O'Mega & WHIZARD: Monte Carlo Event Generator Generation For Future Colliders

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The Linear Collider as New Frontier:

- final states with many tagged weakly interacting particles accessible
- *(in the absence of low energy SUSY:)* physics beyond the standard model may only be accessible in precision tests of standard model processes

$\therefore$ we will need reliable predictions and simulation tools to unleash the full potential of the Linear Collider

  - complete (gauge invariant!) calculations, including irreducible backgrounds
  - polarization must be included

☹ qualitatively more complicated than at LEP1

  - the number of Feynman diagrams explodes combinatorially
  - the algebraic expressions grow much more complicated with the growing number of building blocks (independent momenta and polarizations)
  - the phase space also becomes much more intricate

- $\because$ even if we had enough graduate students and postdocs, we should not waste them on repetitive "assembly line" calculations
- $\therefore$ formalize the calculations so that the repetitive part can be delegated to patient computers. Ideally:

$$\left\{\begin{array}{r}\text{Lagrangian, parameters}\\ \text{final state, cuts}\end{array}\right\} \Longrightarrow \text{efficient unweighted event generator}$$

- ☺ partial solutions exist (CompHEP, Grace, and HELAC), recent progress
  - fast and complete tree level calculations for arbitrary models: O'Mega (T. O. et al.)
  - adaptive phase space generation for many particles: WHIZARD (Wolfgang Kilian), [using VAMP (T. O.)]
- ☹ some essential parts will need a lot more work
  - loops for many particles
  - $\because$ complete one-loop calculations for $2 \to 4$ are the limit of our capabilities

The number of tree Feynman diagrams w/ n legs in vanilla ϕ^3-theory is

$$F(n) = (2n-5)!! = (2n-5)\cdot(2n-7)\cdot\ldots\cdot 3\cdot 1$$

n	$F(n)$	$P(n)$
4	3	3
5	15	10
6	105	25
7	945	56
8	10395	119
9	135135	246
10	2027025	501
11	34459425	1012
12	654729075	2035

☹ computational costs grow beyond all reasonable limits

☹ gauge theory cancellations cause loss of precision

Number of independent momenta

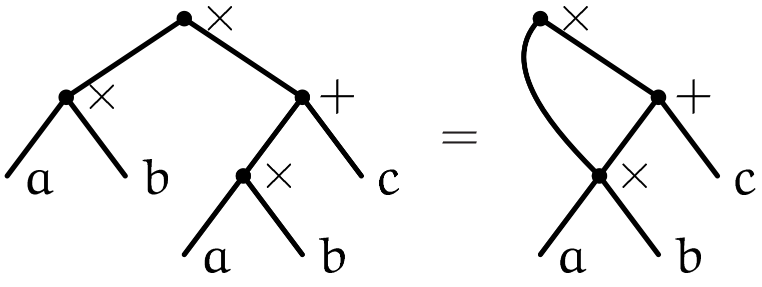
$$P(n) = \frac{2^n-2}{2} - n = 2^{n-1} - n - 1$$

$\therefore$ Feynman diagrams extremely redundant for many particles in the final state!

☹ terms much too large to expect any help from common subexpression elimination by optimizing compilers that don't understand any physics!

Directed Acyclical Graphs (DAGs) are a more efficient representation for arithmetical expressions than the equivalent trees. E. g.:

