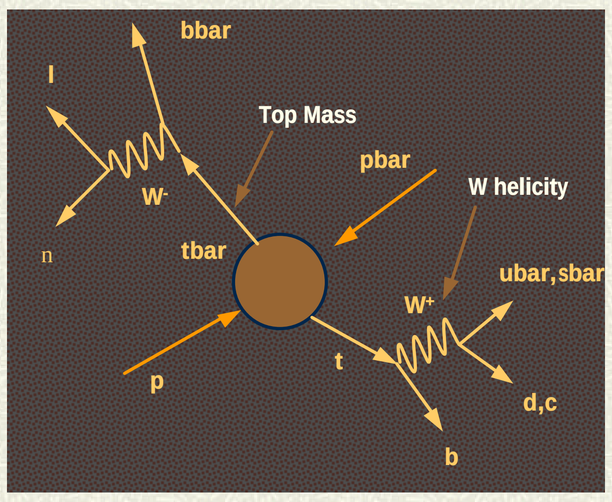
Optimal Use of Information to Measure Top Properties - F. Canelli, J. Estrada

Introduction

Abstract

We present a method developed at DÆ for extracting information from data through a direct calculation of a probability for each event. This probability, which is a function of any parameter of interest, is calculated by convoluting the differential cross section with the resolution and acceptance of the detector. The method is used to re-measure the mass of the top quark and to extract the fraction of longitudinal W bosons in the lepton + jets ttbar sample previously collected by the DÆ experiment during Run I of the Fermilab Tevatron. The new method yields a top mass of M_{top}=180.1 ± 3.6 (stat) ± 4.0 (syst) GeV/c², which corresponds to a significant reduction in the uncertainty on M_{top}. Assuming Standard Model coupling in the tbW vertex, we extract for the first time at DÆ the longitudinal fraction F_0=0.56 ± 0.31 (stat+M_{top}) ± 0.04 (syst).

The top quark was first observed in Run I of the Fermilab TeVatron, in ~100 events/pb of integrated luminosity collected at the CDF and DØ experiments [1,2]. Yet, because of its relatively large mass, and only recent discovery, the properties of the top quark are not well known. Because the top quark is so much more massive than the other fermions, it has been speculated that it may play a unique role within the Standard Model. It is therefore very important to understand the properties of the top quark, their degree of consistency with the Standard Model, and to check whether or not the top quark is truly exceptional.



Top Quark Production

In proton antiproton collisions at Tevatron energies, top quarks are primarily produced in pairs, either via qqbar or gluon fusion. At the Tevatron, the main contribution to the ttbar yield during Run I was from qqbar annihilation. This is purely the result of the fact that the parton distribution functions (PDFs) favor this channel at √s=1.8 TeV. In fact, about 90% of the top quarks are produced through the quark interaction.

Top Production and Decay

The top quark is detected indirectly via its decay products. It decays via the weak interaction, and according to the Standard Model is almost always expected to decay to a b quark and a W boson. This is followed by the W decaying into two quarks or a lepton and a neutrino.

The final state of the ttbar system has different topological classifications that depend on the decay of the W. This analysis uses the "lepton+jets" channel, and corresponds to one W decaying leptonically (into a e or a μ), while the other W decays hadronically. This channel has a branching fraction of about 30%.

Top Quark Mass

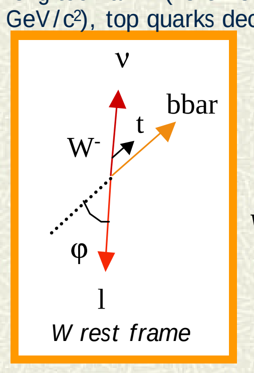
Although its value is not predicted, M_{top} is a fundamental parameter in the Standard Model. The best value of the top quark mass found from combining all channels at the Tevatron is,

$$M_{top}=174.3\pm5.1\,\mathrm{GeV/c^2}$$

The systematic error is dominated by the uncertainty on the jet energy scale. The measurement of M_W will improve significantly in the future, with an uncertainty of 27 GeV/c² being a realistic goal for Run II of the Tevatron. To be able to make maximum use of this precision measurement to constrain the mass of the Higgs, we need to measure the top mass with an uncertainty of less than 3 GeV/c². This will yield a prediction for the Higgs mass with an uncertainty of 40%.

