Froissart bound and chiral limit in QCD

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- Relevance of pions for high energy cross sections addressed before:
  Anselm/Gribov, Bjorken, Tow/Tan, Khoze/Martin/Ryskin

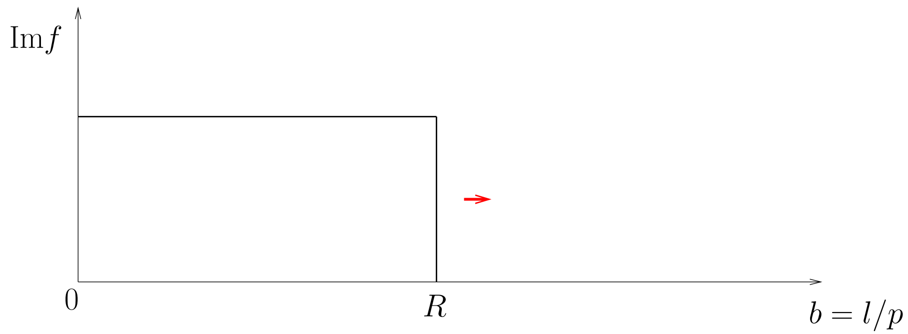
Froissart-Martin bound

Main ingredients:

- Unitarity:

$$|f_l| < 1.$$

Means cross sections can grow infinitely only by expanding in l:



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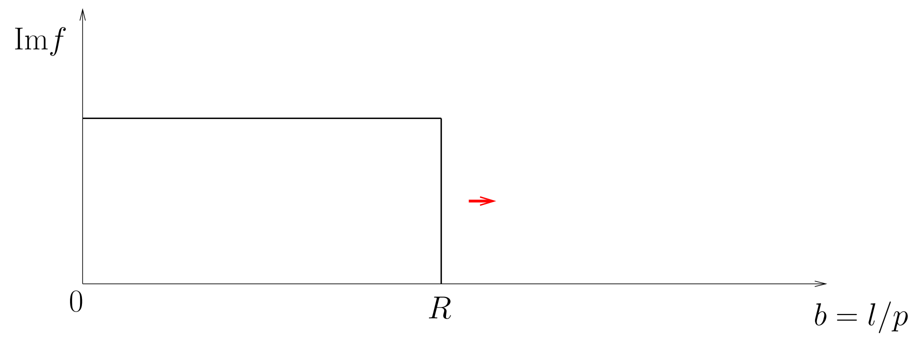
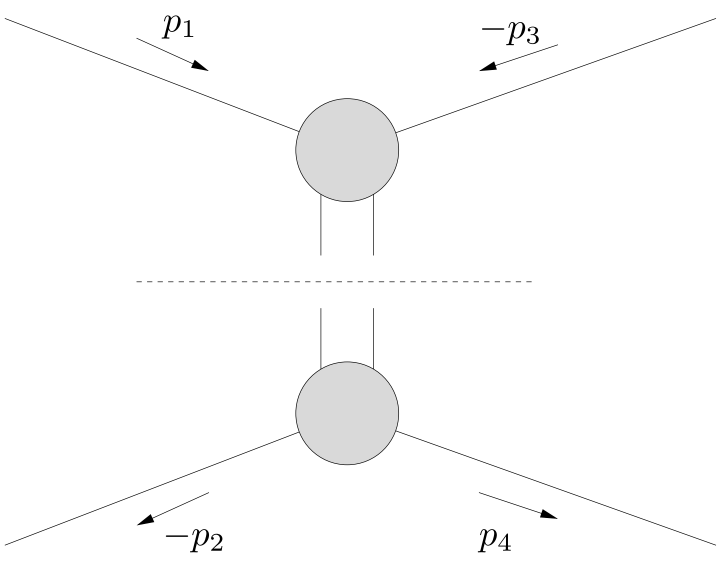


- Analyticity:

$$F(t) = \int d\mu^2 \frac{\rho(\mu)}{\mu^2 - t} + (u\text{-channel}).$$

$$t = (p_1 - p_3)^2$$

$$\rho(\mu) \sim \mathcal{M}(1\bar{3} \to \mu) \cdot \mathcal{M}^*(\bar{2}4 \to \mu).$$