$ab(ab+c) =$

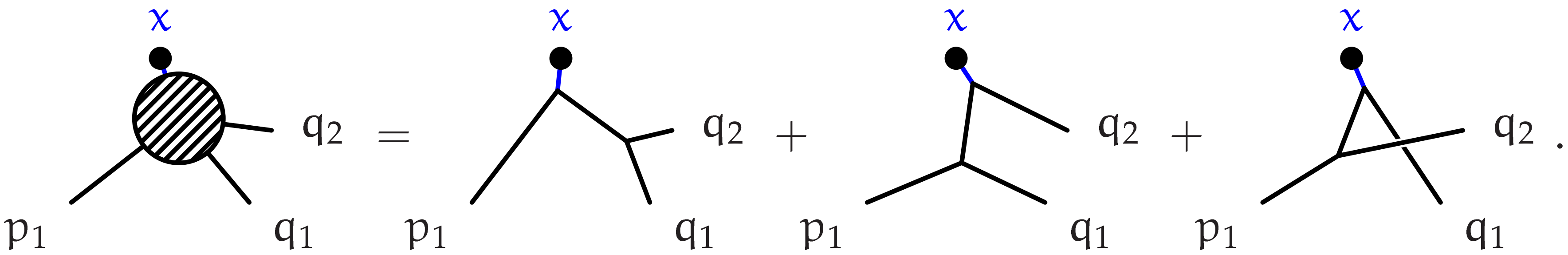


- partial order *"depends on"* prohibits cycles

One particle off-shell wave functions (1POWs) are obtained from Greensfunctions by applying the LSZ reduction formula to all but one line:

$$W(x; p_1, \ldots, p_n; q_1, \ldots, q_m) = \langle \phi(q_1), \ldots, \phi(q_m); \text{out} | \Phi(x) | \phi(p_1), \ldots, \phi(p_n); \text{in} \rangle \,.$$

E. g. $\langle \phi(q_1), \phi(q_2); \text{out} | \Phi(x) | \phi(p_1); \text{in} \rangle$ in unflavored scalar ϕ^3-theory at tree level



- the set of tree level 1POWs forms a DAG and can be constructed recursively
- Theorem: all tree level scattering amplitudes can be represented by combinations of 1POWs (correct combinations are termed keystones)

Grow: starting from the external particles, build the tower of all 1POWs up to a given height (the height is always less than the number of external lines) and translate it to the equivalent DAG D.

Select: from D, determine all possible flavored keystones for the process under consideration and the 1POWs appearing in them.

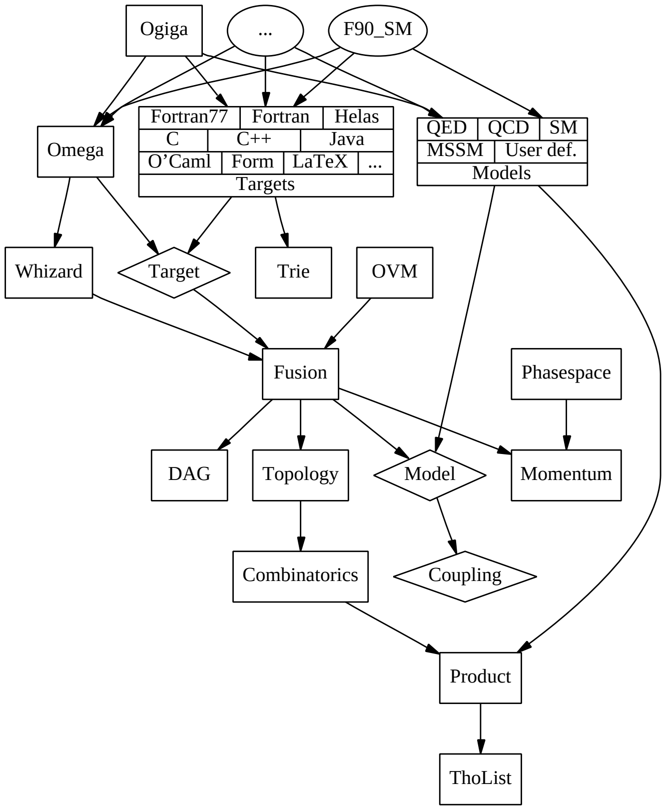
Harvest: construct a sub-DAG $\mathrm{D}^* \subseteq \mathrm{D}$ consisting only of nodes that contribute to the 1POWs appearing in the flavored keystones.

Calculate: multiply the 1POWs as specified by the keystones and sum the keystones.

☺ the resulting DAG contains no more redundancies

☺ the symbolic algorithm contains the numerical methods of Alpha (Caravaglios/Moretti) and HELAC (Kanaki/Papadopoulos) as special cases.

Matrix element compiler O'Mega implemented in Objective Caml (INRIA, France). Its module system supports a flexible architecture with functors building applications from independent modules



The module `Targets` contains implementations of the signature `Target` for each target language and the module `Models` contains implementations of `Model` for each (class of) physics models. E. g. the application writing Fortran95 for the standard model is

```
module O = Omega.Make
             (Fusion.Binary)
             (Targets.Fortran)
             (Models.SM)
let _ = O.main ()
```

- any volunteers for Java and `C++` targets?

