Two-loop QED corrections to Bhabha scattering

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work with Kirill Melnikov, hep-ph/soon

Overview

- Bhabha scattering
  - luminosity determination
  - radiative corrections
- A simple relation between massive and massless scattering amplitudes
  - Mass factorization for $m_e^2 << Q^2$.
- 2-loop QED differential cross section

Bhabha scattering

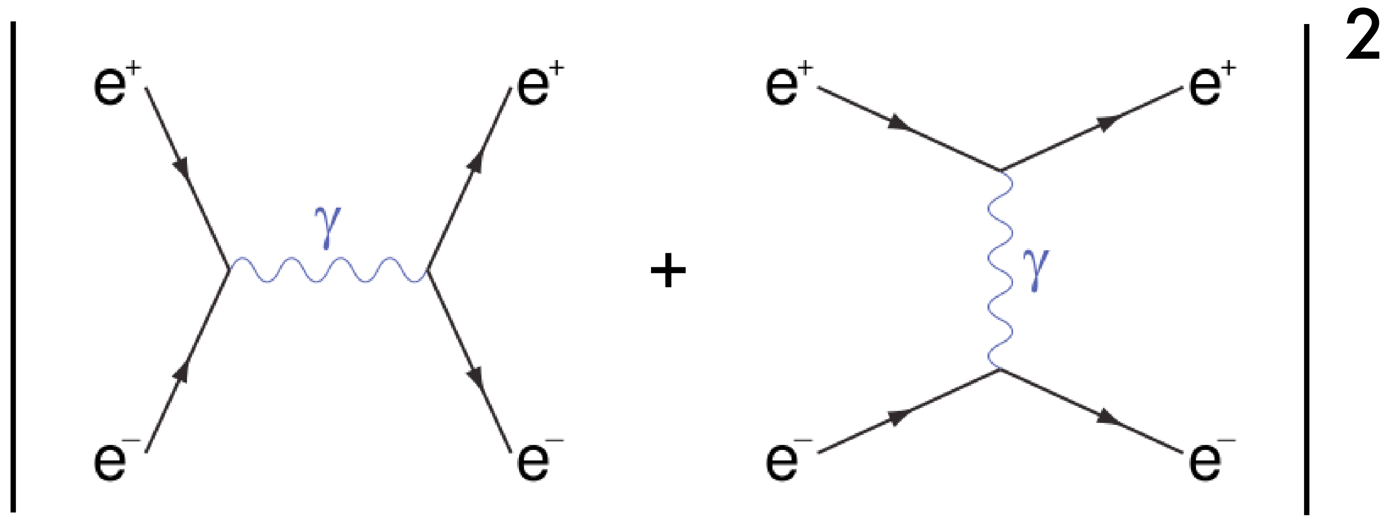
- Used to measure luminosity at e^+e^- colliders

$$\mathcal{L} = \frac{\left.\frac{dN}{dt\,d\Omega}\right|_{\text{measured}}}{\left.\frac{d\sigma}{d\Omega}\right|_{\text{theory}}}$$

← precise prediction crucial

- *Large angle* scattering at low energy meson factories
  - Babar, Belle, BEPC-BES, CLEO-C, Daphne, VEPP-2M, ...
- *Small angle* scattering at high-energy machines
  - LEP, SLD, ILC, ...
  - Electro-weak and new physics at large angles!

Tree level cross section



Homi J. Bhabha '36

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s}\left[\frac{t^2+u^2}{2s^2} + \frac{s^2+u^2}{2t^2} + \frac{u^2}{st}\right]$$

- Cross section diverges as $t \longrightarrow 0$.
  - Even at the Z-pole, small angle scattering is large and dominated by QED.
  - LEP experiments used Bhabha between 20 mrad $< \theta <$ 60 mrad for $\mathcal{L}$ determination

Current precision

- State-of-the art: Monte-Carlo generators that implement NLO and include logarithmically enhanced higher order corrections.
- Small angle scattering
  - MC: 0.05%
  - Exp: LEP 0.035%, Giga-Z 0.02%
- Large angles
  - MC: 0.5% accuracy.
    - New: BABAYAGA@NLO: 0.1%

      Balossini, Calame, Montagna, Nicrosini, Piccini
  - Exp: Cleo-C, BaBar, Belle 1%, Daphne 0.3%

NNLO QED status

- NNLO result for $\theta \rightarrow 0$ known.
  - Only form factor corrections are needed

    Fadin, Kuraev, Lipatov, Merenkov & Trentadue '92
  - Dominant part is included in BHLUMI MC

    Jadach, Placzek, Richter-Was, Ward, Was
- Massless 2-loop virtual corrections calculated

  Bern, Dixon, Ghinculov '01
- Ongoing work on massive NNLO
  - Planar master integrals

    Czakon, Gluza, Riemann '06
  - Electron loop contribution known.

