High-Precision Lattice QCD and Experiment

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Breakthrough?

Before (for 25 years):

- Realistic u/d/s vacuum polarization impossible — small quark masses too expensive.

  $\Rightarrow$ Omit quark loops ("quenched QCD") or include only u, d quarks (no s) but with masses 10-20$\times$ too large.

  $\Rightarrow$ 10-30% systematic errors in almost all lattice QCD results.

Now (since 2000):

- New discretization of quark action that is 50-1000 times faster.

  $\Rightarrow$ Simulations with u, d, s quarks possible, with masses that are 3–5$\times$ smaller than before.

  $\Rightarrow$ Masses small enough to allow accurate extrapolations.

  $\Rightarrow$ High-precision (few %) nonperturbative QCD now!

Essential for Standard Model

E.g., CKM weak interaction parameters
ρ and η from:

B-$\bar{B}$ mixing

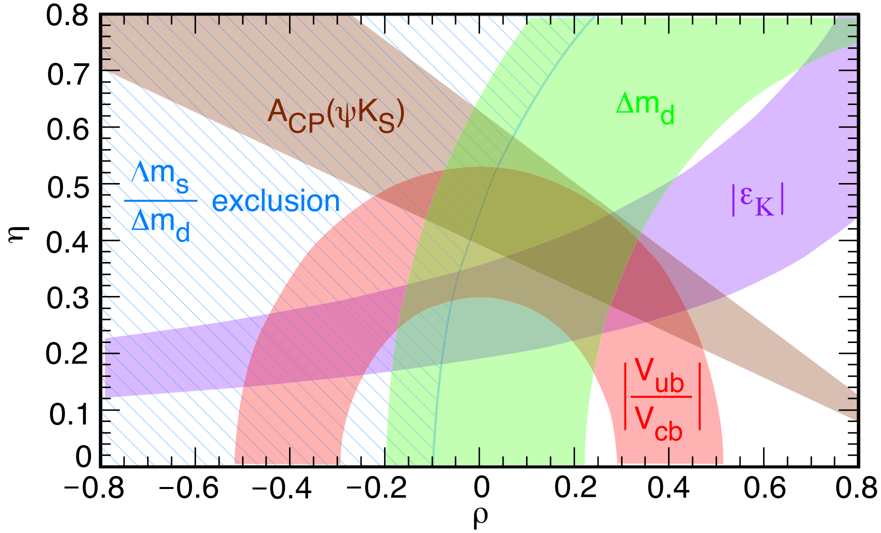
$B \rightarrow \pi l \nu$

K-$\bar{K}$ mixing

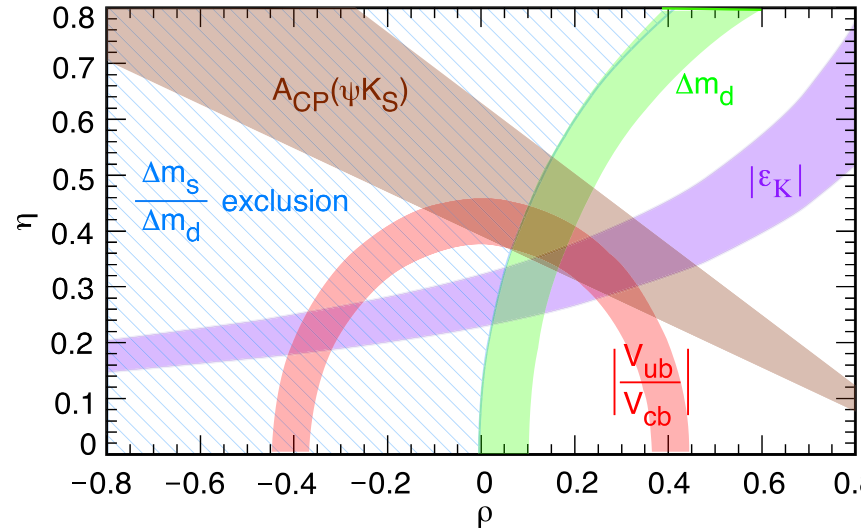
$\dots$

Nonpert've QCD Part $\times$ Weak Int'n Part

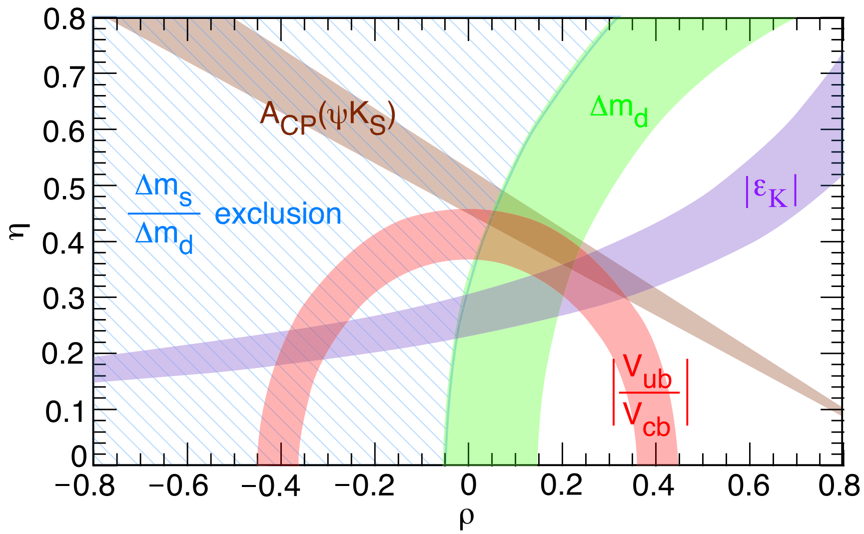
CKM today . . .