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Then use

$$f(b) = \frac{1}{s}\int_0^\infty d\mu^2 K_0(\mu b)\rho(\mu).$$

If $\rho(\mu) \equiv 0$ for $\mu < \mu_0 = 2m_\pi$, then for large b and s

$$f(b) < c(b) e^{-\mu_0 b} s^{N-1}$$

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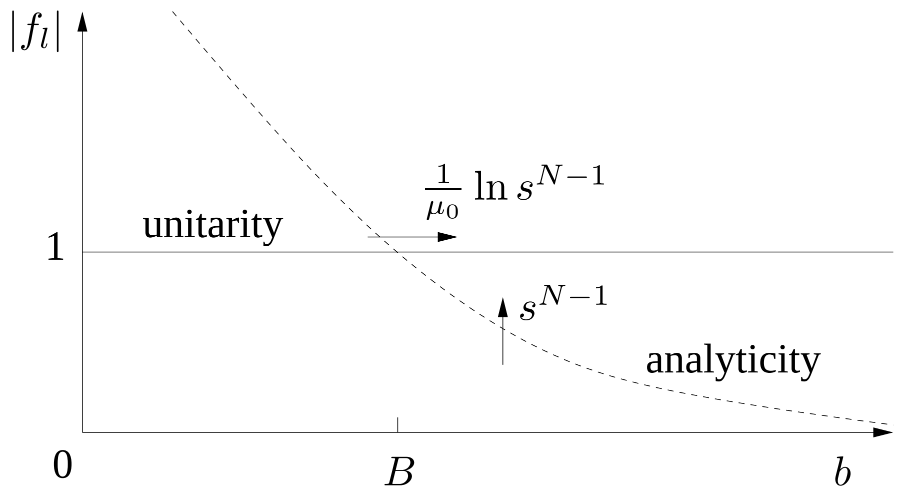


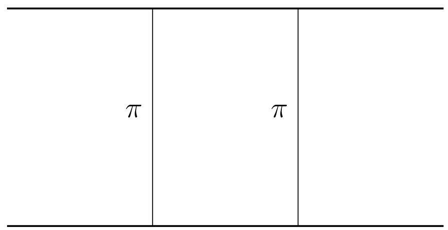
Thus

$$\sigma^{\text{tot}} < \frac{4\pi(N-1)^2}{\mu_0^2}\ln^2 s$$

$$\mu_0 = 2m_\pi, \qquad N = 2$$

ρ near two-pion threshold

Can $\rho(\mu_0)$ grow as s^N?



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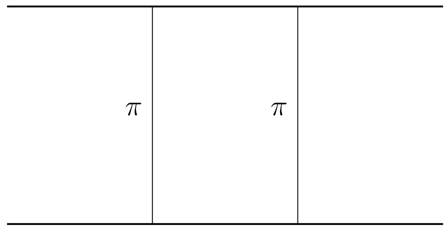

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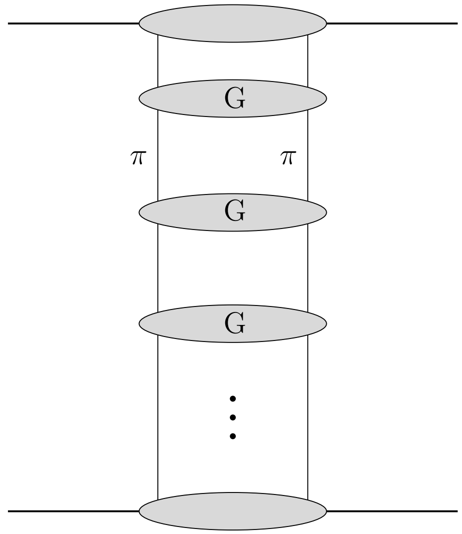

