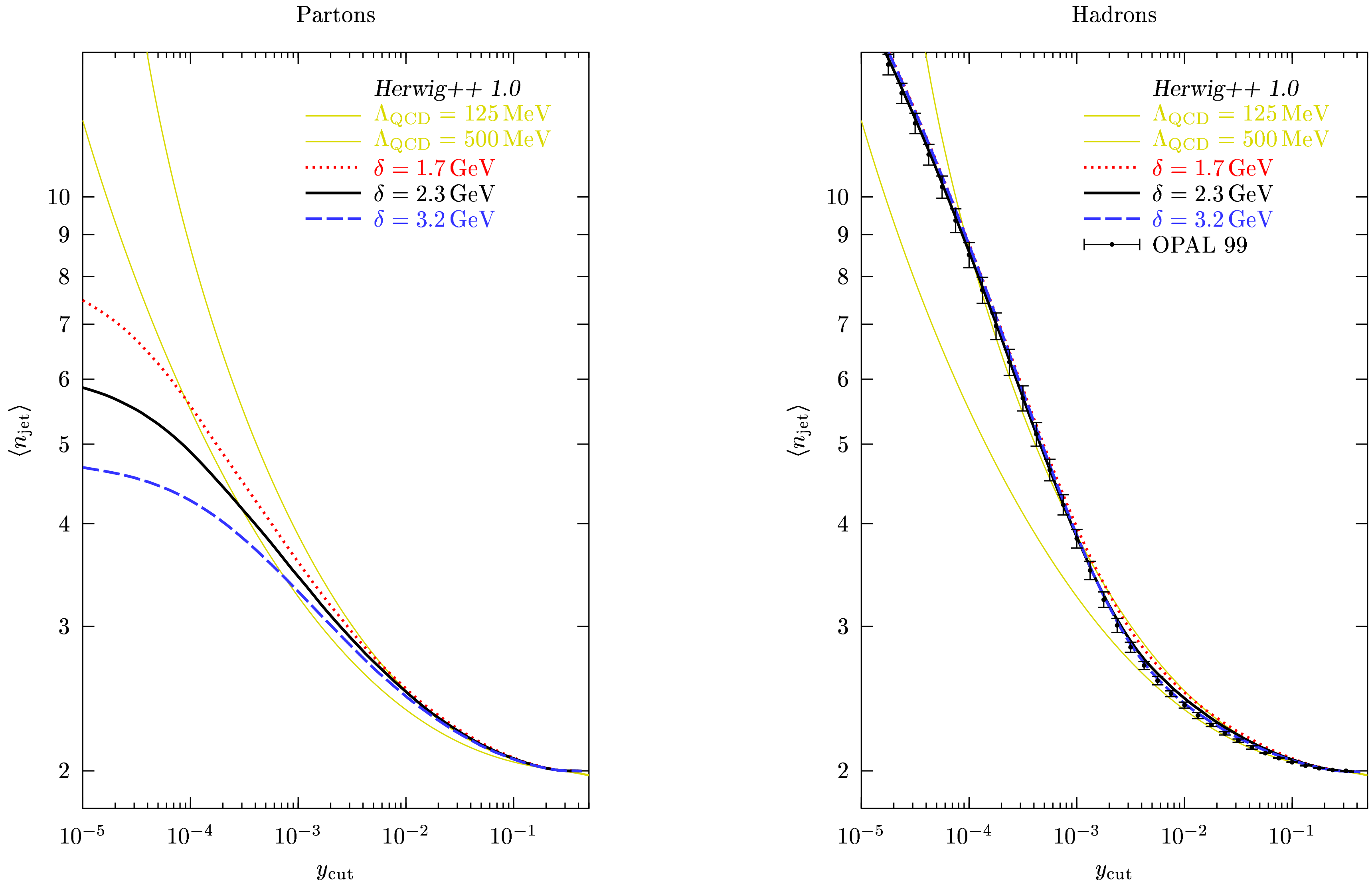
of $*$'s = observables with more than 3σ deviation:

Old Model : Herwig++ : Fortran = 9 : 7 : 13

N.B. No systematic parameter tuning yet.