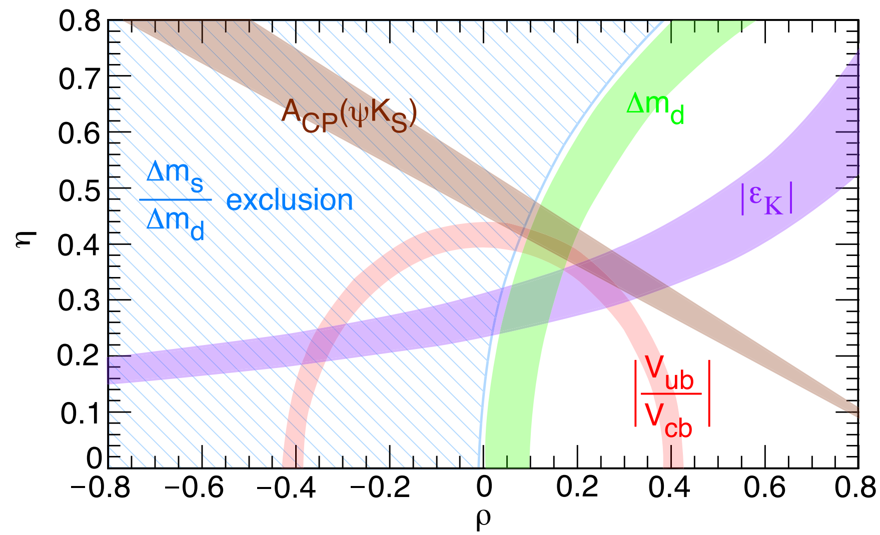


. . . and with 2–3% theory errors.



And with B Factories . . .





95% confidence levels; CLEO-c (2001).

Essential Beyond the S.M.?

2 of 3 known interactions are strongly coupled: QCD, gravity.

Strong coupling is possible (likely?) at the LHC and/or beyond.

- Generic at low energies in non-abelian gauge theories . . .
- . . . unless gauge symmetry spontaneously broken

  $\Rightarrow$ dynamical symmetry breaking

  $\Rightarrow$ strong coupling!

Symanzik-Improved Staggered Quarks

- Staggered quarks + improved discretization.
- Chiral symmetry ⇒ efficient for small quark masses.
- Complication: single quark field $\psi(x)$ creates 4 equivalent species or "tastes" of quark.

  $\Rightarrow$ $\det(D \cdot \gamma + m) \to \det(D \cdot \gamma + m)^{1/4}$.

  $\Rightarrow$ Potential non-locality.
- But:
  - ◇ Fractional roots cause no problem in perturbative QCD.
  - ◇ Anomaly-induced behavior (e.g., $\pi^0 \to \gamma\gamma$), instantons … okay.
  - ◇ Core problem is taste-changing interactions, but these are short-distance and perturbative.
- Careful testing essential!