$\longrightarrow \quad \rho \sim s^{-1} s^{G}$

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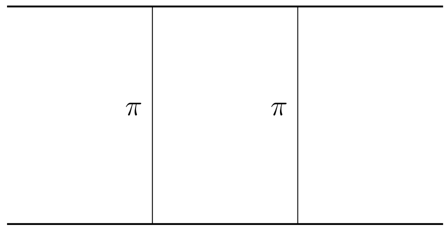

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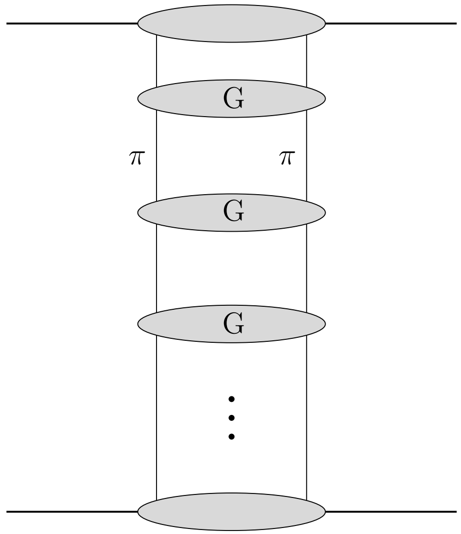

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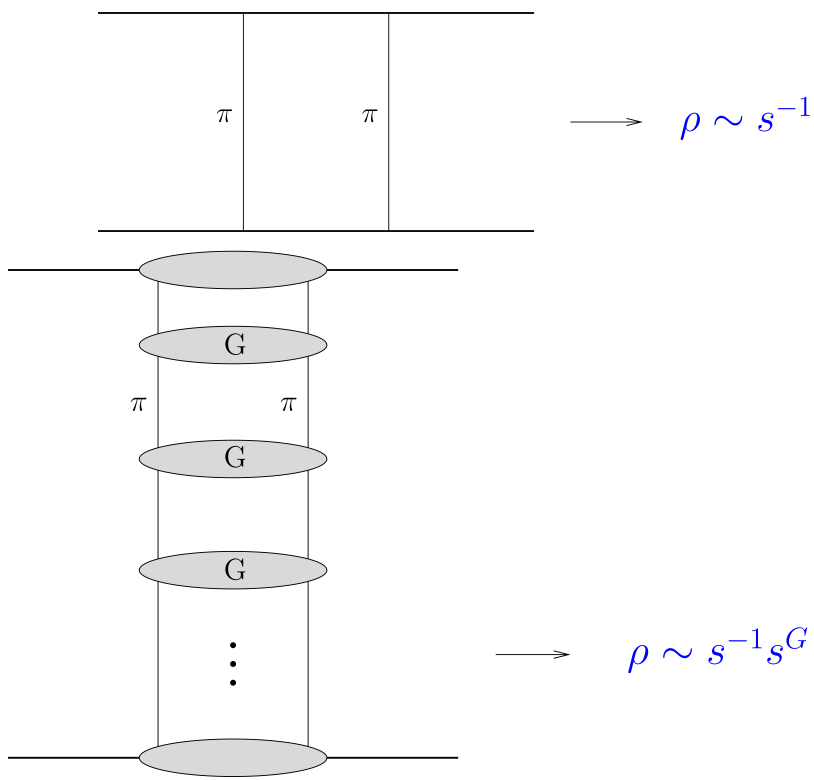

