QCD Pomeron at Linear Colliders

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Abstract

The recent developments in theory on the calculation of $\gamma^*\gamma^*$ reactions at high energies, in the aim of detecting the BFKL Pomeron signals, are briefly reviewed. Special attentions are paid to the BFKL formalism related higher order calculations, the NLO QCD corrections to the Photon Impact Factor.

1 Introduction

The experimental test of the BFKL Pomeron[1] is generally considered to be an important task in strong interaction(QCD) physics. In past years, it has been studied in the small-x region at HERA intensively in many respects[2]. However, because of the non-perturbative effects on both proton and final state hadronization, there still have no definite conclusions been made.

Following the pioneering application of the BFKL formalism to high energy $\gamma^*\gamma^*$ interactions at Linear Colliders by Bartels *et al* and Brodsky *et al*[3], recently much interest has been given to this kind of reaction[4].

[1] E.A. Kuraev, L.N. Lipatov, V.S. Fadin, Sov. Phys. JETP **45** (1977) 199; Ya.Ya. Balitskii and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.

[2] For example: H. Navelet, R. Peschanski, C. Royon, S. Wallow, Phys. Lett. **B385** (1996) 357; J. Bartels, V. Del Duca, A. De Roeck, D. Graudenz, M. Wusthoff, Phys. Lett. **B384** (1996) 300; C. Royon, L. Schoeffel, J. Bartels, H. Jung, R. Peschanski, Hep-ph/0010015, etc.

[3] J. Bartels, A. De Roeck, H. Lotter, Phys. Lett. **B389** (1996) 742; S.J. Brodsky, F. Hautmann, D.E. Soper, Phys. Rev. Lett. **78** (1997) 803, erratum ibid **79** (1997) 3522; ibid. Phys. Rev. D**56** (1997) 6957.

[4] e.g. see: J. Kwiecinski and L. Motyka, Hep-ph/0010029, and references therein.

This process describes the scattering of two small-size projectiles, and its high energy behavior (at not too large energies) is expected to be described by the BFKL Pomeron. Moreover, in $\gamma^*\gamma^*$ scattering process the detection of the BFKL signal has several advantages comparing to that in the deep inelastic scattering or hadron-hadron interactions. One of them is that at Linear Collider one can carefully adjusts the scattering angles and energies of the outgoing leptons in order to chose both interacting photons have the same or similar virtualities, and then get rid of or suppress the effect coming from the DGLAP evolution. In addition, the background in $\gamma^*\gamma^*$ scattering would be expected to be more clean than in other processes. Therefore, a measurement of the reaction $l^+l^- \to l^+l^- + X$ by tagging the outgoing leptons at LEP or Next Linear Colliders (NLC) provides an excellent test of this very important QCD prediction.

So far leading order calculations of the BFKL Pomeron have been compared to the LEP data (both OPAL and L3)[5]. Although the next-to-leading order (NLO) corrections to the BFKL kernel have been obtained two year ago[6], the complete comparison of the NLO BFKL calculations with the known data and making predictions for NLC are still impossible, for there remains the task of calculating also the next-to-leading order corrections of the coupling of the BFKL Pomeron to the external photons, the so-called Photon Impact Factor(PIF).

[5]J. Bartels, C. Ewerz, and R. Staritzbichler, hep-ph/0004029; A. Donnachie, S. Söldner-Rembold, hep-ph/0001035

[6]V.S. Fadin, L.N. Lipatov, Phys. Lett. **B 429** (1998) 127; M. Ciafaloni, G. Camici, Phys. Lett. **B 430** (1998) 349; and references therein.

2 The $\gamma^* \gamma^*$ Scattering in BFKL Formalism

Before the advent of the field theory for strong interaction, the QCD, it is well known that the Regge theory performed excellently in describing the scattering of particles at high energies with strong interaction. The term Pomeron was introduced to stand for the Reggeon whose intercept is slightly larger than 1 and carries vacuum quantum numbers. With getting more and more confidence in QCD being as the correct field theory of strong interaction, people began to investigate the QCD in various aspects in details, and try to compare its predictions with what obtained in Regge theory as well. The BFKL Pomeron is one of the counterparts of the Regge theory in QCD, developed more than twenty years ago.