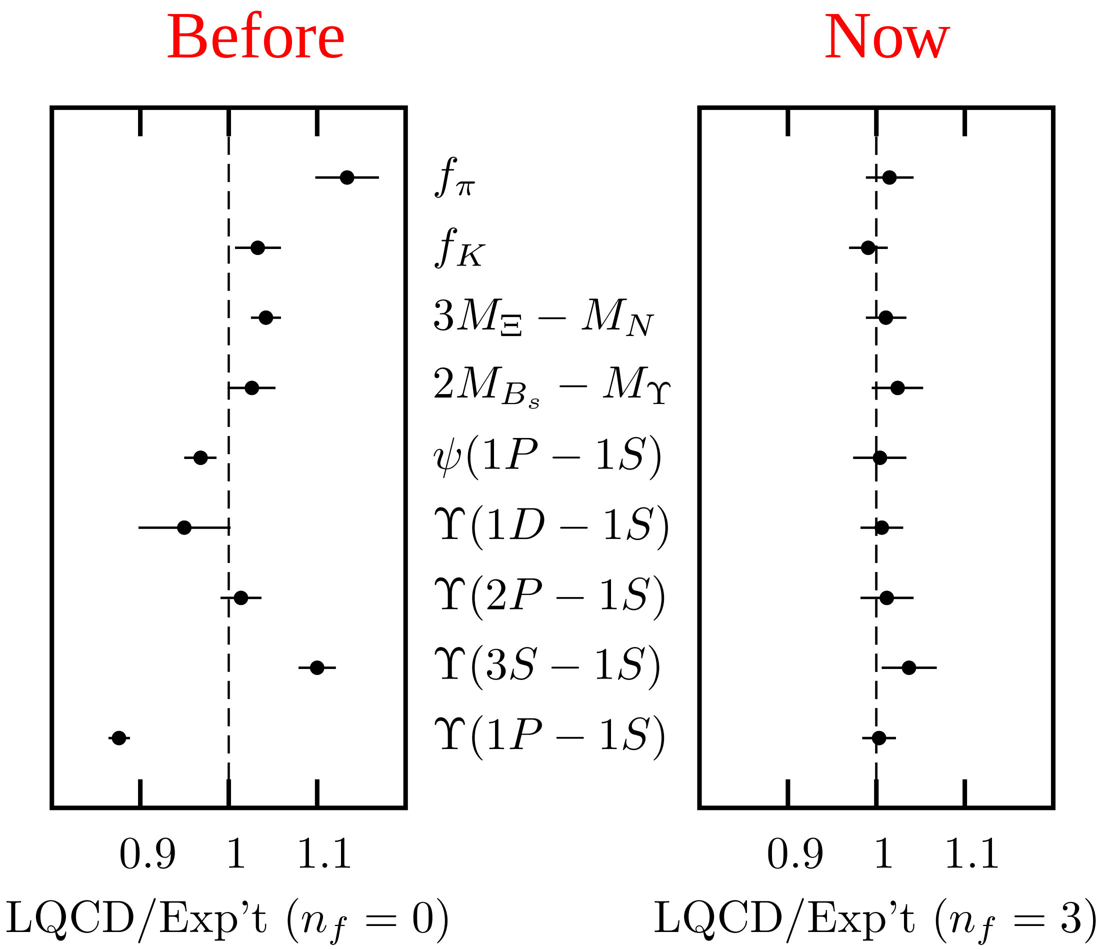
High-Precision Test

1) Tune 5 free parameters (bare $m_u = m_d$, m_s, m_c, m_b and α_s) using m_π, m_K, m_ψ, m_Υ, and $\Delta E_\Upsilon(1P - 1S)$.

2) Compute other quantities and compare with experiment.

Davies et al, hep-lat/0304004. (HPQCD, MILC, Fermilab, UKQCD)

Lattice QCD/Experiment (no free parameters!):



Tests:

- $m_{u,d}$ extrapolation;
- masses and wavefunctions;
- s quark;
- light-quark baryons;
- light-heavy mesons;
- heavy quarks (no potential model. . .);
- improved staggered quark vacuum polarization.

Quark Mass Problem

Too expensive to simulate at realistic $m_{u,d}$.

$\Rightarrow$ Simulate for range of larger $m_{u,d}$ and extrapolate using chiral perturbation theory.

$\Rightarrow$ E.g.,

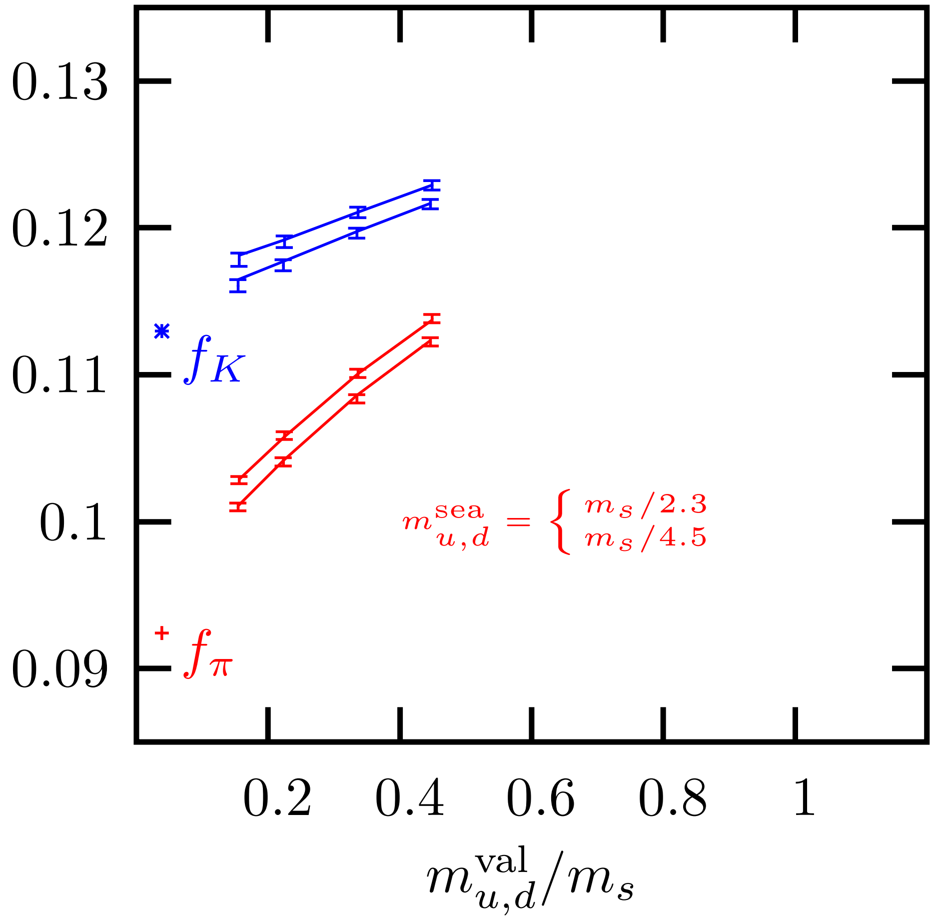
$$f_\pi = f_0 \left(1 + a_0\, x_\pi \log(x_\pi) + a_1\, x_\pi + b\, x_\pi^2 + \cdots\right)$$

where a,b... are $\mathcal{O}(1)$ (fit to simulation) and

$$x_\pi \equiv \frac{m_\pi^2}{1\,\mathrm{GeV}^2} \approx \frac{m_{u,d}}{2m_s} \approx \begin{cases} 0.02 & \text{for real quarks} \\ 0.06\text{–}0.25 & \text{in new simulations} \end{cases}$$

$\Rightarrow$ Keep $m_{u,d} \leq m_s/2$ for high-precision!