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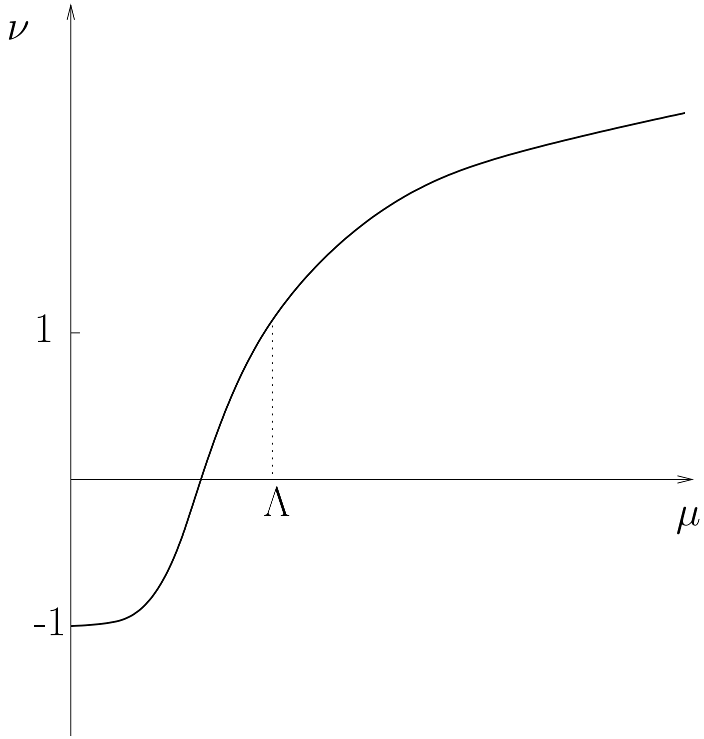
$\rho(\mu)$ begins to grow when μ is larger, and outside of the chiral perturbation theory regime: $\mu > \Lambda_\chi \sim 1$ GeV.

What is the possible growth of σ^{tot}?

Assume that ρ grow as

$$|\rho(\mu)| \sim s^{\nu(\mu)}$$



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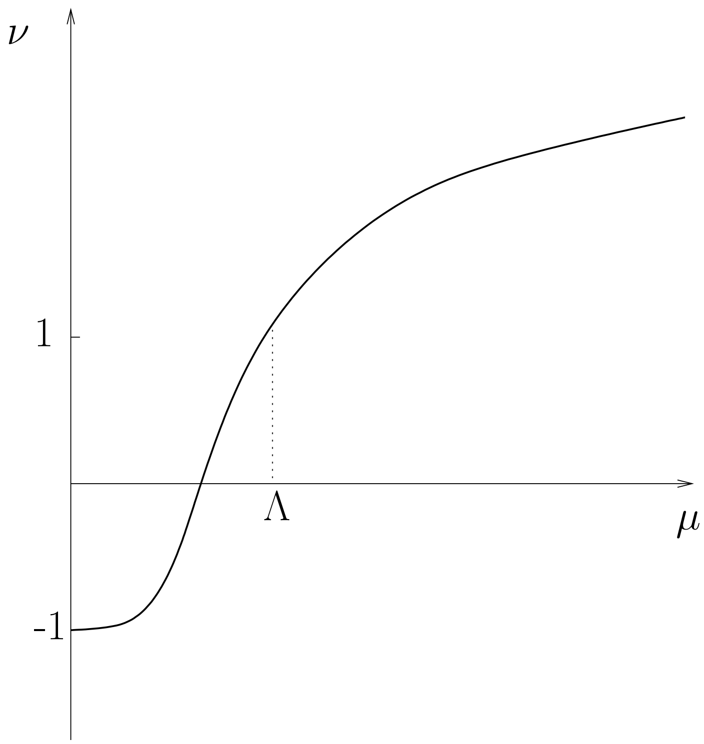

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For large b and s

$$f(b) = \frac{1}{s} \int_0^\infty d\mu^2 K_0(\mu b) \rho(\mu) \sim e^{-\tilde{\mu} b} \, s^{\tilde{\nu}-1}$$

where $\tilde{\mu}$ is either μ_0, or given by the peak in the integrand:

