Neutrino Physics: Open Theoretical Questions

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- What we have learned?
- Open theoretical questions
- Bottom-up
- How we might go...
1. What we have learned?

Neutrino masses and lepton mixing:
Summary
Any problem?

- ~ 2 higher Ar-production rate than Homestake result
- Absence of the upturn of the spectrum

Best fit point:

\[
\Delta m_{12}^2 = 7 \times 10^{-5} \text{ eV}^2 \\
\tan^2 \theta_{12} = 0.4 \\
\sin^2 \theta_{13} \sim 0
\]
Light sterile neutrino

\[ R_\Delta = \frac{\Delta m_{01}^2}{\Delta m_{21}^2} \]

\( \alpha \) - mixing angle of sterile neutrino

Dip in the survival probability:
- reduces the Ar-production rate
- suppresses the upturn of spectrum

Motivation for the low energy solar neutrino experiments
BOREXINO, KamLAND MOON, LENS ...

P. de Holanda, A.S.
\[ \sin^2 2\theta_{23} = 1.0 \]
\[ \Delta m_{32}^2 = 2.0 \times 10^{-3} \text{ eV}^2 \]

SuperKamiokande:
\[ \Delta m_{32}^2 = (1.3 - 3.0) \times 10^{-3} \text{ eV}^2 \]
(90 % C.L.)

\[ \sin^2 2\theta_{23} > 0.9 \]

Confirmed by
MACRO,
SOUDAN
K2K

Combined analysis of CHOOZ,
atmospheric (SK) and solar data:
\[ \sin^2 2\theta_{13} < 0.067 \ (3\sigma) \]

G.L. Fogli et al, hep-ph/p0308055
In the sub-GeV sample

$$\frac{F_e}{F_e^0} - 1 = P_2 (r c_{23}^2 - 1)$$

``screening factor``

$P_2 = P(\Delta m^2_{12}, \theta_{12})$ is the $2\nu$ transition probability

In the sub-GeV sample $r = F_\mu^0 / F_e^0 \sim 2$

The excess is zero for maximal $23$- mixing

Searches of the excess can be used to restrict deviation of the $2-3$ mixing from maximal

Zenith angle and energy dependences of the e-like events
\[
F(\nu_e) = F^0(\nu_e) + p \Delta F^0
\]

\[\Delta F^0 = F^0(\nu_\mu) - F^0(\nu_e)\]

\(p\) is the permutation factor

\(p\) depends on distance traveled by neutrinos inside the earth to a given detector:

\[
d = \begin{cases} 
4363 \text{ km} & \text{Kamioka} \\
8535 \text{ km} & \text{IMB} \\
10449 \text{ km} & \text{Baksan}
\end{cases}
\]

The earth matter effect can partially explain the difference of Kamiokande and IMB: spectra of events

Normal hierarchy is preferable

H. Minakata, H. Nunokawa, J. Bahcall, D. Spergel, A.S.
F. Feruglio, A. Strumia, F. Vissani

Neutrinoless double beta decay

Kinematic searches, cosmology

Both cosmology and double beta decay have similar sensitivities

Sensitivity limit

\[ m_{ee} = \sum_k U_{ek} m_k e^{i\phi(k)} \]
Mass spectrum and mixing

|U_{e3}|^2

|ν_3|

|ν_2|

|ν_1|

Normal mass hierarchy (ordering)

∆m^2_{atm}

∆m^2_{sun}

|U_{e3}|^2

|ν_2|

|ν_1|

|ν_3|

Inverted mass hierarchy (ordering)

∆m^2_{sun}

∆m^2_{atm}

Type of mass spectrum: with Hierarchy, Ordering, Degeneracy

Type of the mass hierarchy: Normal, Inverted

U_{e3} = ?
Leptonic Unitarity Triangle

\[ U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.0 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix} \]

Global fit of the oscillation data $1\sigma$

M.C. Gonzalez-Garcia, C. Pena-Garay

Can we reconstruct the triangle? Can we use it to determine the CP-violating phase? Y. Farsan, A.S.