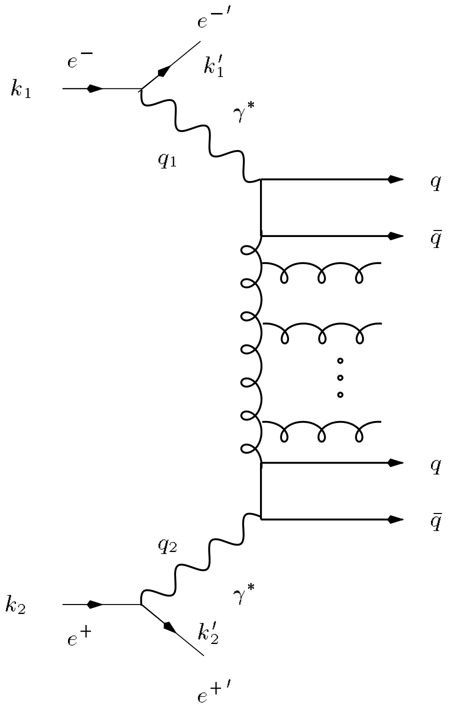

Figure 1: Feynman diagram for the process $e^+e^- \to e^+e^- + X$.

To be more specific, let's have a look on the process $e^+ e^- \to e^+ e^- +$

X. In Regge limit, to leading order in $\text{Log}(\hat{s})$ the interaction can be formatted as, with the idea of Reggeization, a gluon ladder, as depicted in Figure 1. To measure the $\gamma^* \gamma^*$ scattering cross section one needs to make a double tagging of the outgoing leptons close to the forward direction, in this case the virtualities of the two photons would be large enough for the pQCD to be used reliably, but not too large to ensure it is still in the small-x domain. That is

$$Q_1^2,\, Q_2^2 \ll \hat{s}\,, \tag{1}$$

and

$$x_1 = \frac{Q_1^2}{2q_1 \cdot k_2}\,, x_2 = \frac{Q_2^2}{2q_2 \cdot k_1} \ll 1 \tag{2}$$

with $Q_1^2 = -q_1^2$, $Q_1^2 = -q_1^2$, and $\hat{s} = (q_1 + q_2)^2$.

Because the Pomeron has a positive signature, one expects that to leading order in $\text{Log}(\hat{s})$ the the imaginary part of the amplitude square gives the dominant contribution to the cross-section. That means in this accuracy we can get the the cross-section by using the unitarity relation. For instance, the $\gamma^* \gamma^*$ scattering process can be schematically shown as Figure 2,

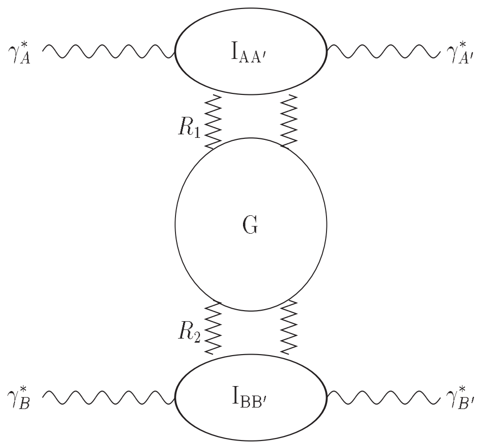

Figure 2: Schematic representation of the process $\gamma_A^* \gamma_B^* \to \gamma_{A'}^* \gamma_{B'}^*$.

Here, the upper and lower blobs represent the Pomeron Photon interactions, in configuration space they are the Photon impact factors, and to leading order can be well understood in the language of Photon wave function[7], while blob G in the middle represents the Green's function for the two interacting Reggeized gluons, the kernel of BFLK equation. To higher order, whether the elegant idea of Photon wave function works or not still remains as an open question.

3 Comparison with the Existing Experimental Data

The comparisons of the BFKL prediction with the experimental data obtained by OPAL and L3 groups at LEP have been done by several groups in details. In both experiments the data lie above the two gluon exchange curve, the Born approximation, but below the BFKL prediction, as shown in figure 3[8], where the $W^2_{\gamma\gamma} = \hat{s}$ and $\sqrt{Q_1^2 Q_2^2} = s_0$.