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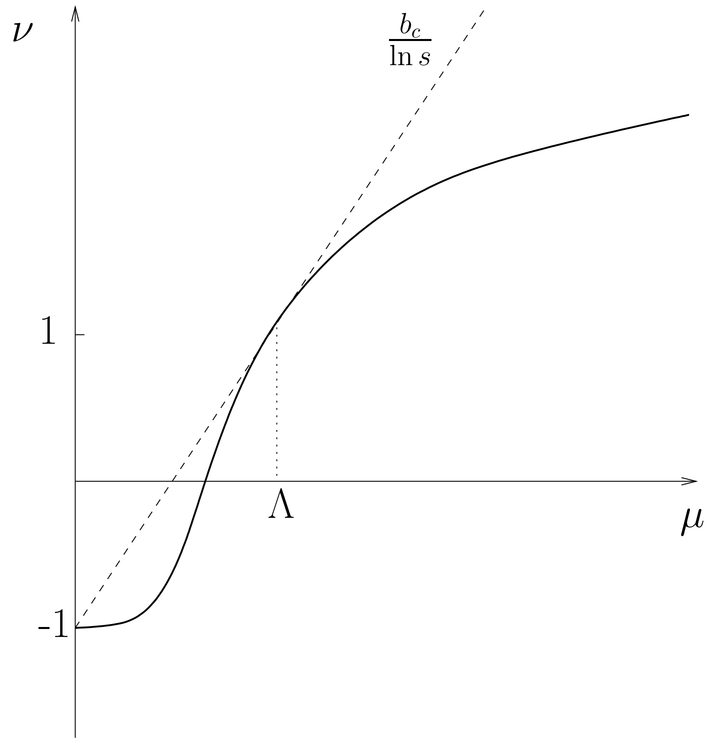

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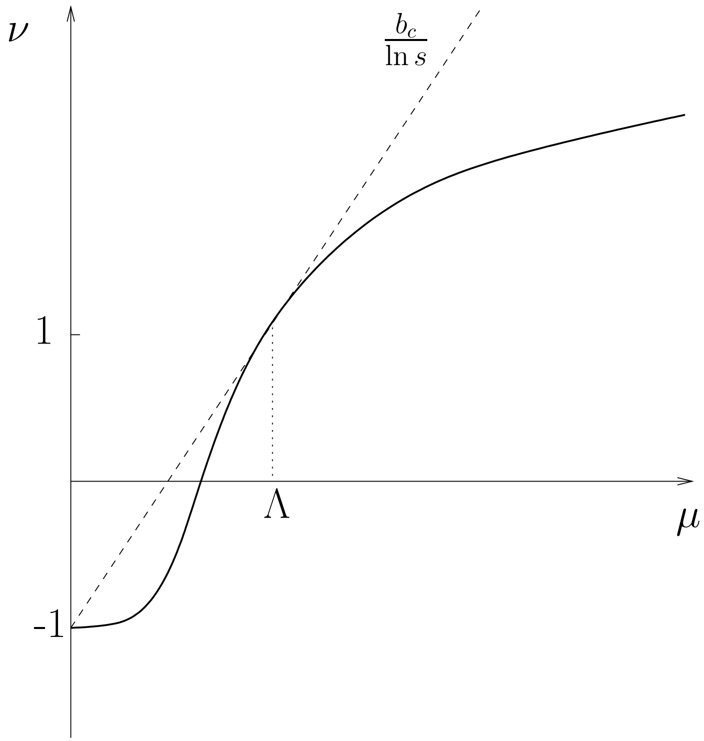

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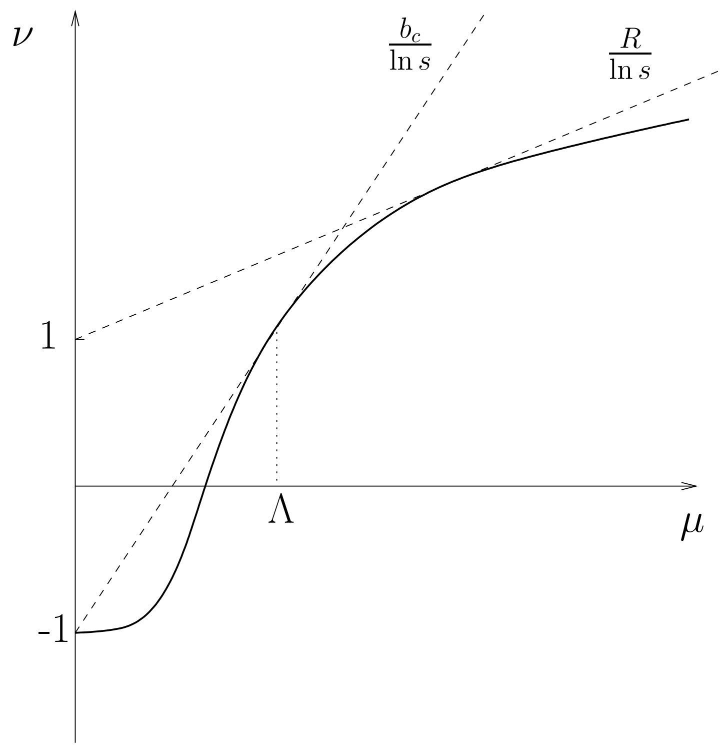

