Collider Signals for Non-Commutative Field Theories

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Motivation for NCQFT:

- Why not? It's interesting!
- String Theory: NCQFT arises through quantization of strings by describing the low-energy excitations of D-branes in background EM fields

Non-Commutative Quantum Field Theories

Generalization of n-dimensional space R^n

Coordinates $x_\mu \Rightarrow$ Operators $\hat{X}_\mu$

$$[\hat{X}_\mu, \hat{X}_\nu] = i\,\theta_{\mu\nu} = \frac{i}{\Lambda_{NC}^2} c_{\mu\nu}$$

Space-time "uncertainty" relation $\Delta\hat{X}_\mu \Delta\hat{X}_\nu \geq \frac{1}{2}|\theta^{\mu\nu}|$

Similar to $\Delta x \Delta p \geq \frac{i}{2}\hbar$

Λ_{NC} = Scale where NC becomes relevant

Most likely value for Λ_{NC} ???

- Probably Planck scale, but which one?

$m_{Pl} \sim 10^{19}$ GeV vs $m_* \sim$ TeV

$c_{\mu\nu}$ = anti-symmetric matrix w/ elements $\mathcal{O}(1)$

Components are identical in all frames

$\Rightarrow$ breaks Lorentz invariance at Λ_{NC}

Formulation of NC Quantum Field Theory

Fields $\phi(x)$ $\Rightarrow$ Operators $\hat{\phi}(\hat{x})$ via Weyl-Moyal Correspondence

Must be careful to preserve ordering in FT!

Introduce Fourier transform pair:

$$\hat{\phi}(\hat{x}) = \int \frac{d^4k}{(2\pi)^4}\, \phi(k)\, e^{ik\hat{x}}$$

$$\phi(k) = \int d^4x\, \phi(x)\, e^{-ikx}$$

Product of 2 Fields $\Rightarrow$ Star Product

$$\begin{aligned}
\hat{\phi}(\hat{x})\,\hat{\phi}(\hat{x}) &= \phi(x) * \phi(x) \\
&= \int \frac{d^4k}{(2\pi)^4} \frac{d^4p}{(2\pi)^4}\, \phi(k)\,\phi(p)\, e^{ik\hat{x}}\, e^{ip\hat{x}} \\
&= \phi(x) \exp\left\{ \tfrac{i}{2}\, \theta^{\mu\nu}\, \overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu \right\} \phi(x) \\
&= \phi(x)\,\phi(x) + \frac{i}{2}\,\theta^{\mu\nu}\,\partial_\mu \phi\, \partial_\nu \phi + \mathcal{O}(\theta^2) + \cdots
\end{aligned}$$

NCQFT = QFT with products $\Rightarrow$ Star Products

+ Moyal brackets

$$[A,B]_{MB} = A*B - B*A$$

with $\int d^4x \, [A(x), B(x)]_{MB} = 0$

NCQFT General Properties

1) Only U(n) Lie algebras are closed under Moyal brackets — Matsubara

$\Rightarrow$ NC gauge theories only based on U(n)

2) Renormalizable, gauge invariant FT remain so — Martin, Sanchez-Ruiz ⋮

(But SSB? Campbell, Kaminsky)

3) Covariant derivatives constructed only for fields w/ $Q = 0, \pm 1$

4) NCQFT w/ space-space NC is $\cancel{CP}$ — Sheikh-Jabbari

NC QED

Hayakawa
Ardolan, Sadooghi
Riad, Sheikh-Jabbari
Martin, Sanchez-Ruiz

$$S_{NCQED} = -\frac{1}{4} \int d^4x\, F_{\mu\nu} * F^{\mu\nu}$$

Gauge invariant under local trans w/

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\,[A_\mu, A_\nu]_{MB}$$

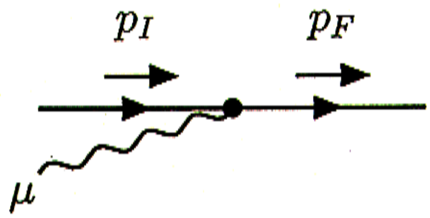
↳ Induces non-abelian terms!

⇒ 3 + 4-pt functions for γ self-coupling

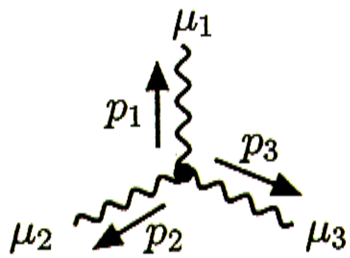
+ interaction vertices pick up momenta dependent phase factors from Fourier transforms

Important for collider tests!

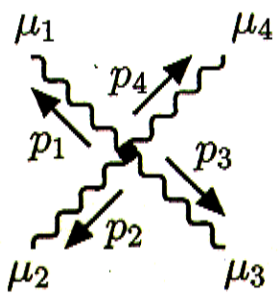
Non-commutative Feynman Rules



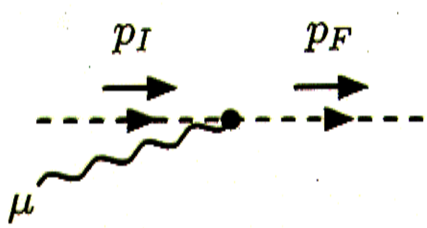
$$= ig\gamma^\mu \exp\left(\tfrac{i}{2} p_I C p_F\right)$$



$$\begin{aligned} &= +2g \sin\left(\tfrac{1}{2} p_1 C p_2\right) \\ &\quad \times [(p_1 - p_2)^{\mu_3} g^{\mu_1\mu_2} \\ &\quad + (p_2 - p_3)^{\mu_1} g^{\mu_2\mu_3} \\ &\quad + (p_3 - p_1)^{\mu_2} g^{\mu_3\mu_1}] \end{aligned}$$



$$\begin{aligned} &= +4ig^2[(g^{\mu_1\mu_3} g^{\mu_2\mu_4} - g^{\mu_1\mu_4} g^{\mu_2\mu_3}) \\ &\quad \times \sin\left(\tfrac{1}{2} p_1 C p_2\right) \sin\left(\tfrac{1}{2} p_3 C p_4\right) \\ &\quad + (g^{\mu_1\mu_4} g^{\mu_2\mu_3} - g^{\mu_1\mu_2} g^{\mu_3\mu_4}) \\ &\quad \times \sin\left(\tfrac{1}{2} p_3 C p_1\right) \sin\left(\tfrac{1}{2} p_2 C p_4\right) \\ &\quad + (g^{\mu_1\mu_2} g^{\mu_3\mu_4} - g^{\mu_1\mu_3} g^{\mu_2\mu_4}) \\ &\quad \times \sin\left(\tfrac{1}{2} p_1 C p_4\right) \sin\left(\tfrac{1}{2} p_2 C p_3\right)] \end{aligned}$$



$$= 2igp_F^\mu \sin\left(\tfrac{1}{2} p_I C p_F\right)$$

Propagators are unchanged

$$\int d^4x\, \hat{\phi}(\hat{x}) * \hat{\phi}(\hat{x}) = \int d^4x\, \phi(x)\, \phi(x)$$

Parameterization of Effect

$$[\hat{x}_\mu, \hat{x}_\nu] = i\,\theta_{\mu\nu} \equiv \frac{i}{\Lambda_{NC}^2}\, c_{\mu\nu}$$

$$c_{\mu\nu} = \begin{pmatrix} 0 & c_{01} & c_{02} & c_{03} \\ -c_{01} & 0 & c_{12} & -c_{13} \\ -c_{02} & -c_{12} & 0 & c_{23} \\ -c_{03} & c_{13} & c_{23} & 0 \end{pmatrix} \quad \text{with } \sum_i |c_{0i}|^2 = 1$$

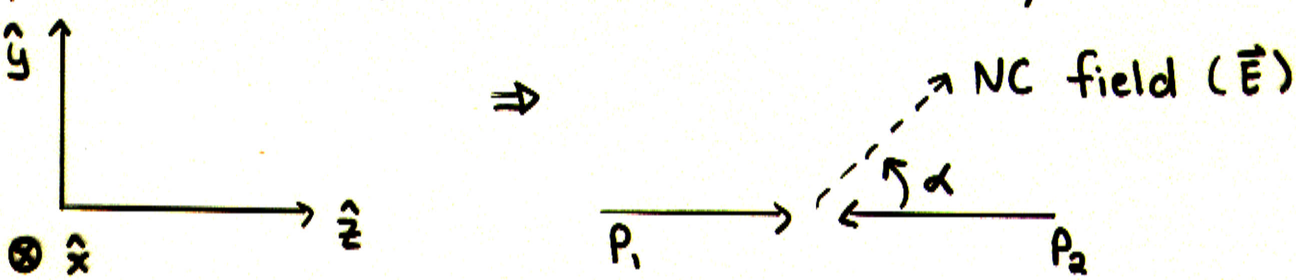
c_{0i} : space-time NC

defines direction of background $\vec{E}$-field

S-matrix is not unitary

$$c_{01} = \sin\alpha \cos\beta$$
$$c_{02} = \sin\alpha \sin\beta$$
$$c_{03} = \cos\alpha$$

β defines origin of ϕ axis : choose $\beta = \pi/2$



C_{ij} : space-space NC

defines direction of background $\vec{B}$-field

S-matrix is unitary

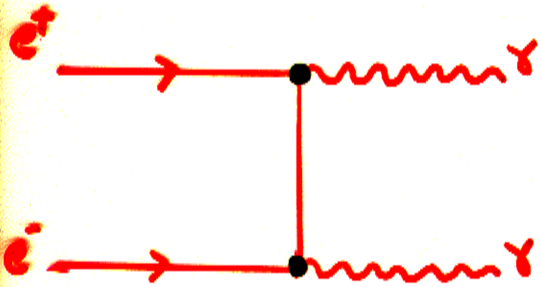
Can choose

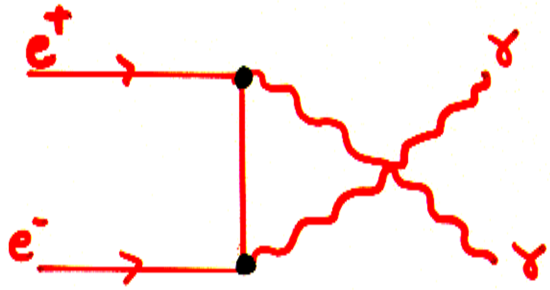
$$C_{12} = \cos\gamma$$

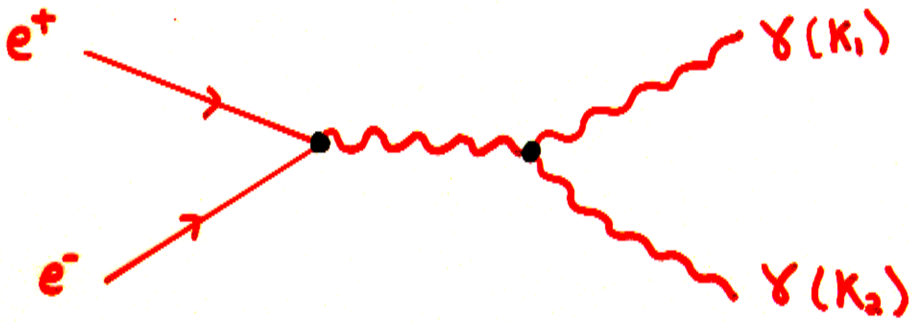
$$C_{13} = \sin\gamma \sin\beta$$

$$C_{23} = -\sin\gamma \cos\beta$$

Pair Annihilation







$$\frac{d\sigma}{d\cos\theta\, d\phi} = \frac{\alpha^2}{4s}\left[\frac{u}{t} + \frac{t}{u} - 4\,\frac{t^2+u^2}{s^2}\sin^2\left(\tfrac{1}{2}\,k_1 \wedge k_2\right)\right]$$

$$\Delta_{NC} \equiv k_1 \wedge k_2 = k_1^{\mu} k_2^{\nu}\, C_{\mu\nu}$$

$$= \frac{-s}{2\Lambda_{NC}^2}\left[c_{01}\sin\theta\cos\phi + c_{02}\sin\theta\sin\phi + c_{03}\cos\theta\right]$$

$\Rightarrow$ only probes space-time NC! [in cm frame]

$$= \frac{-s}{2\Lambda_{NC}^2}\left[\cos\theta\cos\alpha + \sin\theta\sin\alpha\cos(\phi-\beta)\right]$$