    Bonciani et al. '04

Expansion in $m_e^2 \ll s, |t|, |u|$

- Terms suppressed by powers of the electron mass are *negligible* in all applications
  - Condition $\theta \gg \frac{2m}{\sqrt{s}}$ is fulfilled in practice
    - e.g. for $\theta \gg 0.01\,\mathrm{mrad}$ at LEP
- Keep lepton mass at leading power
  - necessary, if isolated leptons are observed rather than "lepton jets" (this is the case for large angle scattering)
  - for easier comparison with exisiting MC's

Expansion in $m_e^2 \ll s, |t|, |u|$

- Expansion of diagrams is nontrivial
  - interplay of different momentum regions
  - need loop integrals to subleading powers to obtain leading power cross section
- Can instead use known massless result:
  - Photonic logarithmic terms $\alpha^2 \ln \frac{m^2}{s}$ derived from divergent part of massless result.

Glover, Tausk and van der Bij '01

  - Complete leading power photonic corrections inferred from massless result and known mass dependence of vector form factor.

Penin '05

"Mass from no mass"

- Penin's derivation of the $m_e^2 \ll s, |t|$ result is somewhat complicated
  - uses photon mass as IR regulator
  - depends on non-renormalization of leading Sudakov log's
- Have much simpler method to restore logarithmic mass dependence of amplitudes

see also Moch and Mitov hep-ph/0612149

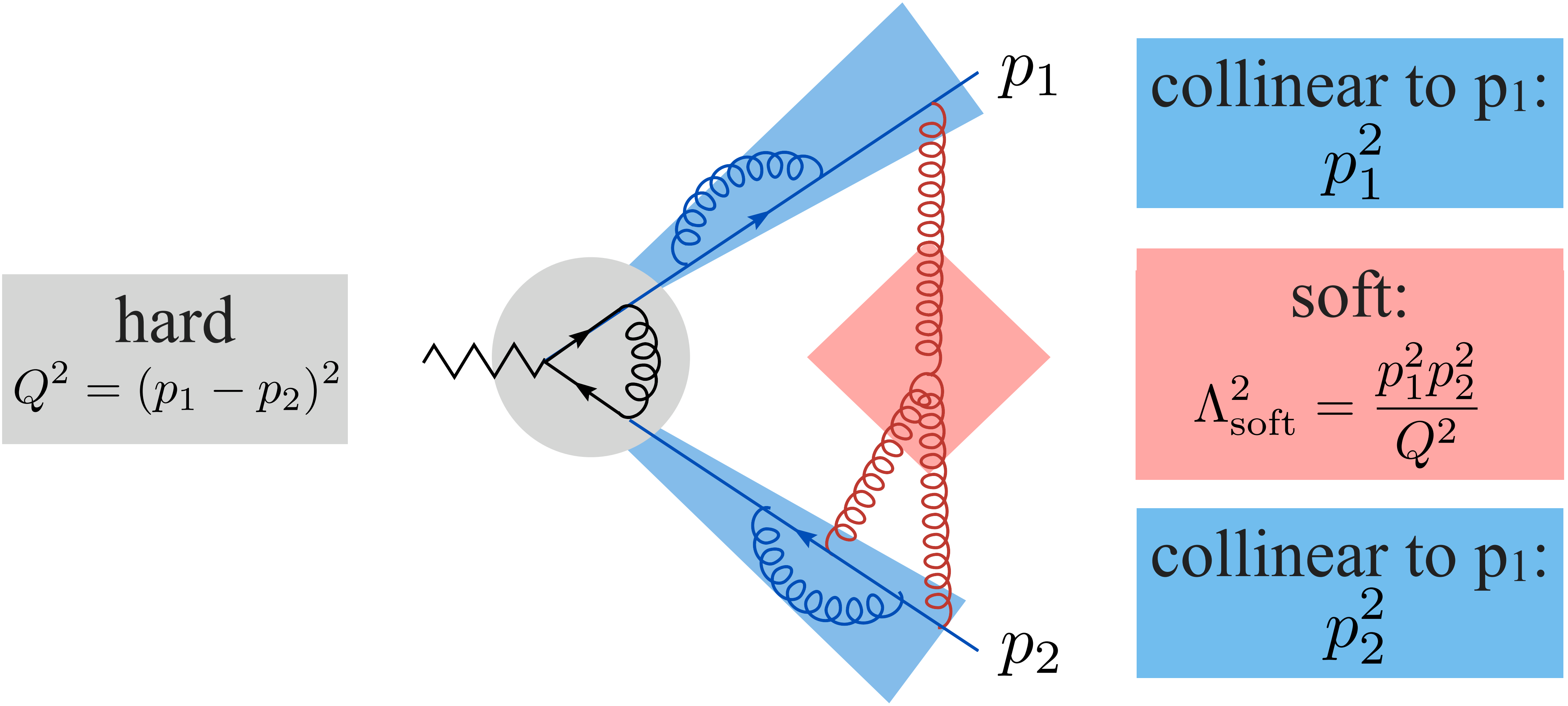
  - Mass effects appear as wave function renormalization on external legs of massless amplitude $\tilde{\mathcal{M}}(\{p_i\})$