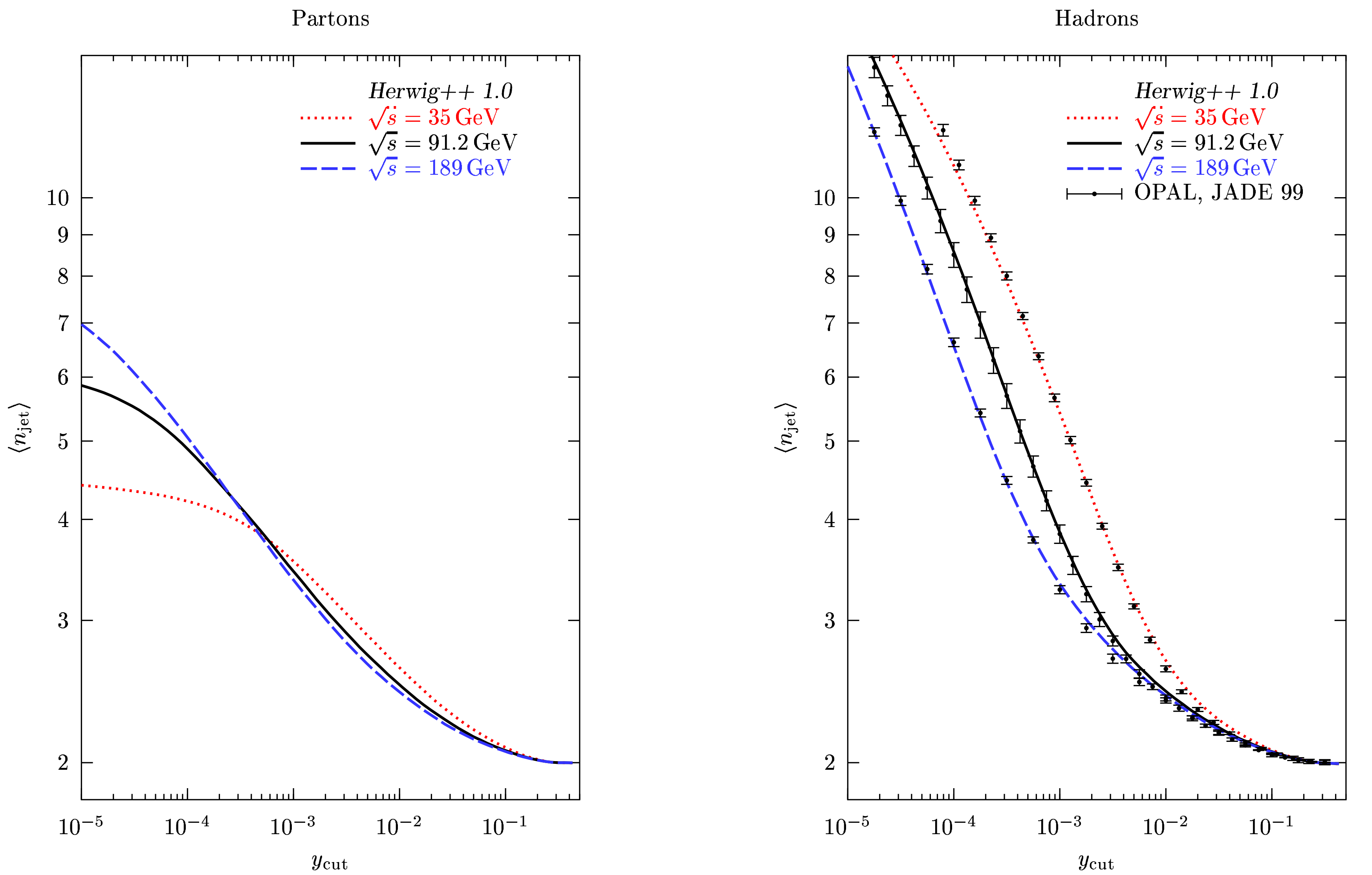
Jet Multiplicity

Cutoff dependence largely cancels between shower and hadronization.