Radiative corrections to four fermion production (standard model, unitarity gauge):

process	Diagrams		O'Mega	
$e^+e^- \to$	#	vertices	#prop.	vertices
$e^+\bar{\nu}_e d\bar{u}$	20	80	14	44
$e^+\bar{\nu}_e d\bar{u}\gamma$	146	730	36	151
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma$	1112	6672	94	468
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma\gamma$	12420	86940	168	1246
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma\gamma\gamma$	138816	1110528	344	3746

- O'Mega amplitudes for up to 7 particles ("$2 \to 5$") tested against MADGRAPH

agreement for random momenta always better than 10^{-11}

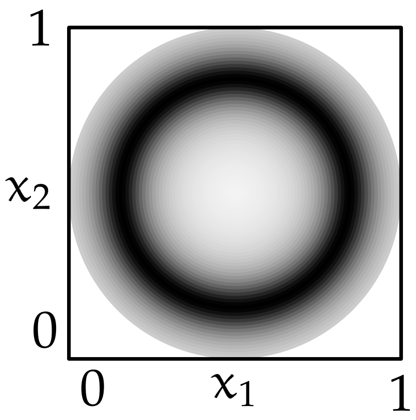
First realistic application

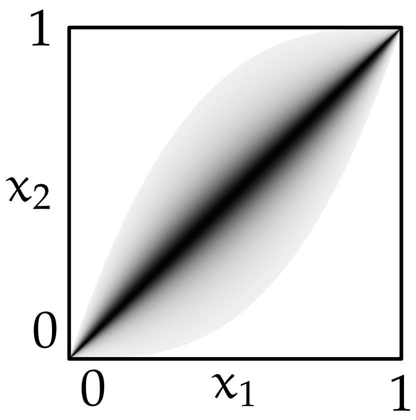
- simulation of six fermion final states in W^+W^- scattering for the TESLA Technical Design Report, using WHIZARD by Wolfgang Kilian as unweighted event generator.

Get it from `http://www.ikp.physik.tu-darmstadt.de/~ohl/omega/`.

- QCD
  - up to two colored particles are already handled
  - factorization for many-jet final states?
- Supersymmetry and MSSM
  - ☺ Jürgen Reuter (Darmstadt) has added unified support for Dirac and Majorana fermions using the Feynman rules of Ansgar Denner et al.
- O'Mega Virtual Machine
  - $\because$ most time is spent in non-trivial vertex evaluations for vectors and spinors, that take $\mathrm{O}(10)$ complex multiplications
  - $\therefore$ virtual vertex evaluation machines can challenge native code and avoid compilations
- O'Giga: O'Mega Graphical Interface for Generation and Analysis
- O'Tera: O'Mega Tool for Evaluating Renormalized Amplitudes

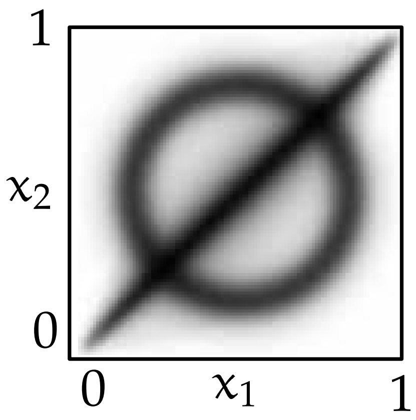
VEGAS' factorized ansatz can deal with





separately after appropriate mapping.

fails for overlapping singularities



which is the common case (if more than one diagram contributes)

∴ adaptive multichannel approach

$$I(f) = \int_M d\mu(p)\, f(p)$$

$$I(f) = \sum_{i=1}^{N_c} \alpha_i \int_0^1 g_i(x) d^n x \, \frac{f(\phi_i(x))}{g(\phi_i(x))}$$

with

$$g = \sum_{i=1}^{N_c} \alpha_i \cdot (g_i \circ \phi_i^{-1}) \left| \frac{\partial \phi_i^{-1}}{\partial p} \right|$$

works with factorized g_i adapted by VEGAS and α_i adapted by variance reduction.

