CKM MATRIX ELEMENT MAGNITUDES

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The present status of experimental results for the magnitudes of Cabibbo-Kobayashi-Maskawa matrix elements is reviewed and used for a unitarity test. The matrix is found to be unitary within ±1.8 standard deviations. The matrix violates CP-symmetry and the size of its CP-violation, as derived from only magnitude measurements and unitarity, is in perfect agreement with the observed CP-violations in K and B meson decays.

1. Introduction

The Standard Model Lagrangian for leptons ν_α, ℓ_α and quarks $\mathrm{u}_\alpha, \mathrm{d}_\alpha$ contains, ignoring right-handed neutrinos, three arbitrary matrices $C^{(\ell)}_{\alpha\beta}$, $C^{(\mathrm{u})}_{\alpha\beta}$, and $C^{(\mathrm{d})}_{\alpha\beta}$ for the coupling of the Higgs doublet to right-handed singlets $\ell_{\mathrm{R}\alpha}$, $\mathrm{u}_{\mathrm{R}\alpha}$, $\mathrm{d}_{\mathrm{R}\alpha}$ and left-handed doublets $(\nu_{\mathrm{L}\beta}, \ell_{\mathrm{L}\beta})$, $(\mathrm{u}_{\mathrm{L}\beta}, \mathrm{d}_{\mathrm{L}\beta})$. This Yukawa structure does not allow simultaneous diagonalization of the two matrices $C^{(\mathrm{u})}$ and $C^{(\mathrm{d})}$ by "rotations" [a] in three-dimensional family space ($\alpha, \beta = 1, 2, 3$), since the doublet partners $\mathrm{u}_{\mathrm{L}\alpha}$ and $\mathrm{d}_{\mathrm{L}\alpha}$ have been coupled by Glashow and, therefore, cannot be rotated separately. Diagonalization of $C^{(\mathrm{u})}$ leads to mass eigenstates $\mathrm{u, c, t}$ with Glashow partners $\mathrm{d', s', b'}$, and diagonalization of $C^{(\mathrm{d})}$ to $\mathrm{d, s, b, u', c', t'}$. The difference V between the two diagonalizing rotations,

$$\begin{pmatrix} \begin{pmatrix} \mathrm{u_L} \\ \mathrm{d'_L} \end{pmatrix} \\ \begin{pmatrix} \mathrm{c_L} \\ \mathrm{s'_L} \end{pmatrix} \\ \begin{pmatrix} \mathrm{t_L} \\ \mathrm{b'_L} \end{pmatrix} \end{pmatrix} = V \begin{pmatrix} \begin{pmatrix} \mathrm{u'_L} \\ \mathrm{d_L} \end{pmatrix} \\ \begin{pmatrix} \mathrm{c'_L} \\ \mathrm{s_L} \end{pmatrix} \\ \begin{pmatrix} \mathrm{t'_L} \\ \mathrm{b_L} \end{pmatrix} \end{pmatrix} , \qquad (1)$$

is the CKM transformation with $VV^+ = 1$. It was introduced in 1973 by Kobayashi and Maskawa,[1] and in recognition of Cabibbo's[2] early work it has been called V_{CKM} since 1987,[3]

$$V = V_{\mathrm{CKM}} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} . \qquad (2)$$

Since the Standard Model allows arbitrary matrices $C^{(\mathrm{u})}$ and $C^{(\mathrm{d})}$, V is allowed to be complex, leading to CP-violation in the standard weak interaction.

The complex matrix elements V_{ij} are not observables because of unobservable phases in the quark fields. Transformations $\mathrm{u}_\alpha \to \mathrm{u}_\alpha \cdot \mathrm{e}^{\mathrm{i}\phi_\alpha}$ and $\mathrm{d}_\beta \to \mathrm{d}_\beta \cdot \mathrm{e}^{\mathrm{i}\phi_\beta}$ lead to $V_{\alpha\beta} \to V_{\alpha\beta} \cdot \mathrm{e}^{\mathrm{i}(\phi_\alpha - \phi_\beta)}$ with arbitrary ϕ_α and ϕ_β. The observables, i.e. invariants under arbitrary phase transformations, are:

- doublets $V_{\mathrm{ij}} V^*_{\mathrm{ij}} = |V_{\mathrm{ij}}|^2$, content of this talk,
- quartets $V_{\mathrm{ij}} V_{\mathrm{kl}} V^*_{\mathrm{il}} V^*_{\mathrm{kj}}$, where modulus and phase are both observable,
- sextets $V_{\mathrm{ij}} V_{\mathrm{kl}} V_{\mathrm{mn}} V^*_{\mathrm{il}} V^*_{\mathrm{kn}} V^*_{\mathrm{mj}}$, and higher n-tets constructed in an analogous way.

The CKM matrix has four observable parameters and an infinite number of choices for these four. One choice is the invariants $|V_{\mathrm{us}}|$, $|V_{\mathrm{cb}}|$, $|V_{\mathrm{ub}}|$, and the phase γ of the quartet $V^*_{\mathrm{ub}} V_{\mathrm{ud}} V^*_{\mathrm{cd}} V_{\mathrm{cb}}$. Wolfenstein has named them[4]

$$|V_{\mathrm{us}}| = \lambda \ , \quad |V_{\mathrm{cb}}| = A \cdot \lambda^2 \ ,$$

$$|V_{\mathrm{ub}}| \cdot \cos\gamma = A \cdot \lambda^3 \cdot \rho \ , \quad |V_{\mathrm{ub}}| \cdot \sin\gamma = A \cdot \lambda^3 \cdot \eta \ , \quad (3)$$

and in addition he has chosen a phase convention where V_{ud}, V_{us}, and V_{cb} are real and positive. In this convention, we have

$$V \approx \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - \mathrm{i}\,\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \overline{\rho} - \mathrm{i}\,\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} \qquad (4)$$

with a precision matched to present experiments and

$$\overline{\rho} = (1 - \lambda^2/2)\rho \ , \quad \overline{\eta} = (1 - \lambda^2/2)\eta \, . \qquad (5)$$

Within the Standard Model, the values of the four parameters A, λ, ρ, and η are arbitrary; they have to be determined by experiment. Measurements of the nine magnitudes of V_{ij} test the validity of $VV^+ = 1$ and determine the four parameter values. Discussions of these measurements are the content of this talk.

[a] unitary transformations

2. $|V_{\rm ud}|$

The magnitude of $V_{\rm ud}$ is determined from super-allowed nuclear β^+ decays, from the β^- decay of polarized neutrons, and from the β^+ decay of π^+ mesons. Super-allowed β^+ decays are nuclear $0^+ \to 0^+$, i.e. pure vector, transitions within the same isospin multiplet. A recent review[5] quotes an average of

$$\mathcal{F}t = (3072.2 \pm 0.9 \pm 1.1)\ {\rm s} \qquad (6)$$

for nine different decays, where $\mathcal{F}t$ is the product of the Fermi function f, the half-life t, and correction factors for internal bremsstrahlung and isospin symmetry breaking. The first error is experimental, the second is an estimate for the correction-factor error. The magnitude of $V_{\rm ud}$ is obtained from

$$|V_{\rm ud}|^2 = \frac{2\pi^3 \ln 2}{2 m_{\rm e} G_{\rm F}^2 (1+\Delta_{RV})\mathcal{F}t}\ , \qquad (7)$$

where $m_{\rm e}$ is the electron mass, $G_{\rm F}$ is Fermi's decay constant taken from muon decay, and $\Delta_{RV} = (2.40 \pm 0.08)\%$ is the electroweak correction. This results in

$$|V_{\rm ud}|_{\rm nuclear} = 0.9740 \pm 0.0005\ , \qquad (8)$$

where I have combined the experimental error of ± 0.0001, the Δ_{RV} error of ± 0.0004, and the $\mathcal{F}/f$ error of ± 0.0003 quadratically.