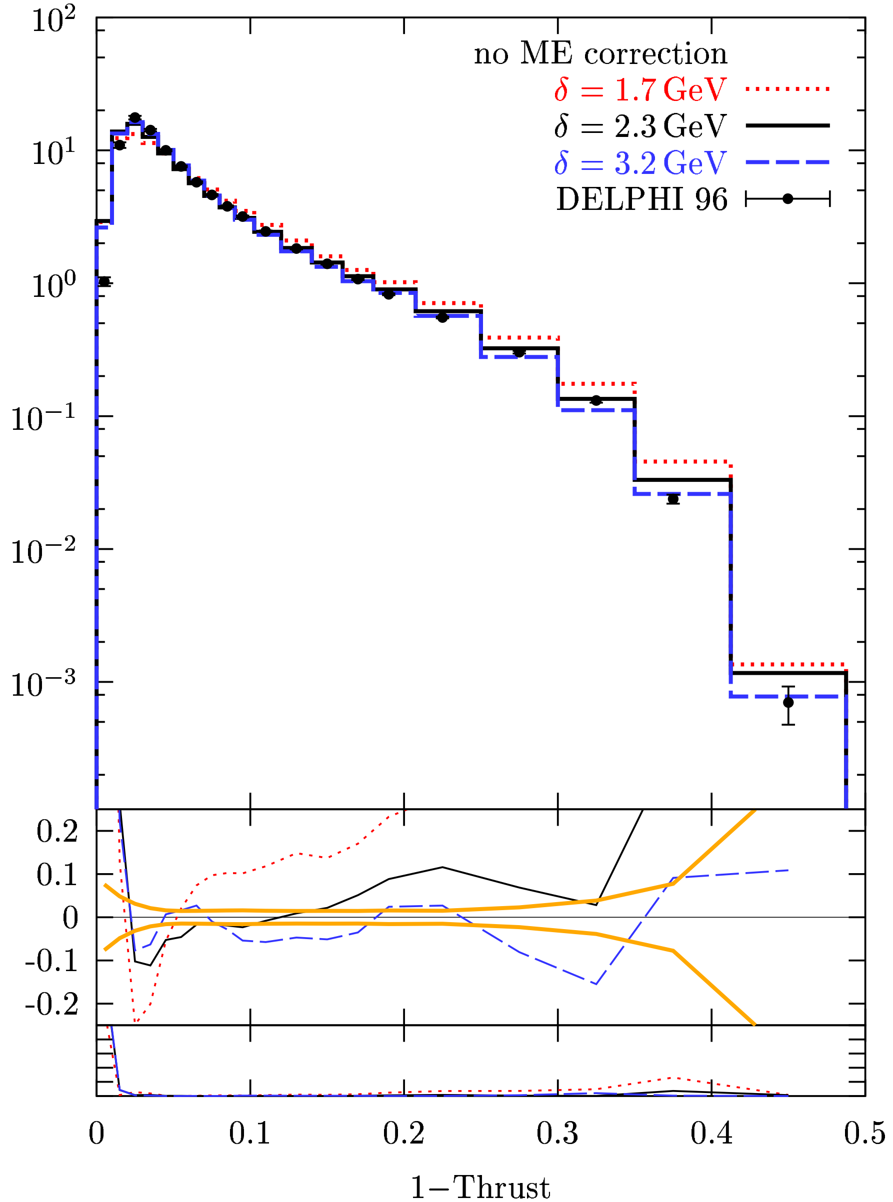


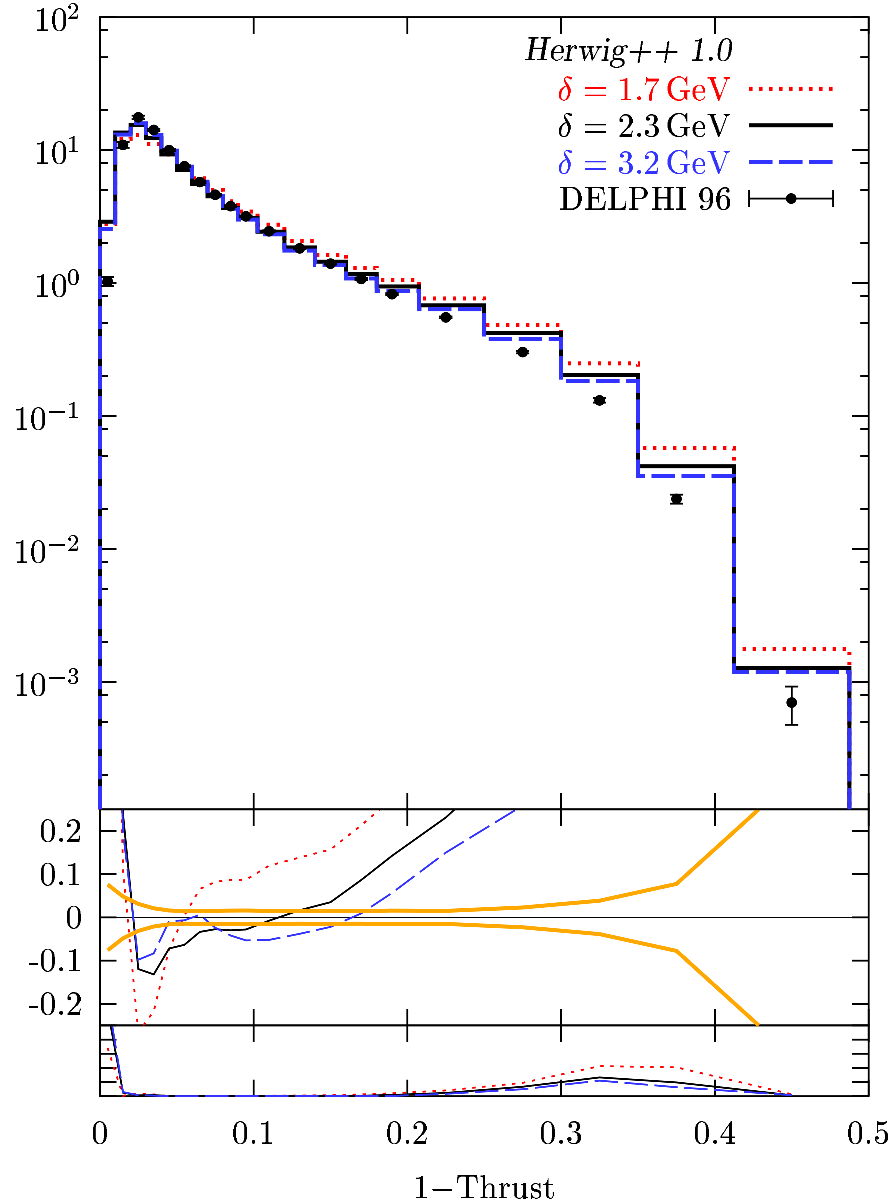
Jet Multiplicity (PETRA, LEP, LEPII)