- in general, $g \circ \phi_i$ does not factorize, even if all g_i factorize.
  - $\pi_{ij} = \phi_j^{-1} \circ \phi_i$: coordinate transformations among coordinate systems in which different singularities factorize.
- pure geometry: economical studies of dependence on cuts and parameters

  $\because$ π_{ij} universal and are calculated automatically by WHIZARD

  $\therefore$ VEGAS can optimize the g_i for each set of parameters and cuts

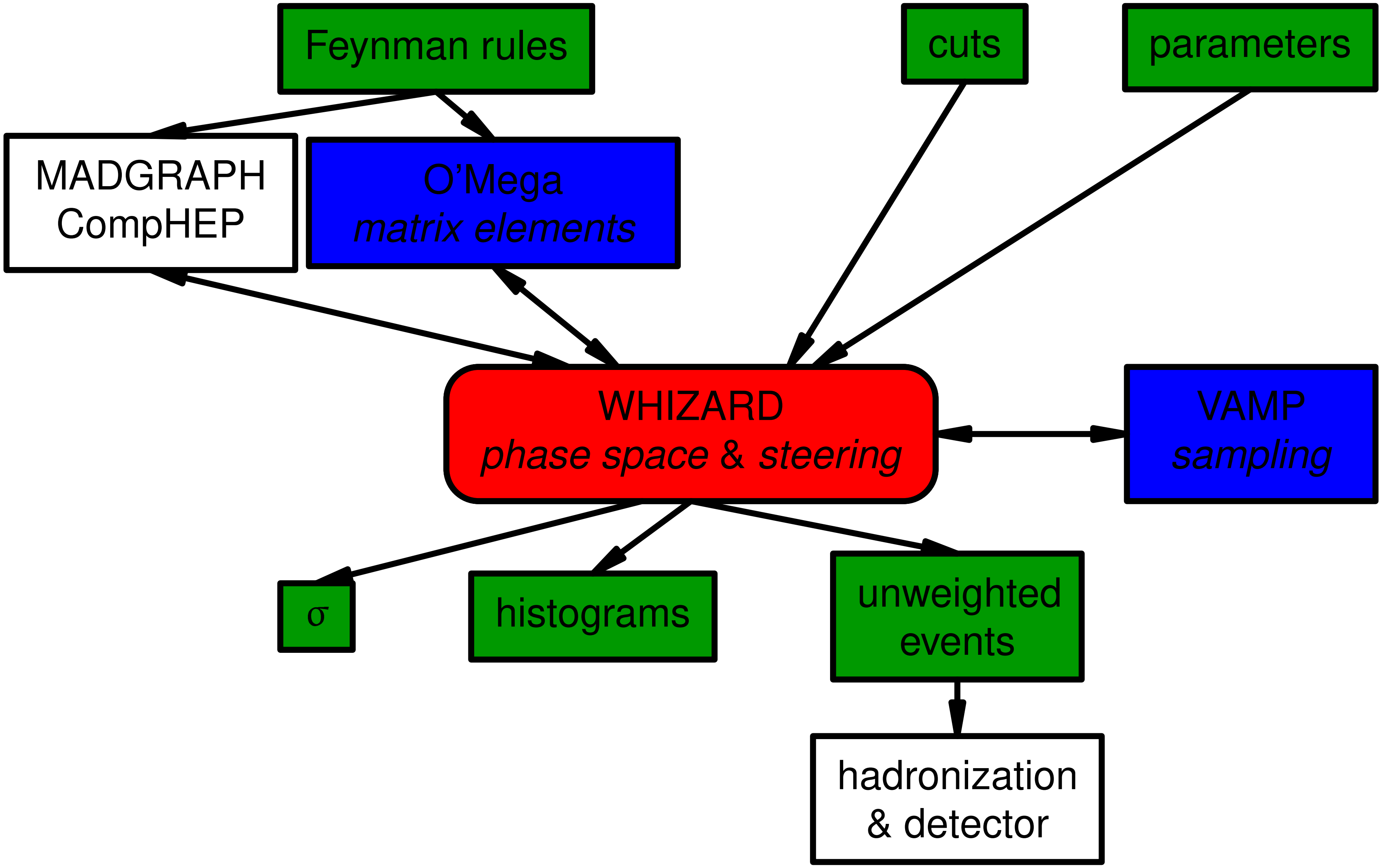
However:

$\because$ singularity structure determined by Feynman diagrams

- naive application brings the combinatorial explosion in through the back door!