$$\mathcal{M}(\{p_i\}, m) = Z_j(m)^{n/2} \tilde{\mathcal{M}}(\{p_i\}) + \mathcal{O}(m^2/Q^2)$$

  - this relation also works for QCD
  - Note: relation involves additional soft part for diagrams with massive fermion loops.

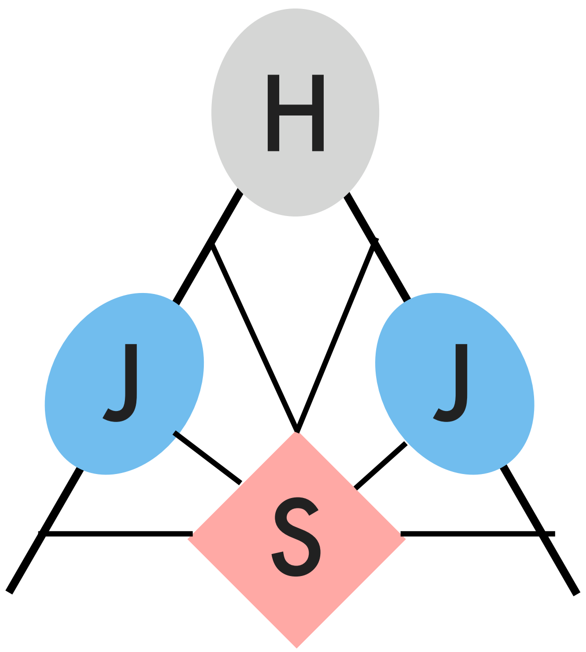
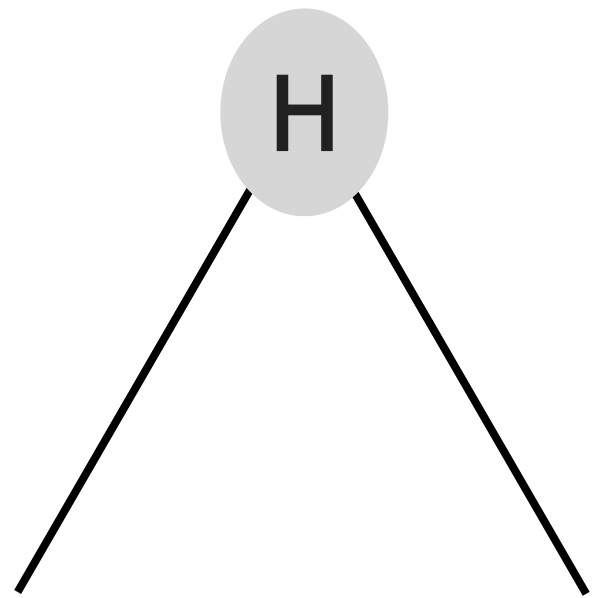
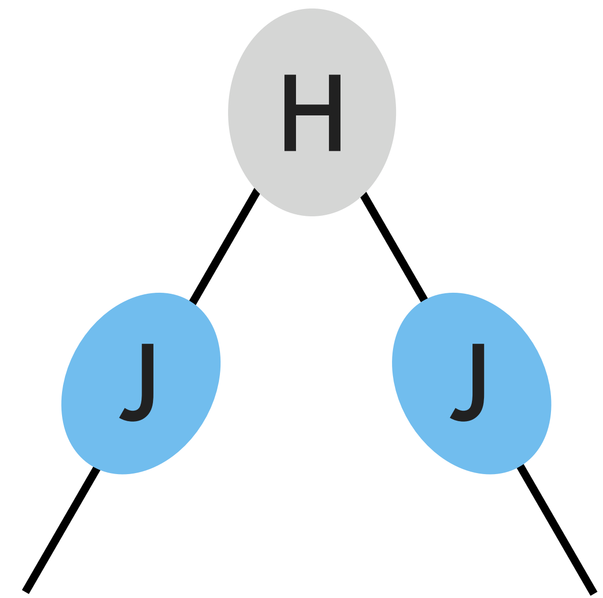
Structure of form factor at large Q^2