$\sqrt{s} = \{35, 91.2, 189\}$ GeV

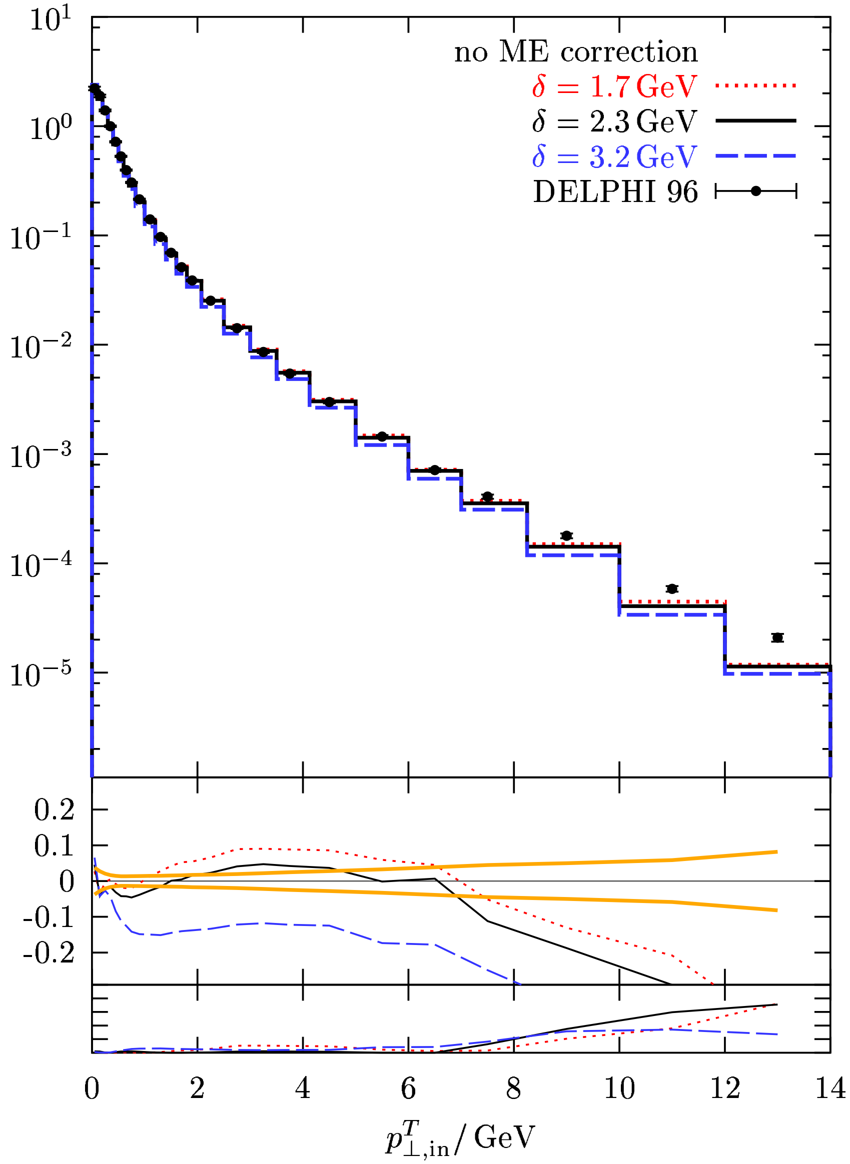


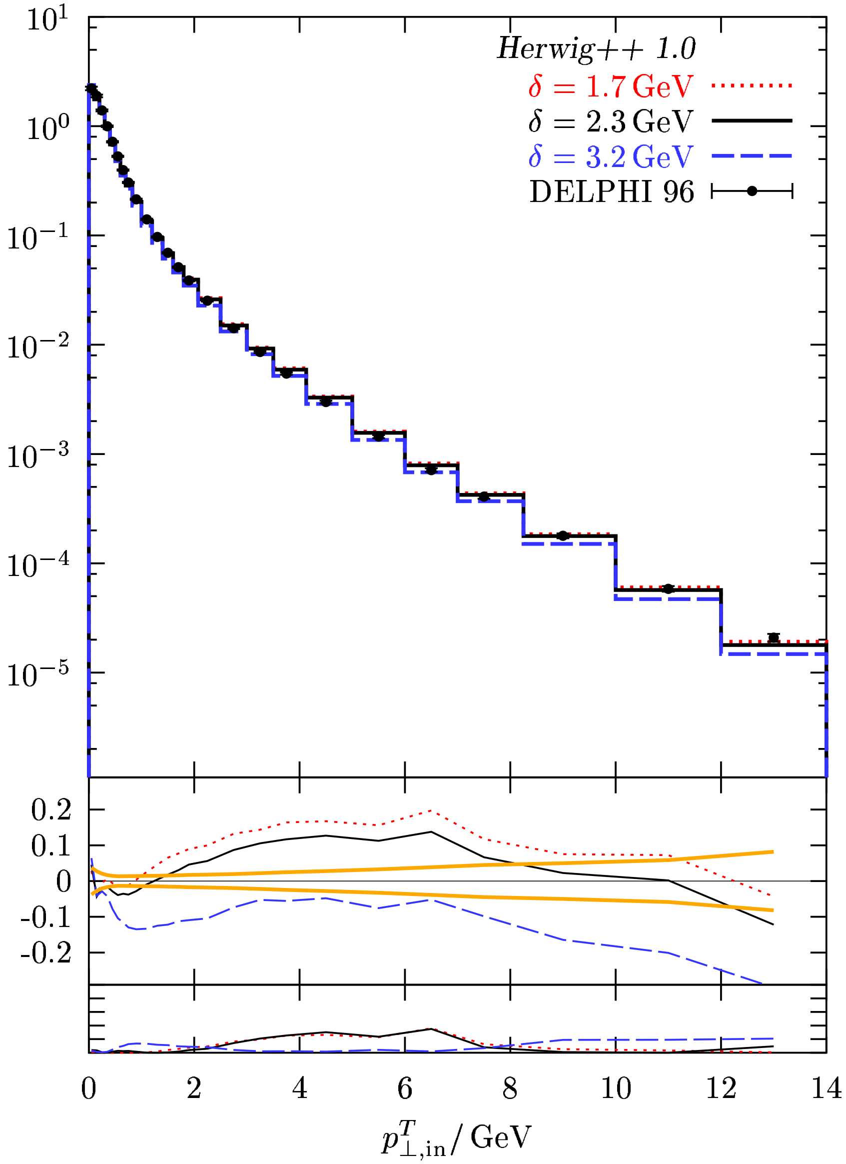
Thrust Distribution — ME Corrections off/on