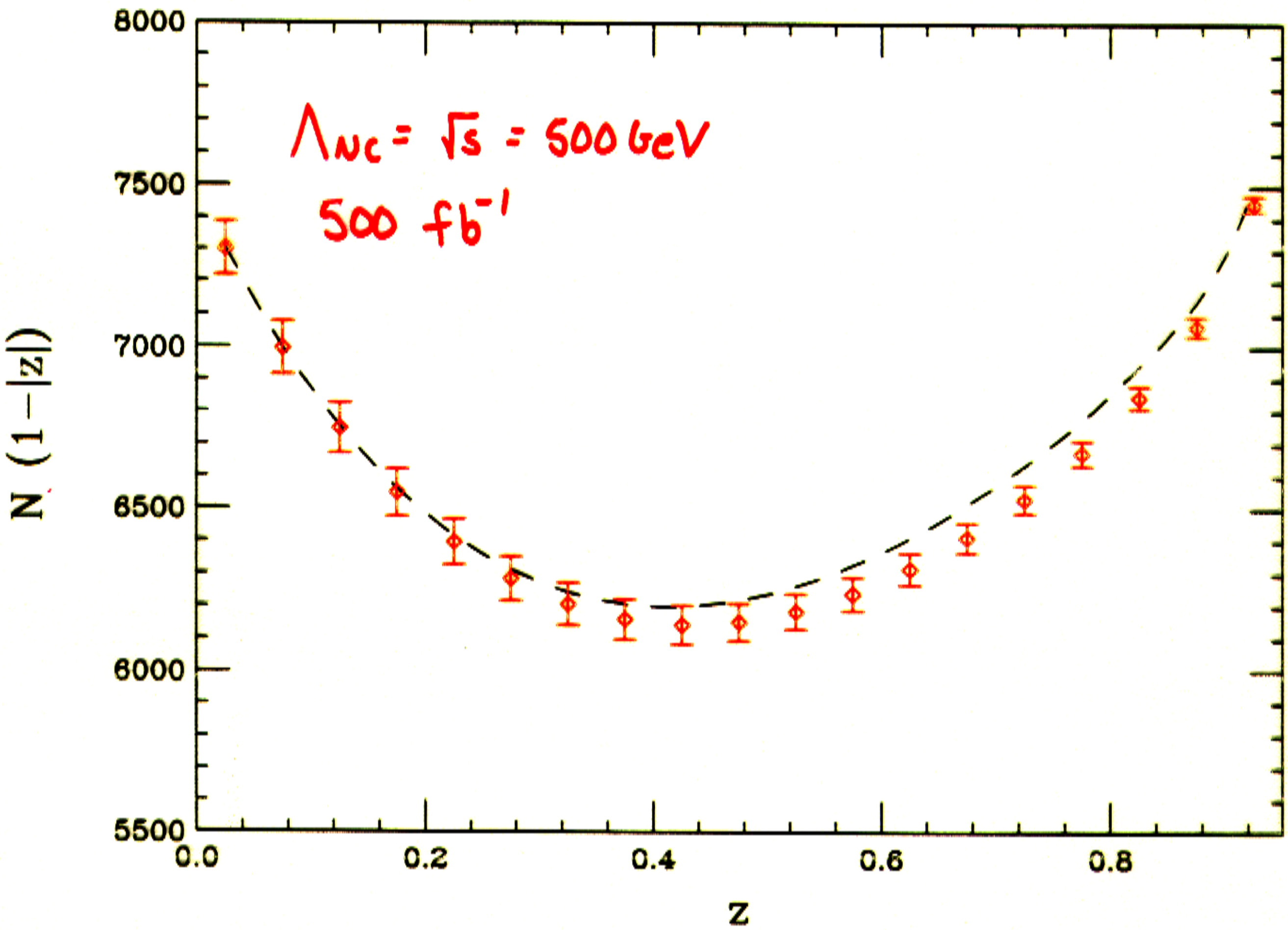
$$= \frac{-s}{2\Lambda_{NC}^2}\cos\theta_{NC}$$

$\longrightarrow \measuredangle$ between $\vec{E}$ & outgoing γ

Bin integrated - normalized angular distribution

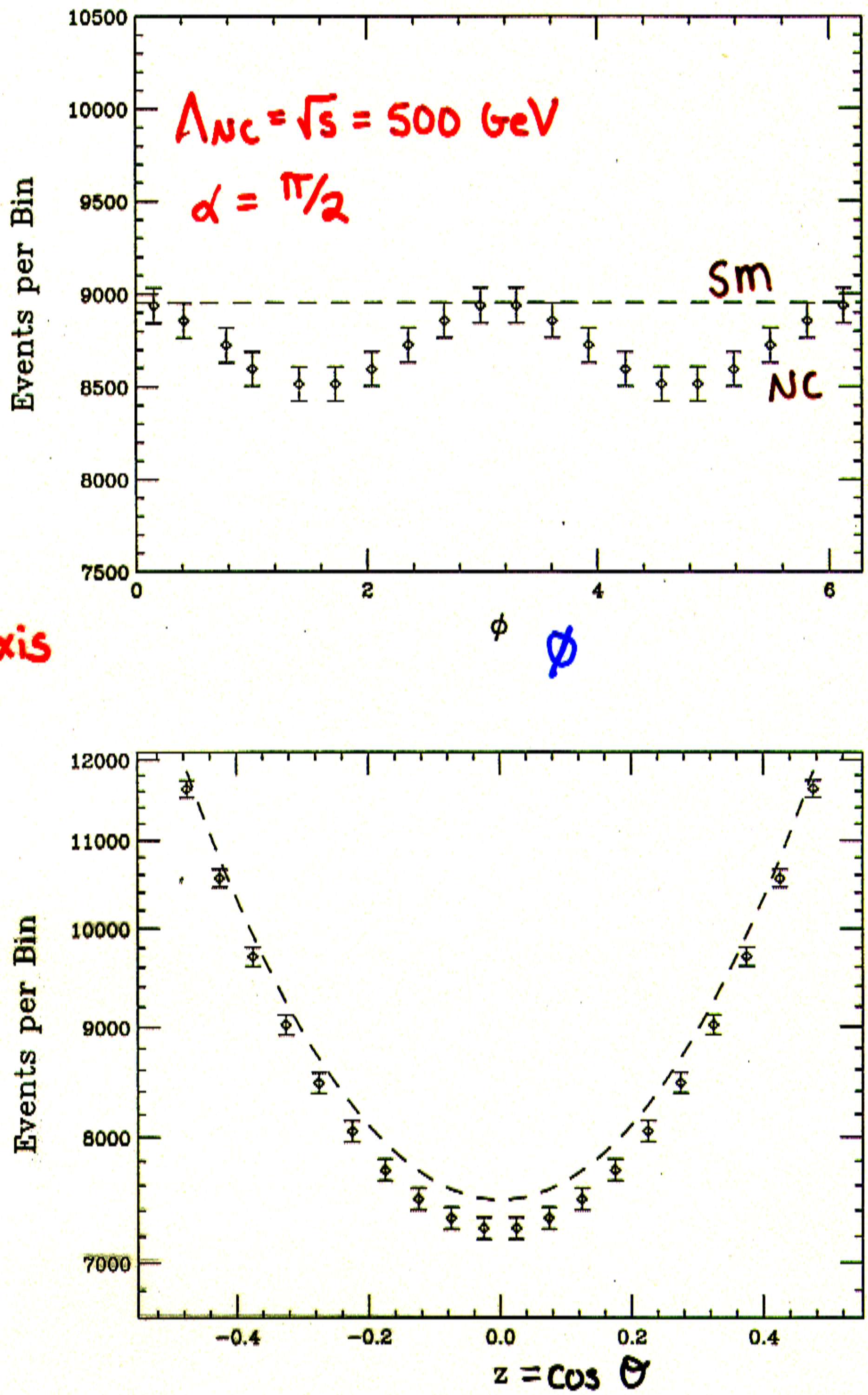
$e^+e^- \to \gamma\gamma$

$\alpha = 0$ $\vec{E}$ // beam



Bin integrated distributions - $e^+e^- \to \gamma\gamma$

\# Events/bin

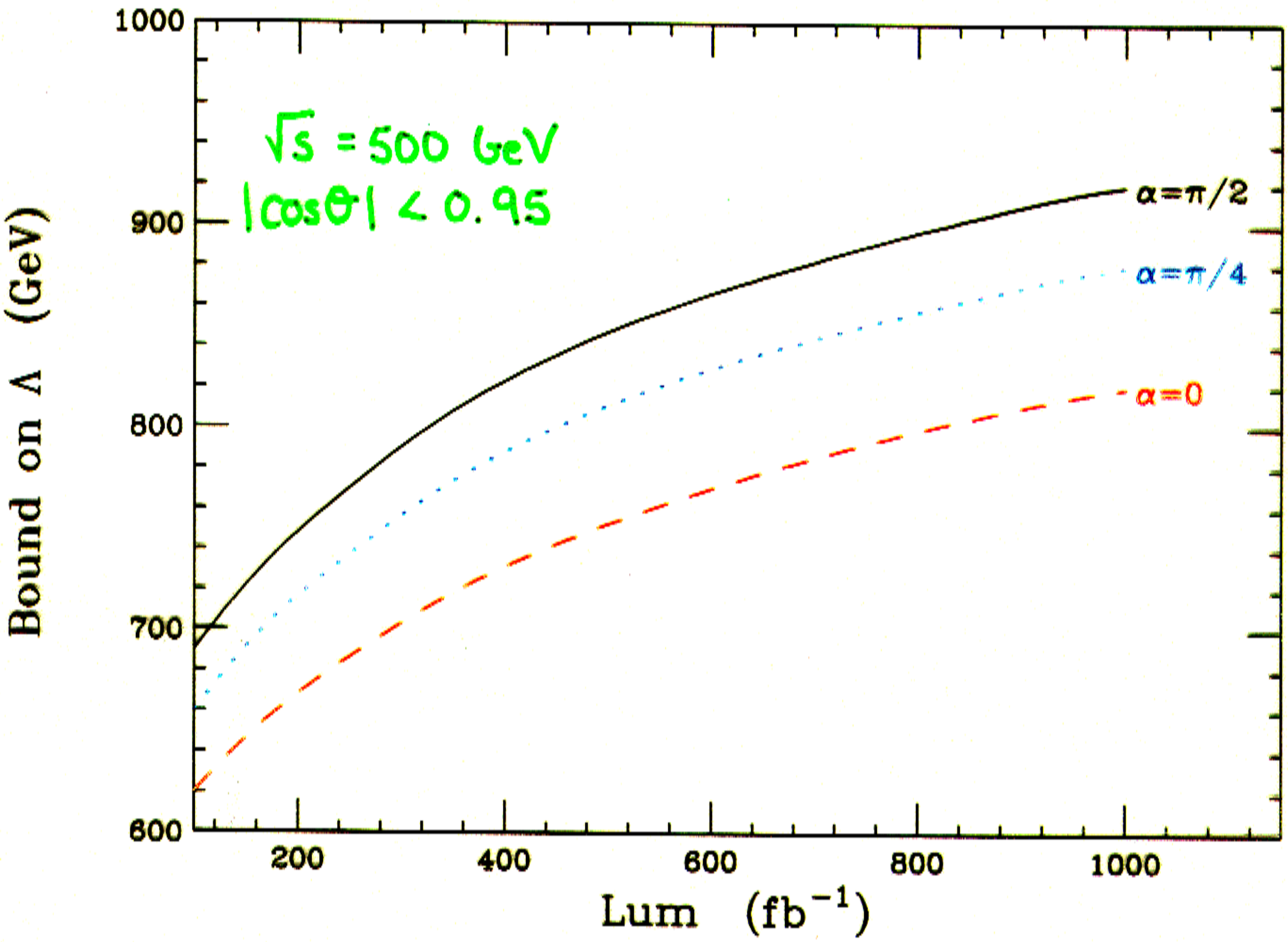


$\vec{E} \perp$ beam axis

Figure 3: ϕ dependence (top) and θ dependence (bottom) of the $e^+e^- \to \gamma\gamma$ cross section for the case $\alpha = \pi/2$. We use $\Lambda = \sqrt{s} = 500$ GeV, luminosity 500 fb^{-1}, parameters relevant for the NLC. In the top panel a cut $|z| < 0.5$ has been employed. The dashed line is the SM expectation.