Neutron β decays are mixed V and A transitions with two couplings $G_{\rm V}$ and $G_{\rm A}$. Conservation of the vector current leads to $G_{\rm V} = G_{\rm F}|V_{\rm ud}|$ with high precision. Since the axial vector current is only partially conserved, $G_{\rm A}/G_{\rm V} = \lambda$ has to be determined experimentally. This is achieved in decays of polarized neutrons, where the angular distribution of decay electrons with respect to the neutron spin direction is given by

$$\frac{{\rm d}N}{{\rm d}\cos\theta} = 1 - \frac{v_{\rm e}}{c} \cdot P \cdot \frac{2\lambda(\lambda+1)}{1+3\lambda^2} \cdot \cos\theta\ , \qquad (9)$$

with the electron velocity $v_{\rm e}$ and the polarization P. The most recent experiment is PERKEO-II[6] at ILL Grenoble. Cold neutrons of temperature 25 K in a beam with transverse polarization $P = (98.9 \pm 0.3)\%$ emit electrons into a 4π detector which separately measures the energy spectra of electrons with $\theta < \pi/2$ and $\theta > \pi/2$. The observed asymmetry results in

$$\lambda = G_{\rm A}/G_{\rm V} = -1.2739 \pm 0.0019\ . \qquad (10)$$

Earlier experiments result in higher values of λ; the quoted errors of all experiments[7] give $\chi^2 = 15.5$ for $N(dof) = 4$. Therefore, I prefer to use only the result in Eq. (10), not only because the experiment is the most recent one and quotes the smallest error, but also because it has the highest polarization and the smallest experimental corrections from the raw measured to the final extracted angular asymmetry. Using the mean life[8] of the neutron $\tau_{\rm n} = (885.7 \pm 0.8)$ s and

$$\frac{1}{\tau_{\rm n}} = \frac{m_{\rm e}^5 G_{\rm F}^2 (1+3\lambda^2)|V_{\rm ud}|^2}{2\pi^3} \cdot f \cdot (1+\delta_{\rm R})(1+\Delta_{\rm RV})\ , \qquad (11)$$

where $\delta_{\rm R}$ is a QED correction and $\Delta_{\rm RV}$ the same electroweak correction as in Eq. (8), leads to[9]

$$|V_{\rm ud}|_{\rm neutron} = 0.9717 \pm 0.0013\ . \qquad (12)$$

Beta decays of π^+ mesons are observed in the recent experiment PIBETA[10] at PSI. The rare decays $\pi^+ \to \pi^0 {\rm e}^+ \nu$ of stopped π^+ are detected by reconstructing $\pi^0 \to \gamma\gamma$ decays in a CsI ball and are normalized by decays $\pi^+ \to {\rm e}^+\nu$. The preliminary result is a $\pi^0 {\rm e}^+ \nu$ branching fraction of $(1.044 \pm 0.007 \pm 0.009) \times 10^{-8}$, leading to

$$|V_{\rm ud}|_{\rm pion} = 0.9765 \pm 0.0056\ . \qquad (13)$$

My combination of the three results in Eqs. (8), (12), and (13) gives

$$|V_{\rm ud}| = 0.9737 \pm 0.0007\ , \qquad (14)$$

where the error includes a scale factor of 1.3. Prospects for improvement: PIBETA is not expected to reach a competitive precision, the final error may go down to ± 0.0030. The precision of the nuclear result is dominated by radiative corrections. Improvements in the near future are only expected from new neutron experiments. Underway are the experiments PERKEO-III and "NEW PERKEO", advanced plans exist also at LANL, SNS, and Gatchina/PSI.

3. $|V_{\rm us}|$

Determinations of $V_{\rm us}$ exist from old and new $K_{\ell 3}$ decays, hyperon β decays, and τ decays. $K_{\ell 3}$ decays $K^+ \to \pi^0 \ell^+ \nu$ and $K^0 \to \pi^- \ell^+ \nu$ are $0^- \to 0^-$, i.e. pure vector transitions with $G_{\rm V} = G_{\rm F} \cdot |V_{\rm us}|$. For $P = K^+$ and $P = K^0$ separately, their rates Γ are

Table 1. K lifetimes and $K_{\ell 3}$ decays with their branching fractions $\mathcal{B}$ and λ values.

Mode	$\mathcal{B}(\%)$	$10^3\,\lambda_+$	$10^3\,\lambda_0$
K^+_{e3}	4.87 ± 0.06	27.8 ± 1.9	
K^0_{e3}	38.79 ± 0.27	29.1 ± 1.8	
$K^+_{\mu 3}$	3.27 ± 0.06	33 ± 10	4 ± 9
$K^0_{\mu 3}$	27.18 ± 0.25	33 ± 5	27 ± 6
$\tau(K^+) = (1.1384 \pm 0.0024) \times 10^{-8}$ s			
$\tau(K^0_L) = (5.17 \pm 0.04) \times 10^{-8}$ s			

described by two form factors $f_+^P(t)$ and $f_0^P(t)$ in the very good approximation

$$f^P_{+,0}(t) = f^P_+(0)\left(1 + \lambda^P_{+,0} \cdot \frac{t}{m_\pi^2}\right) , \quad (15)$$

with $t = (p_P - p_\pi)^2$, resulting in

$$\Gamma(P \to \pi \ell \nu) = \frac{G_F^2 |V_{us}|^2 m_P^5}{192\pi^3} \cdot C_P^2 \cdot |f_+^P(0)|^2 \cdot I(\lambda_+^P, \lambda_0^P) , \quad (16)$$

where the Clebsch-Gordon coefficient C_P is $1/\sqrt{2}$ for both K^+ and K^0_L. Rates (branching fractions/lifetimes) and λ values are determined experimentally, see Table 1. The form-factor values at $t = 0$ have to be obtained theoretically and have essentially remained unchanged[11] since 1984,

$$f_+^{K^0}(0) = 0.961 \ , \ f_+^{K^+}(0) = 0.982 \ . \quad (17)$$

Inclusion of recent radiative corrections by Cirigliano[12] leads to

$$|V_{us}|_{\mathrm{old}\ K\ell 3} = 0.2201 \pm 0.0016 \pm 0.0018. \quad (18)$$

This year heralded $K_{\ell 3}$ results from two new experiments. BNL-E865[13], whose primary aim is searching for rare decays $K^+ \to \pi^+ \mu^+ e^-$, recorded $K^+ \to \pi^0 e^+ \nu$ decays with $\pi^0 \to e^+ e^- \gamma$ in a dedicated run of one week in 1998. Normalizing to $K^+ \to \pi^+ \pi^0$ and $K^+ \to \pi^+ \pi^0 \pi^0$, they obtain the final branching-fraction result

$$\mathcal{B}(K^+ \to \pi^0 e^+ \nu) = (5.13 \pm 0.02 \pm 0.09 \pm 0.04)\% , \quad (19)$$

which is $2.2\,\sigma$ higher than the old K^+_{e3} average shown in Table 1. Their λ_+ value is in perfect agreement with that in the Table. Using the correction factors of Cirigliano,[12] the result in Eq. (19) leads to

$$|V_{us}|_{E865} = 0.2285 \pm 0.0023 \pm 0.0019 , \quad (20)$$

which is, using only the statistical errors, $2.2\,\sigma$ higher than the old K^+_{e3} value and $3.0\,\sigma$ higher than the old $K^+_{\ell 3}, K^0_{\ell 3}$ average given in Eq. (18). The KLOE experiment at the DAΦNE e^+e^- storage ring in Frascati has presented preliminary $K^0_{\ell 3}$ results, with 78 pb^{-1}, using a fraction of their data. At the time of this Symposium, there are no numbers available for $f_+^{K^0}(0) \cdot |V_{us}|$, but the results shown graphically[14] agree well with the old result in Eq. (18) and are in poor agreement with BNL-E865.

Hyperon decays have recently been revisited by Cabibbo *et al.*[15] using experimental data from $n \to p e \nu$, $\Lambda \to p e \nu$, $\Sigma^- \to n e \nu$, $\Xi^- \to \Lambda e \nu$, and $\Xi^0 \to \Sigma^+ e \nu$ including experimental values of the form-factor ratios $g_1(0)/f_1(0)$ for each mode separately. Their result is

$$|V_{us}|_{\mathrm{hyperons}} = 0.2250 \pm 0.0027 , \quad (21)$$

assuming $f_1(0) = 1.000$ without theoretical uncertainty.

