# Electroweak Sudakov Corrections

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Introduction

DL-corrections in Non-Abelian theories

Subleading Corrections in Non-Abelian theories

Leading and Subleading EW Sudakov Logarithms

Comparison with existing calculations

Conclusions

* work based on Phys. Rev. D 61 (2000) 094002, hep-ph/0004056, and ongoing work.

## Introduction

Precision NLC physics (<1%) to disentangle new physics.

At 1 Loop, EW DL's 10-20 %, at 2 Loops few %. Subleading could be significant too.

$\longrightarrow$ Need a complete leading & subl. log analysis through 2 Loops!

In addition:

Theoretical understanding of IR-structure of broken gauge theories.

## Subleading Corrections in Non-Abelian theories

Logarithms at fixed angle, $s \sim |t| \sim |u| \gg \mu^2 \geq m^2$

Double Logarithms from IR evolution equation

Non-Abelian Gribov Theorem $(\boldsymbol{k}_\perp'^2 \gg \boldsymbol{k}_\perp^2 \gg \mu^2)$:

$$\mathcal{M}(p_1,...,p_n;\mu^2) = \mathcal{M}_{\text{Born}}(p_1,...,p_n)$$
$$-\frac{i}{2}\frac{g_s^2}{(2\pi)^4}\sum_{j,l=1,j\neq l}^{n}\int_{s\gg\boldsymbol{k}_\perp^2\gg\mu^2}\frac{d^4k}{k^2+i\epsilon}\,\frac{p_jp_l}{(kp_j)(kp_l)}$$
$$\times\, T^a(j)T^a(l)\mathcal{M}(p_1,...,p_n;\boldsymbol{k}_\perp^2)$$

on the rhs is to be taken on the mass shell, but with the substituted infrared cutoff: $\mu^2 \longrightarrow \boldsymbol{k}_\perp^2$.

The IR-evolution equation is gauge invariant ($k^\mu, k^\nu$ terms give vanishing contribution do to conservation of total color charge $\sum_a T^a = 0$).

For $j \neq l$ we have $\frac{p_j p_l}{k p_j} \sim \frac{E_l}{w}$:

$$\mathcal{M}(p_1, ..., p_n; \mu^2) = \mathcal{M}_{\text{Born}}(p_1, ..., p_n) \\ -\frac{2g^2}{(4\pi)^2} \sum_{l=1}^{n} \int_{\mu^2}^{s} \frac{d\boldsymbol{k}_\perp^2}{\boldsymbol{k}_\perp^2} \int_{|\boldsymbol{k}_\perp|/\sqrt{s}}^{1} \frac{dv}{v} \\ \times C_l \mathcal{M}(p_1, ..., p_n; \boldsymbol{k}_\perp^2)$$

($C_l$ is the eig.val. of the Casimir operator $T^a(l)T^a(l)$ ($C_l = C_A$ for gauge bosons in the adjoint representation of $SU(N)$ and $C_l = C_F$ for fermions in the fundamental representation).

The differential form is thus:

$$\begin{aligned} \frac{\partial \mathcal{M}(p_1, ..., p_n; \mu^2)}{\partial \log(\mu^2)} &= K(\mu^2)\mathcal{M}(p_1, ..., p_n; \mu^2) \\ K(\mu^2) &\equiv -\frac{1}{2} \sum_{l=1}^{n} \frac{\partial W_l(s, \mu^2)}{\partial \log(\mu^2)} \\ W_l(s, \mu^2) &= \frac{g^2}{8\pi^2} C_l \log \frac{s}{\mu^2} \end{aligned}$$

$W_l$ is the probability to emit a soft and almost collinear gauge boson from the particle $l$, subject to the IR cut-off $\mu$ on the transverse momentum.

To logarithmic accuracy, we obtain:

$$\frac{\partial W_l(s,\mu^2)}{\partial \log(\mu^2)} = -\frac{g^2}{8\pi^2} C_l \log\frac{s}{\mu^2}$$

The initial condition is given by:

$$\mathcal{M}(p_1,...,p_n;s) = \mathcal{M}_{\rm Born}(p_1,...,p_n)$$

The solution is thus given by the product of the Born amplitude and the Sudakov form factors:

$$\begin{aligned}\mathcal{M}(p_1,...,p_n;\mu^2) &= \mathcal{M}_{\rm Born}(p_1,...,p_n)\\ &\times \exp\left(-\frac{1}{2}\sum_{l=1}^{n} W_l(s,\mu^2)\right)\end{aligned}$$