$\therefore$ WHIZARD selects the important channels

- s-channel resonances
- $1/t$-poles for massless particles


Feynman rules
cuts
parameters
MADGRAPH
CompHEP
O'Mega
matrix elements
WHIZARD
phase space & steering
VAMP
sampling
σ
histograms
unweighted
events
hadronization
& detector

WHIZARD uses matrix elements from

O'Mega: polarized scattering of many weakly interacting particles, including unstable vector bosons and including (some) deviations from the standard model

MADGRAPH: polarized scattering of colored particles without gauge invariance problems from intermediate vector boson widths

CompHEP: faster for unpolarized scattering of few (possibly colored) particles

Usage:

Process file:

```
ID      In     Out            Method
zh      e1,E1  Z,H            chep
zww     e1,E1  Z,W+,W-        chep
nnbb    e1,E1  n1,N1,b,B      mad
nnucsd  e1,E1  n1,N1,u,C,s,D  omega
```

Compile: `Makefile` performs all necessary steps

- 21 diagrams in 4 groves (gauge invariant subsets): Higgsstrahlung (5), WW-fusion (10), ZZ (4), Z-FSR (2)

- Higgs signal topologies: sss and stt

- background topologies: sss, sst, stt, and ttt

Event generation at $\sqrt{s} = 350$ GeV for $m_H = 120$ GeV.

- In the first pair of steps, VAMP's VEGAS-grids are adapted with fixed relative weights of the channels

- WHIZARD summarizes VAMP's diagnostics

```
! It      Calls    Integral[fb]  Error[fb]  Err[%] Err/Exp Eff[%]    Chi2
!-------------------------------------------------------------------------
! Adapting (fixed weights):  Generating 2 samples of 10000 events ...
   2       20000   5.7019717E+01  1.58E+00    2.76     3.91*  2.31     0.31
```

☺ efficiency not terrible . . .

☹ . . . `Err/Exp` too large

- In the following steps, the relative weights of the channels are allowed to vary

```
! It      Calls    Integral[fb]  Error[fb]  Err[%] Err/Exp Eff[%]   Chi2
!-----------------------------------------------------------------------
! Adapting (var. weights):  Generating 8 samples of 10000 events ...
   3       10000  5.5642224E+01  1.23E+00    2.21    2.21*  7.58
   4       10000  5.9028368E+01  1.06E+00    1.80    1.80*  7.51
   5       10000  5.8586436E+01  8.34E-01    1.42    1.42*  9.82
   6       10000  5.8997829E+01  6.89E-01    1.17    1.17* 12.18
   7       10000  5.8626448E+01  1.04E+00    1.78    1.78  10.78
   8       10000  5.7737567E+01  5.12E-01    0.89    0.89* 17.50
   9       10000  5.7693393E+01  4.75E-01    0.82    0.82* 19.50
  10       10000  5.8216141E+01  5.42E-01    0.93    0.93  14.60
```