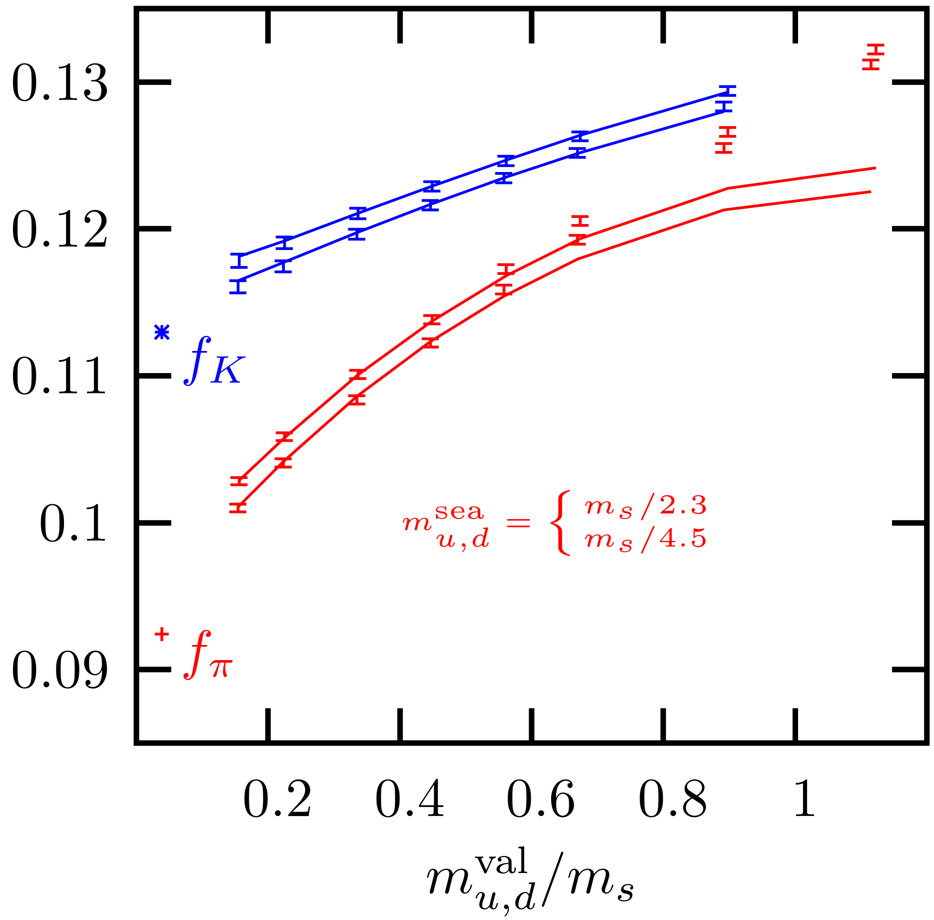
f_π and f_K fits versus valence u, d mass:



Note:

- f_π more sensitive.
- More sensitive to valence mass than sea mass (***).
- Extrapolation correct to within $\pm 2\%$ errors.

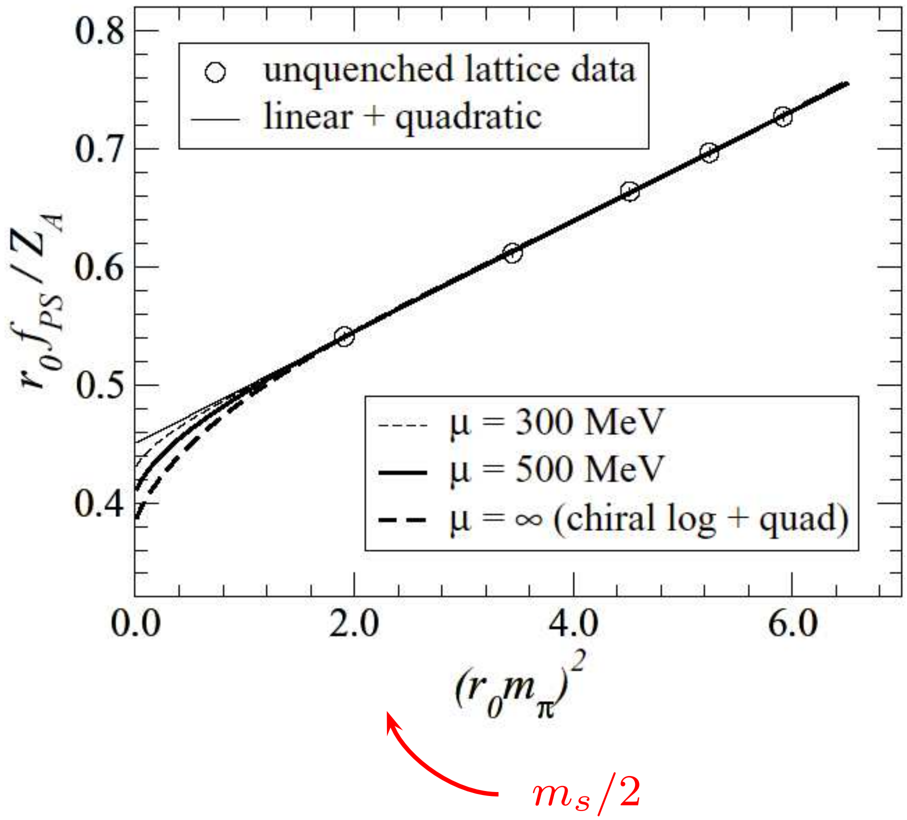
f_π and f_K fits extrapolated to larger masses:



Note:

- Masses $\geq m_s/2$ bad.
- High-mass f_πs linear; extrapolate 10% high.
- Lowest two f_πs extrapolate linearly to within 2%.