Helicity of the W boson

A precise measurement of the helicity of the W boson examines the nature of the decay vertex tbW and provides a stringent test of Standard Model. The standard top quark has a V-A charged-current weak interaction. To conserve angular momentum, the emitted b quark (essentially massless, with its helicity dominantly negative, i.e., spin pointing opposite to its line of flight in the rest frame of the top quark) can point either along the top spin, with the spin projection of the W vanishing (i.e., longitudinally polarized), or in the direction opposite to the top spin, in which case the W must be left-hand polarized (negative helicity). Hence, for massless b quarks, a top quark can decay to a left-handed W (negative helicity W_) or a longitudinal W (zero helicity W_0). Therefore, in the Standard Model (using m_b=0, M_{top}=175 GeV/c² and M_W=80.4 GeV/c²), top quarks decay to longitudinal, left handed and right-handed W bosons with a branching ratio



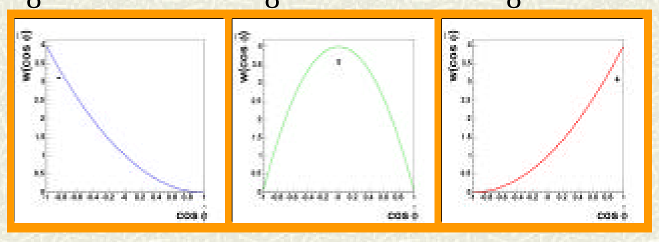
j_{lbbar} is the angle between the lepton and the spin direction of the top quark in the rest frame of the W boson.

$$F_- = \frac{2w}{1+2w} \approx 0.3 \qquad F_0 = \frac{1}{1+2w} \approx 0.7 \qquad F_+=0$$

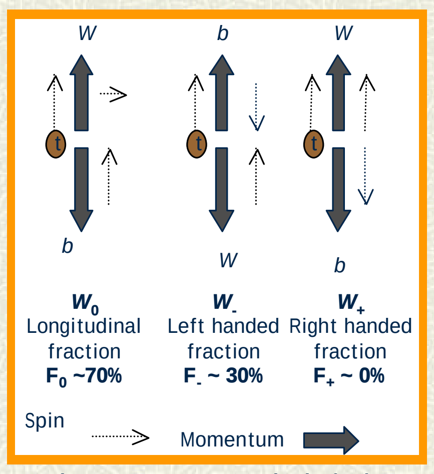
where $\omega = M_W^2/M_{top}^2$

and the angular factor of the matrix element for top decay is contained in

$$w(\cos j_{lbbar}) = F_-\cdot\frac{3}{8}(1-\cos j_{lbbar})^2 + F_0\cdot\frac{3}{8}(1-\cos^2 j_{lbbar}) + F_+\cdot\frac{3}{8}(1+\cos j_{lbbar})^2$$



and similar term for the top decaying to quarks.



Angular momentum conservation in the decay of the top quark.

Matrix Element Method

This method calculates the probability of each event of being signal as a function of the parameter we are measuring (F_0 and M_{top}) [6,7,8]. The probability for each event being background is also calculated. The results are combined in one lkelihood for the sample. These probabilities are functions of all momenta for the measured lepton and jets. In the previous analyses [3], the data was compared with two-dimensional templates and the features of individual events were averaged over the variables not considered in the template.

• The probability of an event as a function of a parameter α that we would like to extract is proportional to the differential production cross section. When the reaction is initiated by partons inside a proton, and the resolution of the detector cannot be ignored, then the cross section has to be folded over the parton distribution functions and detector resolution, and integrated over all the possible parton variables ***y*** leading to the observed set of variables ***x*** (I.e. jets and leptons):

$$\overline{P}_0(x;a) = \frac{1}{s}\int d^n s(y;a)\, dq_1 dq_2\, f(q_1) f(q_2)\, \boldsymbol{W(x,y)}$$

where $d^n s$ is the differential cross section, ***f(q)*** are the parton distribution functions, and ***W(y,x)*** is the probability that a parton level set of variables ***y*** will show up in the detector as the set of variables ***x***.