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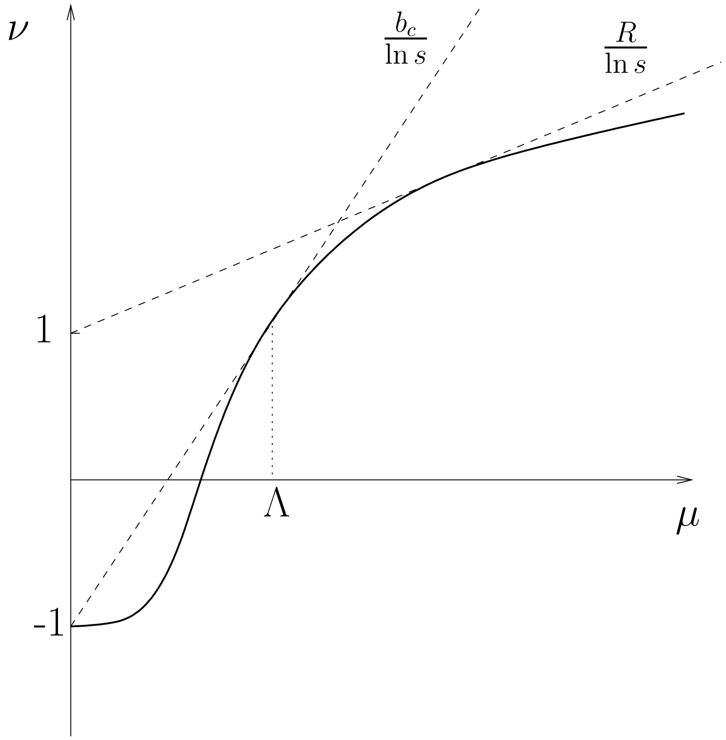
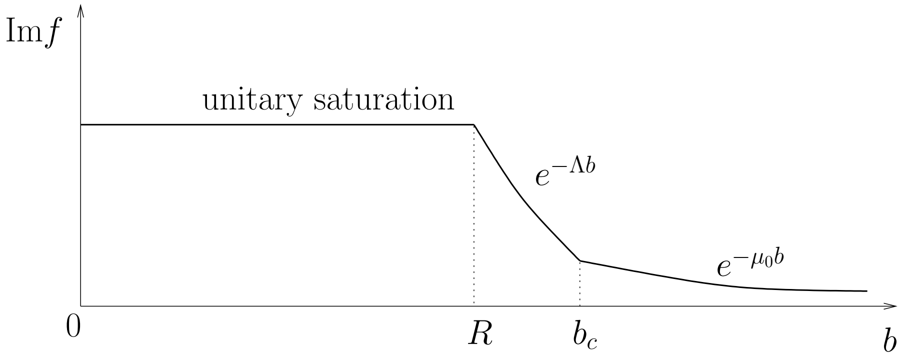
Total cross section (contd.)