- Typical momentum regions / relevant scales:



- Explicit in Soft-Collinear Effective Theory
  - QCD fields are split into soft and collinear fields.
  - Hard part is absorbed into Wilson coefficient.

Form factor in dimensional regularization

off-shell	on-shell *massless*	on-shell *massive*
		
$H \equiv H(Q^2)$ same in all three cases! IR finite.	Jet and soft function scaleless! Soft and collinear divergencies for $d \to 4$	Jet function $J \equiv J(m^2)$ Soft function scaleless! Soft divergencies for $d \to 4$

Jet-function

- Determine Z_j (=J^2) by taking ratio of massive to massless form factor

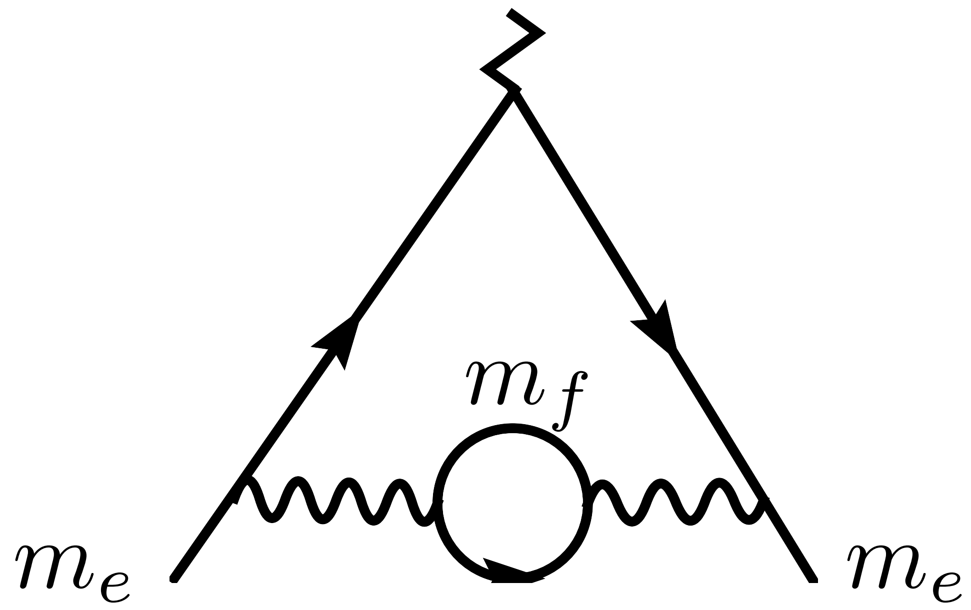
$$Z_j = \frac{F(Q^2, m_e^2, \epsilon)}{F(Q^2, m_e = 0, \epsilon)}$$