95% C.L. Bounds on Λ_{NC}

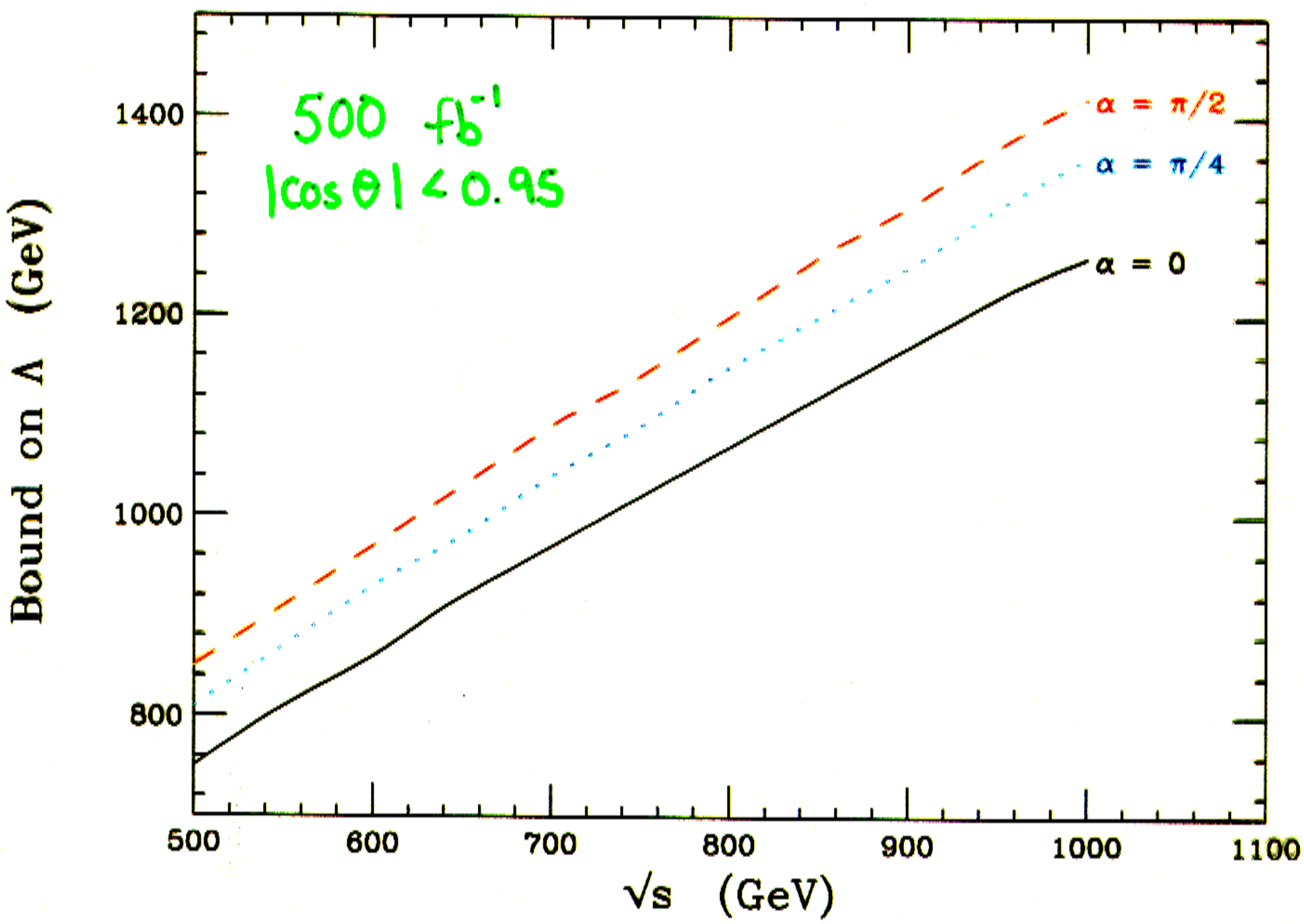
$e^+e^- \to \gamma\gamma$



$\Lambda_{NC}^{max} \sim 1.5\sqrt{s}$

95% CL Bounds on Λ_{NC}

$e^+e^- \to \gamma\gamma$



Moller Scattering

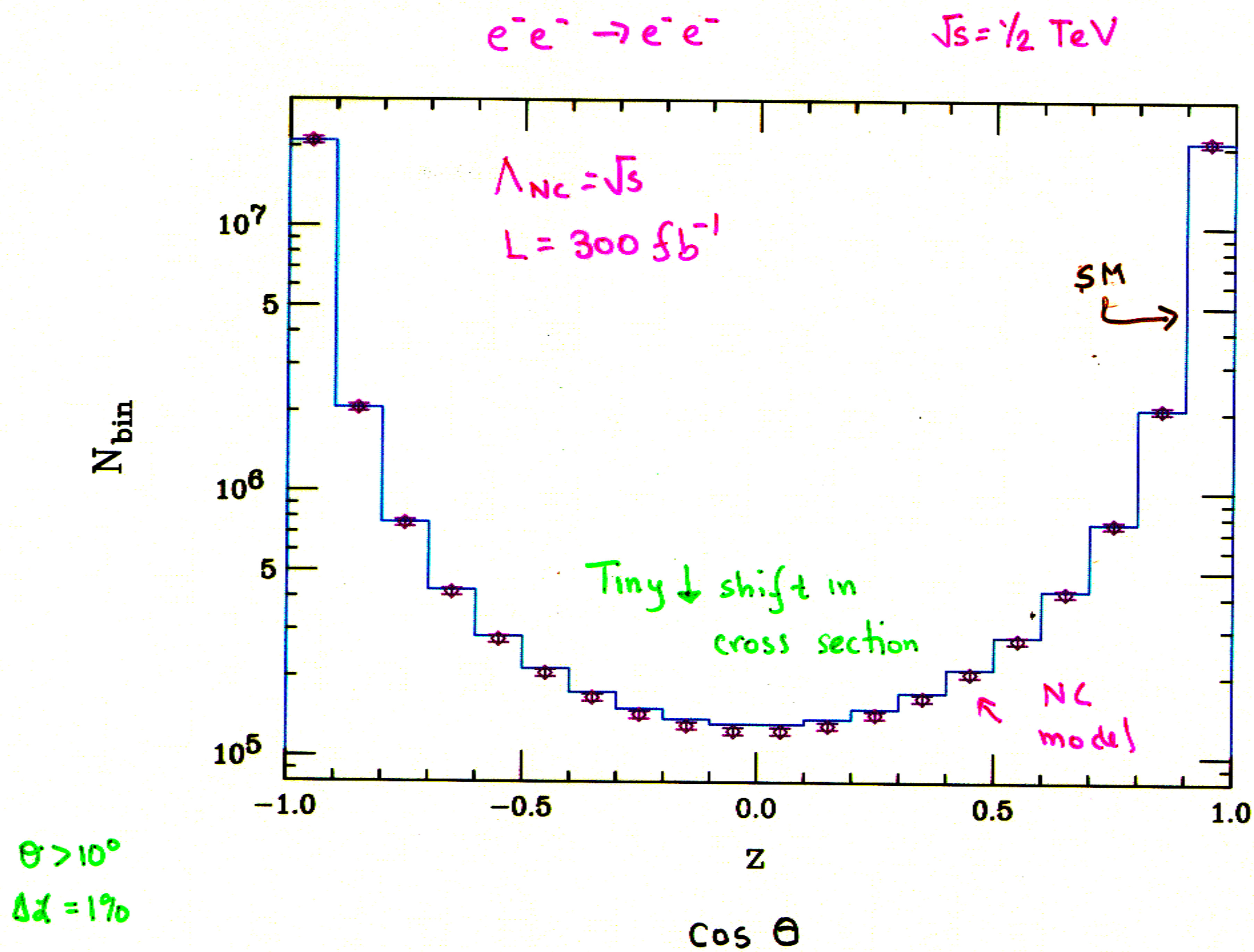
Assume same phase structure for γ/Z

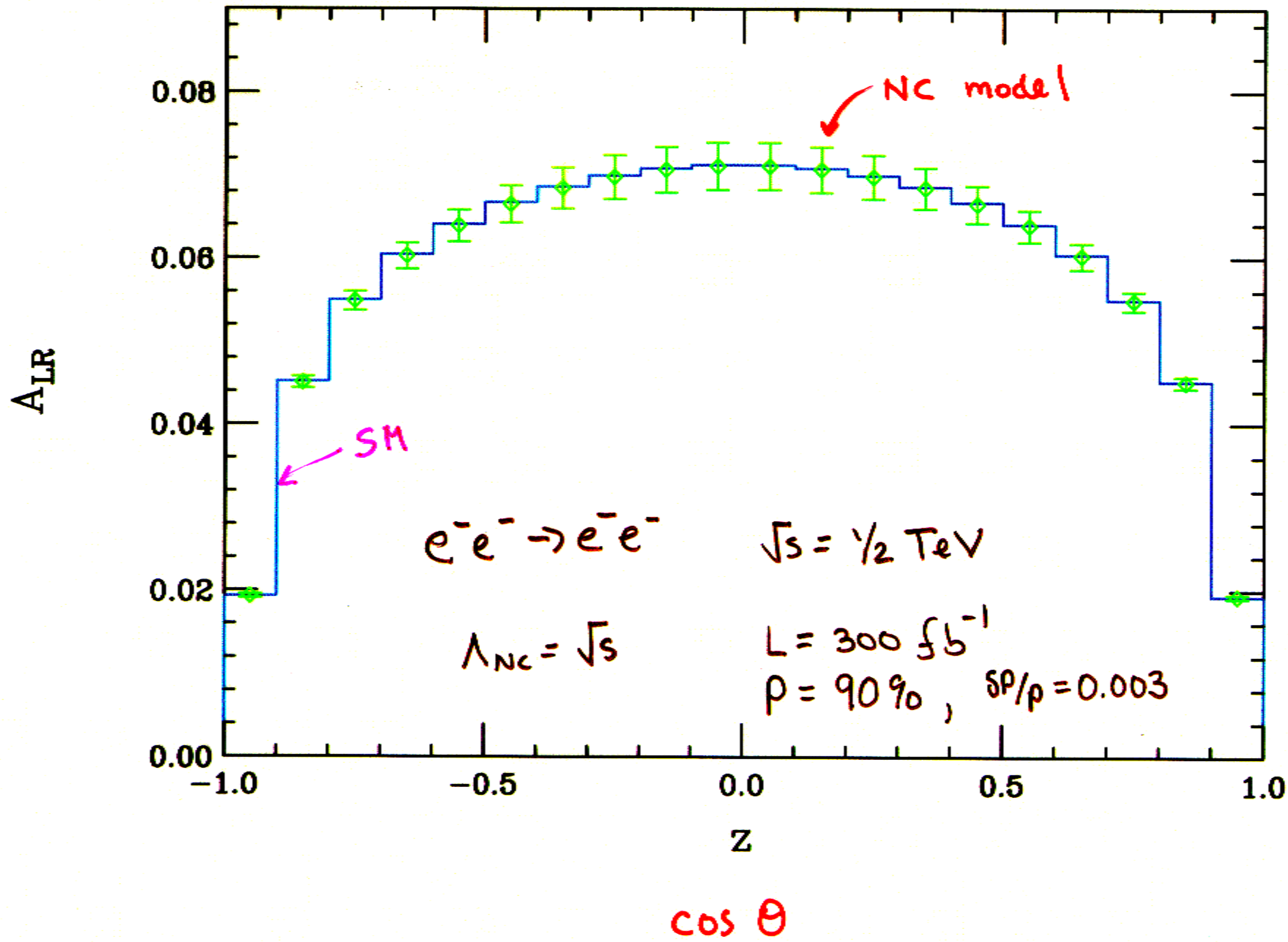
$$\frac{d\sigma}{d\cos\theta\, dz} = \frac{\alpha^2}{4s}\left[(e_{ij}+f_{ij})\left(P^{uu}_{ij}+P^{tt}_{ij}+2P^{ut}_{ij}\cos\Delta_{\text{moller}}\right) + (e_{ij}-f_{ij})\left(\frac{t^2}{s^2}P^{uu}_{ij}+\frac{u^2}{s^2}P^{tt}_{ij}\right)\right]$$

$$\Delta_{\text{moller}} = \phi_u - \phi_t = \frac{-\sqrt{ut}}{\Lambda^2_{NC}}\left[c_{12}\cos\phi - c_{31}\sin\phi\right]$$

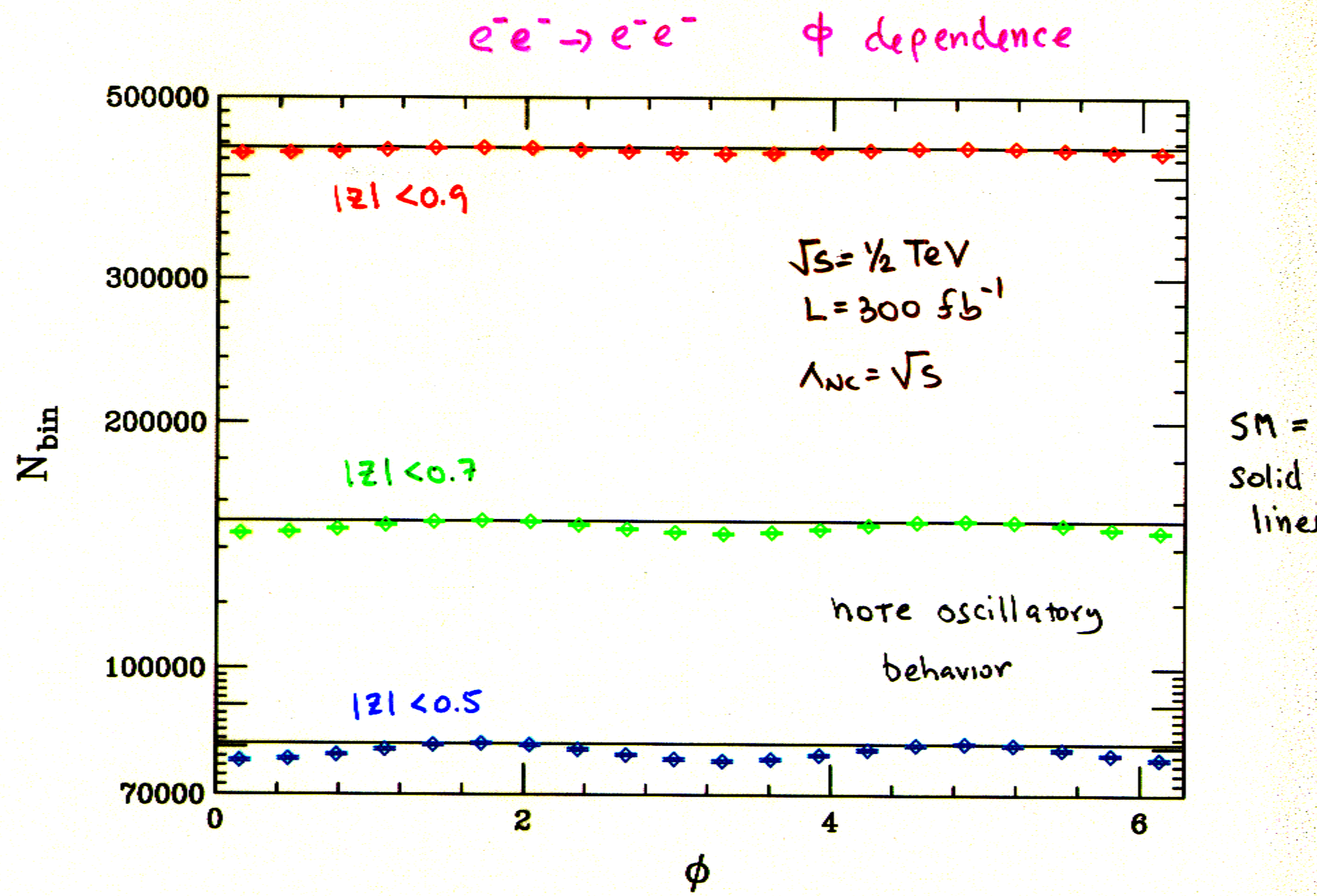
$\Rightarrow$ only probes space-space NC!