$$R = \max\left(\frac{\nu - 1}{\mu}\right)\ln s \sim \frac{\Lambda}{\ln s}$$

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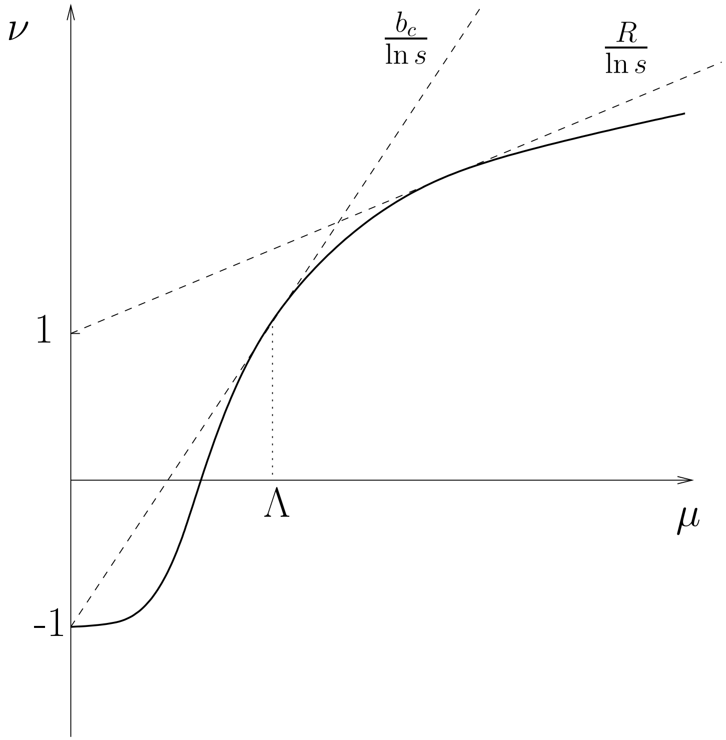

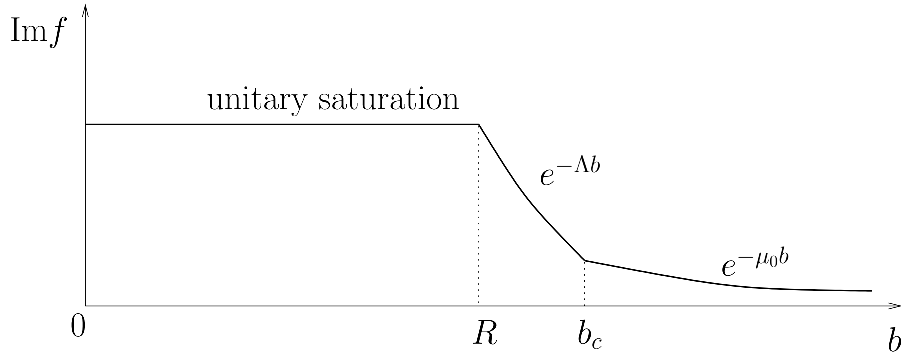

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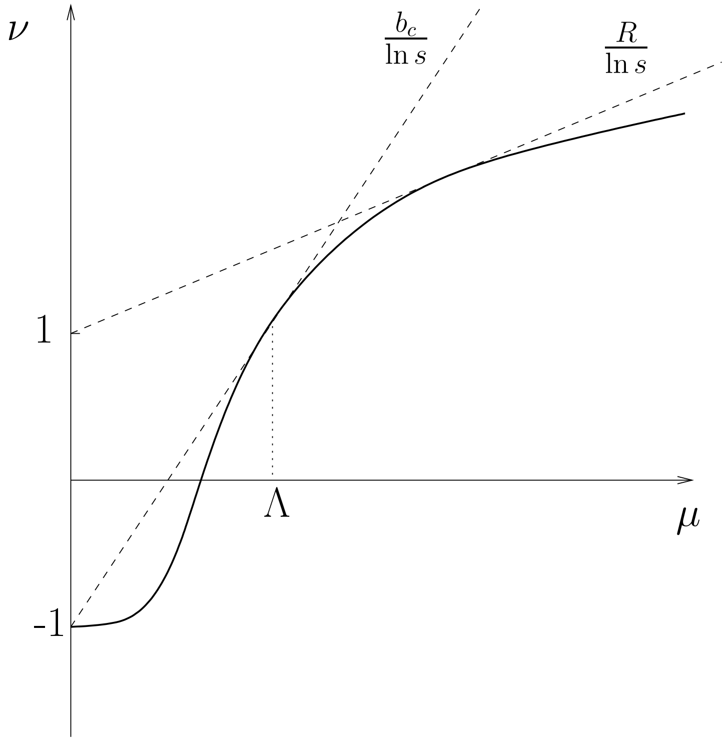

