Universal Torsion Induced Interaction from Large Extra Dimensions

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What is Torsion ?

$$T^{\alpha}{}_{\beta\gamma} = \tilde{\Gamma}^{\alpha}{}_{\beta\gamma} - \tilde{\Gamma}^{\alpha}{}_{\gamma\beta}$$

Usually set to zero in General Relativity since it cannot be removed by a change of coordinates. However :

Metric $\Longleftrightarrow$ Energy-Momentum

Torsion $\Longleftrightarrow$ Spin

Natural to include torsion in the presence of fermions.

With torsion, the connections are:

$$\tilde{\Gamma}^{\alpha}{}_{\beta\gamma} = \Gamma^{\alpha}{}_{\beta\gamma} + K^{\alpha}{}_{\beta\gamma}$$

with

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\delta} \left(\partial_\beta g_{\delta\gamma} + \partial_\gamma g_{\delta\beta} - \partial_\delta g_{\beta\gamma} \right)$$

$$K^{\alpha}{}_{\beta\gamma} = \frac{1}{2} \left(T^{\alpha}{}_{\beta\gamma} - T_{\beta}{}^{\alpha}{}_{\gamma} - T_{\gamma}{}^{\alpha}{}_{\beta} \right)$$

contorsion

Covariant Derivative on Fermions :

$$\tilde{\nabla}_\mu \psi = \partial_\mu \psi + \frac{i}{2} \tilde{\omega}_\mu^{ab} \underbrace{\sigma_{ab}}_{\frac{i}{2}[\gamma_a, \gamma_b]} \psi$$

vierbein

$$\tilde{\omega}_\mu^{ab} = \frac{1}{4}\left(e_{b\alpha} \partial_\mu e^\alpha_a - e_{a\alpha} \partial_\mu e^\alpha_b \right) + \frac{1}{4} \Gamma^\alpha_{\lambda\mu} \left(e^\lambda_a e_{b\alpha} - e^\lambda_b e_{a\alpha} \right) + \frac{1}{4} K^\alpha_{\lambda\mu} \left(e^\lambda_a e_{b\alpha} - e^\lambda_b e_{a\alpha} \right)$$

Comes from the requirement that :

$$\tilde{\nabla}_\mu (\bar{\psi} \gamma^\alpha \psi) = \partial_\mu (\bar{\psi} \gamma^\alpha \psi) + \tilde{\Gamma}^\alpha_{\lambda\mu} (\bar{\psi} \gamma^\lambda \psi)$$

The Model:

1. All Standard Model particles are confined to 4 dimensions.

2. Gravity, with torsion, exists in $4+n$ dimensions.

Action:

$$S = -\frac{1}{\hat{\kappa}^2}\int d^{4+n}x \sqrt{|\hat{g}_{4+n}|}\, \tilde{R}$$

$$+\int d^4x \sqrt{|\hat{g}_4|}\, \frac{i}{2}\left[\bar{\Psi}\gamma^\mu \tilde{\nabla}_\mu \Psi - (\tilde{\nabla}_\mu \bar{\Psi})\gamma^\mu \Psi + 2iM\bar{\Psi}\Psi\right]$$

$$\hat{\kappa}^2 = 16\pi G_N^{(4+n)}$$

$$S = -\frac{1}{\hat{\kappa}^2}\int d^{4+n}x\,\sqrt{|\hat{g}_{4+n}|}\left(R - K^{\hat{\mu}\hat{\nu}\hat{\rho}}K_{\hat{\mu}\hat{\nu}\hat{\rho}}\right)$$

$$+\int d^4x\,\sqrt{|\hat{g}_4|}\;i\bar{\Psi}\left(\gamma^\mu \nabla_\mu - \frac{1}{8}S^\mu\gamma_\mu\gamma_5 + iM\right)\Psi$$

$$S^\mu \equiv i\varepsilon^{\alpha\beta\gamma\mu}T_{\alpha\beta\gamma}$$

Equations of Motion :

$$\begin{cases} \mu = 1,\cdots,4 \\ \hat{\mu} = 1,\cdots,4+n \\ i = 5,\cdots,4+n \end{cases}$$

$$K_{i\hat{\mu}\hat{\nu}} = 0$$

$$S_\mu = i\frac{3}{2}\frac{\sqrt{|\hat{g}_4|}}{\sqrt{|\hat{g}_{4+n}|}}\hat{\kappa}^2\,\bar{\Psi}\gamma_\mu\gamma_5\Psi\;\delta^{(n)}(x)$$

$$R_{\hat{\mu}\hat{\nu}} - \frac{1}{2}g_{\hat{\mu}\hat{\nu}}R = \frac{\hat{\kappa}^2}{2}T_{\hat{\mu}\hat{\nu}} + O(\hat{\kappa}^4)$$