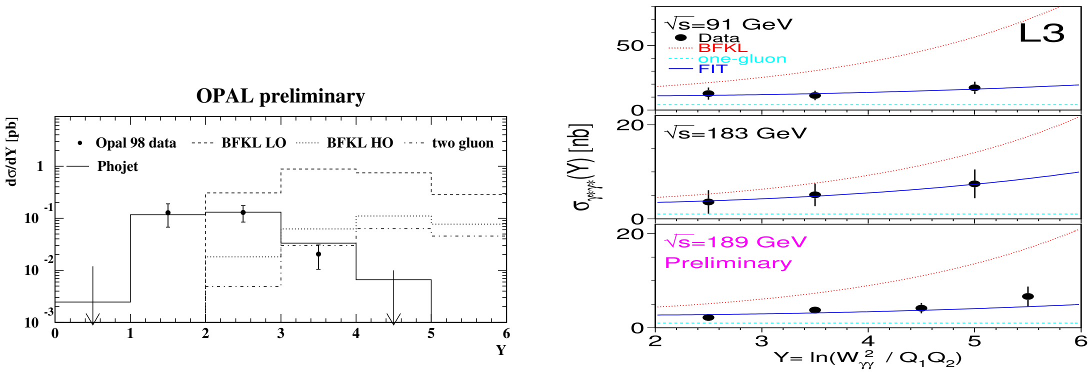

Figure 3: Cross-section of the the process $e^+e^- \to e^+e^-\gamma^*\gamma^* -> e^+e^- + X$ as function of $Y \approx \mathrm{Log}\frac{\hat{s}}{s_0}$.

Because we know the NLO corrections to BFKL is very large, and it will

[7] A.H. Mueller, Nucl. Phys. **B335** (1990) 115; N. Nikolaev, B.G. Zakharov, Z. Phys. **C49** (1991) 607, **C53** (1992) 331; S.J. Brodsky, P. Hoyer, and L. Magnea, Phys. Rev. **D55** (1997) 5585; F. Hautmann, Z. Kunszt, D.E. Soper, Phys. Rev. Lett. **81** (1998) 3333; D. Yu. Ivanov, M. Wüsthoff, Eur. Phys. J. **C8** (1999) 107; S. Gieseke and C.F. Qiao, Phys. Rev. **D61** (2000) 074028.

[8] A. Donnachie and S. Öldner-Rembold, Hep-ph/0001035, and references therein.

lower the theoretical predictions of the cross section, the comparison to go beyond the LO is really necessary. By taking the BLM scale setting scheme[9], Brodsky *et al* first attempted to use the NLO BFKL equation to make the prediction on $\gamma^* \gamma^*$ scattering[10], and the result obtained is encouraging, fits to date pretty well. However, the question still remains that the NLO PIF is still unknow, which for sure would cast some doubts on the result.

Why the QCD corrections to the PIF are important is not only because the corrections may large, but also for consistency reason of the BFKL formalism. One of the uncertainties in making theoretical calculations in leading order BFKL formalism is the energy scale s_0, which value is not determined at leading order and the choice of it is somewhat with arbitrariness, and its influence to the LO predictions is considerably large[11]. However, the uncertainty in s_0 is less serious in the NLO BFKL calculations since dependence of the cross-section on the rapidity, $Y \approx \mathrm{Log}\frac{\hat{s}}{s_0}$, is much weaker. Another major source of uncertainty in the LO calculation is the renormalization scale μ^2, which also incurs large errors in the prediction. e.g, Brodsky *et al* [12] showed that the scale change $\mu^2 \to 4\mu^2$ may alter the results by a factor of 1/4. As well, the scale dependence will be reduced at the NLO level.

[9] S.J. Brodsky, G.P. Lepage and P.B. Machenzie, Phys. Rev. D**28** (1983) 288.

[10] S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, and G.B. Pivovarov, JETP Lett. **70** (1999) 155.

[11] J. Bartels, A. De Roeck, H. Lotter, Phys. Lett. **B389** (1996) 742.

[12] S.J. Brodsky, F. Hautmann, D.E. Soper, Phys. Rev. Lett. **78** (1997) 803, erratum ibid **79** (1997) 3522; ibid. Phys. Rev. D**56** (1997) 6957.

4 The Calculation of QCD Corrections to Photon Impact Factor

The calculations of the NLO corrections to the PIF are still on the way [13]. In the following we sketch the main lines in doing the calculation.