• All the processes that can contribute to the observed final state must be included in the probability. Therefore, we include the background probability

$$\overline{P}_0(x;c_1,\dots,c_K,a) = c_1\overline{\boldsymbol{P}}_{signal}\boldsymbol{(x;a)} + \sum_{i=2}^{K} c_i \overline{\boldsymbol{P}}_{background_i}\boldsymbol{(x)}$$

where i=1,2,…,K represent all possible contributions to the final state under study

• The probability that an event is accepted depends on the characteristics of the event, and not on the process that produced it. When the effects of the detector acceptance are included, the "measured probability ***P(x;a)***" can be related to the "production probability ***P_0(x;a)***":

$$\overline{P}(x;a) = Acc(x)\overline{P}_0(x;a)$$

where ***Acc(x)*** include all conditions for accepting or rejecting an event

• We insert last equations into a likelihood function for ***N*** events, which compares our prediction with the data. The best estimate of a parameter α, will therefore minimize this likelihood function

$$-\ln L(a) = -\sum_{i=1}^{N} \ln \overline{P}(x_i;a) + N\int \boldsymbol{Acc(x)\overline{P}(x;a)dx}$$

Signal Probability $P_{signal}(x;a)$

To calculate the signal probability of ttbar->lepton+jets, $P_{ttbar}(x;\alpha)$, an integration must be performed over 20 variables, corresponding to the vector momenta of the six final-state particles and the longitudinal momenta of the incident partons. Inside the integrals there are 15 δ-functions. Four for total energy and momentum conservation and eleven from the transfer functions (see **Transfer Functions**). The calculation of $P_{ttbar}(x;\alpha)$ therefore involves a five-dimensional integral. The 2 and 3 parton invariant masses, and the absolute value of momentum of one of the quarks were chosen as the integration variables. The reason for this is that the value of the matrix element $|M|^2$ is negligible, except near the four peaks of the Breit-Wigners corresponding to the two top and W decays.

Since we cannot distinguish which jet is associated with which quark, all possible combinations that can lead to the observed final state in the detector are included in the calculation. In addition, there are more than one solution to the neutrino kinematics, and all solutions that are consistent with energy and momentum conservation are taken into account. This effectively increases the ttbar cross section by a factor of 12 and requires the additional factor of 12 in the denominator.

$$P_{ttbar}(x,a) = \frac{1}{12 s_{t\bar t}} \int dr_1\, dm_1^2\, dM_1^2\, dm_2^2\, dM_2^2 \sum_{comb} |M_{ttbar}(a)|^2 \frac{f(q_1)f(q_2)}{|q_1||q_2|} f_6 W_{jet}(x,y)$$

where r_1 is the momentum of one of the jets, $\mathbf{m_1}$, $\mathbf{m_2}$ are the top masses, and $\mathbf{M_1}$, $\mathbf{M_2}$ are the W masses in the event, $\mathbf{f(q_1),f(q_2)}$ represent the parton distribution functions (CTEQ4) for incident partons, $\mathbf{q_1,q_2}$ are the initial parton momenta, f_6 is the six particle phase space, $\mathbf{W_{jet}(x,y)}$ is the probability of measuring **x** when y was produced in the collision (see **Transfer Functions**), $|\mathbf{M}|^2$ is the ttbar->lepton+jets matrix element [4].

Acceptance Integrals

Every criterion used for the selection of the events introduces a bias as a function of the parameter we are measuring, α. The acceptance integrals correct for this bias. They are performed via Monte Carlo method of integration.

Selection Criteria

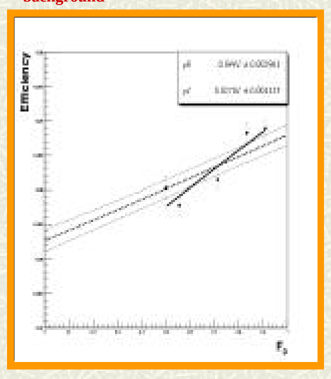
A set of selections was introduced to improve acceptance for lepton+jets from ttbar relative to background [1]. The standard requirements are:

- **Lepton:** E_T>20 GeV, $|\eta^e|$<2, $|\eta^\mu|$<1.7
- **Jets:** ≥4, E_T>15 GeV, $|\eta|$<2
- **Missing E_T** > 20 GeV
- **"E_T^W"** > 60 GeV ; $|\eta_W|$<2