Best f_π analysis without staggered quarks:

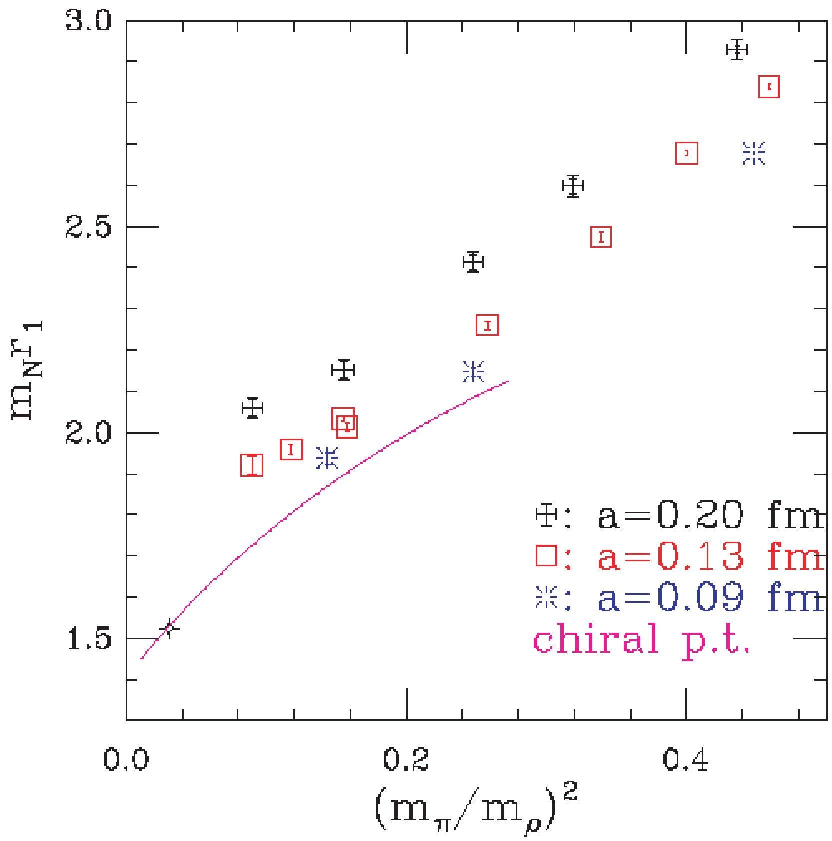


Note:

- Only masses $\geq m_s/2$.
- Very straight line; unexpected, but real physics.
- $\pm 10\%$ extrapolation errors despite 0.5% data errors.
- No s quark in sea ($n_f = 2$).

Aoki et al., hep-ph/307039 (JLQCD Collaboration)

Nucleon mass versus light-quark mass:



Note:

- More sensitive than f_π.
- Finite-a errors $\approx$ 2% for $a = 0.09$ fm.
- Full analysis not complete.

Steve Gottlieb's talk at Lattice '03, July 2003 (MILC Collaboration).

Gold-Plated Quantities

Lattice QCD can't do everything (yet).

- Unstable hadrons strongly affected by finite lattice volume (2.5–3 fm across): e.g., πs in fluctuation $\rho \to \pi\pi \to \rho$ can be on-shell and propagate freely to lattice boundaries.
- Hadrons near decay thresholds, even if stable, fluctuate into nearly on-shell multi-hadron states that again can propagate to the boundaries: e.g., phase space implies ϕ nearly stable, but phase space doesn't limit virtual fluctuations, $\phi \to K\overline{K} \to \phi$.
- Euclidean time $\Rightarrow$ phases in multihadron states subtle.

$\Rightarrow$ Systematic errors of 10% or more (estimate using effective field theory) even with good light-quark masses.

Important to focus theoretical and experimental effort on “gold-plated” quantities:

- hadronic masses, and matrix elements with at most one hadron in initial and/or final states;
- hadrons at least 100 MeV below threshold or with negligible couplings to decay channels (e.g., π, K, D, D_s, $J/\psi\ldots$).

Dozens of gold-plated quantities: e.g.,

- Masses, form factors, decay constants, mixing amplitudes for π, K, p, n (but not ρ, ϕ, $\Delta \ldots$).
- Masses, decay constants, semileptonic form factors, and mixing for D, D_s, B, B_s (but not $D^* \ldots$).
- Masses, leptonic widths, electromagnetic form factors, and mixing for any meson in ψ and Υ families well below D/B threshold.

- Gold-plated quantities for almost every CKM matrix elements (and K-$\overline{K}$ mixing):

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

- Extensive cross-checks for error calibration: Υ, B, ψ, $D\ldots$.

Limitations for Gold-Plated Processes

Systematic errors:

- Finite lattice spacing errors of order 2% or less at $a \leq 0.1$ fm; improved discretizations essential.
- Finite volume errors of order 1–3% (for gold-plated quantities!).
    - In principle, remove using chiral perturbation theory; current volumes probably too small.

- Perturbative (or nonperturbative) matching essential to connect lattice quantities to continuum.
    - ◇ E.g., for f_D use