Lowest order correction to SM is dim-8

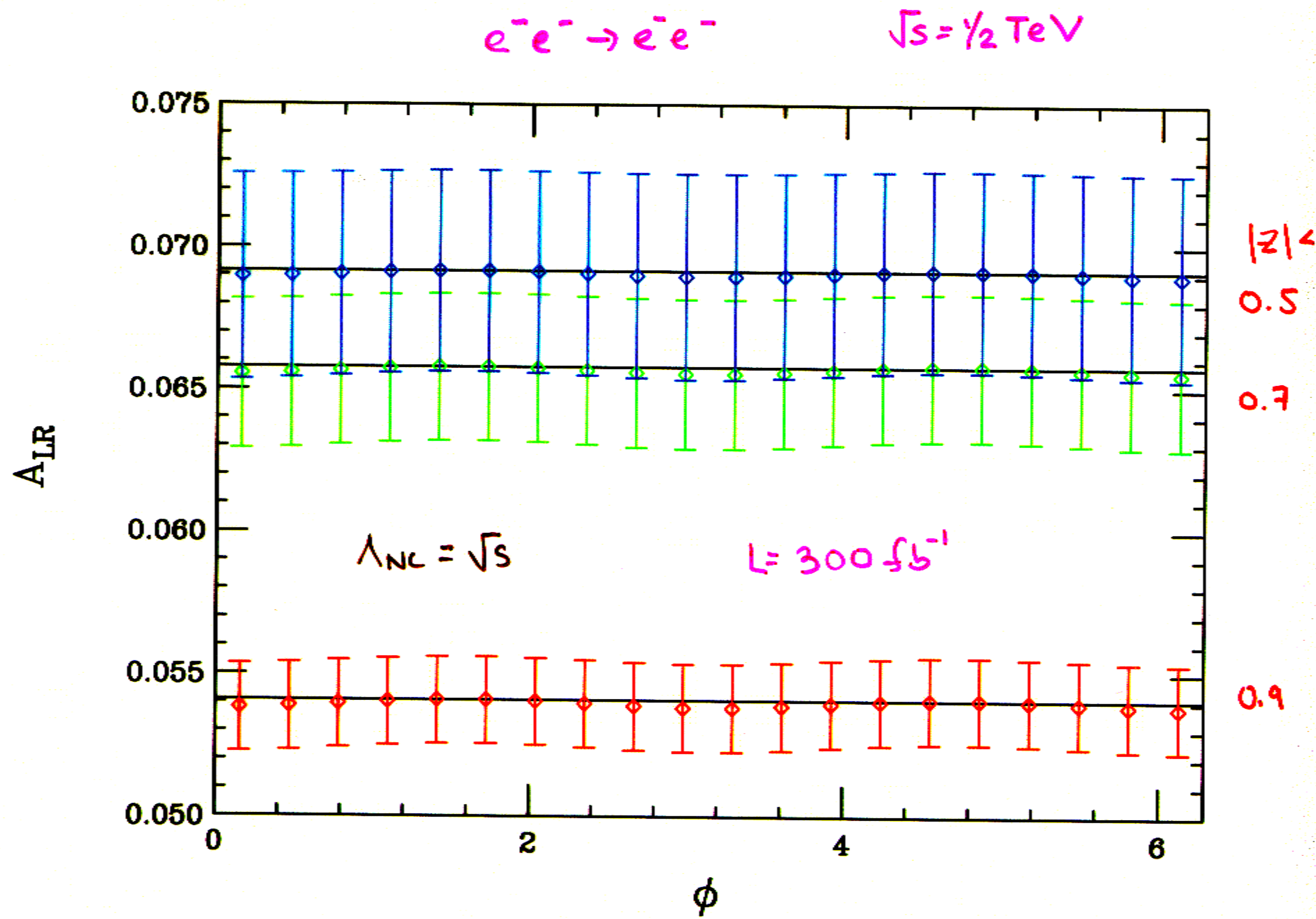

$e^-e^- \to e^-e^-$
$\sqrt{s} = 1/2$ TeV
$\Lambda_{NC} = \sqrt{s}$
$L = 300\ fb^{-1}$
SM
Tiny ↓ shift in cross section
NC model
N_{bin}
10^7
5
10^6
5
10^5
−1.0
−0.5
0.0
0.5
1.0
z
Cos θ
$\theta > 10°$
$\Delta\mathcal{L} = 1\%$



NC model
SM
$e^-e^- \to e^-e^-$
$\sqrt{s} = 1/2$ TeV
$\Lambda_{NC} = \sqrt{s}$
$L = 300\ fb^{-1}$
$P = 90\%$, $\delta P/P = 0.003$
A_{LR}
0.00
0.02
0.04
0.06
0.08
−1.0
−0.5
0.0
0.5
1.0
z
cos θ

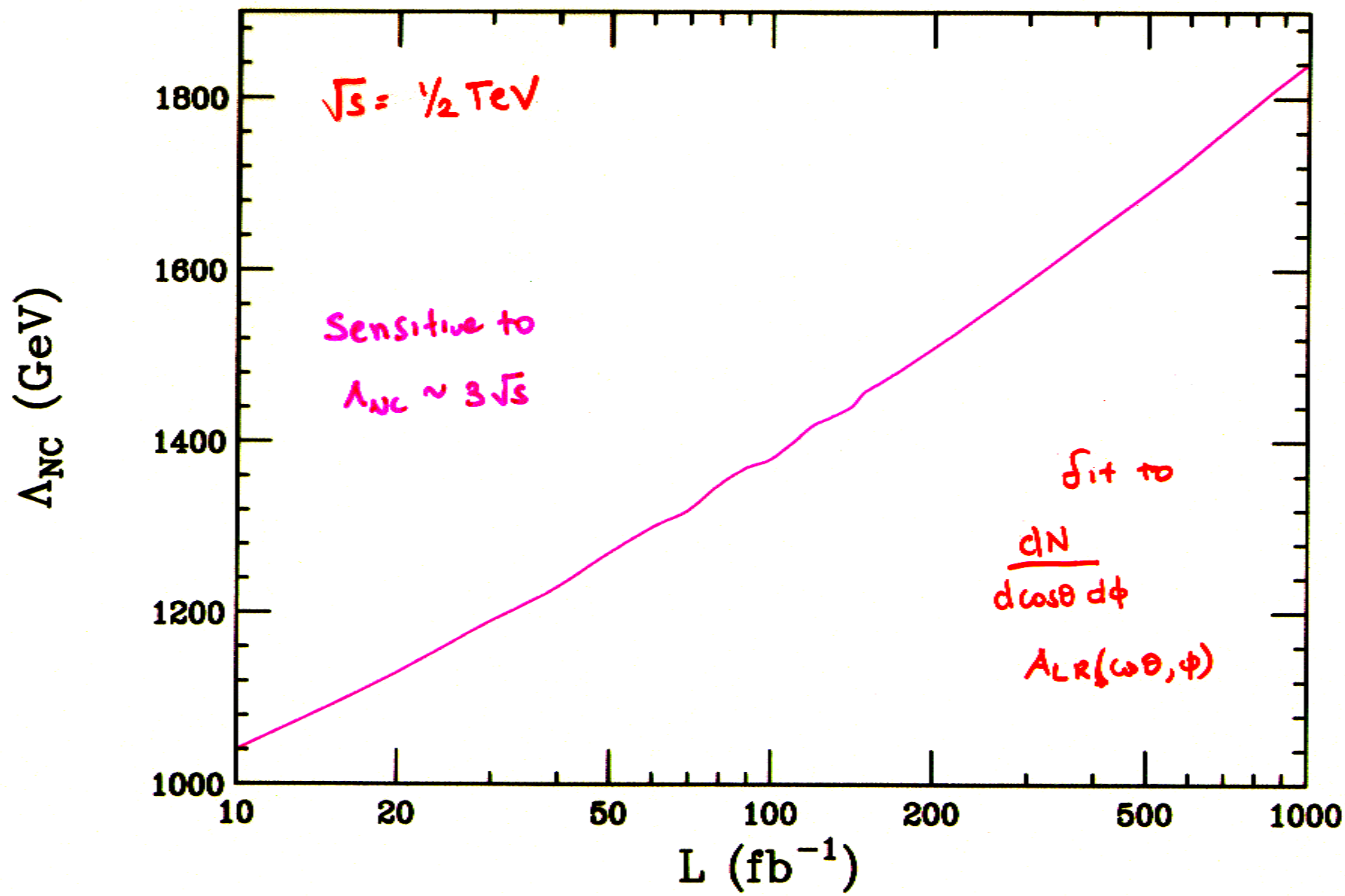


e⁻e⁻ → e⁻e⁻   φ dependence
|z| <0.9
√s = ½ TeV
L = 300 fb⁻¹
Λ_NC = √s
|z| <0.7
note oscillatory behavior
|z| <0.5
SM = solid lines
N_bin
500000
300000
200000
100000
70000
0
2
4
6
φ

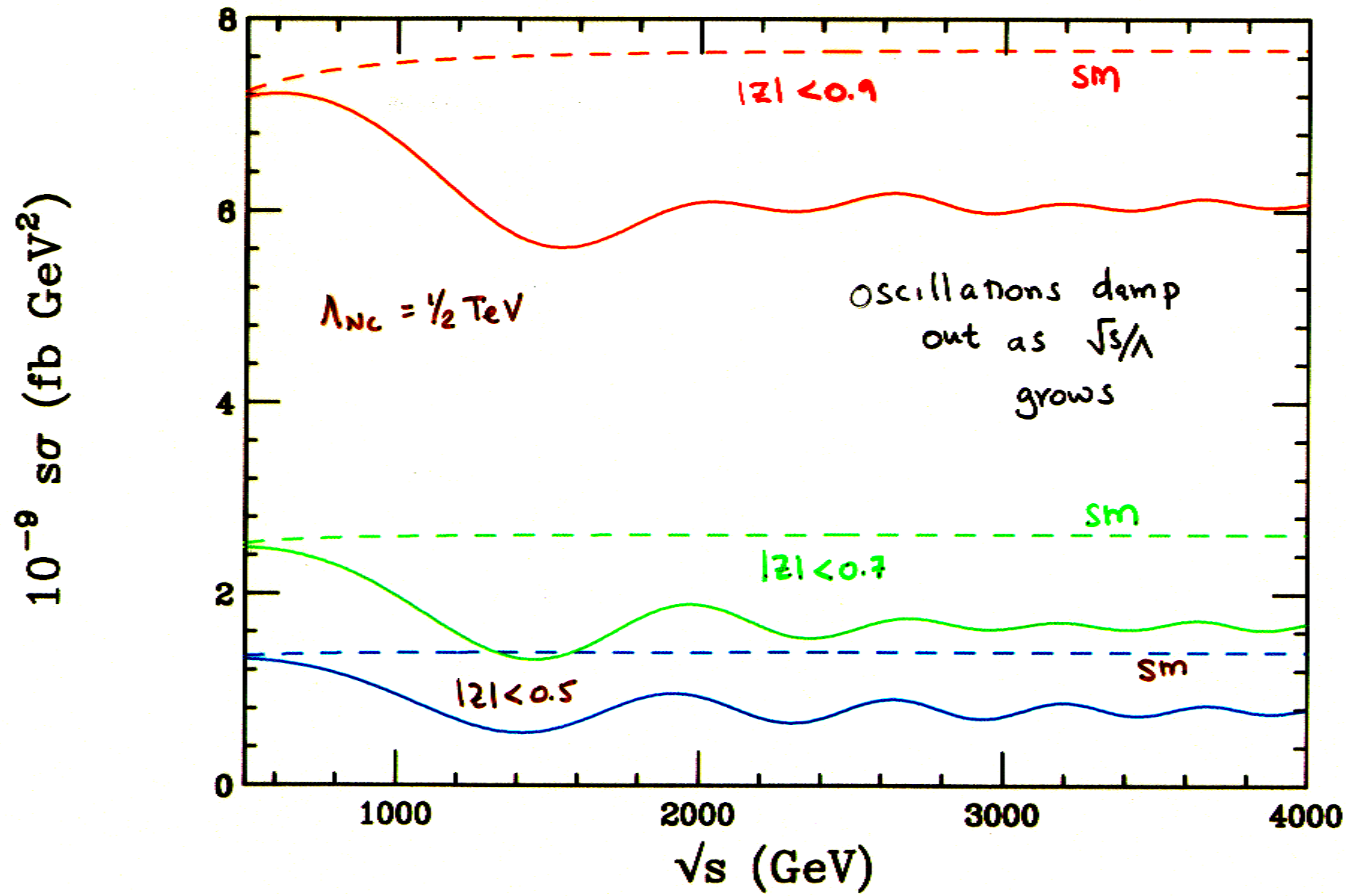


e⁻e⁻ → e⁻e⁻
√s = ½ TeV
0.075
0.070
0.065
0.060
0.055
0.050
A_LR
|z| <
0.5
0.7
0.9
Λ_NC = √s
L = 300 fb⁻¹
0
2
4
6
φ

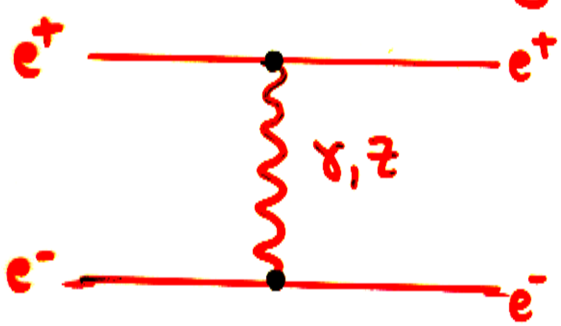
$e^-e^- \to e^-e^-$ 95% CL Lower bound on Λ_{NC}