$$\begin{aligned} Z_j &= 1 + \left(\frac{\alpha}{\pi}\right) m^{-2\epsilon}\left[\frac{1}{2\epsilon^2} + \frac{1}{4\epsilon} + \frac{\pi^2}{24} + 1 + \epsilon\left(2 + \frac{\pi^2}{48} - \frac{\zeta(3)}{6}\right) + \epsilon^2\left(4 - \frac{\zeta(3)}{12} + \frac{\pi^4}{320} + \frac{\pi^2}{12}\right)\right] \\ &+ \left(\frac{\alpha}{\pi}\right)^2 m^{-4\epsilon}\left[\frac{1}{8\epsilon^4} + \frac{1}{8\epsilon^3} + \frac{1}{\epsilon^2}\left(\frac{17}{32} + \frac{\pi^2}{48}\right) + \frac{1}{\epsilon}\left(\frac{83}{64} - \frac{\pi^2}{24} + \frac{2\zeta(3)}{2}\right)\right. \\ &\left. + \frac{561}{128} + \frac{61\pi^2}{192} - \frac{11}{24}\zeta(3) - \frac{\pi^2}{2}\ln(2) - \frac{77\pi^4}{2880}\right] \end{aligned}$$

Q^2 independent ✓

agrees with Moch and Mitov hep-ph/0612149

Fermion loop contributions



- At the leading power, diagram gets contributions from hard, collinear and *soft photon exchange*:

$$S = 1 + \delta S = 1 - (4\pi\alpha_0)^2 \int \frac{d^d k}{(2\pi)^d} \frac{p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k) k^2} \Pi(k^2, m_f^2)$$

$$\delta S = \alpha_0^2 \, m_f^{-4\epsilon} \, \ln\left(\frac{m_e^2}{Q^2}\right) f(\epsilon)$$

Massive Bhabha

- Multiply massless Bhabha amplitude with Z_j^2 and S

$$\mathcal{M}(\{p_i\}, m_e) = Z_j^2(m_e^2)\tilde{\mathcal{M}}(\{p_i\})S(s,t,u) + \mathcal{O}(m/p_i)$$

- Square and add soft radiation with $E_\gamma < \omega \ll m_e$

$$\frac{d\sigma}{d\Omega} = \exp\left(\frac{\alpha}{\pi}F_{\text{soft}}\right) \times \underbrace{Z_j^4 \times |S|^2 \times \frac{d\sigma}{d\Omega}}_{\text{massive virtual}}\Bigg|_{\text{virtual}, m_e=0, m_\mu=0}$$

Input

- Input for the determination of Z_j
  - 2-loop massless FF — e.g. Gehrmann, Huber, Maitre '05
  - 2-loop massive FF — Bernreuther et al. '04
  - heavy fermion contribution — Hoang, Teubner '98; Kniehl '89
- Input to calculate Bhabha scattering
  - 1-loop to $O(\varepsilon^2)$ — Bern, Dixon, Ghinculov '01
  - 2-loop virtual — Bern, Dixon, Ghinculov '01
  - (1-loop) x (1-loop) — inferred from Anastasiou et al. '00
  - F_{soft} to $O(\varepsilon)$ — our own evaluation

Result

- Full agreement with Penin for the photonic two-loop corrections.
  - first independent check of his result
- Agreement with Bonciani et al. for the electron loop contribution.
  - typo in their paper
- Result for the muon contribution is new.

Structure of the result:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{\text{tree}}}{d\Omega}\left[1 + \left(\frac{\alpha}{\pi}\right)\delta_1 + \left(\frac{\alpha}{\pi}\right)^2\delta_2 + \mathcal{O}(\alpha^3)\right]$$

- Collinear logarithms

$$\delta_2 = -\frac{N_f}{9}\ln^3\left(\frac{s}{m_e^2}\right) + \delta_2^{(2)}\ln^2\left(\frac{s}{m_e^2}\right) + \delta_2^{(1)}\ln\left(\frac{s}{m_e^2}\right) + \delta_2^{(0)}$$