Problem: coherence (we deal with coherent states and not mass eigenstates of neutrinos)

\[ |U_{e2} U_{\mu 2}^*|, |U_{e3} U_{\mu 3}^*|, |U_{e1} U_{\mu 1}^*| \]
Ultimate oscillation anomaly?

**CPT-violation**

After KamLAND:
- G. Barenboim,
- L. Borissov, J. Lykken

**Non-standard Interactions**

Disfavored by atmospheric neutrino data, no compatibility of LSND and all-but LSND data below 3σ-level

- K.Babu, S Pakvasa
- M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz

**Sterile neutrino (3 + 1)-scheme (3 + 2)?**

- O. Peres, A.S.
- M. Sorel, J. Conrad, M. Shaevitz
Generic possibility of interest even independently of the LSND result

Generation of large mixing of active neutrinos due to small mixing with sterile state

Produces uncertainty in interpretation of results

The problem is

\[ P \sim |U_{e4}|^2 |U_{\mu 4}|^2 \]

Restricted by short baseline experiments CHOOZ, CDHS, NOMAD

2 - 3\( \sigma \) below the observed probability
3 + 2 scheme

\[ \nu_e \quad \nu_x \quad \nu_s \quad \nu_{\tau} \]

\[ \Delta m^2_{\text{atm}} \quad \Delta m^2_{\text{sun}} \]

\[ \Delta m^2_{\text{LSND}} \quad \Delta m^2_{\text{LSND}}' \]

M. Sorel, J. Conrad, M. Shaevitz

FINeSE
Main features

- Smallness of masses:
  \[ m_\nu < (1 - 2) \text{ eV} \]
  \[ m_\nu > \sqrt{\Delta m_{23}^2} > 0.04 \text{ eV} \]
  \[ m_\nu \ll m_1, m_q \] (at least for one mass)

- Hierarchy of mass squared differences:
  \[ |\Delta m_{12}^2 / \Delta m_{23}^2| = 0.01 - 0.15 \]

- No strong hierarchy of masses:
  \[ |m_2/m_3| > |\Delta m_{12}^2 / \Delta m_{23}^2| = 0.18 + 0.22 - 0.08 \]

- Bi-large or large-maximal mixing between neighboring families (1-2) (2-3):

- Small mixing between remote families (1-3):

\[ \sigma \]

\[ 1\sigma \]

\[ 2-3 \]

\[ 1-2 \]

\[ 1-3 \]

\[ 0 \]

\[ 0.2 \]

\[ 0.4 \]

\[ 0.6 \]

\[ 0.8 \]

| \[ \sin \theta \]|
Several key elements are unknown yet which leads to variety of possible interpretations.

- Absolute mass scale, $m_1$
- Type of spectrum
  - hierarchical
  - partially degenerate
  - quasi degenerate
- CP-violating phases, especially Majorana phases
- Deviation of 2-3 and 1-2 mixings from maximal
- Phenomenological and experimental problems

$\sin \theta_{13}$

Type of mass hierarchy
Ordering of states
(sign of $\Delta m_{13}^2 = m_1^2 - m_3^2$)

- normal
- inverted

Existence of new neutrino states
2. Open theoretical questions

What does all this mean?

(results on neutrino masses and mixing)
What is the origin of neutrino mass?

- we do not know yet the origin of quark and charged lepton masses where information is more complete.
- for neutrinos the problem can be even more complex.
- the hope is that neutrinos can shed some light on whole problem of fermion masses.

Why neutrino masses are small?

- small in comparison with charged leptons and quarks masses.
- what are relations with other mass scales in nature?
  e.g., dark energy scale?
How the observed pattern of the lepton mixing is generated?

- two large mixings and one small (zero)?
- one maximal mixing?
- what are relations between mixing angles?

Why the lepton mixing is large? Why it is so different from quark mixing?

- may be correct question is why the quark mixing is so small?
  In quark sector the smallness of mixing can be related to strong mass hierarchy

Do neutrinos show certain flavor or horizontal symmetry?

- if so, is this symmetry consistent with quark masses and mixing?
- ad hoc introduced symmetries for neutrinos only do not look appealing
Are results on neutrino masses and mixing consistent with

- quark-lepton symmetry?
- Grand Unification?