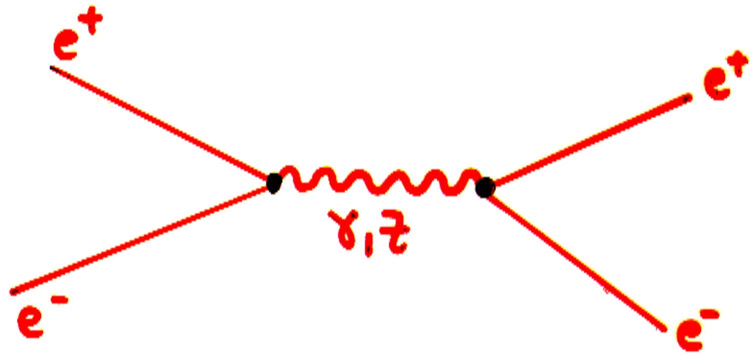


$e^-e^- \to e^-e^-$ hi-Energy behavior



Bhabha Scattering



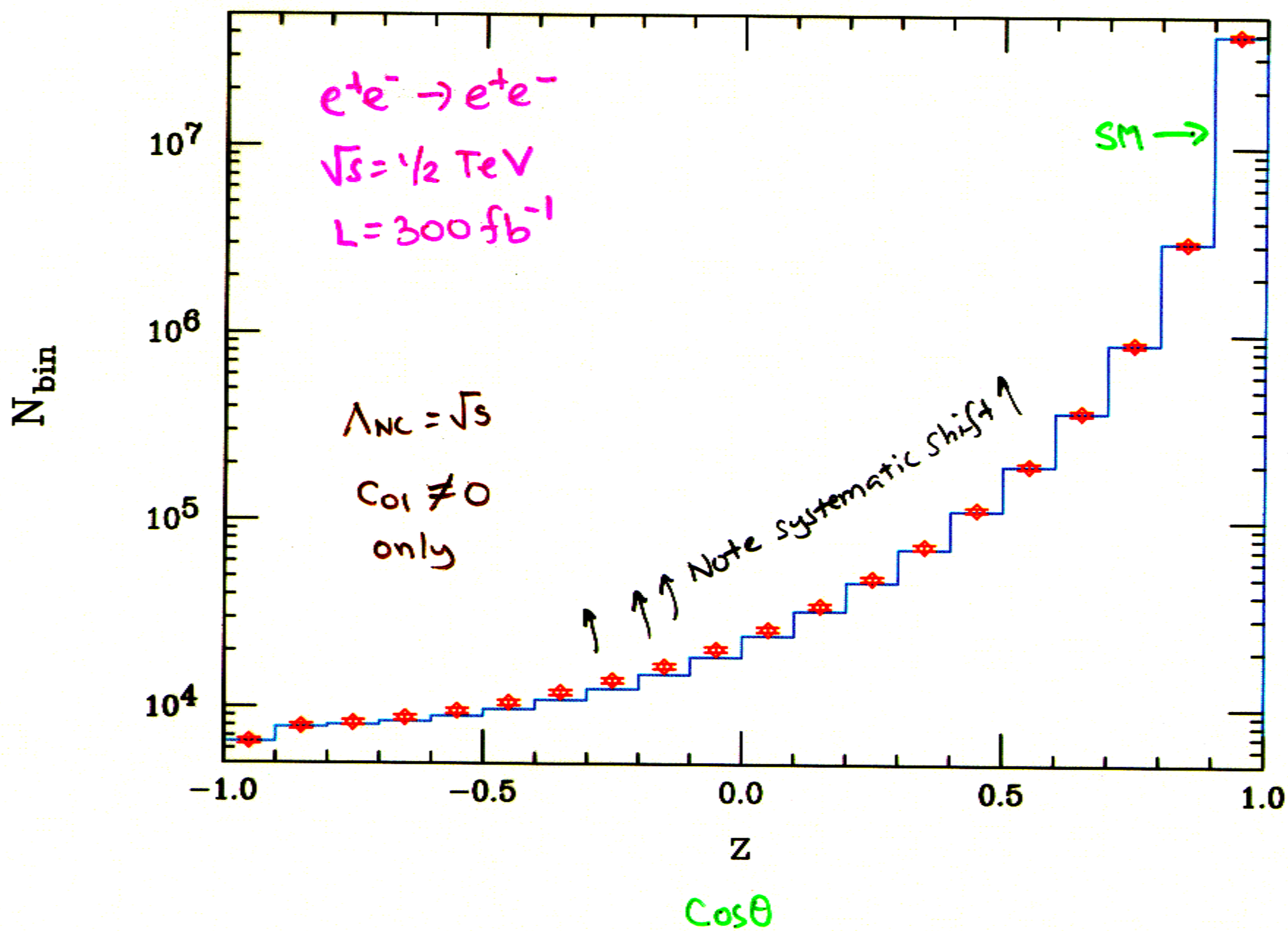


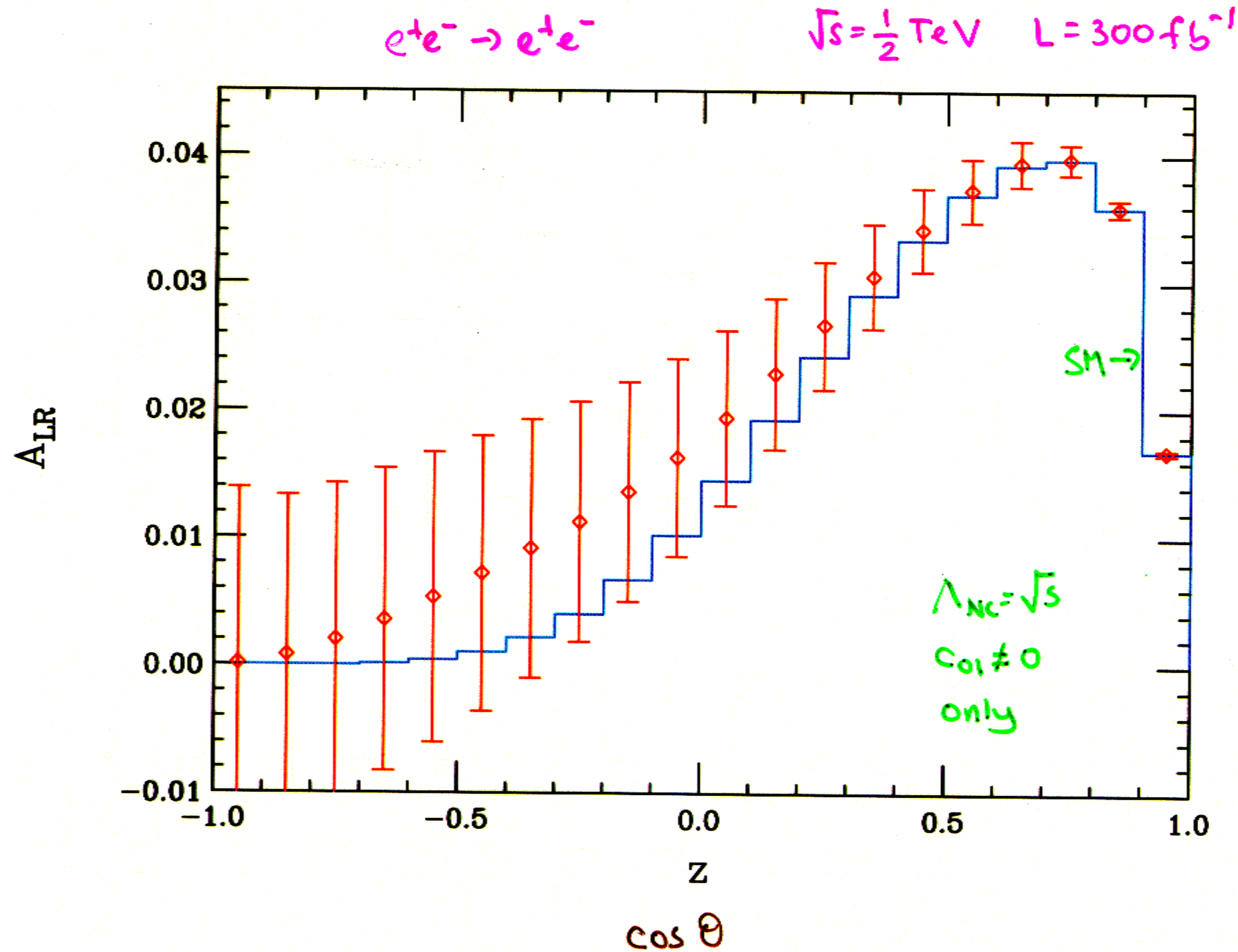
Assume same phase structure for γ, Z

$$\frac{d\sigma}{dz\,d\phi} = \frac{\alpha^2}{2s}\left[(e_{ij}+f_{ij})\left(\rho^{ss}_{ij}+\rho^{tt}_{ij}+2\rho^{st}_{ij}\frac{u^2}{s^2}\cos\Delta_{Bhabha}\right) + (e_{ij}-f_{ij})\left(\rho^{ss}_{ij}\,t^2/s^2+\rho^{tt}_{ij}\right)\right]$$

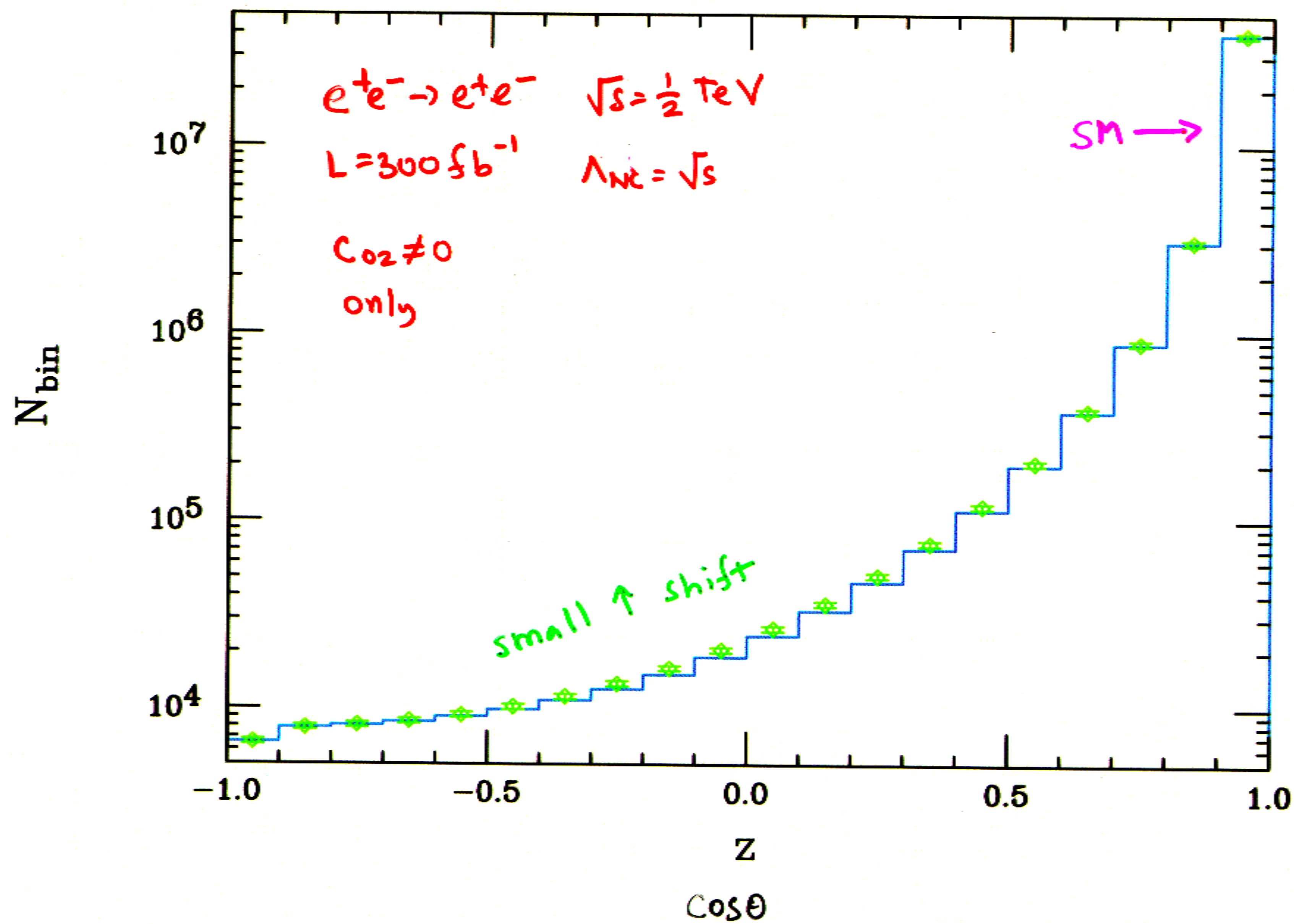
$$\Delta_{Bhabha} = \phi_s - \phi_t = \frac{-1}{\Lambda^2_{NC}}\left[c_{01}t + \sqrt{ut}\,(c_{02}\cos\phi + c_{03}\sin\phi)\right]$$

Only probes space-time NC !

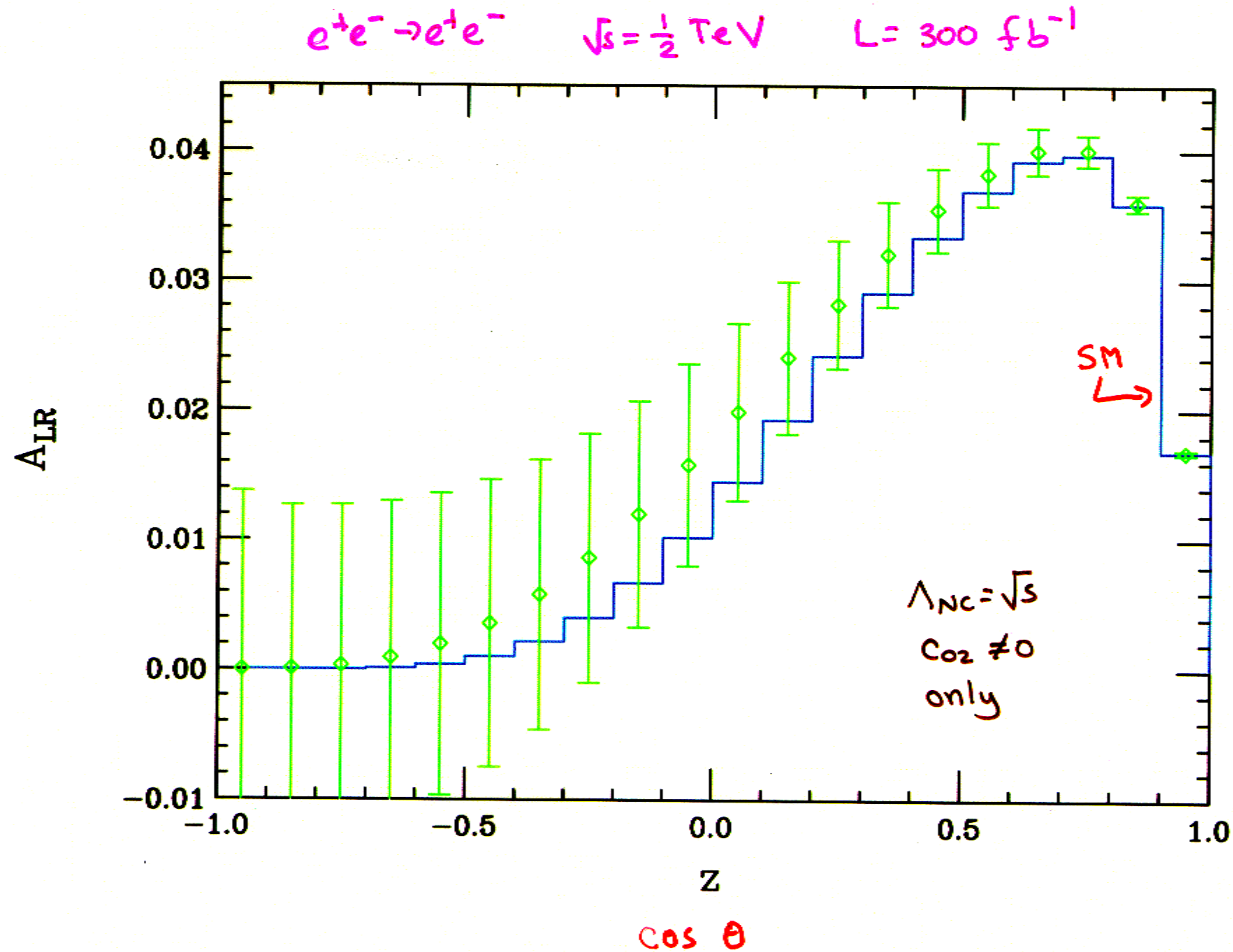

e+e- → e+e-
√s = 1/2 TeV
L = 300 fb-1
ΛNC = √s
C01 ≠ 0
only
Note systematic shift
SM →
Nbin
10^7
10^6
10^5
10^4
−1.0
−0.5
0.0
0.5
1.0
z
Cosθ