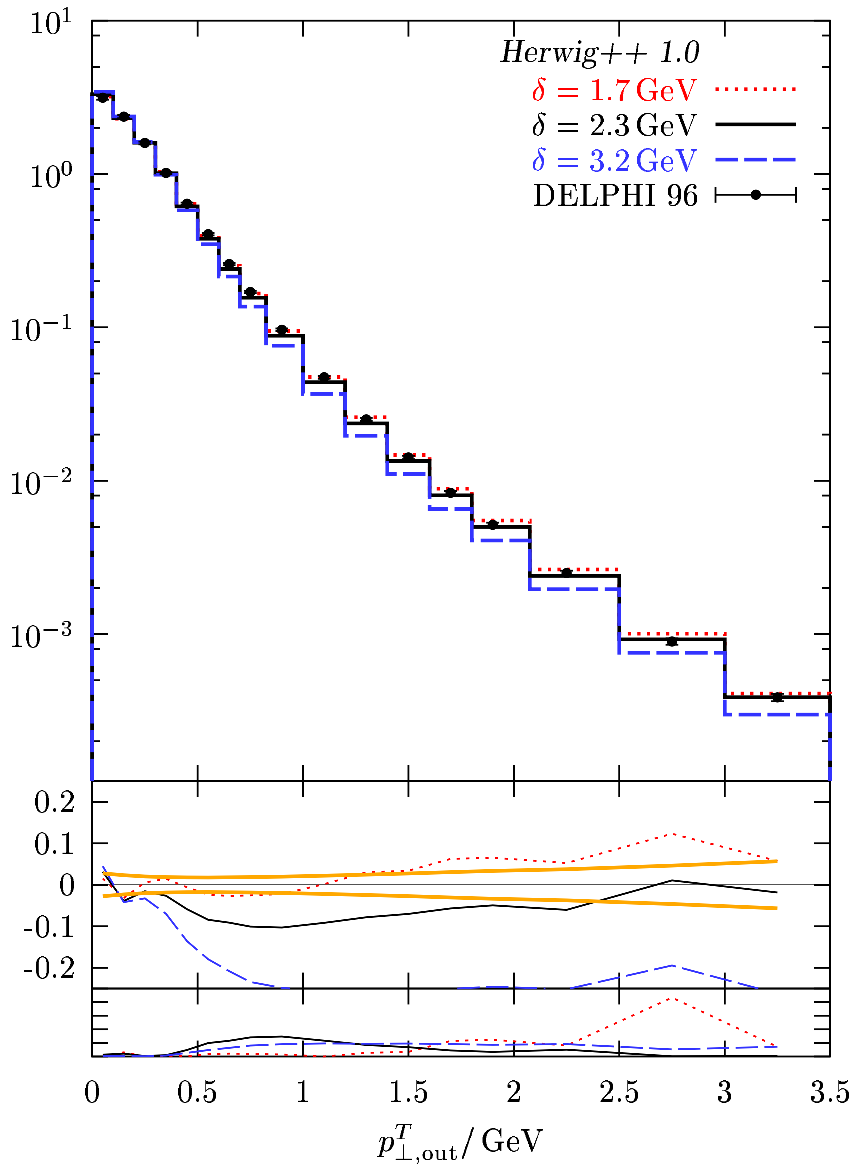


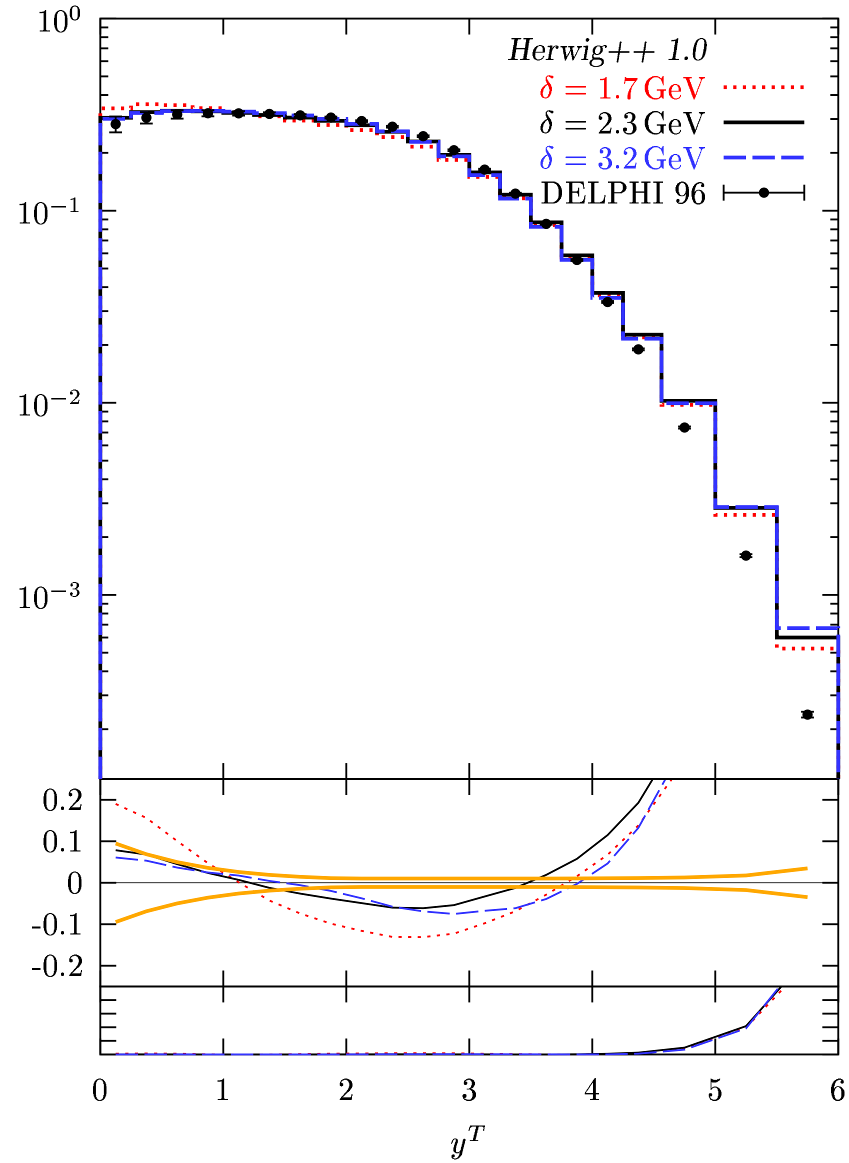
Single particle distributions: $p^T_{\perp,\mathrm{in}}$