If new light (sterile) neutrino(s) exist

- what is their nature?
- why they are light?
What are implications of the neutrino results for

- GUT
- SUSY
- Extra Dimensions?
- Strings

vice versa

What these theories can tell us about neutrinos?
3. Bottom-Up
Experimental results on \( \Delta m_{ij}^2 \), \( \theta_{ij} \), and \( m_{ee} \)

Mass matrix unifies information contained in masses and mixing

Reconstruct the neutrino mass matrix in the flavor basis

Identify symmetry and underlying dynamics

Identify symmetry scale and symmetry basis

Renormalization group effects

\[
m = U^* m_{\text{diag}} U^+ 
\]

\[
m_{\text{diag}} = \text{diag}(m_1 e^{-2i\rho}, m_2, m_3 e^{-2i\sigma}) 
\]

\[
U = U(\theta_{ij}, \delta) 
\]

\[
m_2 = \sqrt{m_1^2 + \Delta m_{21}^2} 
\]

\[
m_3 = \sqrt{m_1^2 + \Delta m_{31}^2} 
\]
\[ m_3 / m_2 = 5 \]
\[ m_1 = 0.006 \text{ eV} \]
\[ \sin^2 2\theta_{23} = 1 \]
\[ \sin \theta_{13} = 0.1 \quad \delta = 0 \]

\[
\begin{pmatrix}
0 & 0 & \lambda \\
0 & 1 & 1 \\
\lambda & 1 & 1
\end{pmatrix}
\]

a).

\[
\begin{pmatrix}
0 & \lambda & 0 \\
\lambda & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}
\]

b).

\[
\begin{pmatrix}
\lambda^2 & \lambda & \lambda \\
\lambda & 1 & 1 \\
\lambda & 1 & 1
\end{pmatrix}
\]

\[
\lambda \sim 0.2
\]

c).

\[
\begin{pmatrix}
q^4 & q^3 & q^2 \\
q^3 & q^2 & q \\
q^2 & q & 1
\end{pmatrix}
\]

\[ q \sim 0.7 \]
Normal ordering

\[ m_3 / m_2 = 2 \]
\[ m_1 = 0.027 \text{ eV} \]
\[ \sin^2 \theta_{23} = 1 \]
\[ \sin \theta_{13} = 0.1 \quad \delta = 0 \]

Flavor alignment

\[
\begin{pmatrix}
q^4 & q^3 & q^2 \\
q^3 & q^2 & q \\
q^2 & q & 1
\end{pmatrix}
\]
\[ q \sim 0.7 \]
Quasi-degeneracy

\[ m_3 / m_2 = 1.01 \]
\[ m_1 = 0.35 \text{ eV} \]
\[ \sin^2 2\theta_{23} = 1 \]
\[ \sin \theta_{13} = 0.1 \quad \delta = 0 \]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0 \\
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix}
\]

M. Frigerio, A.S
\( m_3 / m_2 = 0.5 \)
\( m_3 = 0.029 \text{ eV} \)
\( \sin^2 2\theta_{23} = 1 \)
\( \sin \theta_{13} = 0.1 \quad \delta = 0 \)
Inverted hierarchy

\[ m_3 / m_2 = 0.1 \]
\[ m_3 = 0.005 \text{ eV} \]
\[ \sin^2 2\theta_{23} = 1 \]
\[ \sin \theta_{13} = 0.1 \quad \delta = 0 \]

a).
\[
\begin{pmatrix}
0.7 & 1 & 1 \\
1 & 0.1 & 0.1 \\
1 & 0.1 & 0.1
\end{pmatrix}
\]

b).
\[
\begin{pmatrix}
1 & < 0.1 & < 0.1 \\
< 0.1 & 0.5 & 0.5 \\
< 0.1 & 0.5 & 0.5
\end{pmatrix}
\]
1). Large variety of different structures is still possible, depending strongly on unknown $m_1$, type of mass hierarchy, Majorana phases $\rho$ and $\sigma$, weaker dependence is on $\sin\theta_{13}$ and $\delta$.

2). Generically the hierarchy of elements is not strong: within 1 order of magnitude. Although, matrices with one or two zeros are possible.