Tau decays are sensitive to $|V_{us}/V_{ud}|$ in their branching ratio $\Gamma(\tau^- \to \bar{u} s \nu)/\Gamma(\tau^- \to \bar{u} d \nu)$. Using ALEPH data on $\Gamma(\tau \to K n \pi \nu)/\Gamma(\tau \to \mathrm{hadrons}\ \nu)$ and $m_S(2\ \mathrm{GeV}) = (105 \pm 20)$ MeV, Gamiz *et al.*[16] find

$$|V_{us}|_\tau = 0.2179 \pm 0.0044 \pm 0.0009 , \quad (22)$$

where the first error is experimental and the second from theory. Note that the first error dominates. Future measurements with a larger number of tau decays could give smaller errors on $|V_{us}|$ and, by determining moments of the hadron-mass spectrum, also an independent input value for the s quark mass m_s.

Final results from KLOE, also on $K^+_{\ell 3}$ rates, are expected soon. Also NA48 and KTeV could analyse these decay modes. For hyperons, more theoretical work on $f_1(0)$ would be welcome. For tau decays, BABAR and BELLE have recorded 10^8 $\tau\tau$ events and should look into their potential to get $\Gamma(\tau^- \to \overline{u} s \nu)$ and hadron-mass moments in this inclusive decay mode. My average from Eqs. (18), (20), and (22) is

$$|V_{us}| = 0.2210 \pm 0.0023 . \quad (23)$$

4. $|V_{cd}|$

There is no new information on this matrix element. Dimuon production by neutrinos and antineutrinos on nuclei has given[17]

$$|V_{\rm cd}| = 0.224 \pm 0.016 \ . \qquad (24)$$

New potentials will be opened by the experiment CLEO-c.

5. $|V_{\rm cs}|$

The magnitude of $V_{\rm cs}$ is obtained from the vector transitions $D^+ \to \overline{K}^0 \ell^+ \nu$ and $D^0 \to K^- \ell^+ \nu$, in complete analogy to the $K_{\ell 3}$ transitions in Eq. (16), and from decays of real W bosons. The first method gives,[18] using $f_+(0) = 0.7 \pm 0.1$,

$$|V_{\rm cs}| = 1.04 \pm 0.16 \ . \qquad (25)$$

The second method is much more precise. Since it requires results from the third quark family, it will be discussed later in Sec. 11.

6. Unitarity Check of the udsc Submatrix

Using the averages for $|V_{\rm ud}|$, $|V_{\rm us}|$, $|V_{\rm cd}|$, and $|V_{\rm cs}|$ in Eqs. (14), (23), (24), and (25), we may check if the mixing matrix of the first two quark families is unitary. The results are

$$\begin{aligned} |V_{\rm ud}|^2 + |V_{\rm us}|^2 &= 0.9969 \pm 0.0017 \ , \\ |V_{\rm cd}|^2 + |V_{\rm cs}|^2 &= 1.13 \pm 0.33 \ , \\ |V_{\rm ud}|^2 + |V_{\rm cd}|^2 &= 0.9983 \pm 0.0073 \ , \\ |V_{\rm cd}|^2 + |V_{\rm cs}|^2 &= 1.13 \pm 0.33 \ , \\ |V_{\rm ud}V_{\rm cd}| - |V_{\rm us}V_{\rm cs}| &= -0.012 \pm 0.039 \ , \\ |V_{\rm ud}V_{\rm us}| - |V_{\rm cd}V_{\rm cs}| &= -0.018 \pm 0.040 \ . \end{aligned} \qquad (26)$$

With the exception of only the first line, which fails by $-1.8\,\sigma$, all other checks fulfill unitarity with better than $\pm 0.5\,\sigma$. Within the present experimental precision, we could not predict the existence of a third family from the results discussed so far. A unitarity-constrained fit to the 2×2 matrix gives

$$\lambda_{\rm Wolfenstein} = 0.2235 \pm 0.0033 \ , \qquad (27)$$

where the error contains a scaling factor of 1.8.

7. $|V_{\rm cb}|$

Sources for the determination of $|V_{\rm cb}|$ are inclusive and exclusive semileptonic B meson decays. The inclusive rate $\Gamma(B \to X e \nu)$ is proportional to $|V_{\rm cb}|^2$. It is determined from measurements of the B meson lifetime and the branching fraction $\mathcal{B}(B \to X e \nu)$, where both are averages from the mixture of B^+ and B^0 in Υ(4S) decays. The lifetimes of B^+ and B^0 are very well known, recent best values of the Heavy Flavor Averaging Group[19] (HFAG) from measurements of ALEPH, BABAR, BELLE, CDF, DELPHI, L3, OPAL, and SLD are

$$\begin{aligned} \tau(B^0) &= (1.534 \pm 0.013) \text{ ps} \ , \\ \tau(B^+) &= (1.653 \pm 0.014) \text{ ps} \ ; \end{aligned} \qquad (28)$$

both have reached a precision of $\pm 0.8\%$.

Table 2. Branching fractions of inclusive decays $B \to X \ell \nu$ on the Υ(4S) resonance. These decays are tagged by $B \to X \ell \nu$ decays (ℓ), $B \to X e \nu$ (e), or full reconstruction (rec) of the second B.

Experiment	Year	Ref.	Tag	Result (%)
ARGUS	1993	20	ℓ	$9.75 \pm 0.50 \pm 0.39$
BABAR	2002	21	rec	$10.40 \pm 0.50 \pm 0.46$
BELLE	2002	22	ℓ	$10.90 \pm 0.12 \pm 0.49$
BABAR	2003	23	e	$10.91 \pm 0.18 \pm 0.29$
BELLE	2003	24	rec	$11.19 \pm 0.20 \pm 0.31$
CLEO	2003	25	ℓ	$10.91 \pm 0.08 \pm 0.30$
Average	07/03	19		10.90 ± 0.23

Table 2 lists all results of $\mathcal{B}(B \to X \ell \nu)$ measurements on the Υ(4S) resonance which is known to decay[26] with $(49.0 \pm 1.8)\%$ into $B^0\overline{B}^0$ and with $(51.0 \pm 1.8)\%$ into B^+B^- pairs. Tagging the semileptonic B decay with either fully reconstructed or semileptonically decaying second B mesons in the event has the advantage that the reconstructed flavor of the second B meson allows one to separate primary and secondary leptons in the signal B decay.[20] Figure 1 shows the spectrum of primary ($B \to X e \nu$) and secondary ($B \to X_c Y$, $X_c \to Z e \nu$) electrons from about 10^6 tagged B decays.[22] Extrapolation of the primary spectrum leads to $\mathcal{B}(B \to X \ell \nu)$. For a determination of $|V_{\rm cb}|$, we have to subtract the small fraction with which the b quark decays into $u \ell \nu$, $\mathcal{B}(B \to X_u \ell \nu) \approx (0.20 \pm 0.05)\%$. The error of this correction is negligible, therefore we have a $\pm 1.1\%$ contribution to the precision of $|V_{\rm cb}|$ from the branching fraction and a $\pm 0.4\%$ contribution from the B meson lifetime. The remaining error is from theory, but here the QCD approximation as an effec-

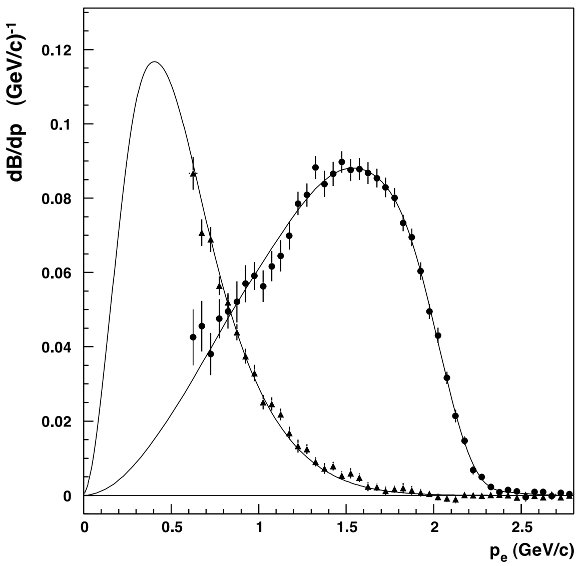

Figure 1. Primary (B $\to$ $Xe\nu$) and secondary (B $\to$ X_cY, $X_c \to Ze\nu$) electrons as measured by BELLE.[22]

tive theory with heavy quarks has brought considerable progress during the last few years.