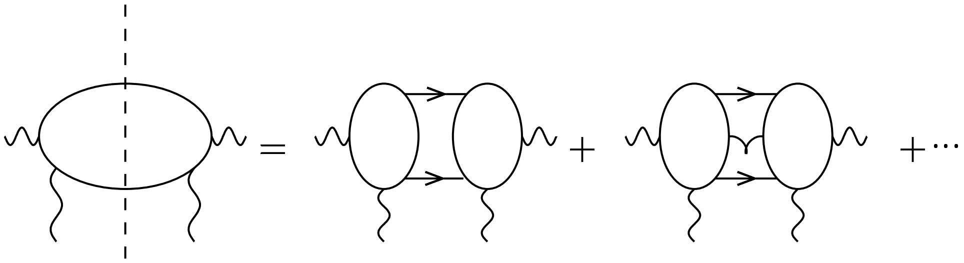

Figure 4: Contributions to the photon impact factor.

As shown in Figure 4, the photon impact factor can be obtained from the energy discontinuity of the amplitude $\gamma^* + Reggeon \to \gamma^* + Reggeon$. In leading order α_s this discontinuity is simply the square of the scattering amplitude $\gamma^* + Reggeon \to q\bar{q}$ in the tree approximation, and the Reggeon, i.e. the Reggeized gluon, can be identified with the elementary t-channel gluon (with a particular helicity). In the next-to-leading order new contributions have to be calculated. For the $q\bar{q}$ intermediate state we need the NLO corrections to the $\gamma^* + Reggeon \to q\bar{q}$ amplitude either on the lhs or on the rhs of the discontinuity line, and the $q\bar{q}g$ intermediate state requires with leading order amplitudes $\gamma^* + Reggeon \to q\bar{q}g$ on both sides of the energy discontinuity line. The task of calculating the NLO corrections to the PIF therefore can be organized in three steps, (i) the calculation of the NLO corrections to the $\gamma^* + Reggeon \to q\bar{q}$ vertex, (ii) the vertex $\gamma^* \to q\bar{q}g$ in leading order, and (iii) the integration over the phase space of the intermediate

[13] J. Bartels, S. Gieseke, and C.-F. Qiao, Hep-ph/9909102; V. Fadin, D. Ivanov, and M. Kotsky, hep-ph/0007119.

states.

Up to now, the first of those three steps has been finished, that is the NLO corrections to the Reggeon-Photon coupling. To get the NLO Reggeon-Photon Coupling, for simplicity we can consider the process $\gamma^* + q \to q\bar{q} + q$ in the high energy scattering limit, as shown in Figure 5.

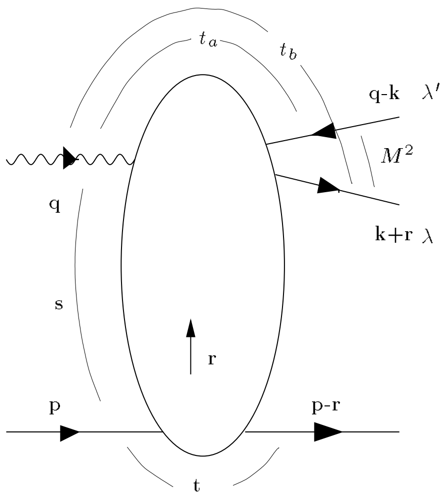

Figure 5: Kinematics of the process $\gamma^* + q \to q\bar{q} + q$.

At high energy Regge limit, that is

$$t,\ Q^2,\ t_a,\ t_b,\ M^2\ \ll s\ , \tag{3}$$

with the help of helicity amplitude formalism[14] and Sudakov decomposition, e.g.,

$$k = \alpha q' + \beta p + k_\perp\ , \tag{4}$$

all the next-to-leading diagrams, depicted as Figure 6 and their charge conjugation partners are calculated. To proceed, the technique and results of Campbell *et al*'s[15] are emploied. In order to apply for the

[14] A. Ballistrero and E. Maina, Phys. Lett. **B350** (1995) 225.

[15] J.M. Campbell, E.W.N. Glover, and D.J. Miller, Nucl. Phys. **B 498** (1997) 397.

process of $\gamma^*\gamma^*$ reaction high energy limit is properly taken with the help of the computer algebra system Mathematica and the package FeynCalc[16].