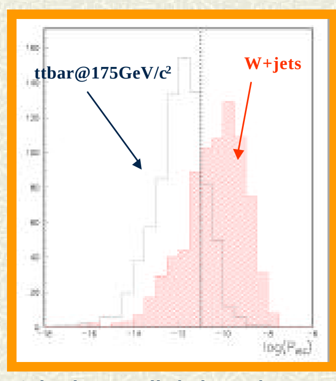
We also applied specific selections for this analysis:

• Since we use a leading-order matrix element in our calculation of the probabilities, we restrict our sample to **only 4 jets**

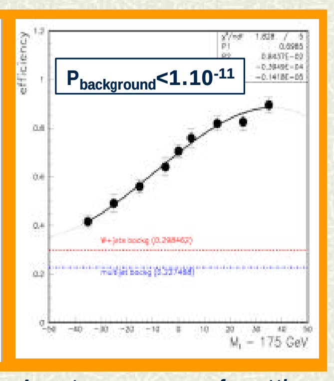
• To increase the purity of signal, we apply a selection in the background probability, $P_{background}$<1.10⁻¹¹



Acceptance curves for ttbar events as a function of F_0, after all the selections were applied.



Selection applied in order to increase the purity of the signal. Only those events on the left of the dashed line, are kept for further analysis.



Acceptance curves for ttbar, W+jets and multijet events as a function of the top mass, M_{top}, after $P_{background}$<1.10⁻¹¹ was applied.

Approximations in the probabilities definitions (things to do better with more statistics)

- **Only qqbar production**: it does not include 10% of ttbar events that are produced by gluon fusion
- **Only W+jets background**: that is ~85% only of the background
- **Leading-Order ttbar matrix element**: no extra jets, constrains our sample to have only 4 jets

After these approximations, the likelihood function is

$$-\ln L(a) = -\sum_{i=1}^{N} \ln\left[c_1 P_{ttbar}(x_i;a) + c_2 P_{W+jets}(x_i)\right] + N\int A(x)\left[c_1 P_{ttbar}(x;a) + c_2 P_{W+jets}(x)\right]dx$$

where c_1 and c_2 are minimized at each point of α.

Transfer Functions W(x,y)

W(x,y) is the probability of measuring a **set of variables *x*** when a **set of variables *y*** was produced (***x*** jet variables, ***y*** parton variables). It is taken as a δ function for quantities that correspond to well measured objects. Due to the excellent granularity of the DØ calorimeter, angles are considered well measured. Also, since energies of electrons are measured much better than for jets, the momenta of electrons will also be considered well measured. For W->μν, the muon momentum is often not well measured, so the muon momentum resolution has to be included. The effect is taken into account by integrating numerically over the resolution of the muons. For the e+jets final states, we write:

$$W(x,y) = d^3(p_e^y - p_e^x)\prod_{j=1}^{4} W_{jet}(E_j^y, E_j^x)\prod_{i=1}^{4} d^2(\Omega_i^y - \Omega_i^x)$$

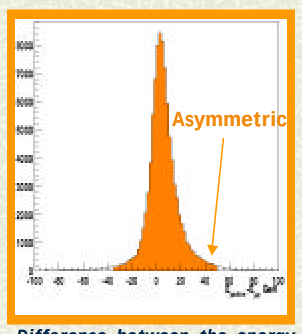
where p_e^y and p_e^x are the produced and measured electron momenta, E^y and E^x are the parton and jet energies, and W^y and W^x_j are the quark and jet angles.

Mapping between jet and parton energies - W_{jet}(Eparton,Ejet)

The mapping between parton and jet energies is determined by the transfer function $W_{jet}(E_{parton},E_{jet})$. This function models the smearing in jet energies from effects of *radiation, hadronization, measurement resolution, and jet reconstruction algorithm,* taking into account the shape of the $\delta E = E_{parton} - E_{jet}$ to avoid underestimation the jet energy. The function is obtained from Monte Carlo event sample, and parameterized using 2 Gaussians, one to account for the peak and the other to fit the asymmetric tails,