Twenty years ago, we described the spectrum in Fig. 1 with the ACCMM model[27] which gave the partial rate as $\mathrm{d}\Gamma/\mathrm{d}p_\ell = |V_{\mathrm{cb}}|^2 \cdot f(m_\mathrm{b}, m_\mathrm{c}, p_\mathrm{F}, \alpha_\mathrm{s})$ and allowed $|V_{\mathrm{cb}}|$ to be determined with a precision in the order of 10%. The parameter p_F described the "Fermi motion" of the b quark in the B meson. Today, heavy quark effective QCD[28] with its tools Operator Product Expansion (OPE) and Heavy Quark Expansion (HQE) replaces p_F by a more strictly defined matrix element λ_1, uses α_s and quark masses m_b and m_c in a given renormalization scheme, and introduces a few more parameters like the matrix element λ_2 describing QCD magnetism which was absent in ACCMM.

HQE expresses the total rate $\Gamma(\mathrm{B} \to X_\mathrm{c}\ell\nu)$ and moments of the partial rate $\mathrm{d}^2\Gamma(\mathrm{B} \to X_\mathrm{u}\ell\nu)/\mathrm{d}m_X^2\mathrm{d}E_\ell$ as functions of the parameters $|V_{\mathrm{cb}}|$, m_b, m_c, λ_1, λ_2 and more. Since the dependence on these parameters is different for different moments, a sufficiently large number of observed moments determines all parameters, and a larger number allows to test the consistency of the description. Up to now, measurements have been presented for the "zeroth moments",

$$R(E_0) = \int_{E_0}^{E_{\max}} \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\mathrm{d}E_\ell \ , \qquad (29)$$

n-th lepton-energy moments with $n = 1, 2, 3$,

$$M_{En}(E_0) = \frac{1}{R(E_0)} \int_{E_0}^{E_{\max}} E_\ell^n \frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell}\mathrm{d}E_\ell \ , \qquad (30)$$

and n-th hadronic-mass moments with $n = 1, 2, 3$,

$$M_{mn}(E_0) = \frac{1}{R(E_0)} \int_{E_0}^{E_{\max}} \int_{m^2_{\min}}^{m^2_{\max}} m_X^n \frac{\mathrm{d}^2\Gamma}{\mathrm{d}m_X^2\mathrm{d}E_\ell}\mathrm{d}E_\ell\mathrm{d}m_X^2 . \qquad (31)$$

DELPHI[29] has measured four mass moments ($n = 1, 2, 4, 6$) and three energy moments ($n = 1, 2, 3$) for $E_0 = 0$. One set of parameters fits all observations very well, and in the kinetic renormalization scheme[30] they obtain

$$|V_{\mathrm{cb}}| = 0.0429 \times (1 \pm 0.012 \pm 0.019 \pm 0.010) \ , \qquad (32)$$

where the central value is rescaled with $\mathcal{B} = 10.9\%$, the first error originates from the lifetime and branching fraction $\mathcal{B}$, the second from the HQE parameter fit, and the third one is an estimate of the HQE theoretical uncertainty.

Because of the high B meson boost in Z^0 decays, DELPHI is able to determine the six moments in full phase space, i.e. with $E_0 = 0$. This is not possible in the Υ(4S) experiments CLEO and BABAR because of difficulties with lepton identification and background separation at low lepton energies. In 2001, CLEO presented two independent HQE analyses. Using $M_{m2}(1.5\,\mathrm{GeV})$ and the first photon-energy moment in b $\to$ sγ decays which is only sensitive to the HQE parameter m_b, they find[31]

$$|V_{\mathrm{cb}}| = 0.0414 \cdot (1 \pm 0.012 \pm 0.022 \pm 0.020) \ , \qquad (33)$$

where the three errors are defined as in Eq. (32) and where I also rescaled with $\mathcal{B} = 10.9\%$. Using $M_{E1}(1.5\,\mathrm{GeV})$ and $R(1.7\,\mathrm{GeV})$/R$(1.5\,\mathrm{GeV})$, CLEO finds[32]

$$|V_{\mathrm{cb}}| = 0.0418 \cdot (1 \pm 0.012 \pm 0.012 \pm 0.022) \ , \qquad (34)$$

with the same comment on errors and the central value. I am not aware of a combined fit with all four CLEO observations, but the extracted HQE parameters m_b and λ_1 in the two fits agree very well.

This summer, BABAR[33] presented E_0-dependent $n = 2$ hadronic-mass moments from a sample of 89 M $\mathrm{B\overline{B}}$ pairs in which one B meson is fully reconstructed in a non-leptonic mode. Requiring that the other B ("signal B") has a lepton (e or μ) with $E_\ell > 0.9$ GeV and good agreement between E_{miss}

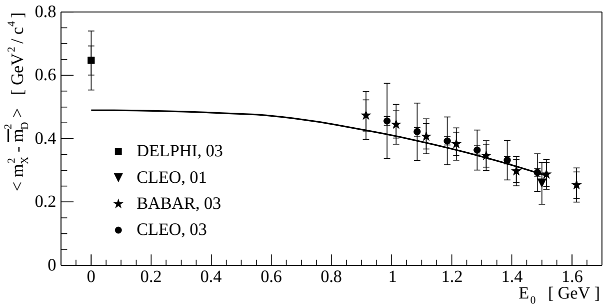

Figure 2. Measurements of hadronic-mass moments M_{m2}, $\overline{m}_D^2 = [(m_D + 3\,m_{D^*})/4]^2$ subtracted, in decays $B \to X_c \ell\nu$ from CLEO,[31,35] DELPHI,[29] and BABAR.[33] The curve is taken from a HQE fit[33,34] to the BABAR points.

and $|\vec{p}_{\rm miss}|$ as a neutrino signature, the invariant mass of all remaining tracks and neutrals in the event is taken as m_X in a $B \to X\ell\nu$ decay of the signal B. Detailed Monte Carlo studies show that this reconstructed m_X is on average 74% of the true m_X and that the relation between the mean reconstructed and the mean true hadronic mass is very well described by a linear function. Using this Monte Carlo m_X "calibration", BABAR finds the M_{m2} moments as shown in Fig. 2. HQE parameters and $|V_{\rm cb}|$ are fitted in three renormalization schemes. In the 1S scheme, using the HQE of Bauer *et al.*,[34] the result is

$$|V_{\rm cb}| = 0.0418 \cdot (1 \pm 0.025 \pm 0.017)\ , \qquad (35)$$

where the first error combines the errors from τ_B, $\mathcal{B}$, and the HQE parameter fit, and the second one estimates the theoretical error of the HQE ansatz for the fit. For this Symposium, CLEO[35] has produced a set of $M_{m2}(E_0)$ values from 10 M $B\overline{B}$ events. They are in good agreement with the BABAR values, see Fig. 2, but have not yet been used for a new determination of $|V_{\rm cb}|$. To conclude this discussion of inclusive $|V_{\rm cb}|$ determinations, there is surprisingly good agreement between the HQE description of all observed moments. This description results in a ±3.0% precision on $|V_{\rm cb}|$. My average of the results in Eqs. (32), (33), (34), and (35) is

$$|V_{\rm cb}|_{\rm incl} = 0.0421 \pm 0.0013\,. \qquad (36)$$

Determinations of $|V_{\rm cb}|$ with exclusive semileptonic B decays are, and have been for a long time, dominated by analyses of $B^0 \to D^*\ell\nu$ decays, since $B \to D\ell\nu$ is experimentally less background-free and theoretically less certain, knowledge on $B \to D^{**}\ell\nu$ and $B \to D^{(*)}\pi\ell\nu$ is very limited, and $B^+ \to D^*\ell\nu$ requires very good photon and π^0 reconstruction.

Table 3. Branching fractions $\mathcal{B}$ of the decay $B^0 \to D^{*-}\ell^+\nu$, slopes $\rho_{A_1^2}$ and values at $w = 1$ of the function $A_1(w) \cdot |V_{\rm cb}|$ for this decay from two new experiments presented at this Symposium. $A_1(1) = F(1)$.