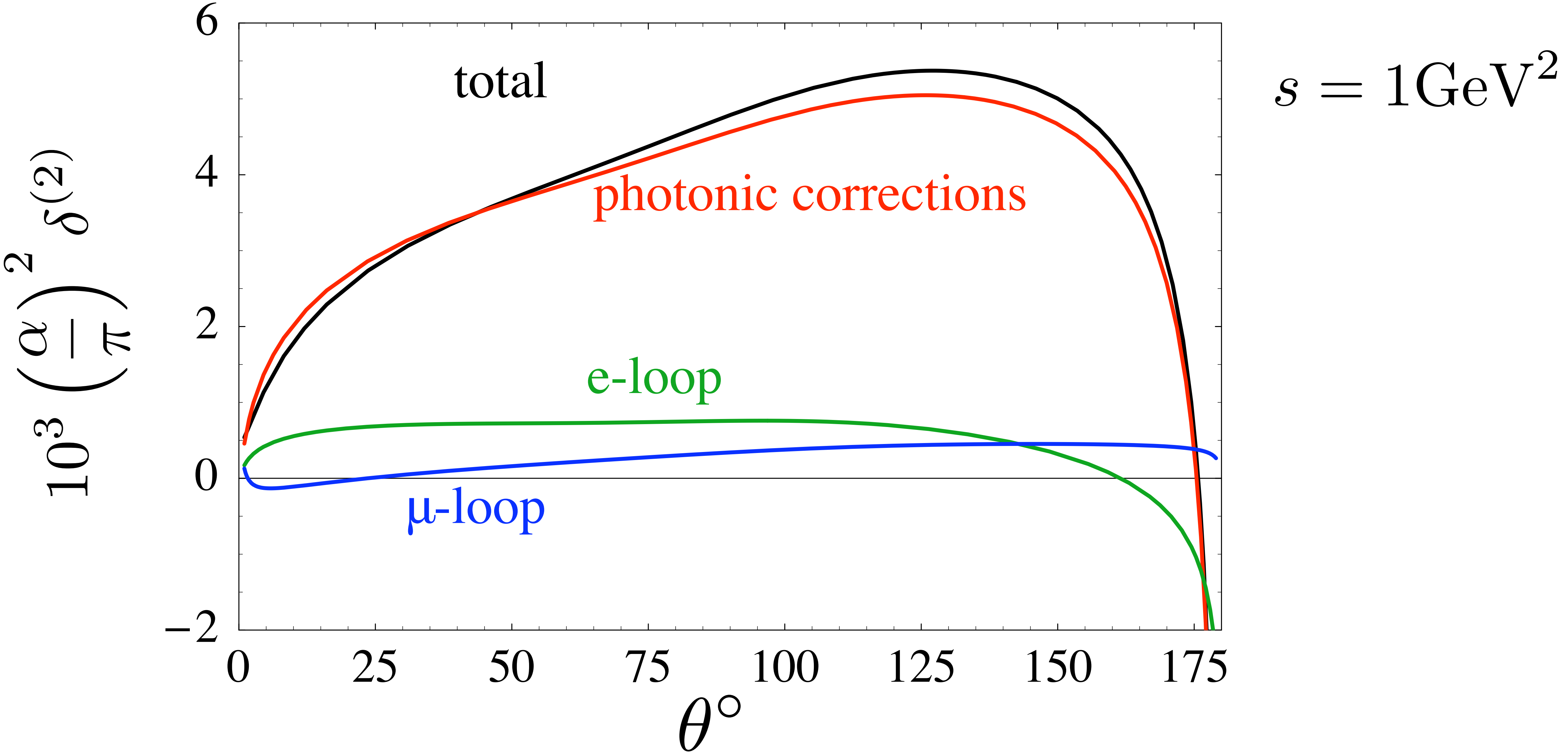
- Muon mass logarithms

$$\delta_2 = M_2\ln^2\frac{m_\mu^2}{m_e^2} + M_1\ln\frac{m_\mu^2}{m_e^2} + M_0$$

- Soft logarithms $E_\gamma < \omega$