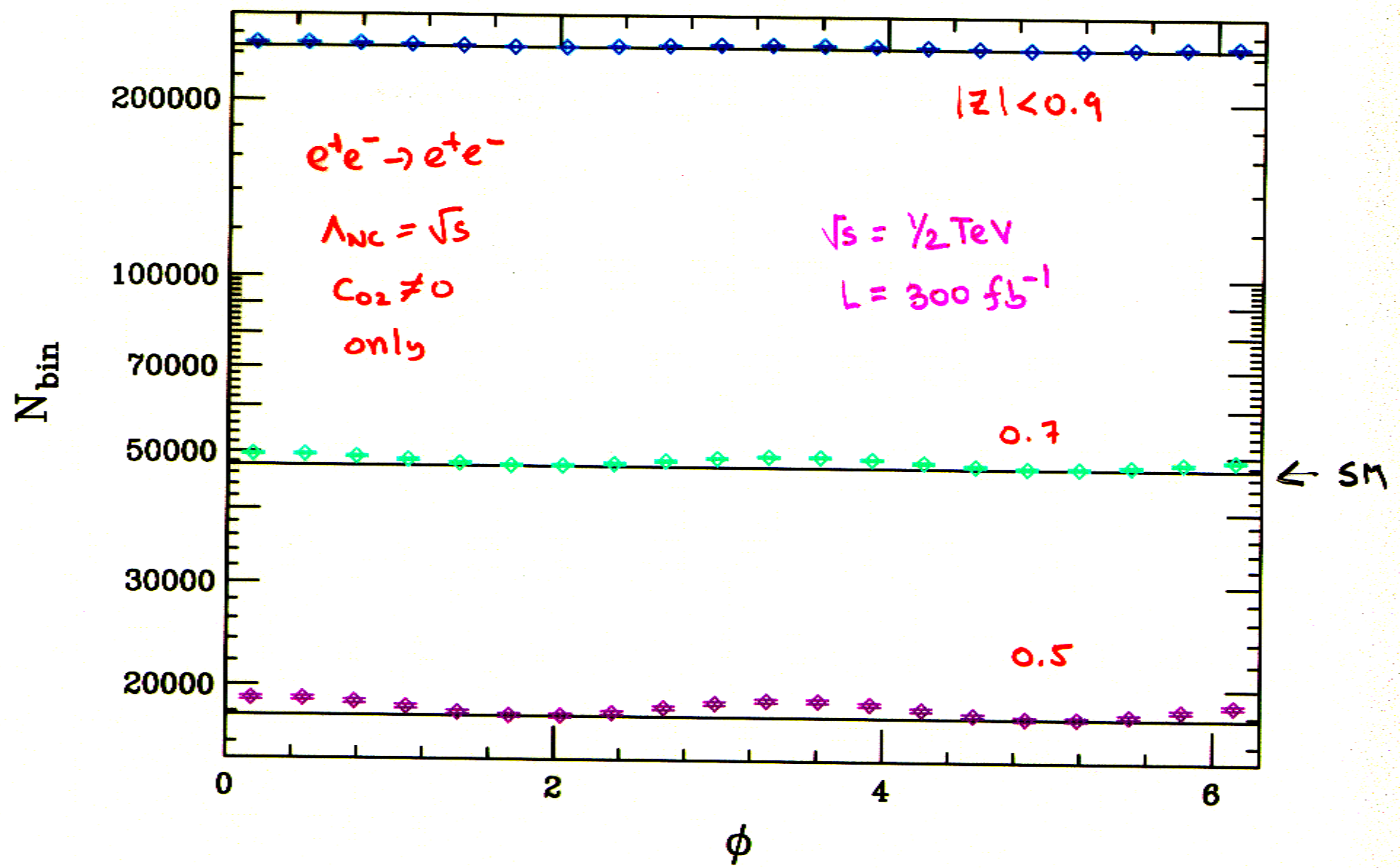
e+e- -> e+e-
√s = 1/2 TeV
L = 300 fb^-1
0.04
0.03
0.02
0.01
0.00
−0.01
A_LR
−1.0
−0.5
0.0
0.5
1.0
z
cos Θ
SM ->
Λ_NC = √s
c_01 ≠ 0
only



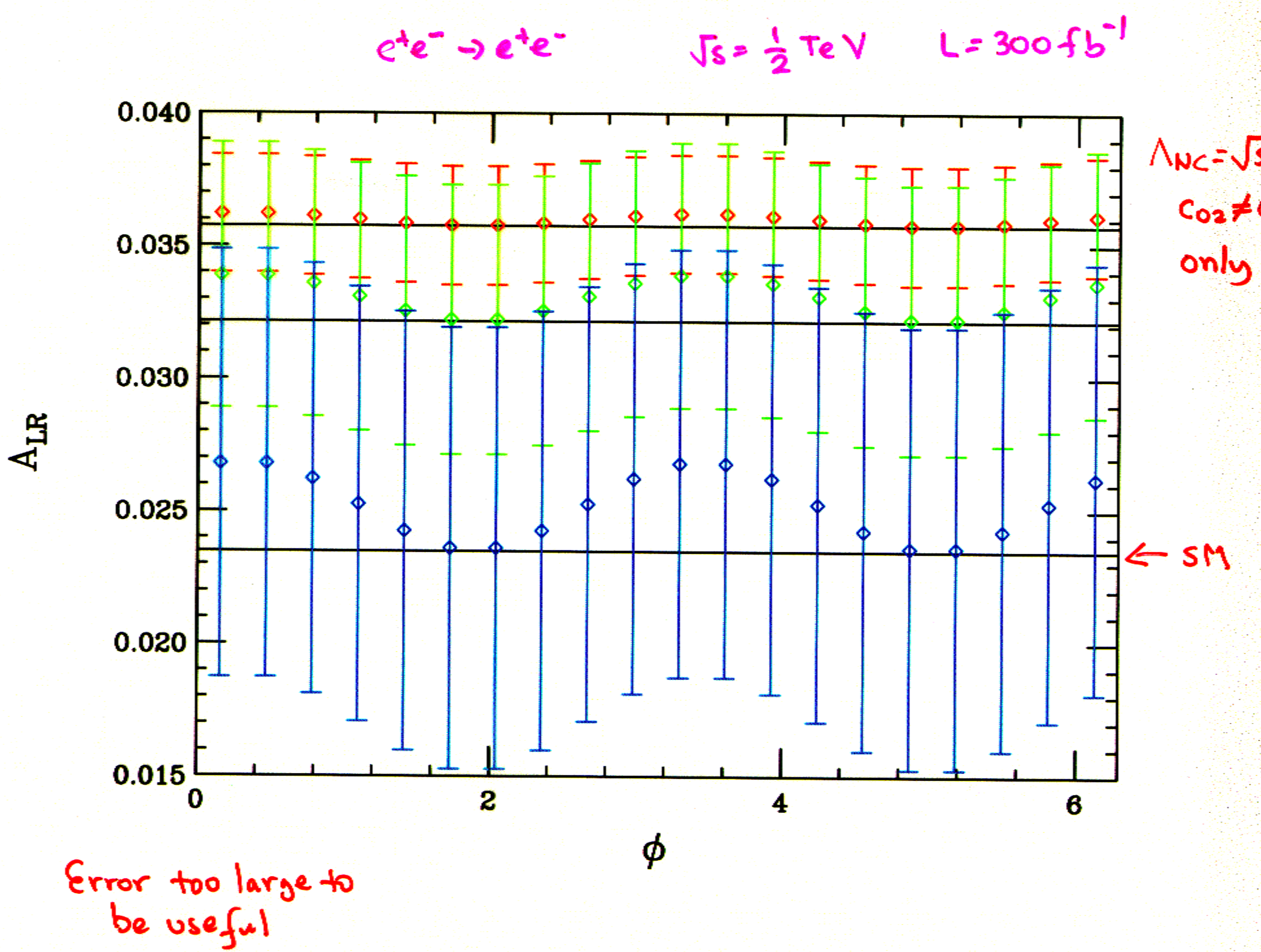
$e^+e^- \to e^+e^-$
$\sqrt{s} = \frac{1}{2}$ TeV
$L = 300\ fb^{-1}$
$\Lambda_{NC} = \sqrt{s}$
$c_{02} \neq 0$ only
SM
small ↑ shift
10^7
10^6
10^5
10^4
N_{bin}
−1.0
−0.5
0.0
0.5
1.0
z
Cosθ



$e^+e^- \to e^+e^-$
$\sqrt{s} = \frac{1}{2}$ TeV
$L = 300\ fb^{-1}$
A_{LR}
0.04
0.03
0.02
0.01
0.00
−0.01
−1.0
−0.5
0.0
0.5
1.0
z
cos θ
SM
$\Lambda_{NC} = \sqrt{s}$
$c_{02} \neq 0$
only

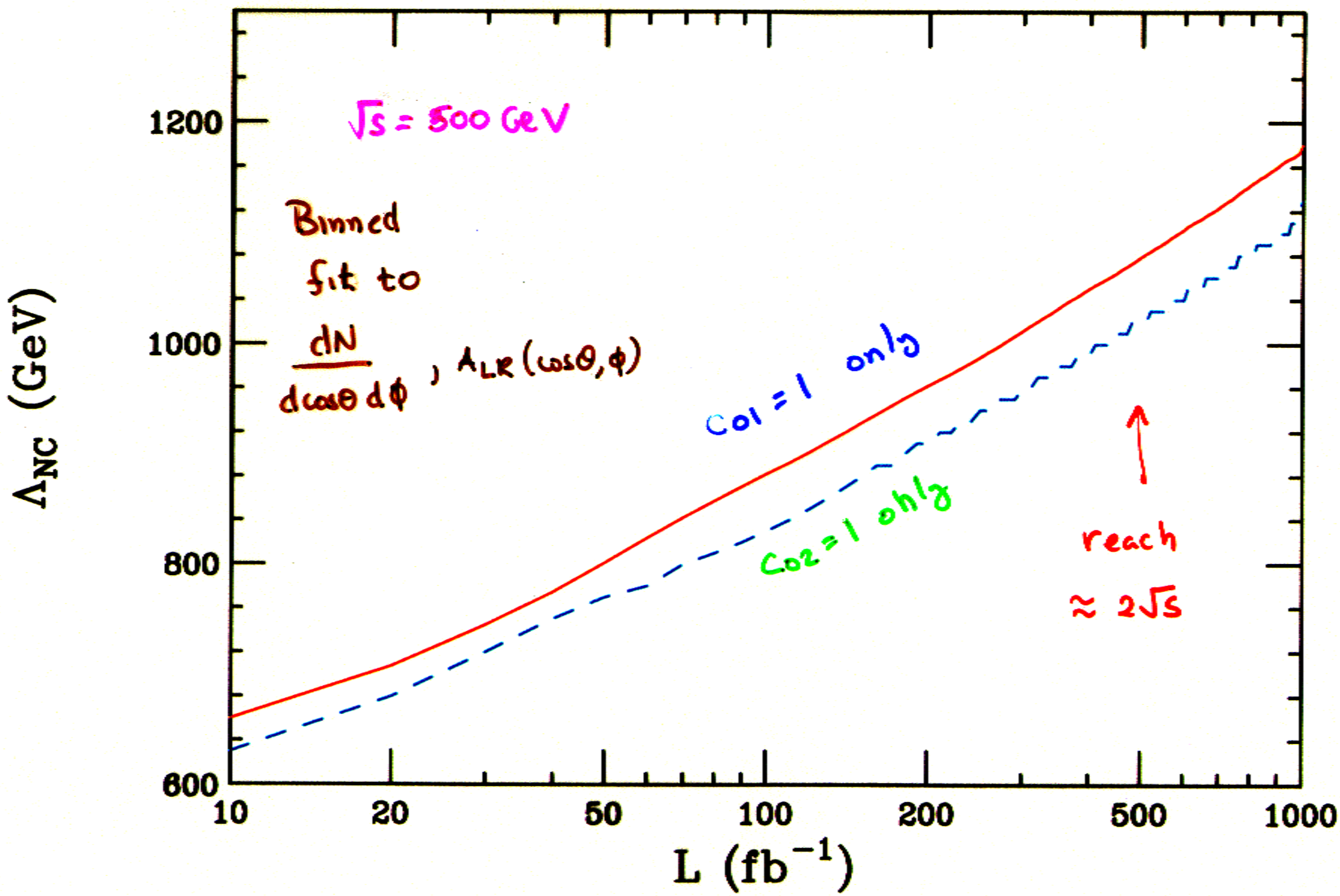


N_bin
200000
100000
70000
50000
30000
20000
0
2
4
6
φ
e+e- → e+e-
Λ_NC = √s
C_02 ≠ 0
only
|z| < 0.9
√s = 1/2 TeV
L = 300 fb^-1
0.7
0.5
← SM