$$J_{\rm cont} = Z\, J_{\rm latt} + a^2 \Delta J$$

where

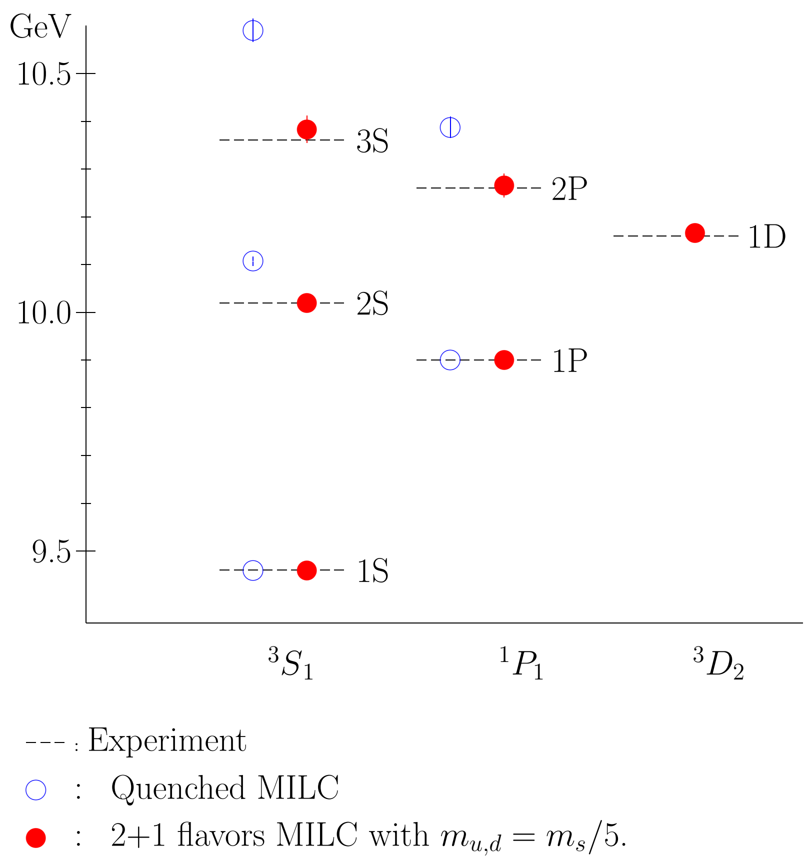
$$Z = 1 + c_1\, \alpha_s(\pi/a) + c_2\, \alpha^2(\pi/a) + \cdots$$

and $\alpha_s \approx 0.25$ for current as ($\Rightarrow$ need 2nd order for few % errors!).

    - ◇ (Super) Computer automation is essential (e.g., 3-loop calculations by Mason and Trottier with improved actions).
    - ◇ Usually the dominant error.

Sampler of Recent Calculations

Υ Spectrum

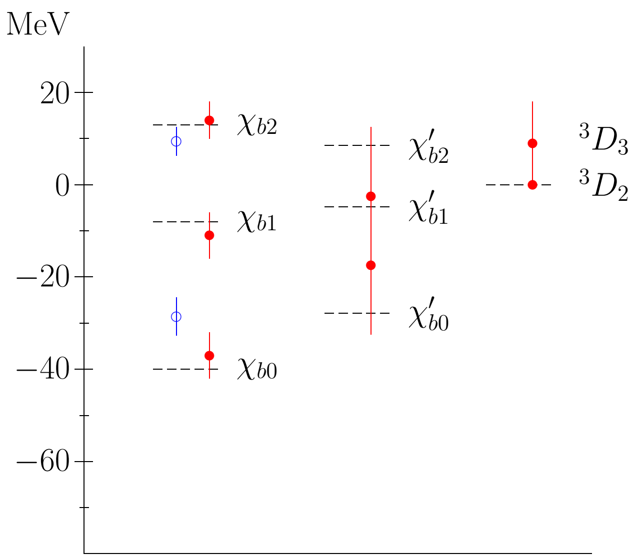


Note:

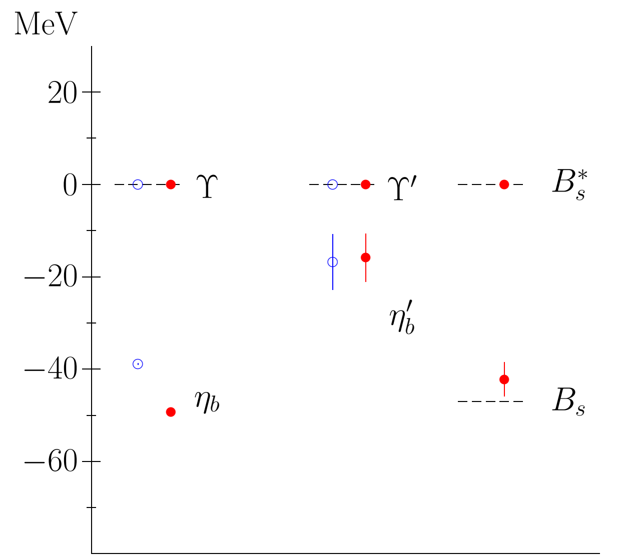
- Direct from QCD path integral; no potential model. . . .
- Tests/tunes b quark action for use in B physics $\Rightarrow$ overconstrained.
- Other tests: leptonic widths, photon transitions, fine structure.
- Statistical and systematic errors of 2–3%; 1S and 1P used in tuning.

Davies, Gray et al. (HPQCD, 2002).

Υ Fine Structure



--- : Experiment
○ : Quenched
• : 2+1 flavours MILC with $m_{u,d} = m_s/5$.