	DELPHI[36]	BABAR[37]
$\mathcal{B}$ (%)	$5.90 \pm 0.22 \pm 0.48$	$4.68 \pm 0.03 \pm 0.29$
$\rho_{A_1^2}$	$1.32 \pm 0.15 \pm 0.33$	$1.23 \pm 0.02 \pm 0.28$
$10^3 \lvert V_{\rm cb}\rvert \cdot F(1)$	$39.2 \pm 1.8 \pm 2.2$	$34.0 \pm 0.2 \pm 1.3$

There are two new analyses presented to this Symposium, from DELPHI and BABAR. DELPHI[36] uses 3.4 M Z^0 decays with 1688 decays of neutral B mesons into $D^{*\pm}\ell\nu$, $\ell = e, \mu$, and BABAR[37] 86 M Υ(4S) with 55700 decays into the same modes. The dominant background in both analyses originates from $B \to D^{**}\ell\nu$ and $B \to D^{(*)}\pi\ell\nu$ events where one or more extra pions, in addition to the well-reconstructed $D^{*\pm}$, are present in a semileptonic decay. DELPHI uses hemisphere and vertex requirements for removing these extra pions, and BABAR uses the kinematic check if $\vec{p}(D^*)+\vec{p}(\ell)$ is compatible with $|\vec{p}(\nu)| = E(B) - E(D^*) - E(\ell)$ and $|\vec{p}(B)|$.

Both analyses determine the partial rate $d\Gamma/dw$, where w is the four-vector product of $p(B)$ and $p(D^*)$. This partial rate is determined by the available phase space and by three form factors $F_i(w)$ which are related to each other and can be parametrized by heavy quark effective QCD (HQET). QCD also predicts $F(1)$, where $F(w)$ is one of the three form factors. Its most precise value is obtained with the help of lattice QCD,[38]

$$F(1) = 0.913 \begin{array}{l} +0.030 \\ -0.035 \end{array}\ . \qquad (37)$$

Since phase space forces $d\Gamma/dw$ to vanish at $w = 1$, $F(1)$ has to be determined by an extrapolation of the observed $F(w)$, essentially $d\Gamma/dw$ divided by phase space, to $w = 1$. The results of the two new experiments are given in Table 3, where $\rho^2_{A_1}$ is the slope parameter of the form factor $A_1(w)$ in the HQET parametrization of Caprini *et al.*.[39] The slope parameters agree whereas the results for $|V_{\rm cb}|$ and $\mathcal{B}$ disagree by about $2\,\sigma$. Figure 3 shows the fit results of all $D^*\ell\nu$ analyses selected by HFAG. Earlier work like that of ARGUS[40] is missing because it used a

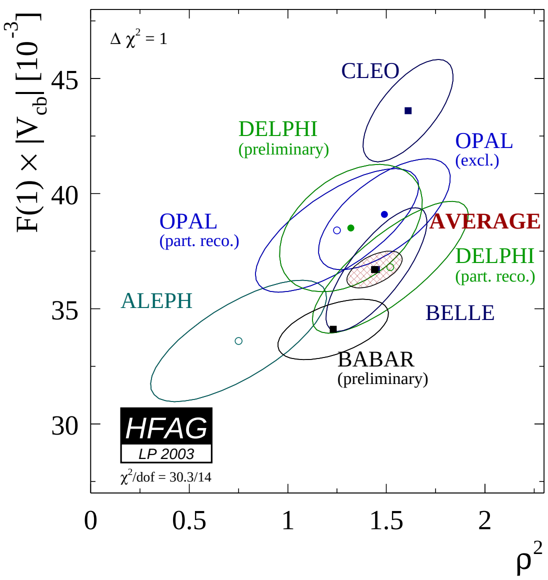

Figure 3. Slopes $\rho^2_{A_1}$ and values at $w = 1$ of the function $A_1(w) \cdot |V_{\rm cb}|$ from eight experiments as compiled by HFAG[19] for this Symposium. The dashed ellipse is the HFAG average with unscaled errors.

different form factor convention. The agreement between the eight results in Fig. 3 is not very good. The average of the $|V_{\rm cb}|$ results is

$$F(1) \cdot |V_{\rm cb}| = 0.0367 \pm 0.0013 \ , \qquad (38)$$

where I have increased the error by a scale factor of 1.7 since the one-dimensional fit for $F(1) \cdot |V_{\rm cb}|$ gives $\chi^2 = 19.5$ for $N(dof) = 7$. The lattice QCD result in Eq. (37) leads to

$$|V_{\rm cb}|_{\rm excl} = 0.0402 \pm 0.0020 \ , \qquad (39)$$

and combined with the compatible inclusive result in Eq. (36) leads to my final best value of

$$|V_{\rm cb}| = A\lambda^2 = 0.0415 \pm 0.0011 \ . \qquad (40)$$

8. $|V_{\rm ub}|$

Sources for $|V_{\rm ub}|$ are inclusive and exclusive semileptonic B meson decays into charmless final states. Exclusive $|V_{\rm ub}|$ determinations have been published by two experiments, BABAR[41] with $\rm B^0 \to \rho^- e^+ \nu$ and $\rm B^+ \to \rho^0 e^+ \nu$ decays from 55 M Υ(4S) events, and CLEO[42] with decays into $\pi^- \ell^+ \nu$, $\pi^0 \ell^+ \nu$, $\rho^- \ell^+ \nu$, $\rho^0 \ell^+ \nu$, and $\omega \ell^+ \nu$ from 10 M Υ(4S) events. Both combine modes with the help of isospin and quark-model constraints,

$$\Gamma(\rho^0 \ell\nu) = \Gamma(\omega \ell\nu) = \Gamma(\rho^- \ell\nu)/2 \ ,$$
$$\Gamma(\pi^0 \ell\nu) = \Gamma(\pi^- \ell\nu)/2 \ . \qquad (41)$$

Even in the subsamples with high energy leptons, $E_\ell > 2.3$ GeV, there are substantial backgrounds from non-$\rm B\overline{B}$ ("continuum") events, from crossfeed between different signal modes and from "downfeed" where $\rm b \to u \ell \nu$ decays with additional pions are reconstructed in a signal mode. BABAR uses five different form-factor calculations for an extraction of $|V_{\rm ub}|$. They give compatible results, and BABAR[41] quotes an average of

$$|V_{\rm ub}| = (3.64 \pm 0.22 \pm 0.25 \begin{array}{c} +0.39 \\ -0.56 \end{array}) \times 10^{-3} \ , \qquad (42)$$

where the first error is statistical, the second describes the experimental systematics, and the third one describes the spread between the five different form-factor calculations.

Since the CLEO detector is operated at an energy symmetric $\rm e^+e^-$ storage ring, it has a larger solid angle coverage in the center-of-mass system than BABAR. This larger hermeticity allows better "neutrino reconstruction" and better determination of $q^2 = (p_\nu + p_\ell)^2$. CLEO is therefore able to use events with lower momentum leptons and to divide the data into two samples with $q^2 < 16\,{\rm GeV}^2$ where light cone sum rules are expected to predict model-independent form factors, and with $q^2 \geq 16\,{\rm GeV}^2$ where lattice QCD calculations predict the form factors well. Both subsamples for both $\pi\ell\nu$ and $\rho\ell\nu$ decays result in four compatible extractions of $|V_{\rm ub}|$; CLEO[42] quotes an average of

$$|V_{\rm ub}| = (3.17 \pm 0.17 \begin{array}{c} +0.16 \\ -0.17 \end{array} \begin{array}{c} +0.53 \\ -0.39 \end{array}) \times 10^{-3} \ , \qquad (43)$$

with the same error definitions as above in Eq. (42). My average of the two exclusive experiments is

$$|V_{\rm ub}|_{\rm excl} = (3.40 \begin{array}{c} +0.24 \\ -0.33 \end{array} \pm 0.40) \times 10^{-3} \ . \qquad (44)$$

Three methods have been tried so far for the inclusive determination of $|V_{\rm ub}|$: (a) the "endpoint" method requiring $\rm B \to X e \nu$ events with $E_\ell > 2.3\,{\rm GeV}$ in order to suppress leptons from $\rm B \to X_c e \nu$ decays; (b) the "low-mass" method allowing a wider range of E_ℓ, e.g. $E_\ell > 1.0\,{\rm GeV}$, and requiring $m_X < 1.5\,{\rm GeV}$ in order to suppress hadronic masses from $\rm B \to X_c e \nu$ events; and (c) the "high-q^2" method which combines (b) with an additional requirement of high values of $q^2 = (p_\ell + p_\nu)^2$ since $\mathrm{B} \to X_\mathrm{c} \mathrm{e}\nu$ decays are not only limited to $E_\ell < 2.3\,\mathrm{GeV}$ and $m_X > m_\mathrm{D}$ but also to $q^2 < 12\,\mathrm{GeV}^2$.

