Abstract
The equations of state for degenerate electron and neutron gases are studied in the presence of magnetic fields. After including quantum effects, it is found that some magnetized stars can be unstable based on the criterium of stability of pressures. It is shown that strongly magnetized white dwarfs should collapse producing a supernova type Ia, whilst hypermagnetized neutron stars cannot stand their own super strong magnetic field and must implode, too. A comparison of our results with some of the available observational data for other stars is also presented.

Introduction
Many stellar objects are known to be endowed with large magnetic fields.
- white dwarfs with surface $B$ from $10^{12}$ to $10^{15}$ G have been discovered (C. Kamper et al. 1976).
- Magnetic fields of strength of order $10^{15}$ G have been suggested to exist in the core of neutron stars at pulsars (Chakrabarti et al. 1987).

Electrons (e) and neutrons ($n$) gases in strong magnetic fields are considered with the aim to study the equation of state (EoS) of white dwarfs, neutron stars and supernovae. It was found some effect, which is described below, which can be understood as manifestations of the Lorentz force in the extreme quantum regime. Such effects open the possibility of an instability which may lead to a magnetic quantum collapse of the gas under consideration. This effect appears as a critical curve relating the density of the star and its magnetic field.

Why appear this effect?
The loss of mirror symmetry of the particle spectrum $\rightarrow$ an anisotropy in the thermodynamic properties of the system $\rightarrow$ an instability in its presence.
If the symmetry is axial, the instability is axial too.
That means $P_{\parallel} \neq P_{\perp}$
Two cases:
- It is a sufficient condition for collapse to occur if $P_{\parallel} \leq P_{\perp}$
- Quantum/Casimir effect may occur if $P_{\parallel} \geq P_{\perp}$

Energy-Momentum Tensor and Pressures
The Energy-Momentum tensor of matter $T_{\alpha\beta}$, $F_{\alpha\beta}$ is the electromagnetic field tensor, is

$$T_{\alpha\beta} = \frac{1}{2} (\partial \phi / \partial x^\alpha) (\partial \phi / \partial x^\beta) = -\pi_{\alpha\beta} + \frac{1}{4} F_{\gamma\delta} F^{\gamma\delta} \eta_{\alpha\beta} - f_{\alpha\beta}\Omega$$

$i = n, p, e$, the spatial components are

$$T_{\alpha\beta} = P_{\alpha\beta} - \Omega_{\alpha\beta} = -\pi_{\alpha\beta} - f_{\alpha\beta}\Omega$$

Quantum Case

$$\Lambda > 0 \rightarrow P_{\alpha\beta} < P_{\parallel}$$

with leads to a ferromagnetic behavior in the sense that $M = (1/2)B$ is now linear (but no spin-phonon coupling is assumed).

- charged fermions (mesons) used for describing White Dwarfs, Churchill et al. in PRL 84, 5261 (2000)

Our model of stars assume;
- Neutron gravitational potential (no grand unification) compress the pressure emanated by quantum gas.
- Relativistic Quantum Statistical Physics: Dense gas of fermions with magnetic moment $\neq 0$.

Calculation Procedure: to obtain
- the particle spectrum
- thermodynamic potential of the system
- magnetization of the system

these quantities allow to infer the equation of state (EoS) of the system

Degenerate Magnetized E-gas

Thermodynamical Potential

$$\Omega_e = \frac{k}{\nu} \sum_{\nu=\pm 1} \int \left( \frac{2}{\nu} m e^{-\nu m / kT} - B_{\nu} - (m^2 + 2\nu B_m) \right)$$

Chemical Potential

$$\mu_e = \frac{\sqrt{m^2 - e^2} - e^2}{\sqrt{m^2 - e^2}}$$

Density of Particles

$$\rho_e = \frac{2}{\nu} \sum_{\nu=\pm 1} \int \left( \sqrt{m^2 - e^2} + e^2 \right)$$

Magnetization

$$\rho_e = \frac{\sqrt{m^2 - e^2} - e^2}{\sqrt{m^2 - e^2}}$$

$$\psi = \frac{1}{\nu} \sum_{\nu=\pm 1} \int \left( \sqrt{m^2 - e^2} + e^2 \right)$$

Neutron Gas in a background of protons and electrons

For free neutrons (with the anomalous magnetic moment) in a B field the Dirac equation reads (Tenessen et al. 1998)

$$\left( D_{\alpha\beta} \psi_{\alpha\beta} + m + eA_{\alpha\beta} \psi_{\alpha\beta} \right) = 0$$

The significance of the equation is that the movement of the neutron magnetic moment is similar to the spin current

$$E(n, p, e) = \frac{1}{2} \left( E_n^2 + E_p^2 + E_e^2 + \sqrt{E_n^2 + E_p^2 + E_e^2} \right)$$

Magnetized neutron gas

Most of the observed neutron stars are polarized, i.e., fast rotating neutron stars with strong magnetic fields. They consist mostly of neutron matter with a high central density. As in the case of WDs, a similar study can be performed for neutron stars if we assume they are well described by a degenerate neutron gas.

Conclusion remarks
- We have presented a consistency theorem which could be applied directly the stability of main sequence and compact remnant stars whose structure is dictated by a combination of quantum and magnetic effects.
- We have shown that in general results of the theory are consistent with the current observational data. A major outcome is the possibility of some special configurations of highly magnetized WDs could collapse and trigger explosions similar to SNIa.
- Another consequence is that some neutron stars endowed with superstrong magnetic fields would be naturally unstable and therefore should collapse for their unperturbed magnetic field.

EOS neutron gas

The solution of the equation $\Delta = 0$ gives

$$N_e = 3.25 \times 10^{34} \Omega_9^{-3}$$

The instability region determined by the $(N_e, B)$ plane for magnetized neutron gas. The labelled points of quantum neutron stars (by $\lambda = 0.012, 0.025, 0.05, 0.1, 0.25, 0.5, 1.0$, respectively) are included in EOS neutron gas.