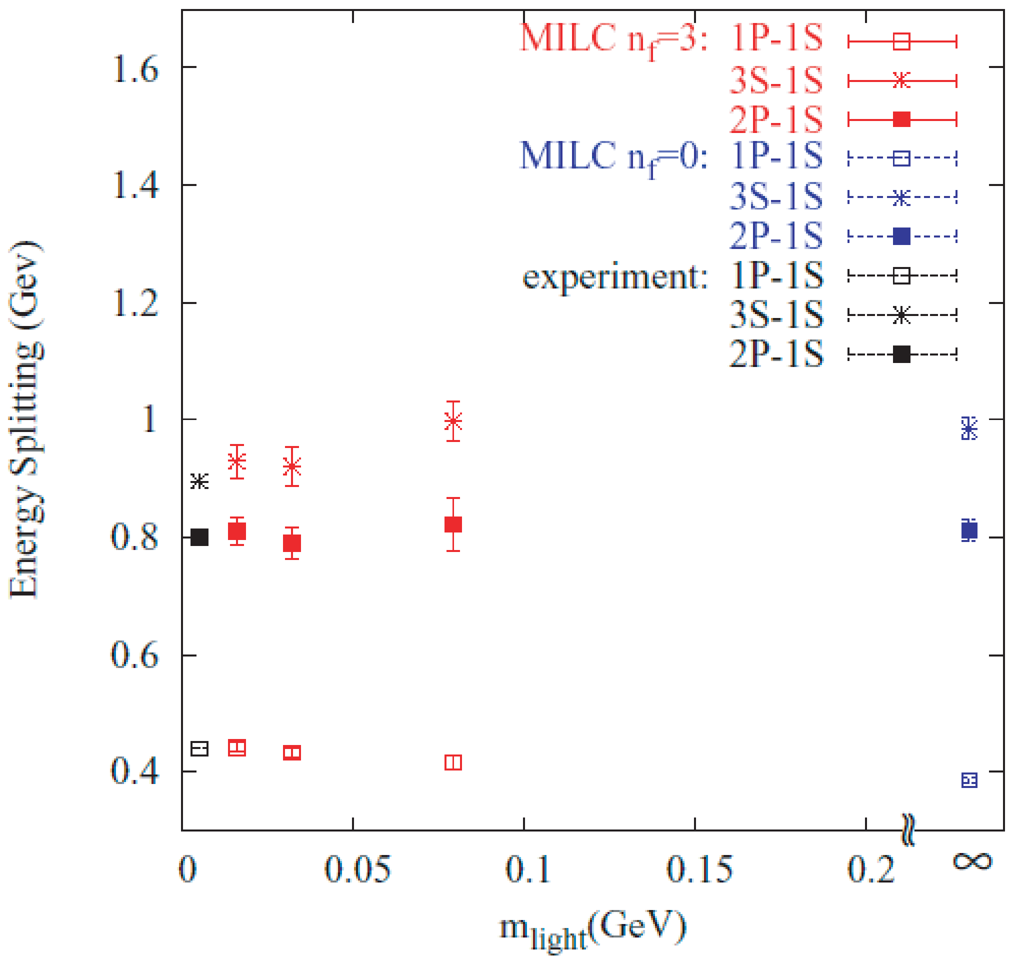


--- : Experiment
○ : Quenched
• : 2+1 flavours MILC with $m_{u,d} = m_s/5$.

Note: 20–30% systematic error due to use of tree-level pert'n theory.

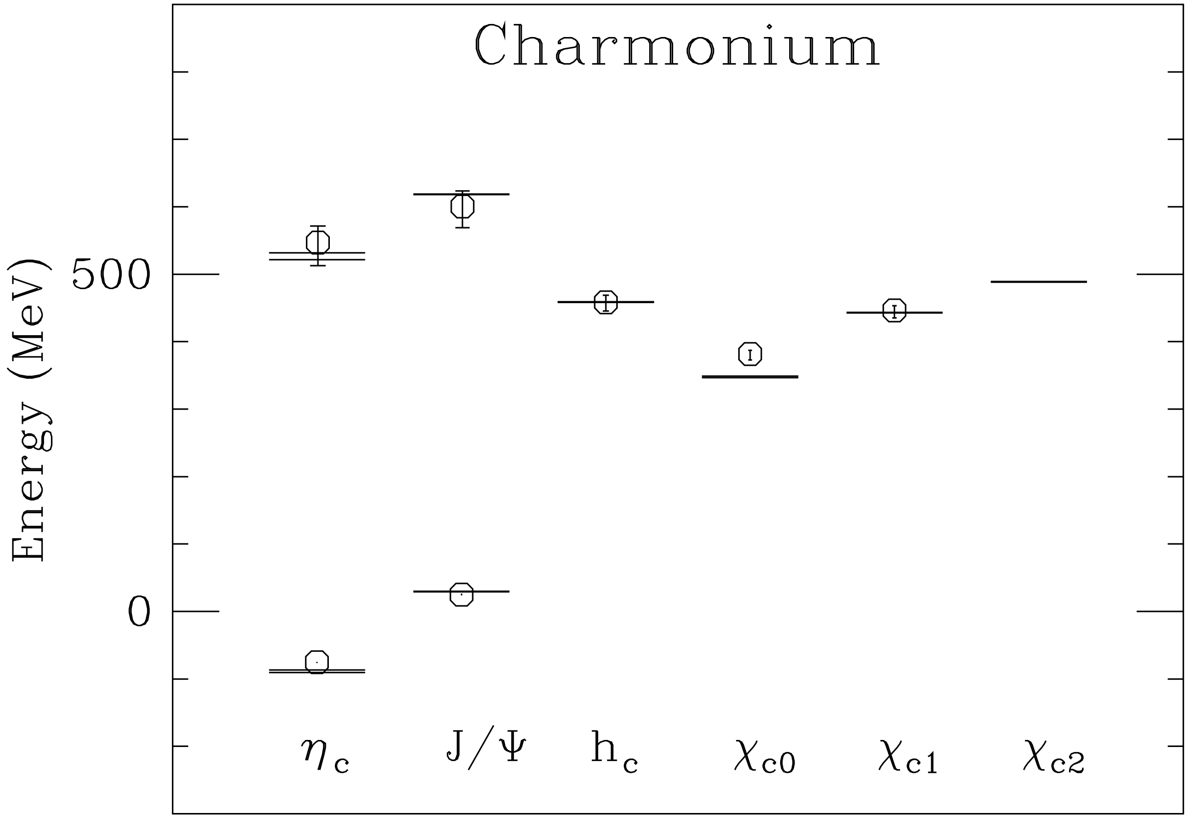
Davies, Gray et al. (HPQCD, 2002).