3). Structures (in the flavor basis):
   - with dominant diagonal elements ($\sim I$), or dominant $\mu\tau$-block,
   - with dominant $e$-row elements, ($ee-, \mu\tau-, \tau\mu$-) elements, etc.,
   - democratic structures,
   - with flavor alignment,
   - non-hierarchical structures with all elements of the same order
   - with flavor disordering,
   - with zeros and various equalities of matrix elements.

4). Typically hierarchical structures appear for $\rho$ and $\sigma$ near $0, \pi/2, \pi$.

5). The structures can be parameterized in terms of power of small parameter $\lambda = 0.2 - 0.3$ consistent with Cabibbo mixing.
Do neutrino results on masses and mixing or the neutrino mass matrix show some symmetry?

Is the neutrino mass matrix consistent with symmetries suggested for quarks?

- \( L_e - L_\mu - L_\tau \)
- Discrete symmetries: \( A_4, S_3, Z_4, D_4 \)
- \( U(1) \) charges: discrete free parameters, also coefficients \( \sim O(1) \) in front of
- \( SU(2) \)
- \( SU(3) \)

Treat quarks and leptons differently.

- in the Froggatt-Nielsen context can describe mass matrices both quarks and leptons.
- Complicated higgs sector to break symmetry too restrictive...
4. How we might go ...
Minimalistic approach:

Relate features of the neutrino masses and mixing with already known difference of neutrino and quarks and charged leptons.

- Neutrality $Q_{\gamma} = 0$, $Q_c = 0$
- Right handed components, if exists, are singlet of $SU(3) \times SU(2) \times U(1)$
- Unprotected by this symmetry
- Possibility to be a Majorana particle (Majorana mass term)
- Can mix with singlets of the SM symmetry group
- Can propagate in extra dimensions
- Can have large Majorana masses $M_R \gg V_{EW}$

$q - 1$, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

Minimal number of new concepts

Properties of mass spectrum and mixing

Is this enough?
Seesaw

\[
\begin{pmatrix}
0 & m_D \\
m_D & M_R
\end{pmatrix}
\]

\[
m_\nu = - m_D^T M_R^{-1} m_D
\]

(type I)

\[
m_D = Y v_{EW}
\]

\[
M_R = f_S <S> = f_S v_R
\]

If the SU(2) triplet, \(\Delta_L\), exists with interaction \(f_\Delta l^T l \Delta_L + \text{h.c.}\), then \(f_\Delta l^T l \Delta_L + \text{h.c.}\)

\[
m_\nu = m_L - m_D^T M_R^{-1} m_D
\]

(type II)

If \(\Delta_L\) is heavy, induced VEV due to the interaction with doublet \(<\Delta_L> = \langle H\rangle^2 / M\)

In SO(10): \(\Delta_L\) and S are in the same 126, \(f_\Delta = f_S = f\)

\[
m_\nu = f \lambda \frac{v_{EW}^2}{v_R} - m_D^T f^{-1} m_D = \frac{v_{EW}^2}{v_R} (f \lambda - Y^T f^{-1} Y)
\]

Flavor structure of two contributions correlates
The number of the RH neutrinos can differ from 3 limit when one of $\nu_R$ is very heavy $M \sim M_{Pl}$ motivated by horizontal SU(2)$_H$

- one massless neutrino
- less number of parameters

Beyond SM: many heavy singlets...

...string theory

R.N. Mohapatra
J. Valle

Three additional singlets S which couple with RH neutrinos

$\mu \ll M_D$
$\mu \gg M$
$\mu \sim M_{GU}$, $M \sim M_{Pl}$ explains intermediate scale

$m_\nu = - m_D^T M_D^{-1T} \mu M_D^{-1} m_D$
GUT provide large scale comparable to the scale of RH neutrino masses

One can argue that GUT (+ seesaw) can naturally lead to large lepton mixing, or inversely, that large lepton mixing testifies for GUT

1. Suppose that all quarks and leptons of a given family are in a single multiplet $F_i$ (as 16 of SO(10)).

2. Suppose that all yukawa couplings are of the same order thus producing matrices with large mixing.

3. If Dirac masses are generated by a unique higgs multiplet, (10 of SO(10)), the mass matrices of the up- and down-components of the weak doublets will be identical, and so will be diagonalized by the same rotation.