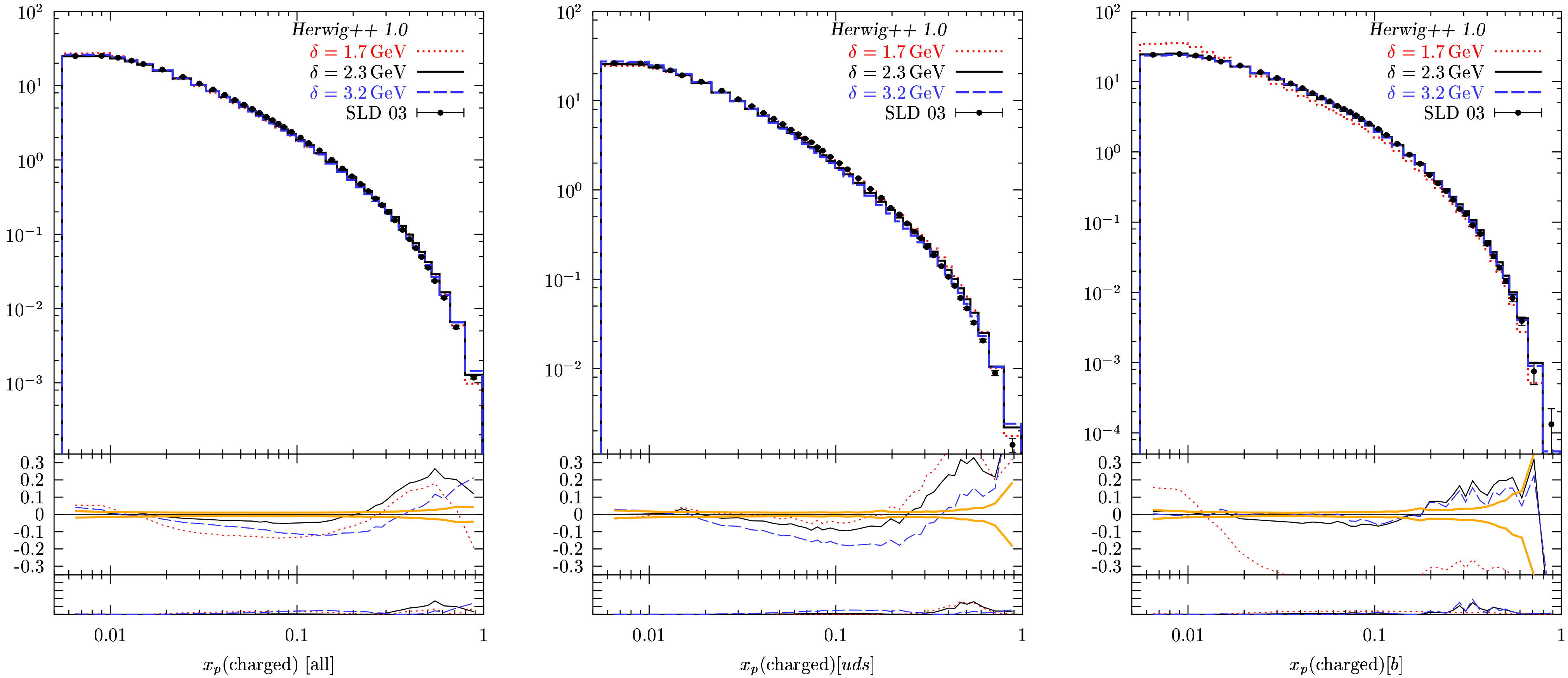


$p^T_{\perp,\mathrm{out}}$ and y^T

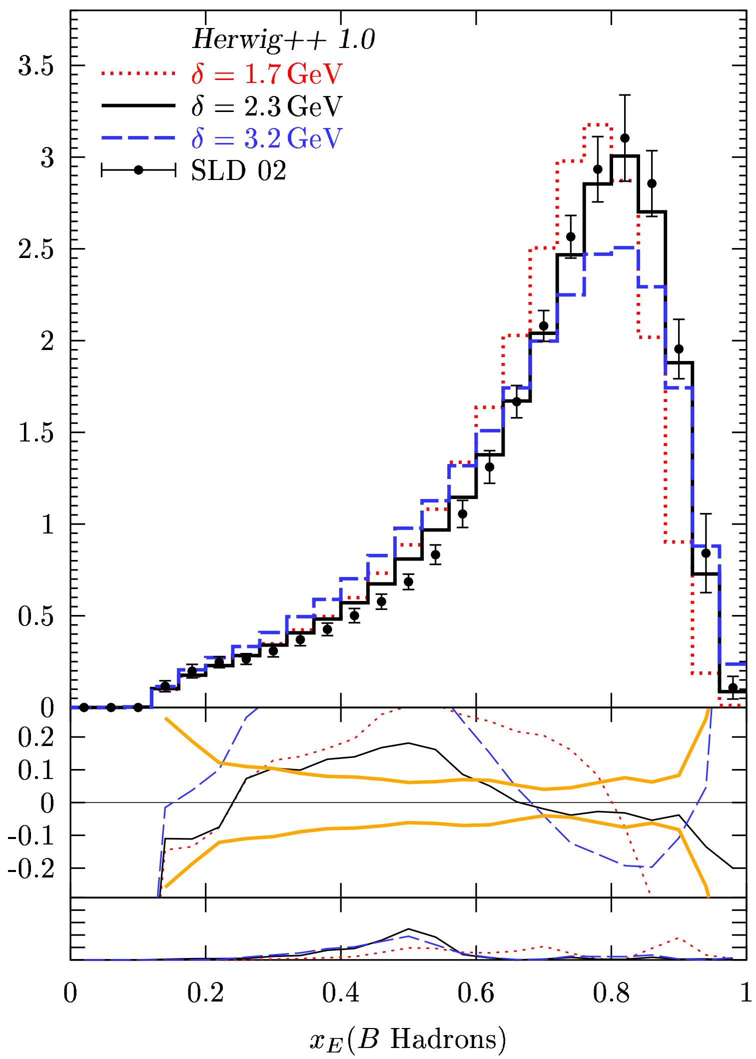


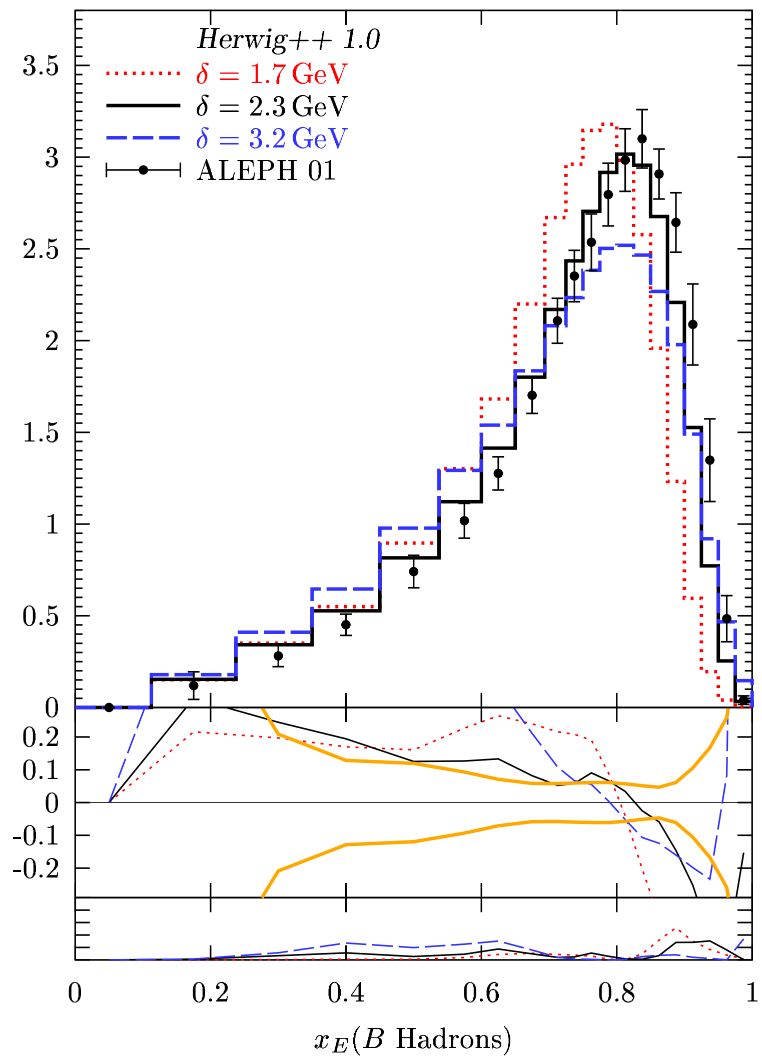