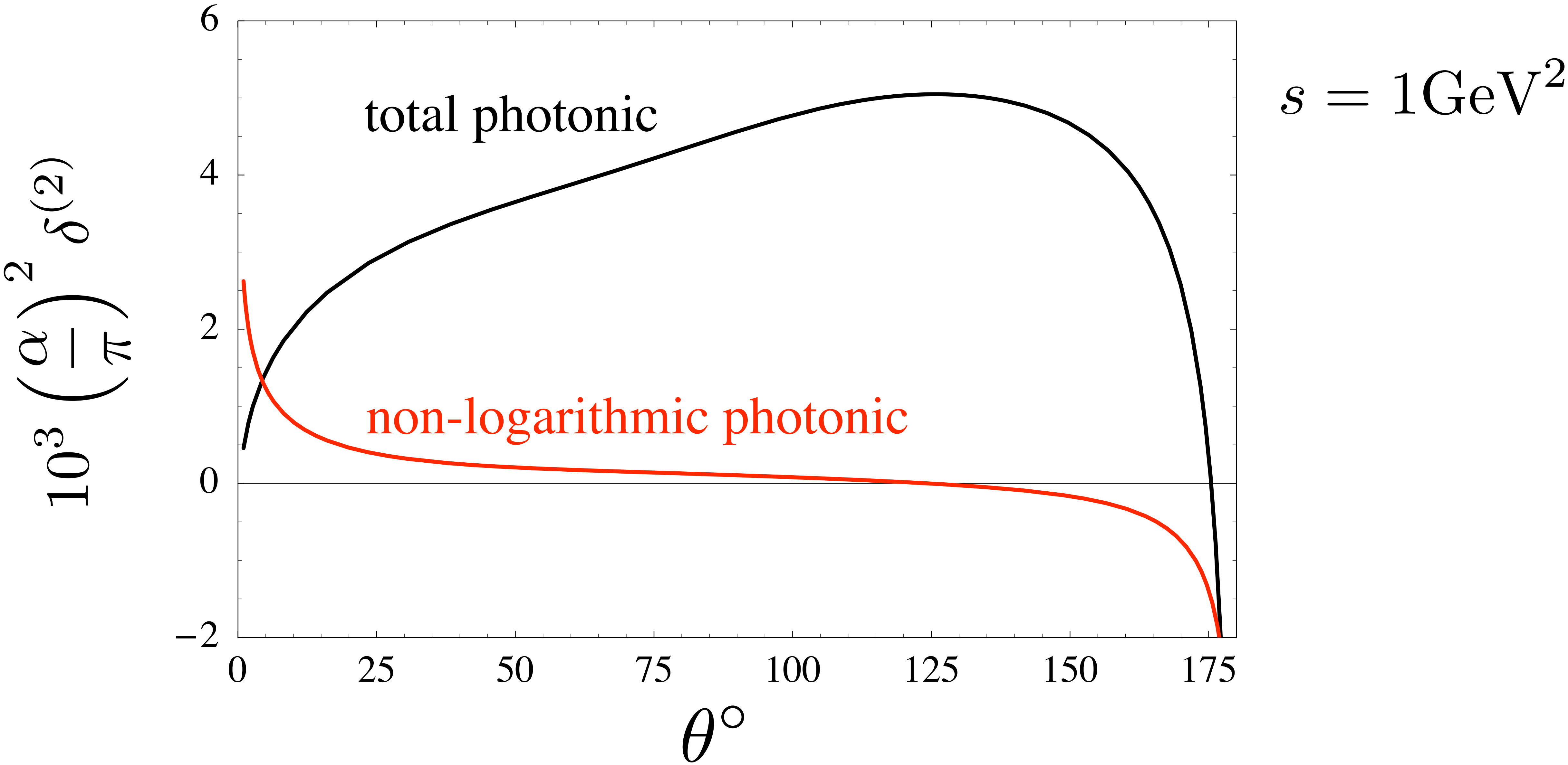
$$\delta_2 = S_2\ln^2\left(\frac{2\omega}{\sqrt{s}}\right) + S_1\ln\left(\frac{2\omega}{\sqrt{s}}\right) + S_0$$