As a result,
- no mixing appears for quarks
- masses of up and down components will be proportional each other

4. In contrast to other fermions RH neutrinos have additional yukawa interactions (with 126) which generate the Majorana mass terms.

5. Since those (Majorana type) Yukawa couplings are also of generic form they produce $M$ with large mixing which leads then to large lepton mixing.

Need to be slightly corrected
Strong hierarchy of the quark and charged lepton masses

In this scenario $m_D = \text{diag}(m_u, m_c, m_t)$

$$m_v = m_L - m_D^T M_R^{-1} m_D$$

Then for generic $M_R$ the seesaw of the type I produces strongly hierarchical matrix with small mixing

Possible solutions:

- Type II seesaw: no dependence on $m_D$
- Special structure of $M_R$ which compensate strong hierarchy in $m_D$
- Substantial difference of Dirac matrices of quarks and leptons $m_D(q) \neq m_D(l)$
Can the same mechanism (seesaw) which explains a smallness of neutrino mass also explain large lepton mixing? Large lepton mixing is an artifact of seesaw?

Quark-lepton symmetry
\[ m_D \sim m_{\text{up}}, \ m_l \sim m_d, \] small mixing in Dirac sector

Special structure of \( M_R \)

Large lepton mixing

Two possibilities:

Strong (``quadratic'') hierarchy of the right handed neutrino masses:
\[ M_{iR} \sim (m_{i\text{up}})^2 \]

Strong interfamily connection (pseudo Dirac structures)
\[ M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix} \]
Leptogenesis gives strong restrictions

In the hierarchical case the lower bound on the lightest mass

\[ M_1 > 4 \times 10^8 \text{ GeV} \]

W. Buchmuller
P. di Bari M. Plumacher,
S. Davidson, A. Ibarra

Only in particular cases with strong degeneracy: \( M_1 = M_2 \) required asymmetry can be produced

E. Kh. Akhmedov,
M. Frigerio, A.S.
Structure of the mass matrix generated by the type II (triplet) seesaw can be related to quark and lepton masses.

Generates neutrino masses \( m_L \)

\[ m_L \sim Y_{126} \]

126 of SO(10)

gives contribution to quark and lepton masses (Georgi-Jarlskog relation)

\[ m_b - m_\tau \sim Y_{126} \]

(subtraction of 10-contribution)

Large 2-3 mixing needs \( b - \tau \) unification

\( b - \tau \) unification: element \( (Y_{126})_{33} \sim (Y_{126})_{23} \ll 1 \)

--> large 2-3 lepton mixing

Successful leptogenesis is possible with participation of the scalar triplet

K. Babu, R. Mohapatra, Matsuda, B. Bajc, G. Senjanovic, F. Vissani, R. Mohapatra, Goh, Ng

T. Hambye, G. Senjanovic
Large mixing from the Dirac neutrino mass matrix

\[ m_D = m \begin{pmatrix} * & * & \varepsilon \\ * & * & 1 \\ * & * & 1 \end{pmatrix} \]

\[ M_R^{-1} = M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

(* \ll \varepsilon)

Seesaw gives:

\[ m_\nu = \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \varepsilon & 1 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix} \]

In another version it may coincide with seesaw enhancement:
Single RH neutrino dominance is realized when other RH neutrinos are heavy = strong hierarchy
Large mixing follows from charged lepton mass matrix.

Non-symmetric mass matrices.

No contradiction with GUT:
- In SU(5): LH components of leptons are unified with RH components of quarks: \( 5 = (d^c, d^c, d^c, l, \nu) \)

\[
\begin{pmatrix}
\eta & 0 & 0 \\
0 & 0 & \varepsilon \\
0 & -\varepsilon & 1
\end{pmatrix}
\quad \quad 
\begin{pmatrix}
0 & \delta & d' \\
\delta & 0 & \sigma + \varepsilon \\
\delta' & -\varepsilon & 1
\end{pmatrix}
\]

\( \eta \ll \delta, \delta' \ll \varepsilon \)

Also possible in SO(10) if it is broken via SU(5).

Double lopsided (for both large mixings).