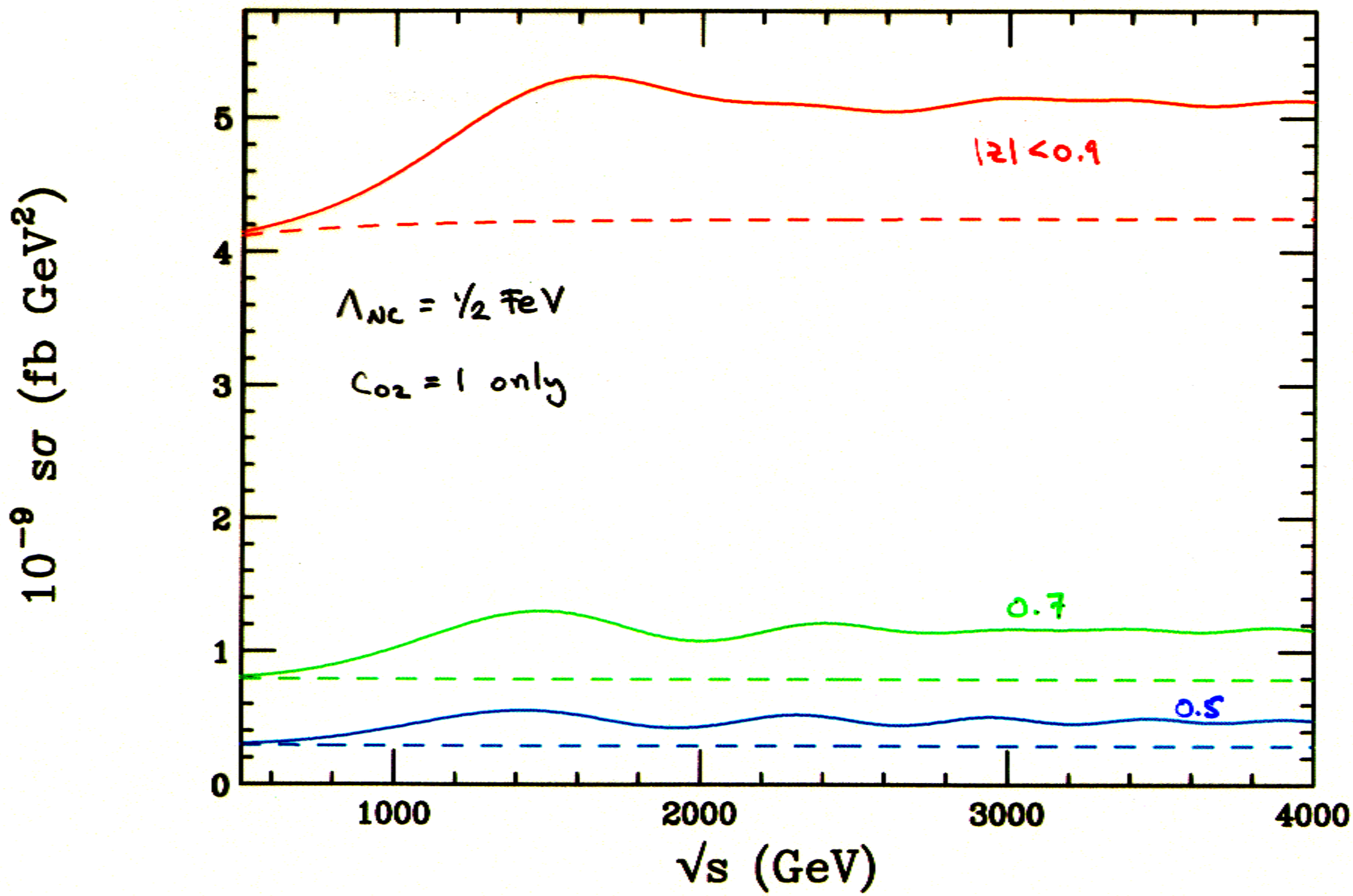


$e^+e^- \to e^+e^-$
$\sqrt{s} = \frac{1}{2}$ TeV
$L = 300 fb^{-1}$
$\Lambda_{NC} = \sqrt{s}$
$c_{02} \neq 0$
only
← SM
0.040
0.035
0.030
0.025
0.020
0.015
0
2
4
6
A_{LR}
ϕ
Error too large to be useful

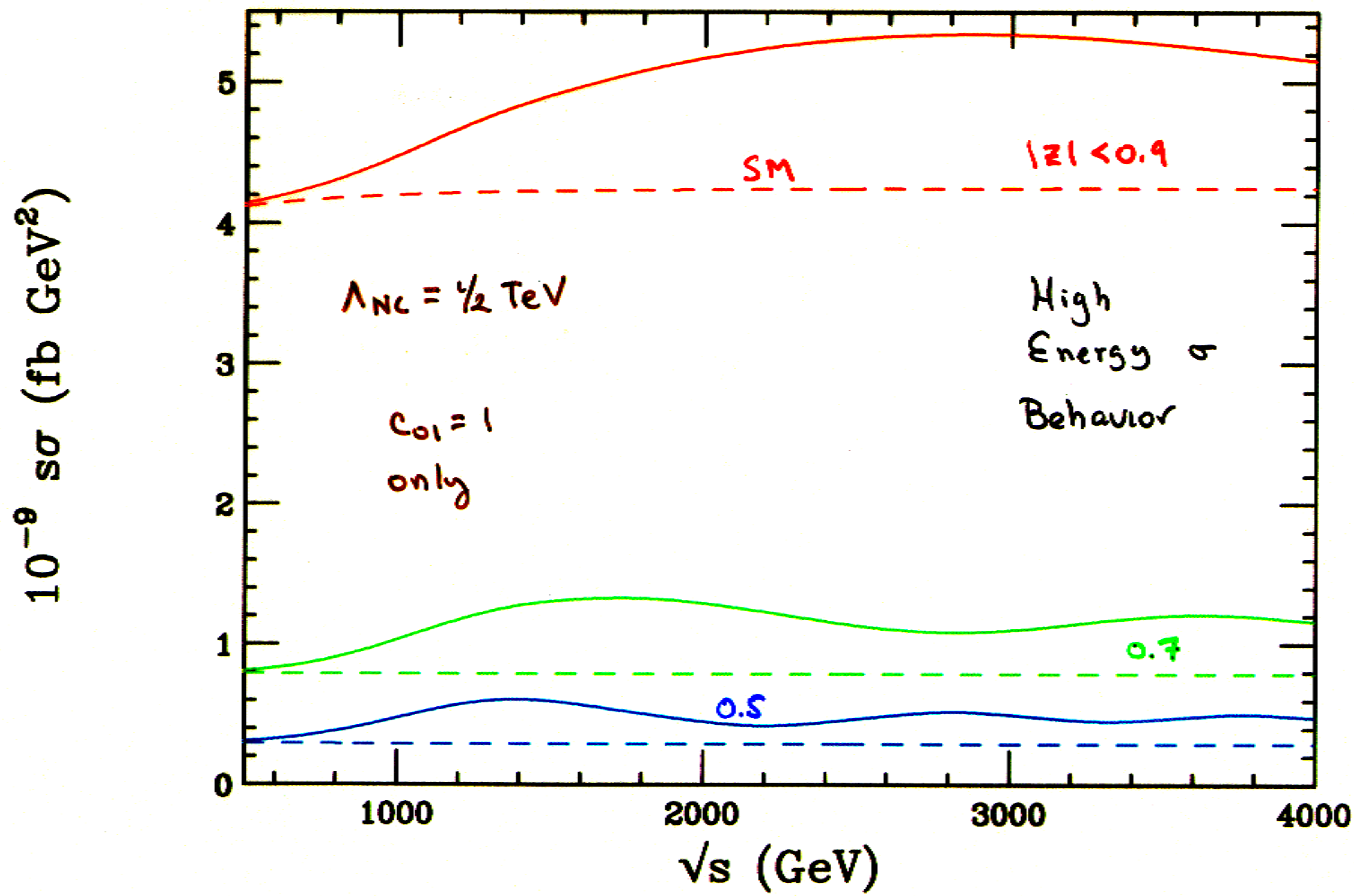
Bhabha Scattering Λ_{NC} 95% CL bound



High Energy Bhabha scattering



Bhabha scattering $e^+e^- \to e^+e^-$



$\gamma\gamma \rightarrow \gamma\gamma$ - 4 NC Contributions

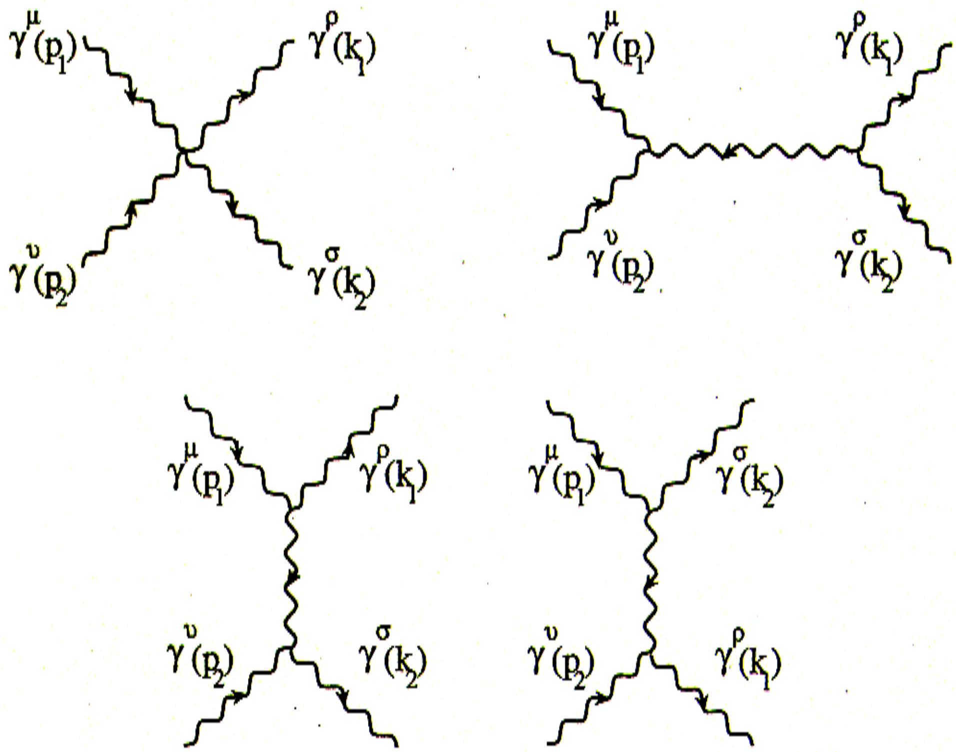
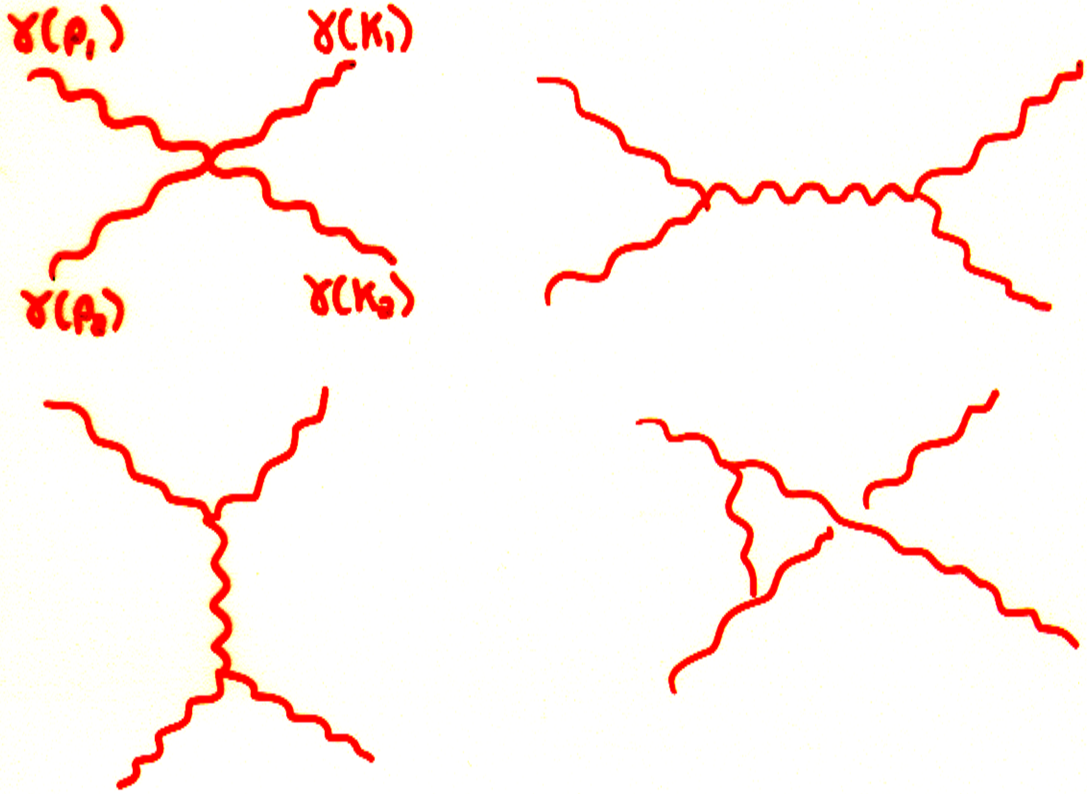


Figure 6: The tree level contributions to $\gamma\gamma \rightarrow \gamma\gamma$ in NCQED.