Scaled momentum (all, uds, b)



B-fragmentation function

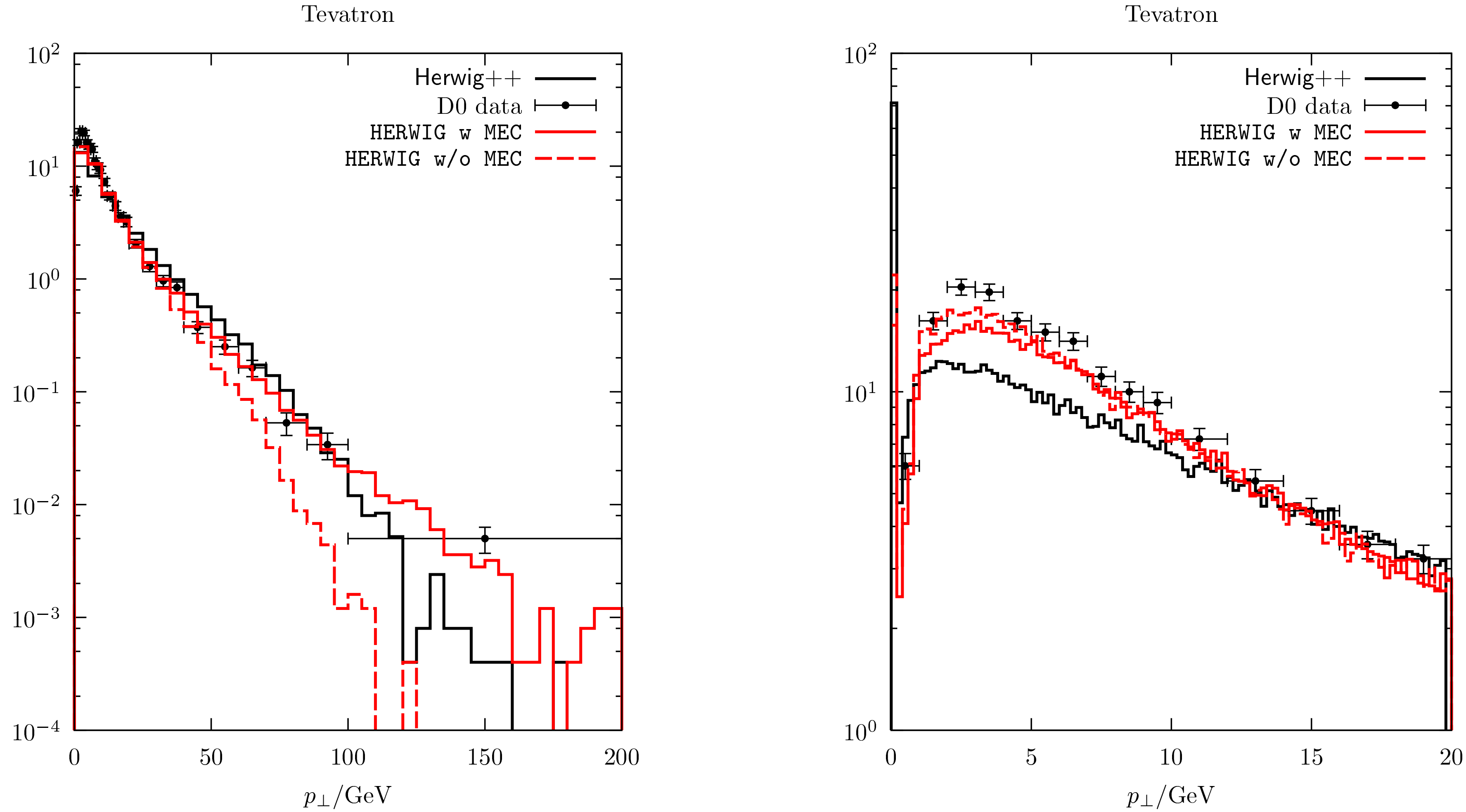




Only parton shower parameters varied!