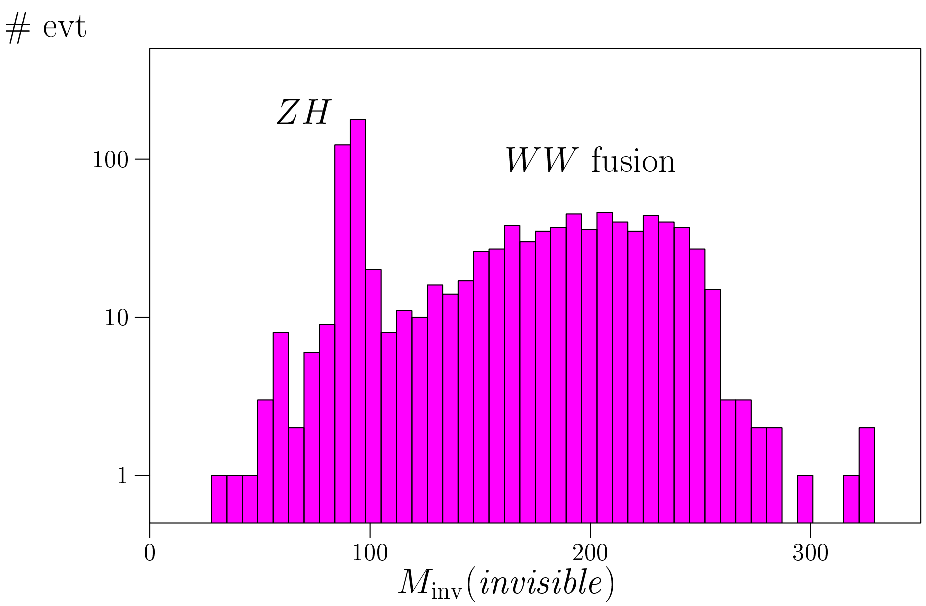
☺ significantly larger efficiency and very good `Err/Exp`

- Finally generate some events

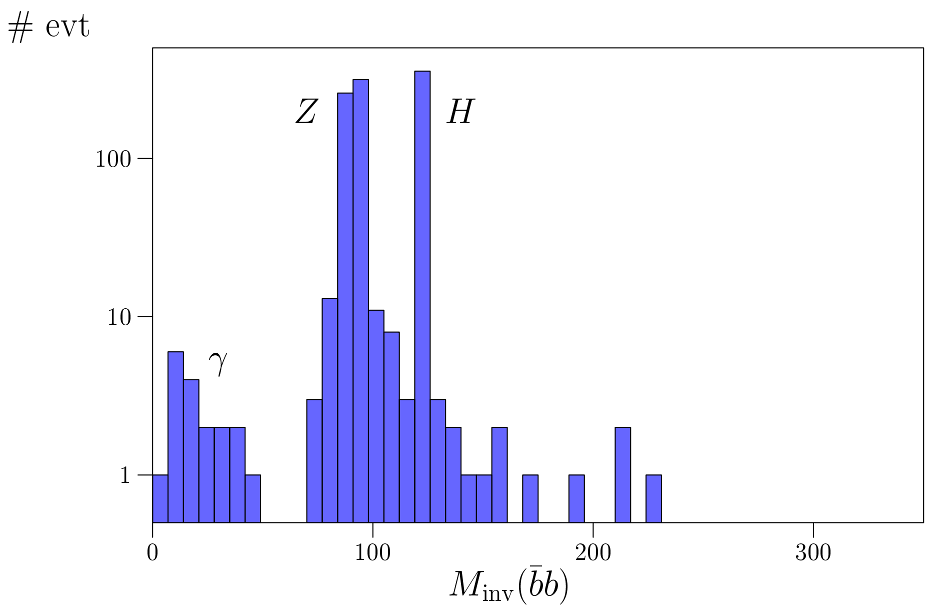
```
! Integrating (fixed w.):  Generating 2 samples of 10000 events ...
  12       20000  5.8910540E+01  4.25E-01    0.72    1.02  11.64    0.05
```

☺ 15 min for adaptation, 10 min for 10,000 unweighted events on a Pentium II 233 MHz.

missing mass



invariant $b\bar{b}$-mass



Observations:

- adaption typically takes a bit longer than event generation
- ☺ adapted grids and weights can be saved and reloaded if the cuts and parameters are changed only slightly

WHIZARD will be available from
`http://www-ttp.physik.uni-karlsruhe.de/~kilian/whizard/` soon.

Further On Up The Road

- make O'Mega more complete (QCD, MSSM)
- add better interaction of O'Mega and WHIZARD to avoid redundancies
  - O'Mega purely symbolical: values of masses, couplings, energies and cuts still unspecified
  - $\therefore$ channel selection has to be done in WHIZARD
- create generic interface to JETSET and friends (non-trivial physics!)
- efficient incoherent jet-like sums, avoiding combinatorial explosion
- loops (holy grail)
  - effective actions in O'Mega
  - ☹ straightforward, but tedious
  - numerical approach
  - ☹ hard problem, others have failed
  - ☺ challenge!