$\gamma\gamma \to \gamma\gamma$ - 4 NC Contributions



+ SM

6 non-vanishing NC amplitudes

$$\mathcal{M}^{NC}_{+--+} = -32\pi\alpha \frac{t}{s}\left[\sin\left(\tfrac{1}{2} p_1 \wedge k_1\right)\sin\left(\tfrac{1}{2} p_2 \wedge k_2\right) + \tfrac{t}{u}\sin\left(\tfrac{1}{2} p_1 \wedge k_2\right)\sin\left(\tfrac{1}{2} p_2 \wedge k_1\right)\right]$$

$$\mathcal{M}^{NC}_{++++} = \text{BLOB}$$

$$\mathcal{M}^{NC}_{----} = \mathcal{M}^{NC}_{++++}$$

$$\mathcal{M}^{NC}_{-++-}(k_1, k_2) = \mathcal{M}^{NC}_{+-+-}(k_2, k_1) = \mathcal{M}^{NC}_{-+-+}(k_2, k_1) = \mathcal{M}^{NC}_{+--+}(k_1, k_2)$$

$P_1 \wedge P_2 \sim C_{03}$

$P_1 \wedge K_1 \sim F(C_{03}, C_{01}, C_{02}, C_{13}, C_{23})$

$P_2 \wedge K_1 \sim F(C_{03}, C_{01}, C_{02}, C_{13}, C_{23})$

$P_1 \wedge K_2 \sim F(C_{03}, C_{01}, C_{02}, C_{13}, C_{23})$

$\Rightarrow$ Probes both space-space + space-time NC

Take $P_e = 90\%$, $P_\ell = 1$

$|\cos\theta| \leq 0.8$

No dependence on C_{12} $\Rightarrow$ $\vec{B}$ // beam axis are unobservable

Space-space $\neq 0$

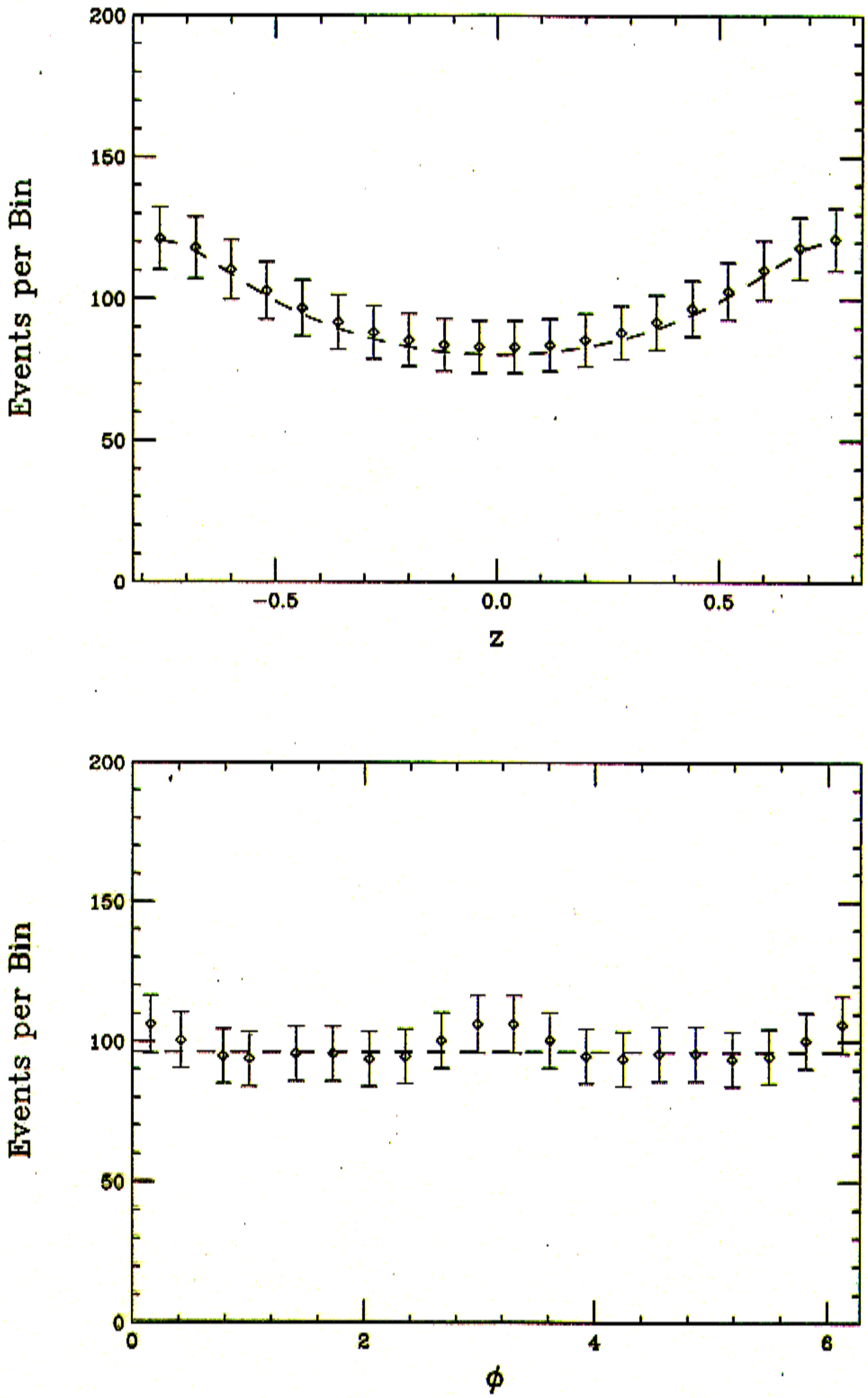


Figure 9: θ dependence (top) and ϕ dependence (bottom) of the $\gamma\gamma \to \gamma\gamma$ cross section for the case $c_{13} = 1$. We again use $\Lambda = \sqrt{s} = 500$ GeV, luminosity $500\,\mathrm{fb}^{-1}$.

space-time ≠ 0

$\vec{E}$ // beam

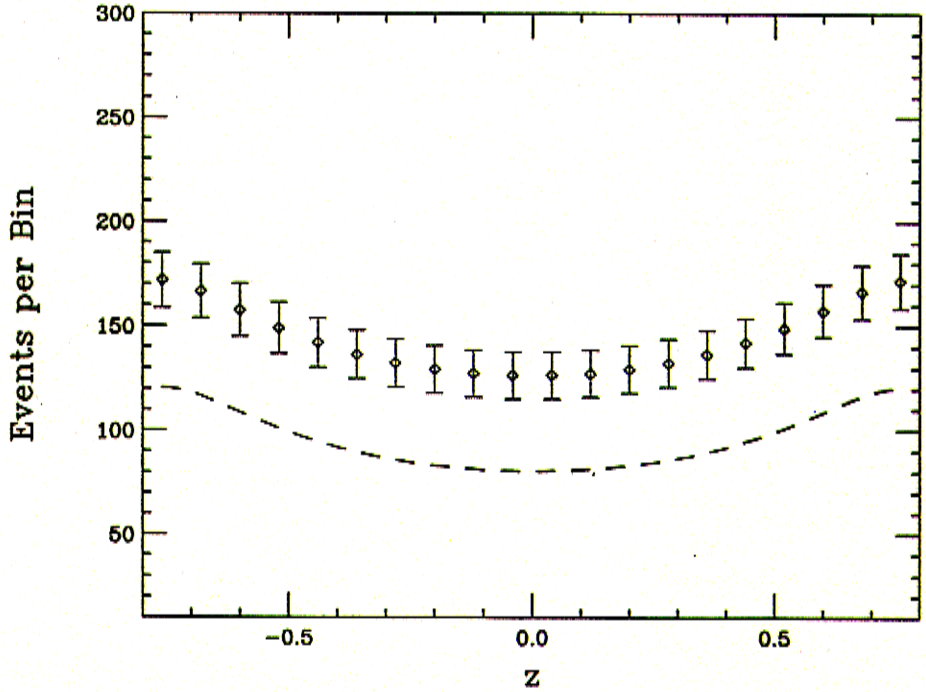


Figure 7: θ dependence of the $\gamma\gamma \to \gamma\gamma$ cross section for the case $c_{03} = 1$. We use $\Lambda = \sqrt{s} = 500$ GeV, luminosity $500\,\mathrm{fb}^{-1}$.

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Appendix

In this appendix we present the SM amplitudes and photon distribution functions relevant for the process $\gamma\gamma \to \gamma\gamma$. For a more detailed discussion the reader is referred to [4, 5, 6, 7].

The one loop contributions to $\gamma\gamma \to \gamma\gamma$ come from W boson and fermion loops; at high energies, which we are considering, there is only one non-negligible independent helicity amplitude; the approximate amplitudes for each contribution are

$$\frac{\mathcal{M}^{(W)}_{++++}(s,t,u)}{\alpha^2} \approx 12 + 12\left(\frac{u-t}{s}\right)\left[\ln\left(\frac{-u-i\varepsilon}{m_W^2}\right) - \ln\left(\frac{-t-i\varepsilon}{m_W^2}\right)\right]$$

space-time $\neq 0$

$\vec{E} \perp$ beam

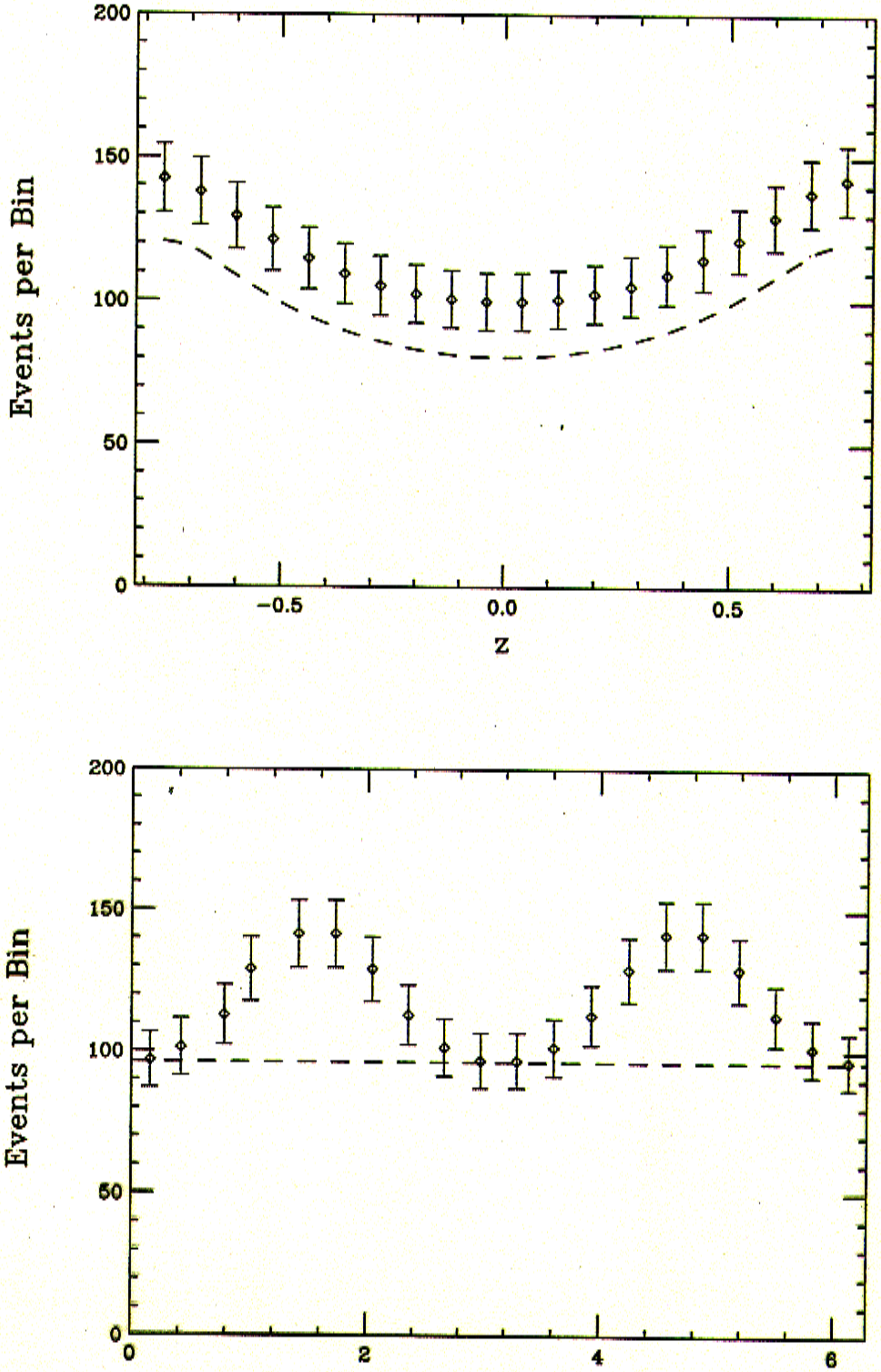

Figure 8: θ dependence (top) and ϕ dependence (bottom) of the $\gamma\gamma \to \gamma\gamma$ cross section for the case $c_{02} = 1$. We again use $\Lambda = \sqrt{s} = 500$ GeV, luminosity $500\,\mathrm{fb}^{-1}$.

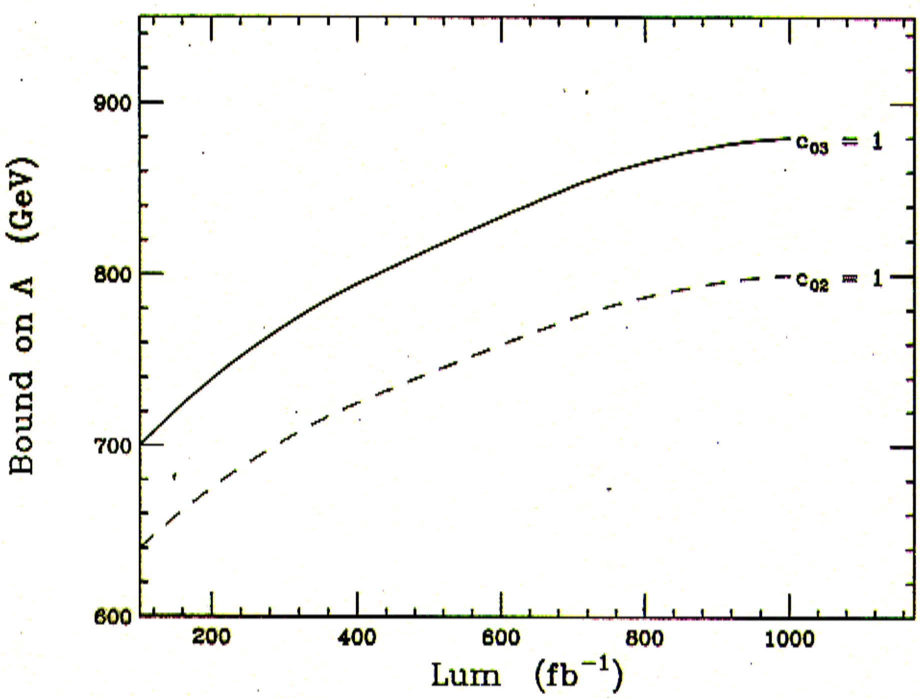

Figure 10: Lower bound on Λ as a function of luminosity. We set $\sqrt{s} = 500$ GeV.

Conclusions

NCQFT provides novel probes of structure of space-time manifold.

Process	Probe	Reach
$e^+e^- \to \gamma\gamma$	Space-Time	$\sim 1.5\sqrt{s}$
$e^+e^- \to e^+e^-$	Space-Time	$\sim 2\sqrt{s}$
$e^+e^- \to e^+e^-$	Space-Space	$\sim 3\sqrt{s}$
$\gamma\gamma \to \gamma\gamma$	Space-Space + Space-Time	$\sim 1.5\sqrt{s}$