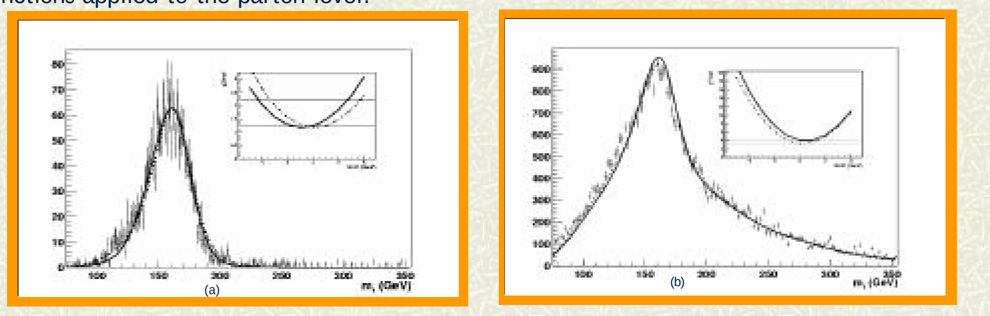
$$W_{jet}(E_{parton},E_{jet}) = \frac{1}{\sqrt{2p}\,(p_1+p_2p_5)}\left[\exp\frac{-(d_E-p_1)^2}{2p_2^2} + p_3\exp\frac{-(d_E-p_4)^2}{2p_5^2}\right]$$

where $p_i = a_i + b_i E_{parton}$



Difference between the energy of the reconstructed jet and the associated parton in Monte Carlo simulated ttbar events.

We compare the three invariant masses calculated directly using Monte Carlo HERWIG jets after **full DØ reconstruction**, using the standard criteria, with predictions based on the transfer functions applied to the parton level.



Three jet invariant mass for the decay products of the top quark. The fully simulated and reconstructed Monte Carlo HERWIG events (histogram) are compared with a prediction using parton model and transfer functions (solid line). The dashed line corresponds to a transfer function with a variant parameterization. (a) Only the right combination among jets is considered. (b) Three invariant mass using all possible combination among jets and restricting the sample to only 4 jets without requiring jet-parton matching.

Background Probability $P_{background}$(x)

- It is defined only in terms of the main background (W+jets, 85%), $P_{W+jets}(x)$, which proves to be an adequate representation for multijet background
- The background probability for each event is calculated using VECBOS subroutines for W+jets [5]
- It uses the same transfer functions for modeling the jet resolutions as used for signal events
- All the permutations are considered, together with all possible values of the *z* component of the momentum of the neutrino
- The integration is done over the jet energies

Results - M_{top} and F_0

DÆ Run I Data

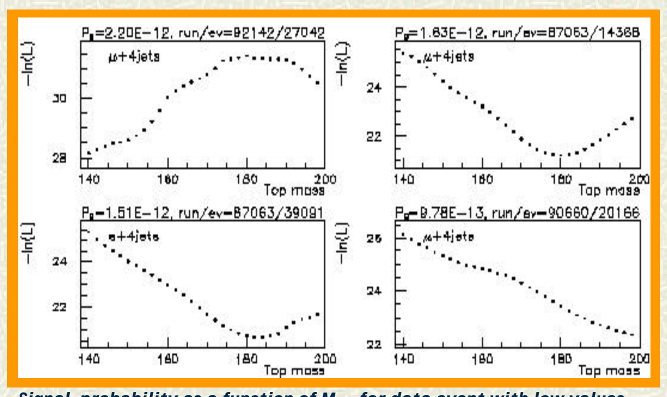
DØ statistics from Run I corresponds to 125 pb⁻¹. After applying the standard selection criteria, **91** ttbar candidates remained. Only **71** of these remaining events have 4 jets. And from this sample only **22** have a background probability larger than 1.10⁻¹¹. We proceed to extract M_{top} and F_0 from these 22 events.

The minimization of the –ln *L* as a function of only c_1 and c_2 results in

$$n_{ttbar} = 12 \pm 3$$

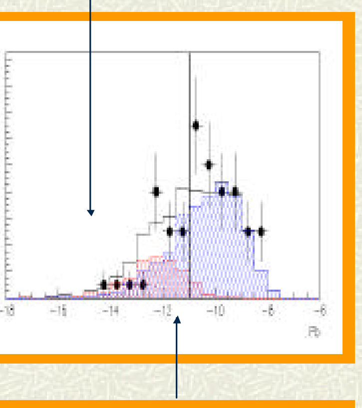
and a purity = $n_{ttbar}/(n_{ttbar}+n_{W+jets})$=0.54