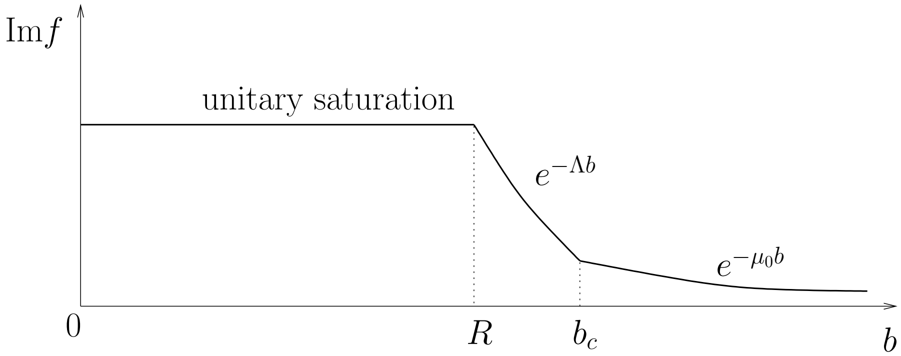

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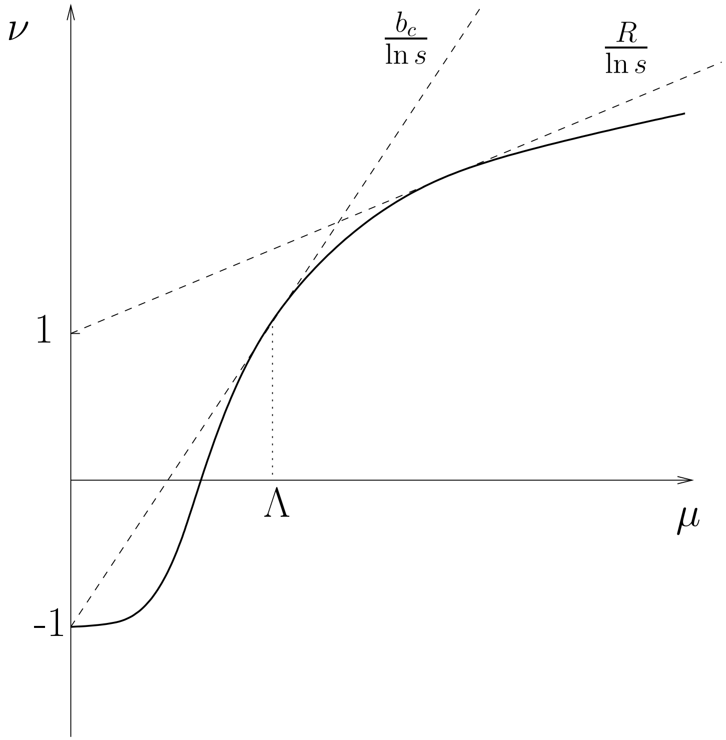

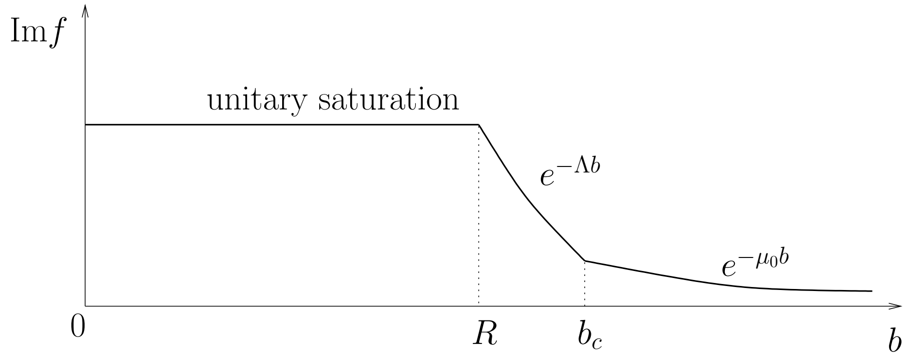

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Experimentally: $C \sim 1$ for $\Lambda \sim 1$ GeV.

Can one see the crossover of slope in $f(b)$? Perhaps pp2pp can tell. (Guryn's talk)