The first $|V_\mathrm{ub}|$ measurements[43,44] had been performed with method (a) which has the big disadvantage that extrapolation to $E_\ell = 0$ has large model dependence and little access to stricter QCD estimates. Method (b) is less QCD-dependent, but requires good knowledge of the hadronic mass which can be obtained at the $\Upsilon(4\mathrm{S})$ by tagging the semileptonically decaying B with a fully reconstructed second B. At present, this reduces the number of observed semileptonic decays by a factor of about 10^3. Method (c) is even less QCD-dependent but requires even higher numbers of tagged B decays.

Two new results have been presented to this Symposium. BELLE[45] uses method (a) with 29 M $\Upsilon(4\mathrm{S})$ events and determines the partial rate $\Delta\Gamma(\mathrm{B} \to X \mathrm{e}\nu,\, 2.3 < E_\ell < 2.6\,\mathrm{GeV})$. Extrapolation to $E_\ell = 0$ is achieved with the help of the "shape function" concept,

$$\Gamma(\mathrm{B} \to X_\mathrm{u} \mathrm{e}\nu) = \Delta\Gamma(2.3 < E_\ell < 2.6\,\mathrm{GeV})/\mathrm{f_u}\ , \quad (45)$$

where the extrapolation factor f_u is obtained using the shape of the partial rate

$$f(E_\gamma) = \mathrm{d}\Gamma(\mathrm{b} \to \mathrm{s}\gamma)/\mathrm{d}E_\gamma\,, \quad (46)$$

which has recently been measured by CLEO.[46] The shape of $\mathrm{d}\Gamma(\mathrm{B} \to X_\mathrm{u}\ell\nu)/\mathrm{d}E_\ell$ in the endpoint region is related to $f(E_\gamma)$ with the help of nonlocal operators ("twists") in heavy quark effective QCD.[47,48] This recipe has already been used in the $|V_\mathrm{ub}|$ endpoint analyses of CLEO[49] and BABAR.[50] In fact, the new BELLE analysis uses CLEO's value and error for f_u in Eq. (45), and BELLE obtains[45]

$$|V_\mathrm{ub}| = (3.99 \pm 0.17 \pm 0.16 \pm 0.59) \times 10^{-3}\,. \quad (47)$$

The first error is statistical, the second systematic from the experiment, and the third one estimates the uncertainty from using the shape-function concept.

BABAR[51] presents a "low-mass" analysis from 89 M $\Upsilon(4\mathrm{S})$ events resulting in 32000 tagged decays $\mathrm{B} \to X \mathrm{e}\nu$ with a fully reconstructed B as the tag and with $E_\ell > 1.0\,\mathrm{GeV}$. Reconstruction of the hadronic mass m_X in these decays gives about 600 events with $m_X < 1.6\,\mathrm{GeV}$, 400 of which are estimated to be $\mathrm{B} \to X_\mathrm{u}\ell\nu$ signal events. Again using a shape-function concept, BABAR fits the observed m_X spectrum and extracts

$$|V_\mathrm{ub}| = (4.62 \pm 0.28 \pm 0.27 \pm 0.40 \pm 0.26) \times 10^{-3}\,. \quad (48)$$

The first two errors are experimental, statistical and systematic, the third estimates the shape-function uncertainty, and the last one estimates the uncertainty in translating $\Gamma(\mathrm{B} \to X_\mathrm{u}\ell\nu)$ into $|V_\mathrm{ub}|$.

Table 4. Results for $|V_\mathrm{ub}|$ as compiled by HFAG[19] for this Symposium.

Experiment	Method	$10^3 \cdot |V_\mathrm{ub}|$
ALEPH		$4.12 \pm 0.67 \pm 0.76$
L3		$5.70 \pm 1.00 \pm 1.40$
DELPHI		$4.07 \pm 0.65 \pm 0.61$
OPAL		$4.00 \pm 0.71 \pm 0.71$
LEP Average		$4.09 \pm 0.37 \pm 0.56$
CLEO	endpoint	$4.08 \pm 0.22 \pm 0.61$
BABAR	endpoint	$4.43 \pm 0.26 \pm 0.67$
CLEO	m_X and q^2	$4.05 \pm 0.61 \pm 0.65$
BELLE	m_X	$5.00 \pm 0.64 \pm 0.53$
BELLE	m_X and q^2	$3.96 \pm 0.47 \pm 0.52$
BABAR	m_X	$4.62 \pm 0.38 \pm 0.49$
BELLE	endpoint	$3.99 \pm 0.25 \pm 0.59$

Table 4 summarizes all inclusive $|V_\mathrm{ub}|$ results except very early ones. It includes a high-q^2 result from BELLE which will not be discussed here because of its preliminary status. This method, however, seems to be very promising for the future when 3 to 10 times more tagged decays will be available. Finding an objective best value from the entries in Table 4 is difficult, my estimate gives

$$|V_\mathrm{ub}|_\mathrm{incl} = (4.26 \pm 0.13 \pm 0.50) \times 10^{-3}\,, \quad (49)$$

where the first error is meant to have a gaussian shaped likelihood and the second one a rectangular likelihood. Combining this result with the exclusive best estimate in Eq. (44) leads to a best estimate of

$$|V_\mathrm{ub}| = (3.80 \begin{array}{c}+0.24\\-0.13\end{array} \pm 0.45) \times 10^{-3}\,, \quad (50)$$

as sketched in Fig. 4.

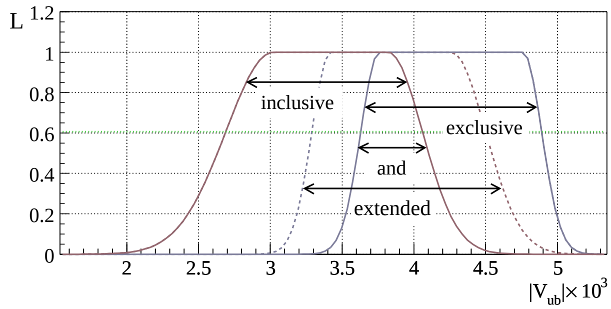

Figure 4. Likelihood functions for $|V_{\rm ub}|$ from exclusive (Eq. (44)) and inclusive (Eq. (49)) measurements. The "and" of both functions is so narrow that an "extended" product, the flat part of which is defined by the centres of $\mathcal{L}_{\rm excl}$ and $\mathcal{L}_{\rm incl}$, is used as combined result (Eq. (50)).

9. First Way to $|V_{\rm td}|$

The experimental precision on the mass difference $\Delta m_{\rm d}$ of the two mass and lifetime eigenstates of the $\rm B^0_d\overline{B}^0_d$ system has nearly reached 1%. Using all time-dependent measurements of $\rm B^0_d\overline{B}^0_d$ oscillations, the present best value[7] is

$$\Delta m_{\rm d} = (0.502 \pm 0.007)\,/\,{\rm ps}\,. \qquad (51)$$

Using this value, the t quark mass measurement from FNAL[52] and

$$f_{\rm B_d}\sqrt{B_{\rm B_d}} = (233 \pm 13 \pm 12)\,{\rm MeV} \qquad (52)$$

from lattice QCD,[53] where $f_{\rm B_d}$ is the purely leptonic decay constant of the $\rm B^+$ meson, and $B_{\rm B_d}$ is the additional non-perturbative "bag factor" in the Standard Model description of $\rm B^0_d\overline{B}^0_d$ oscillations, gives the result

$$|V_{\rm td}| \cdot |V_{\rm tb}| = (9.2 \pm 1.4 \pm 0.5) \times 10^{-3}\,. \qquad (53)$$

The errors are dominated by those in Eq. (52). The first one is the statistical error from the lattice calculation precision with a Gaussian likelihood, and the second one is an estimate of the lattice QCD precision with a rectangular likelihood.