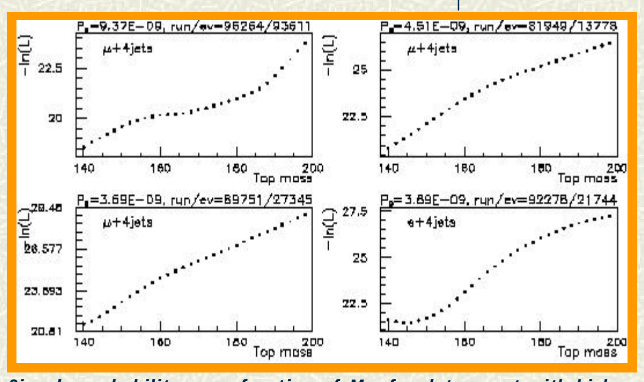
P_{ttbar} as a function of M_{top}



Signal probability as a function of M_{top} for data event with low values of background probability (signal like events).

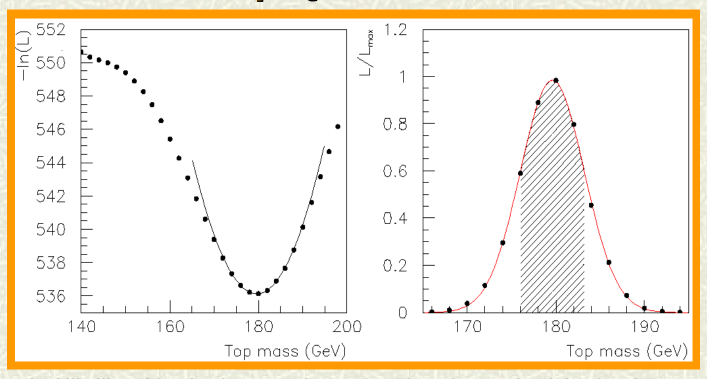
Background probability for the data sample (circles). Only those events on the left of the vertical line, are kept for further analysis. The black histogram corresponds to a Monte Carlo simulation with the measured fractions of signal (red) and background (blue) events.





Signal probability as a function of M_{top} for data event with high values of background probability (background like events).

Top Quark Mass



Final likelihood for the data sample as a function of M_{top}. The right plot shows the Gaussian fit to the likelihood from which we extract the most probable mass and its statistical uncertainty.

$$M_{top} = 180.1 \pm 3.6_{stat} \pm 4.0_{syst}\ \mathrm{GeV/c^2}$$ assuming F_0=0.70

- This new technique improves the statistical error on M_{top} from 5.6 GeV/c² [PRD 58, 52001, (1998)] to 3.6 GeV/c²
- Decrease in the statistical error is equivalent to a factor of 2.4 in the number of events
- 0.5 GeV/c² shift has been applied, based on Monte Carlo studies

Systematic Uncertainties for top quark mass

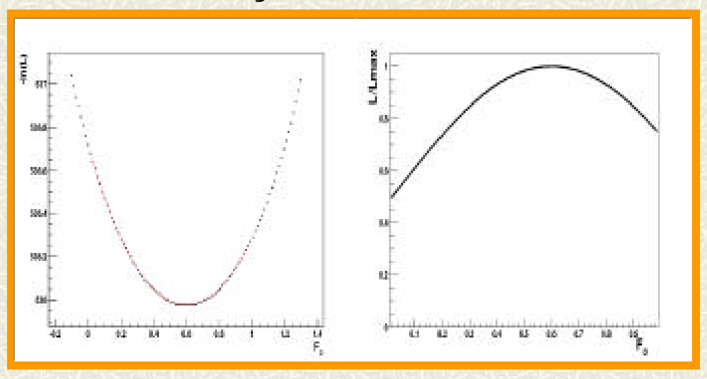
Determined from MC studies with large event samples:

Signal model	1.5 GeV/c²
Background model	1.0 GeV/c²
Noise and multiple interactions **PRD 58 52001, (1998)**	1.3 GeV/c²

Determined from data:

Jet Energy Scale	3.3 GeV/c²
Parton Distribution Function	0.2 GeV/c²
Acceptance Correction	0.5 GeV/c²

Helicity of the W Boson



Final likelihood for the data sample as a function of F_0.