The final results are quite complicated, while the further investigations show that they possess some intrinsic natures as expected. For example, the t_a t_b symmetry, the gauge invariance and the large M^2 asymptotic behavior, etc.

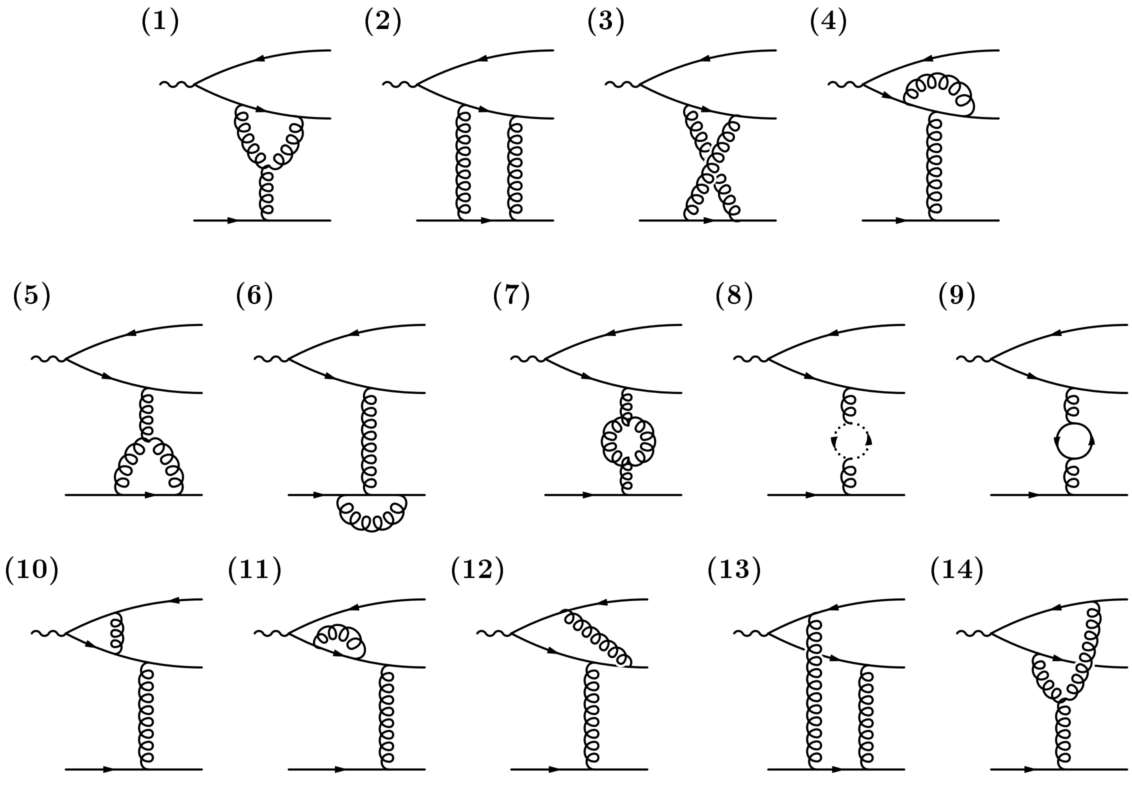

Figure 6: Feynman diagrams for the process $\gamma^* + q \to q\bar{q} + q$.

Because of complexity of the results in NLO Reggeon-Photon vertex, the following two steps left in calculating the NLO PIF are obviously not an easy task. Now, at least two groups are working on it.

[16]R. Mertig, M. Böhm and A. Denner, Comput. Phys. Commun. **64**, (1991) 345.

5 Summary and Conclusions

In this talk we have simply reviewed the theoretical status in the research of finding QCD Pomeron signal in $\gamma^* \gamma^*$ process. The current comparisons of theoretical calculations with the LEP data are encouraging, but still pre-mature, uncertainties lie in both theory and experiment. People expect that at the NLC, with high luminosity amd larger center of mass energy of up to 1 TeV, an undoubted test on BFKL Pomeron can be proceed.

In the end we would like to point out that the LEP date for the total $\gamma^* \gamma^*$ cross section clearly indicate that the LO BFKL and two-gluon model are not sufficient to describe the data. To make a consistent NLO prediction for NLC or comparison with the LEP data, the still unknown PIF with NLO QCD corrections is an inevitably ingredient.