10. $|V_{\rm ts}|$

Information on $|V_{\rm ts}| \cdot |V_{\rm tb}|$ is available from $\rm B^0_s\overline{B}^0_s$ oscillations and from $\rm B \to X_s\gamma$ decays. The existence of $\rm B^0_s\overline{B}^0_s$ oscillations is well established. Inclusive time-integrated measurements of e. g. $\Gamma(\rm b\overline{b} \to \ell^\pm\ell^\pm)/\Gamma(b\overline{b} \to \ell^\pm\ell^\mp)$ at LEP lead to

$$f_{\rm s} \cdot \chi_{\rm s} = \overline{\chi} - f_{\rm d} \cdot \chi_{\rm d} = 0.0509 \pm 0.0060\,, \qquad (54)$$

which is more than $8\,\sigma$ above zero; notations and numbers are taken from Schneider.[54] The strength of $\rm B^0_s\overline{B}^0_s$ oscillations is unknown. Pitts[55] has discussed the status at this Symposium and has presented limits on $\Delta m_{\rm s}$ as well as improvement prospects. The HFAG 95% limit is now[19]

$$\Delta m_{\rm s} > 14.4\,{\rm ps}^{-1}\,. \qquad (55)$$

Using the lattice QCD result[56]

$$\xi = \frac{f_{\rm B_s}\sqrt{B_{\rm B_s}}}{f_{\rm B_d}\sqrt{B_{\rm B_d}}} = 1.24 \pm 0.04 \pm 0.06\,, \qquad (56)$$

where $f_{\rm B_s}$ and $B_{\rm B_s}$ have the same meaning for the $\rm B_s$ meson as in Eq. (52), this $\Delta m_{\rm s}$ limit leads to

$$|V_{\rm ts}| \cdot |V_{\rm tb}| > 0.003\,. \qquad (57)$$

The branching fraction of inclusive radiative decays $\rm B \to X_s\gamma$ has been determined by CLEO, ALEPH, BABAR, and BELLE. Ali and Misiak[57] use the mean value for an information on $|V_{\rm ts}|$ and derive

$$|V_{\rm ts}| \cdot |V_{\rm tb}| = 0.047 \pm 0.008\,. \qquad (58)$$

11. Second Way to $|V_{\rm cs}|$

As already mentioned in Sec. 5, decays of real W bosons at LEP-II[58] allow a better determination of $|V_{\rm cs}|$ than semileptonic decays of D mesons into K mesons. Using $\alpha_{\rm s}$ corrections, the measured branching ratio $\Gamma(\rm W \to hadrons)/\Gamma(W \to e\nu)$ can be translated into

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2|V_{\rm cd}|^2 + |V_{\rm cs}|^2 + |V_{\rm cb}|^2 = 2.039 \pm 0.026\,. \qquad (59)$$

Since the five other determinations have a much smaller absolute error than the result on $|V_{\rm cs}|$ in Eq. (25), this LEP-II result can be used to extract

$$|V_{\rm cs}| = 0.995 \pm 0.014\,, \qquad (60)$$

which is much more precise than the $\rm K \to D\ell\nu$ result.

12. Unitarity Check of the Full Matrix

Using all final estimates for $|V_{\rm ud}|$, $|V_{\rm us}|$, $|V_{\rm ub}|$, $|V_{\rm cd}|$, $|V_{\rm cs}|$, and $|V_{\rm cb}|$ presented so far, we can check three of the six unitarity constraints which the CKM matrix has to fulfill. I obtain

$$|V_{\rm ud}|^2 + |V_{\rm us}|^2 + |V_{\rm ub}|^2 = 0.9969 \pm 0.0017\,, \qquad (61)$$

$$|V_{\rm cd}|^2 + |V_{\rm cs}|^2 + |V_{\rm cb}|^2 = 1.042 \pm 0.026\,, \qquad (62)$$

$$|V_{\rm ud}V_{\rm cd}|-|V_{\rm us}V_{\rm cs}|\pm|V_{\rm ub}V_{\rm cb}| = -0.002\pm0.016\,. \quad (63)$$

The sum of squares in the first row is equal to one with $-1.8\,\sigma$. The sum in the second row, highly correlated with that in the first row because of using Eq. (59), is one within $+1.6\,\sigma$. Eq. (63) estimates the real part of $V^*_{\rm ud}V_{\rm cd} + V^*_{\rm us}V_{\rm cs} + V^*_{\rm ub}V_{\rm cb}$ with any phase γ of the quartet $V^*_{\rm ub}V_{\rm cb}V_{\rm ud}V^*_{\rm cd}$, see Eq. (3). This upper limit estimate is compatible with zero within $\pm0.12\,\sigma$.

In conclusion, the observed CKM matrix element magnitudes fulfill unitarity reasonably well. That means, the strengths of all processes which we call "weak" are consistently described by the Standard Model weak interaction in which the CKM matrix is necessarily unitary.

Now assuming unitarity, we can use the relation $|V_{\rm tb}|^2 = 1 - |V_{\rm cb}|^2 - |V_{\rm ub}|^2$ in order to obtain

$$|V_{\rm tb}| = 0.99913 \pm 0.00009\,; \quad (64)$$

i.e. the only unmeasured matrix element has the smallest error. With this result, Eq. (53) translates into

$$|V_{\rm td}| = (9.2 \pm 1.4 \pm 0.5) \times 10^{-3}\,. \quad (65)$$

And from the relation $|V_{\rm ts}|^2 = |V_{\rm cb}|^2 - |V_{\rm ub}|^2 - |V_{\rm td}|^2$ we then obtain

$$|V_{\rm ts}| = 0.0406 \pm 0.0023\,, \quad (66)$$

which is more precise than the result in Eq. (58).

13. $|V_{\rm td}|$ Again

The unitarity result in Eq. (66) and the ratio of $\rm B_d\overline{B}_d$ and $\rm B_s\overline{B}_s$ oscillation strengths,

$$\frac{\Delta m_{\rm d}}{\Delta m_{\rm s}} = \frac{m_{\rm B_d}}{m_{\rm Bs}} \cdot \frac{|V_{\rm td}|^2}{|V_{\rm ts}|^2} \cdot \xi^2\,, \quad (67)$$

with ξ as defined in Eq. (56), allows a second determination of $|V_{\rm td}|$. The HFAG compilation[19] of $A(\Delta m_{\rm s})$ results allows the approximation

$$\mathcal{L}(\Delta m_{\rm s}) = {\rm e}^{-(A-1)^2/2\sigma_A^2} \text{ for } \Delta m_{\rm s} < 20\,{\rm ps}$$
$$= 1 \text{ for } \Delta m_{\rm s} > 20\,{\rm ps}\,, \quad (68)$$

for the likelihood of $\Delta m_{\rm s}$, where A is the amplitude of $\rm B\overline{B}$ oscillations as a function of Δm as taken from HFAG. Combining this likelihood function with Eqs. (66), (67), and with my first $|V_{\rm td}|$ result in Eq. (65) gives $|V_{\rm td}|$ with likelihood contours as shown in Fig. 5 in the $\overline{\rho},\overline{\eta}$ plane.

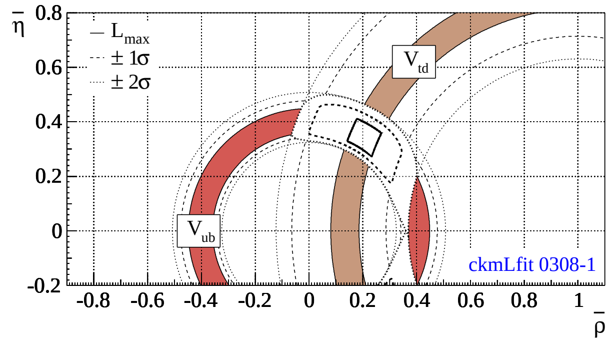

Figure 5. Likelihood contours[60] for $|V_{\rm ub}|$ and $|V_{\rm td}|$ in the $\overline{\rho}$, $\overline{\eta}$ plane. Solid curves are for $\mathcal{L} = \mathcal{L}_{\rm max}$, dashed for $\mathcal{L} = \mathcal{L}_{\rm max} \times {\rm e}^{-1/2}$ ($\pm1\,\sigma$), and dotted for $\mathcal{L} = \mathcal{L}_{\rm max} \times {\rm e}^{-2}$ ($\pm2\,\sigma$). Also plotted (in white) is the product likelihood for both measurements. It excludes $\overline{\eta} = 0$ with $2.0\,\sigma$.