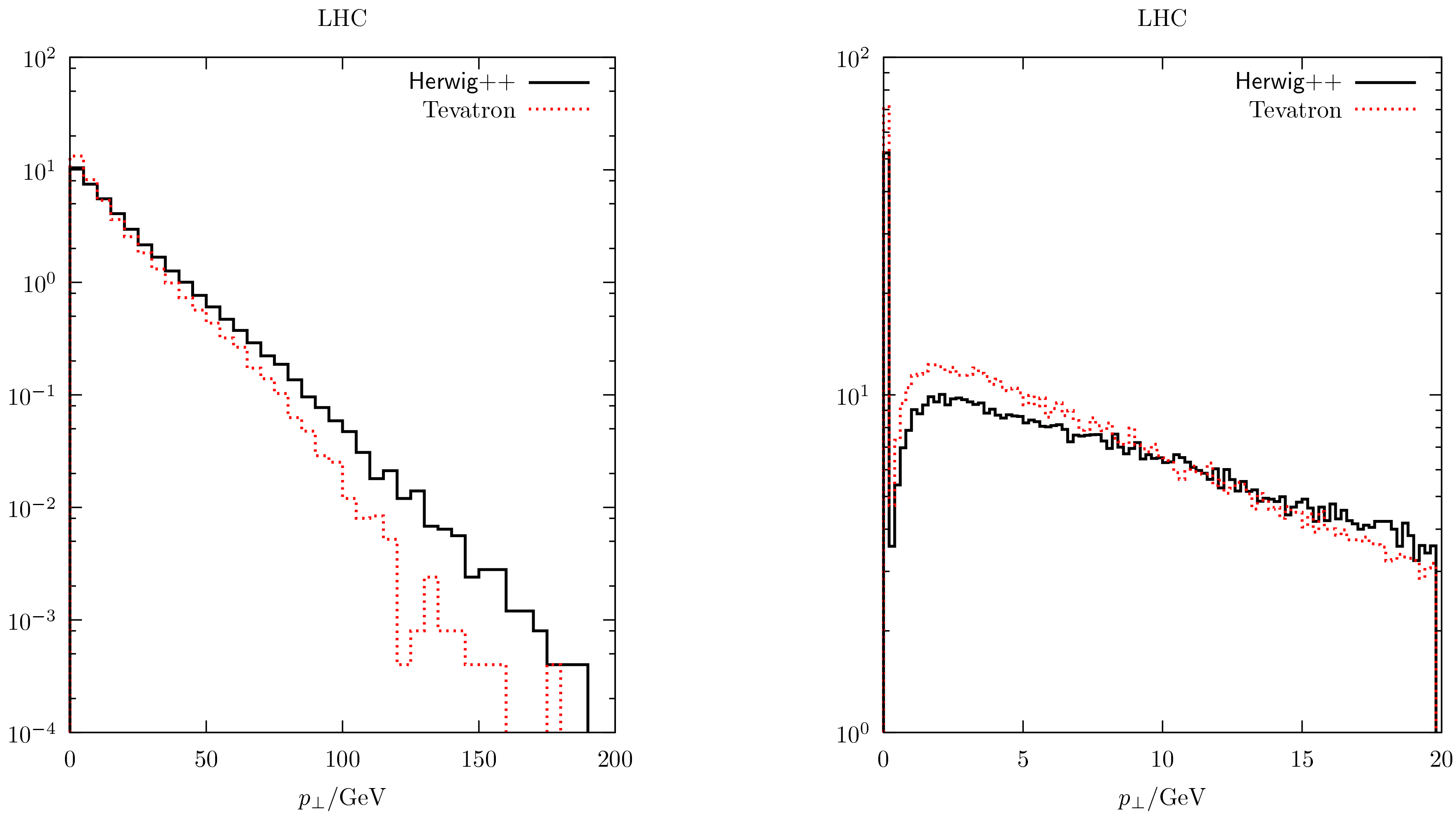
Drell-Yan Process at Tevatron

- p_t of Z^0 in $p\bar{p} \to Z^0 X$ (no intrinsic p_t, no matrix element correction)



Drell-Yan Process at LHC

- p_t of Z^0 in $pp \to Z^0 X$ (no intrinsic p_t, no matrix element correction)



Status of Herwig++

S. Gieseke, A. Ribon, P. Richardson, M.H. Seymour, P. Stephens, B.R. Webber
(Cambridge, CERN, Durham, Manchester)

http://www.hep.phy.cam.ac.uk/theory/Herwig++

Hard Matrix Elements

- ✓ Only simple $2 \to 2$ ME so far.
- ✓ Hard and soft ME corrections for $e^+e^- \to q\bar{q}g$.
- ✓ We have a working interface to AMEGIC++. For e^+e^- this will do the job for up to 6 jets.
- ★ CKKW ME+PS matching algorithm will be implemented.
- ★ More processes straightforward.
- ★ Users can easily and safely include their own matrix elements.

Parton Shower

- ✓ New parton shower developed.
- ✓ New evolution variables for better treatment of heavy quarks and smooth coverage of phase space.
- ✓ Extension to spacelike shower for $\bar{p}p$ & pp now working.
- ★ Multiscale shower designed for treatment of unstable particles (no physics implementation yet).

Status of Herwig++ (ctnd')

Hadronization

- ✓ Cluster hadronization is designed and implemented completely.
- ✓ Improved cluster decays implemented and tested.
- ✓ Works very well, further thorough tests ongoing.
- ★ Lund string fragmentation model implemented in Pythia7 will work together with Herwig++.

Decays

- ✓ Fortran HERWIG decays are reproduced with class `Hw64Decayer` using the same ME's as before.
- ✓ `DecayerAMEGIC` gets final states for decays (eg. t decay, SUSY in future) directly from `AMEGIC++`
- ★ *NEW* web-based C++ particle data and decay matrix element database under test (P Richardson).
- ★ More to come (`EvtGen`,. . .)?

What's next for Herwig++?

Near Future. . .

- Initial state shower:
  - Complete testing with Drell-Yan processes.
- Refine e^+e^-:
  - Full CKKW ME+PS matching.
  - Precision tune to LEP data using Richardson particle database.
- With IS and FS showers running:
  - Jet, Higgs and heavy quark production in pp collisions.
  - Cross-check with Tevatron data and finally make predictions for the LHC.
- Underlying event:
  - UA5 model ready in C++
  - Multiple-interaction model (JIMMY) being developed.
- Hadronic decays: spin correlations.
- New ideas: NLO, multiscale, SUSY . . .

Schedule?

- Limited range of pp processes ready for Tevatron next year.
- Full generator ready for LHC!