Hybrid possibilities: large 2-3 mixing from charged lepton mass matrix, large 1-2 mixing from neutrino mass matrices.
Mixing is small at the Unification scale (similar to quark mixing) running to low energies enhancement of mixing.

Requirement:

Large mixing $\leftrightarrow$ Quasi-degenerate spectrum

$$\frac{d \sin \theta_{23}}{dt} \sim (\sin \theta_{12} U_{\tau 1} D_{31} - \cos \theta_{12} U_{\tau 2} D_{32})$$

$t = 1/8\pi^2 \log q/M$

$$D_{ij} = (m_i + m_j)/(m_i - m_j)$$

Enhancement when neutrinos become more degenerate

Requires fine tuning of the initial mass splitting and radiative corrections

In MSSM both 1-2 and 2-3 mixings can be enhanced. In SM ?

If masses from Kahler potential: large mixing infrared fixed point

Generation of small elements radiatively: $\Delta m_{12}^2$, $\sin \theta_{13}$


J.A. Casas, J.R. Espinosa, I. Navarro

S. Petcov, A.S. A. Joshipura, M. Lindner
For hierarchical RH neutrino spectrum gives bound on

- Mass of the lightest RH neutrino $M_{1R}$
- Effective parameter $\tilde{m}_1$
  which determines the washout effect
  Probe of $(Y Y^*)_{ij}$

$M_{1R} > 4 \times 10^8 \text{ GeV}$

$m_\nu < 0.1 \text{ eV}$ excluding degenerate spectrum (?)

for type II seesaw: still possible

G. Senjanovic
T. Hambye

Renormalization effects of RH neutrinos

Renormalization effects between the scale $M_{iR}$ and GUT e.g., on $m_b - m_\tau$ mass relation

F. Vissani, A.S.,
H. Murayama, R. Rattazzi
A. Brignole
Superpotential

$$W_{lep} = e^c Y e^c y^1 H_1 + \nu^c Y \nu y^1 H_2 + \frac{1}{2} \nu^c M_R \nu^c$$

Structures relevant for seesaw \((Y, M_R)\)

Imprinted into structure of SUSY (slepton) sector

Assumptions:

1. Universal soft masses \((m_0^2, A_0)\) at high scale \(M_X\)
2. No new particles apart from those in MSSM

Contribution to the low energy left handed slepton mass matrices:

$$\begin{align*}
(m_s^2)_{ab} &= m_a^2 \delta_{ab} - \frac{1}{8\pi^2} (3m_0^2 + A_0^2) (Y^+)_{ai} (Y)_{ib} \log(M_X / M_{iR}) \\
\end{align*}$$

diagonal part

If large lepton mixing originates from the Dirac matrix (lopsided models, versions of SRHN dominance) \((Y)_{\mu i}, (Y)_{ie}\) are large

\[
B(\mu \rightarrow \gamma \ e) \sim 10^{-11} - 10^{-12}
\]

At the level of present bound

\[
(m_S^2)_{\mu e} = \frac{1}{8\pi^2}(3 m_0^2 + A_0^2) (Y^+)_{\mu i} (Y)_{ie} \log(M_X/M_{iR})
\]
Other mechanisms

Radiative mechanisms
- Zee (one loop, generalized)
- Zee-Babu (two loops)
- Trilinear R-violating couplings

Bi-linear R-parity violation

Extra Dimensions
- Large extra D (ADD)
- Warped extra D (RS)
- Infinite extra D (Dvali-Poratti)
- ...

Dynamical symmetry breaking
Technicolor

Little Higgs

Deconstruction

Can accommodate neutrino masses
produce some interesting features
Main open question: what is behind obtained results? Preference? Probably seesaw, and probably associated with Grand Unification. Although other mechanisms are not excluded and can give important or sub-leading contributions.

How to check our ideas about neutrinos? Future experiments will perform precision measurements of neutrino parameters. Apart from that we will need results from non-neutrino experiments:
- from astrophysics and cosmology
- from searches for proton decay and rare decays
- from future high energy colliders.

Enormous progress in determination of the neutrino masses and mixings, studies of properties of mass matrix. Still large freedom in possible structures exists which leads to very different interpretations.