14. CP-Violation in the CKM Matrix

Figure 5 also shows the likelihood contours of $|V_{\rm ub}|$ as presented at the end of Sec. 8. The two circular bands for $|V_{\rm ub}|$ and $|V_{\rm td}|$ have two intersection regions, one with $\overline{\eta} \approx +0.35$ and one with $\overline{\eta} \approx -0.35$. Both solutions violate $\overline{\eta} = 0$, i.e. CP-symmetry in the Standard Model weak interaction, with 2.0 standard deviations.

Figure 6 superimposes the $\overline{\rho}$, $\overline{\eta}$ bands of the two well-known CP-violating effects in $\rm K^0 \to \pi\pi$ decays ($\epsilon_{\rm K}$) and in $\rm B^0 \to (c\overline{c})K$ decays ($\sin 2\beta$). For the $\sin 2\beta$ band, I have used here the new world average

$$\sin 2\beta = 0.737 \pm 0.048\,, \quad (69)$$

including the newest result from BELLE as presented by Browder[59] at this Symposium. All four bands intersect perfectly in one solution, i.e. the CP-violation concluded from the magnitudes of $V_{\rm ub}$ and $V_{\rm td}$ describes quantitatively the CP-violation in K and B decays.

15. Final Fit for A, λ, ρ, and η

Also shown in Fig. 6 is a simultaneous fit to the four quantities ($\epsilon_{\rm K}$, $|V_{\rm ub}|$, $|V_{\rm td}|$ and $\sin 2\beta$) with the help of the program ckmLfit.[60] With the $\sin 2\beta$ value in Eq. (69), the fit results are

$$\overline{\rho} = 0.21 \pm 0.08 \pm 0.05\,,$$
$$\overline{\eta} = 0.35 \pm 0.04 \pm 0.02\,. \quad (70)$$

The $\sin 2\beta$ input reduces the error on $|V_{\rm ub}|$ and gives $|V_{\rm ub}|$ a gaussian shaped likelihood since the error on

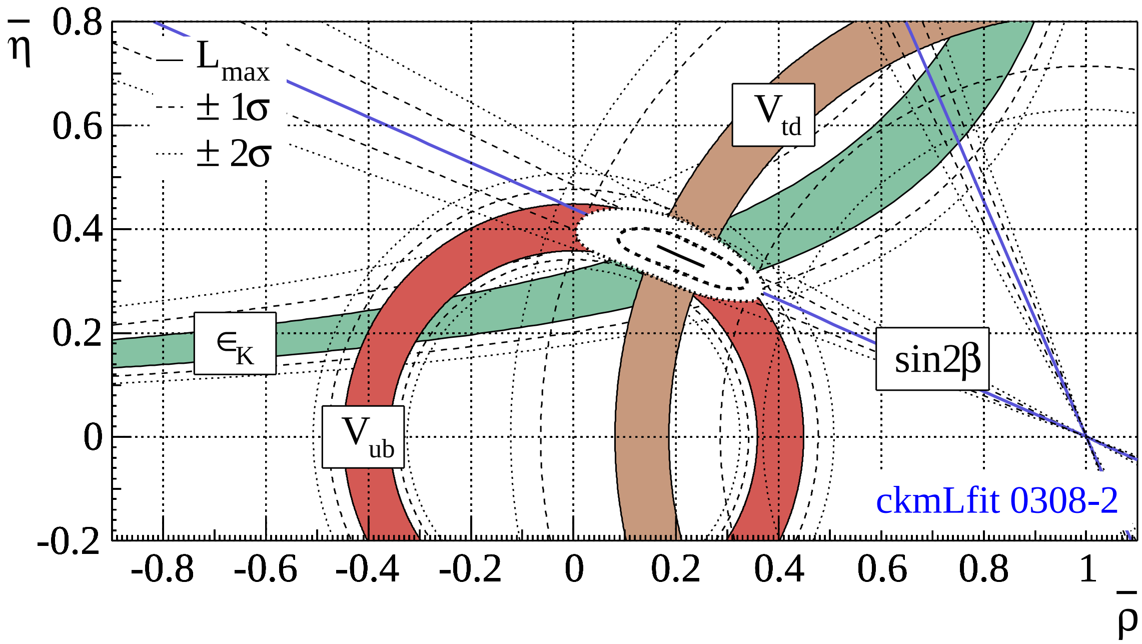

Figure 6. Likelihood contours[60] for ϵ_K and $\sin 2\beta$ superimposed to the contours of Fig. 5. The contours around the white region in the centre show the product of all four likelihoods, leading to the final fit result in Eqs. (70) and (71).

$\sin 2\beta$ is dominated by statistics. Because of the high correlation between the two parameters $\overline{\rho}$ and $\overline{\eta}$, I want to give the final fit results in a different and also extended form:

Taking all magnitude results of this review, combining them with the two explicitly CP-violating results for ϵ_K and $\sin 2\beta$, and constraining them by unitarity, the CKM matrix is given by the four parameters:

$$\begin{aligned} \lambda &= 0.2235 \pm 0.0033\,, \\ A\lambda^2 &= 0.0415 \pm 0.0011\,, \\ A\lambda^3\sqrt{\rho^2+\eta^2} &= (3.85 \pm 0.33)\,10^{-3}\,, \\ \mathrm{atan2}(\eta/\rho) &= (58 \pm 19)^{\mathrm{o}}\,. \end{aligned} \tag{71}$$

The relative errors on these four parameters are $\pm 1.5\%$, $\pm 2.7\%$, $\pm 9\%$, and $\pm 5\%$ of 360^{o}, respectively. At this moment, we know $A\lambda^2$ better than A. For a long time, we were used to the hierarchy of magnitudes 1, λ, λ^2, λ^3 also being a hierarchy of precision. This is no longer so. Therefore, the four parameters in Eq. (71), i.e. the magnitudes $|V_{\mathrm{us}}|$, $|V_{\mathrm{cb}}|$, $|V_{\mathrm{ub}}|$, and the phase of the quartet $V^*_{\mathrm{ub}}V_{\mathrm{ud}}V^*_{\mathrm{cd}}V_{\mathrm{cb}}$ may, in future, be four better suited choices than the parameters A, λ, ρ, and η.

Acknowledgements

I would like to thank my BABAR colleagues, especially Th. Brandt, O. Buchmüller, R. Dubitzky, U. Langenegger, and V. Lüth, for many helpful discussions during the preparation of this talk. I also express my thanks to R. Nogowski for his help with the figures and to E. Maly and F. Roschk for reading the final manuscript.

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DISCUSSION

Ikaros Bigi (Univ. of Notre Dame): You listed some homework for theorists concerning the calculation of $\Gamma(\mathrm{B} \to l\nu X)$. Those items with $O(1/(m_b)^3)$ contributions and $(\alpha_s)^2$ terms have been done already in our scheme in a paper that has been published this year. The limiting factor theoretically is $O(\alpha_s)$ corrections to the non-perturbative contributions.

Alberto Sirlin (NYU): In the case of V_{us}, there is a recent work by Bijnens and Jalavora. They consider two-loop contributions in chiral perturbation theory in the isospin limit. Their result depends on two unknown constants that may, in principle, be determined from the slope and curvature of $\mathrm{K}_{\mu 3}$ form factors.

Klaus Schubert: As far as I know this has been included.

Sheldon Stone (Syracuse Univ.): Since Ikaros Bigi wants more homework, let me say that the error due to the “Duality” assumption is still an open question and many of us therefore do not consider it legitimate to average the inclusive and exclusive determinations of V_{cb}, the exclusive having well understood theoretical errors.

Vera Luth (SLAC): Measurements of exclusive charmless semileptonic decays require knowledge of form factors. Experiments will be able to measure rates as a function of q^2. Theoretical predictions over the full range in q^2 would be very welcome, especially for lattice calculations!