Energy Momentum Tensor

Eliminate torsion from action:

$$S = -\frac{1}{\hat{\kappa}^2}\int d^{4+n}x \sqrt{|\hat{g}_{4+n}|}\, R$$

$$+ \int d^4x \sqrt{|\hat{g}_4|}$$

$$\times \left[\bar{\psi}(i\gamma^\mu \nabla_\mu - M)\psi + \underbrace{\frac{3}{32}\frac{\sqrt{|\hat{g}_4|}}{\sqrt{|\hat{g}_{4+n}|}}\hat{\kappa}^2 (\bar{\psi}\gamma_\mu\gamma_5\psi)^2 \delta^{(n)}(0)}_{\text{Torsion Induced Four Fermion Interaction !!}} \right]$$

Regularize δ-function

brane width $\approx$ planck mass in 4+n

$$\delta^{(n)}(x) \rightarrow \frac{1}{(2\pi)^n}\int_0^{M_S} d^n k = \frac{M_S^n}{2^{n-1}\pi^{n/2}\, n\, \Gamma(n/2)}$$

$$\hat{\kappa}^2 = 16\pi (4\pi)^{n/2}\Gamma(n/2)\, M_S^{-(n+2)}$$

$$\Delta S = \int d^4x \frac{3\pi}{n M_S^2}\left[\sum_j \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j\right]^2$$

sum over all flavors

$$\frac{3\pi}{n M_s^2}\left[\sum_j \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j\right]^2$$

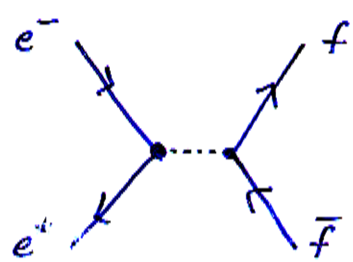
NO SUPPRESSION DUE TO ENERGY OR BRANE RECOIL

Universal !!

U(45) invariant

Can we constrain the size of this interaction ?

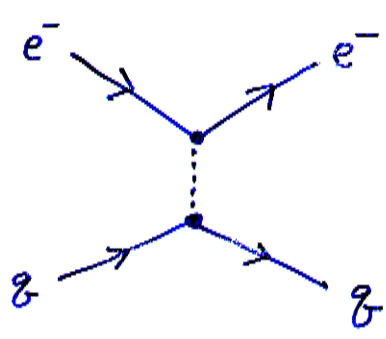
OPAL



$\sqrt{n}\, M_s \geq 10.3$ TeV

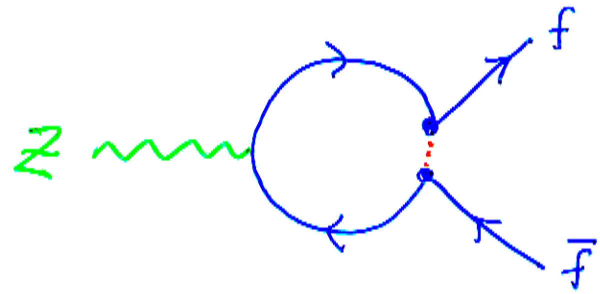
HERA

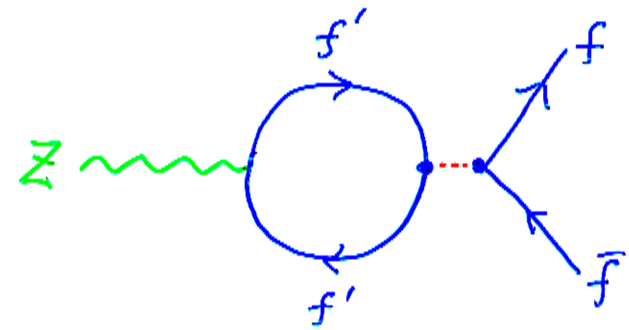
etc.



$\sqrt{n}\, M_s \geq 5.3$ TeV

Precision Electroweak:





~~Universal Multiplicative Correction~~

Common Universal Shift

Cancel in ratios of coupling constants.

$$\Downarrow$$

$$\sqrt{n}\, M_s \geq 39 \text{ TeV}$$

$$\begin{cases} n=2 & M_s \geq 28 \text{ TeV} \\ n=4 & M_s \geq 19 \text{ TeV} \\ n=6 & M_s \geq 15 \text{ TeV} \end{cases}$$

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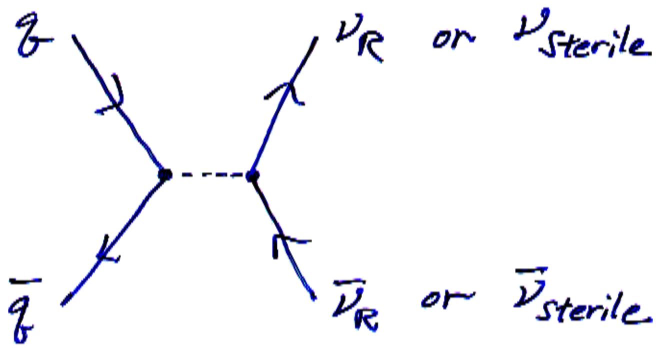

$$\sqrt{n}\, M_S \geq 210 \text{ TeV}$$

Conclusion:

(NOT NECESSARILY ALL)

☆ As long as fermions are confined to 4D, torsion will necessarily induce a universal contact interaction which can be tightly constrained.

☆ Must impose the extra assumption that the connection is symmetric to avoid this.