Υ Splittings: Insensitive to u, d Mass



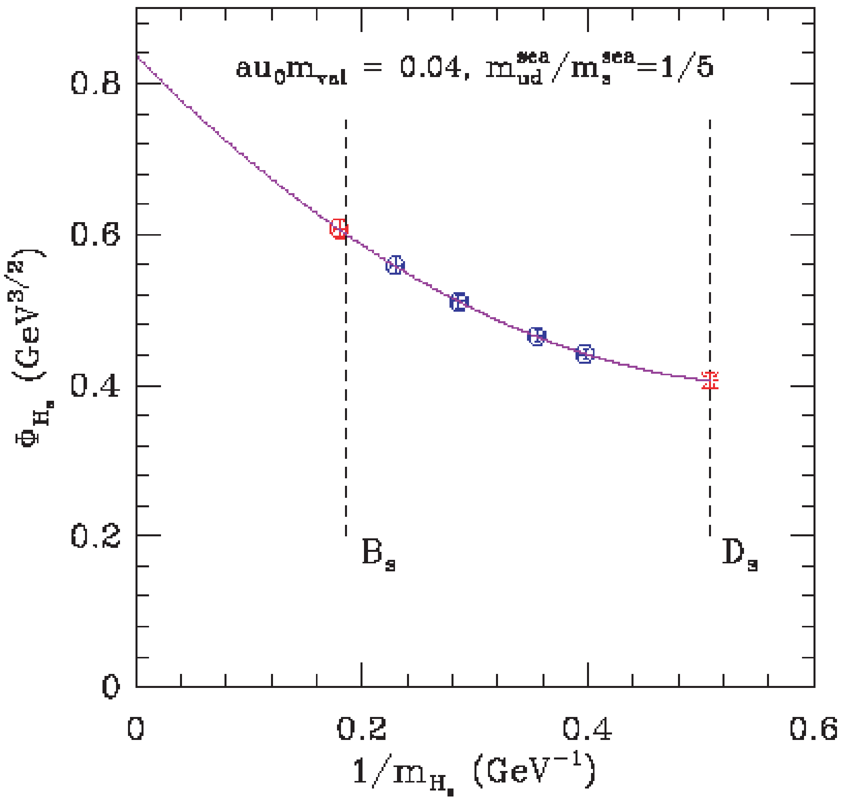
Davies, Gray et al. (HPQCD, 2002).

J/ψ Spectrum



A. Kronfeld's talk at Lattice '03 (Fermilab Collaboration, 2003).

$\Phi_{H_s} \equiv f_{H_s} \sqrt{m_{H_s}}$ versus $1/m_{H_s}$



Implies:

$$f_{B_s} = 262 \pm 28 \text{ MeV}$$
$$f_{D_s} = 289 \pm 41 \text{ MeV}$$

where dominant error is due to use of 1st-order perturbation theory (***).

M. Wingate's talk at Lattice '03 (HPQCD, 2003).

D_s Spectrum: First $n_f = 3$ Results

$$D_s(0^+) - D_s(0^-) = \begin{cases} 370\,(27)\,\text{MeV} & \text{Fermilab collaboration} \\ 360\text{–}400\,\text{MeV} & \text{HPQCD collaboration} \\ 350\,\text{MeV} & \text{Experiment} \end{cases}$$

$$D_s(1^+) - D_s(1^-) = \begin{cases} 388\,(20)\,\text{MeV} & \text{Fermilab collaboration} \\ 351\,\text{MeV} & \text{Experiment} \end{cases}$$

But $D_s(0^+, 1^+)$ close to threshold

$\Rightarrow$ *not* gold-plated

$\Rightarrow$ lattice results should be 5–15% high

$\Rightarrow$ $0^+, 1^+$ are likely cs P-states.

Lattice QCD results: P. Mackenzie and C. Davies. See also Bardeen et al, hep-ph/0305049.

Not Covering…

- New 3-loop accurate determination of $\alpha_{\overline{\rm MS}}(M_Z)$ from $n_f = 3$ lattice QCD (Mason, Trottier et al, HPQCD 2003).
- Preliminary studies of semileptonic form factors for Bs and Ds with $n_f = 3$ and small quark masses; moving NRQCD for high-recoil. (Fermilab and HPQCD collaborations)
- Quenched and only somewhat unquenched calculations of large variety of quantities in heavy-quark physics, QCD thermodynamics, hadronic physics. Technology well developed; needs $n_f = 3$ and small light-quark masses. (See earlier reviews.)
- Domain-wall and GW fermion algorithms — potentially very important in long-term.

Conclusion

Few percent precision $\Rightarrow$ superb opportunity for lattice QCD to have an impact on particle physics.

- LQCD essential to high-precision B/D physics at BaBar, Belle, CLEO-c, Fermilab…
- *Predicting* CLEO-c, BaBar/Belle results $\Rightarrow$ much needed credibility for LQCD.
- Critical to focus on gold-plated quantities.
- Landmark in history quantum field theory: quantitative verification of nonperturbative technology (c.f., 1950s).
- Ready for beyond the Standard Model, strong coupling beyond QCD?