$F_0 = 0.60 \pm 0.30_{stat}$ assuming M_{top}=175 GeV/c²

$F_0 = 0.56 \pm 0.31_{stat+Mtop} \pm 0.04_{syst}$ integrating over M_{top}

- First F_0 measurement done at DÆ
- **CDF** measurement of F_0 = 0.91 ± 0.37 (stat) ± 0.13 (syst) (*PRL 84,216*) using 108 leptons (70% signal) [PRL 84,216, (2000)]
- Corrected for response

Systematic Uncertainties W boson helicity

Determined from MC studies using Run I statistics:

Signal model	0.020
Background compositeness	0.010
Noise and multiple interactions	0.006

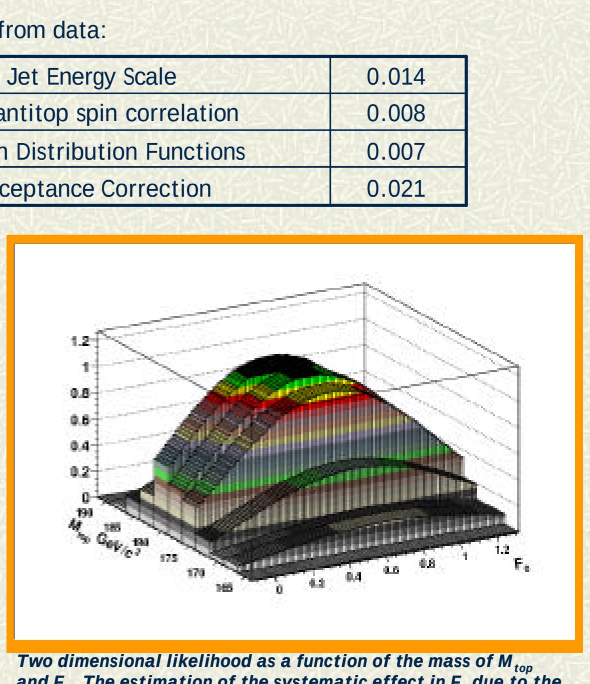
Determined from data:

Jet Energy Scale	0.014
Top-antitop spin correlation	0.008
Parton Distribution Functions	0.007
Acceptance Correction	0.021

P_{ttbar} as a function of F_0 and M_{top}

Because the extracted F_0 is required to lie inside the physical region, its total uncertainty cannot be greater than 0.34, that is the 68.27 interval for the region between 0 and 1. Adding all the separate components in quadrature, may in fact, overestimate the total uncertainty, and perhaps bring it above the limit of 0.34 for the case of no information whatsoever on F_0. A fully Bayesian approach, integrating over all uninteresting parameters, would provide the best way of estimating the uncertainty, but this is not always possible. Therefore, if it is possible we should calculate the uncertainties "internally" so this limit could be attained.

We calculate the the systematic error due to the uncertainty on the top mass, integrating over it (from 165 GeV/c² to 190 GeV/c²) and using no prior knowledge of the top quark mass. Therefore, the statistical error in F_0 and the systematic effect of the uncertainty in M_{top} are estimated simultaneously by projecting this likelihood onto the F_0 axis.



Two dimensional likelihood as a function of the mass of M_{top} and F_0. The estimation of the systematic effect in F_0 due to the uncertainty in top mass is obtained by integrating over the top mass.

Conclusions

Using LO approximation and parameterized showering, we calculated the event probabilities, and measured:

M_{top} (preliminary) =180.1 ± 3.6 (stat) ± 4.0 (syst) GeV/c²

⇒Significant improvement to our previous analysis, and is equivalent to having 2.4 times more data

F_0 (preliminary) = 0.56 ± 0.31(stat+M_{top}) ± 0.04 (syst)

=> First F_0 measurement done at DÆ

We have a method that allows us to optimize information to extract M_{top} and F_0. The statistical power comes from:

- ✓ Correct permutation is always considered (along with the other eleven)
- ✓ All features of individual events are included, thereby well measured events contribute more information than poorly measured events
- ✓ The probability depends on **all measured quantities** (except for unclustered energy)
- ✓ This method offers the possibility of increasing the statistics for F using **both W decay branches**
- ✓ For higher statistics, one clearly needs to improve the calculation of the probabilities, but this method is a **better way** to do the analysis

To consider for the future:

- A very general method (other top quark properties, Higgs searches, etc.)

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