Size of the two-loop corrections



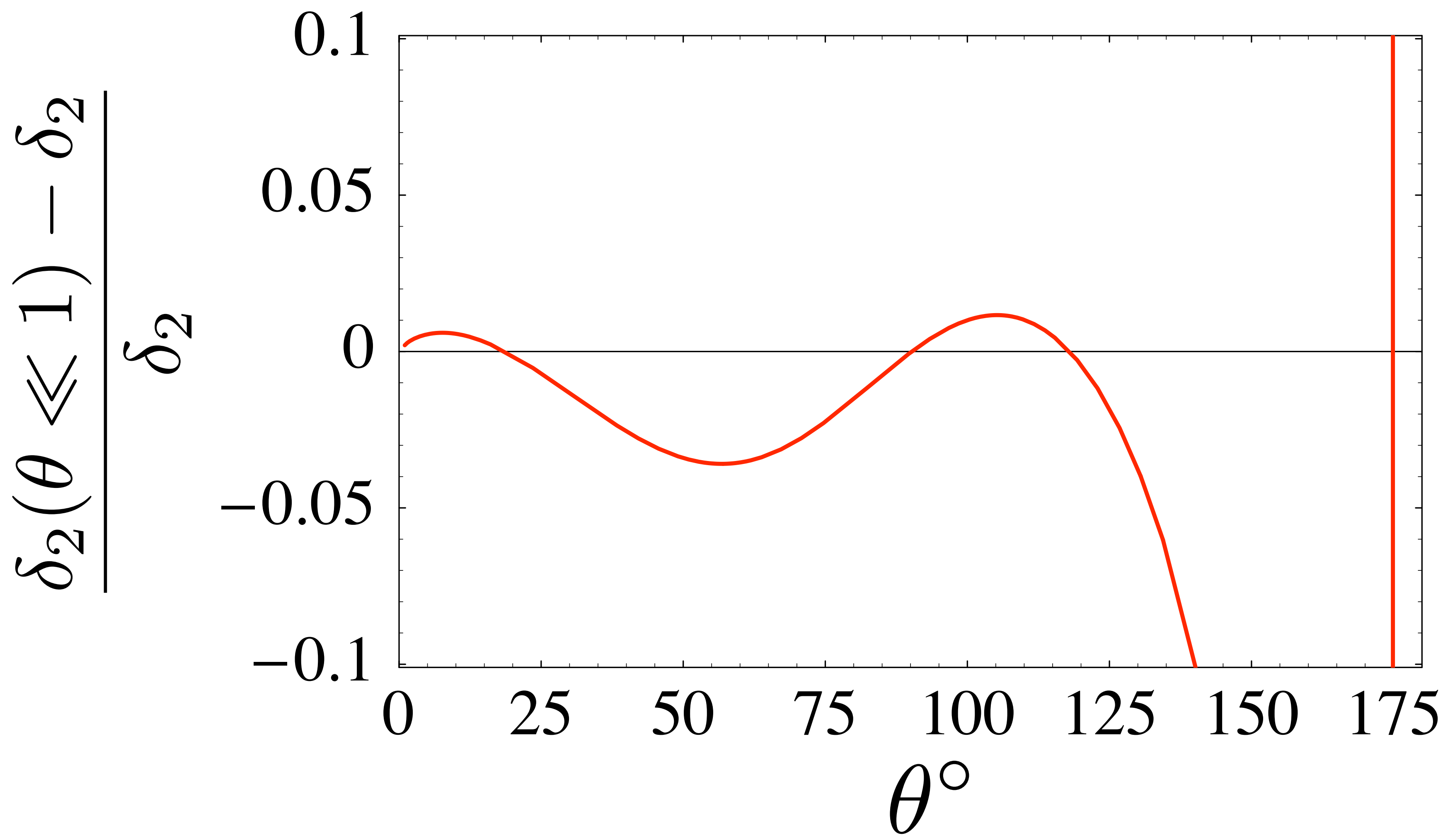
- Assume MC takes care of soft radiation.
  - Set $L_{\text{soft}} = \ln(2E_\gamma^{\text{soft}}/\sqrt{s}) \to 0$

Non-logarithmic corrections



- Assumes MC takes care of soft radiation and implements correct $\ln(m_e^2/s)$ terms.

Small angle expansion



- Small angle expansion work up to large angles!
- (Plot shows expansion of full result, not comparison with Fadin et al.)

Summary

- Have established a simple relation between massless and massive amplitudes at large momentum transfers.
- Have applied it to Bhabha scattering at NNLO
  - rederivation of results of Penin for photonic corrections and of Bonciani et al. for electron loops.
    - first independent check of these results
  - new result for μ-loop contribution
- Same relation can also be used for QCD processes, such as heavy quark production.

see Moch and Mitov hep-ph/0612149