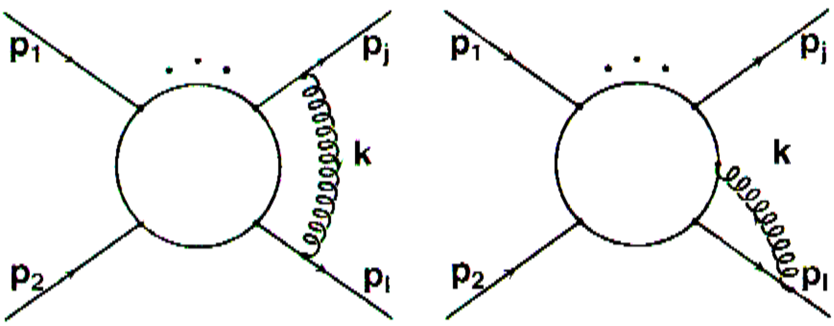
$\longrightarrow$ exactly analogous Sudakov exponentiation for the gauge group $SU(N)$ to the Abelian case.

In massless case, no subleading *soft* log.:

$$C_0(s/\mu^2) \equiv$$

$$\int \frac{d^4k_{[\boldsymbol{k}_\perp^2>\mu^2]}}{(2\pi)^4(k^2+i\varepsilon)(k^2+2p_jk+i\varepsilon)(k^2-2p_lk+i\varepsilon)}$$

$$= \frac{i}{2(4\pi)^2 s}\log^2\frac{s}{\mu^2} \quad + \quad const.$$




$\longrightarrow$ Subleading log.: Collinear or RG

$\longrightarrow$ Use virtual contr. to A.P. splitting functions:

$P_{BA}$ describes probability of $B$ inside $A$ with fraction $z$ of energy of $A$ with probability $\mathcal{P}_{BA}$:

$$d\mathcal{P}_{BA}(z) = \frac{\alpha_s}{2\pi} P_{BA} dt$$

where $t = \log \frac{s}{\mu^2}$

$$d\mathcal{P}_{BA}(z) = \frac{\alpha_s}{2\pi} \frac{z(1-z)}{2} \overline{\sum_{spins}} \frac{|V_{A \longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2} d \log \boldsymbol{k}_\perp^2$$

$V_{A \longrightarrow B+C}$: elementary vertices

$$P_{BA}(z) = \frac{z(1-z)}{2} \overline{\sum_{spins}} \frac{|V_{A \longrightarrow B+C}|^2}{\boldsymbol{k}_\perp^2}$$

Calculate *virtual* contributions:

$$
\begin{aligned}
P_{qq}^V(z) &= C_F\left(-2\log\frac{s}{\mu^2}+3\right)\delta(1-z)\\
P_{gg}^V(z) &= C_A\left(-2\log\frac{s}{\mu^2}+\frac{4}{C_A}\beta_0^{\mathrm{QCD}}\right)\delta(1-z)
\end{aligned}
$$

These fulfill Altarelli-Parisi Equations:

$$
\begin{aligned}
\frac{\partial q(z,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y} q(z/y,t)P_{qq}^V(y)\\
\frac{\partial g(z,t)}{\partial t} &= \frac{\alpha_s}{2\pi}\int_z^1 \frac{dy}{y} g(z/y,t)P_{gg}^V(y)
\end{aligned}
$$

$$
q(1,t) = q_0 \exp\left[-\frac{\alpha_s(s)C_F}{2\pi}\left(\log^2\frac{s}{\mu^2}-3\log\frac{s}{\mu^2}\right)\right]
$$

$$
g(1,t) = g_0 \exp\left[-\frac{\alpha_s(s)C_A}{2\pi}\left(\log^2\frac{s}{\mu^2}-\frac{4}{C_A}\beta_0^{\mathrm{QCD}}\log\frac{s}{\mu^2}\right)\right]
$$

where $\beta_0^{\mathrm{QCD}} = \frac{11}{12}C_A - \frac{1}{3}T_F n_f$ with $C_A = 3$, $T_F = \frac{1}{2}$

Universality leads to general scattering amplitude:

$$
\mathcal{M}(p_1,\ldots,p_n,g_s,\mu) = \mathcal{M}(p_1,\ldots,p_n,g_s(s)) \times \exp\left(-\frac{1}{2}\sum_{j=1}^{n_q} W_j^q(s,\mu^2) - \frac{1}{2}\sum_{l=1}^{n_g} W_l^g(s,\mu^2)\right)
$$

with $n_q + n_g = n$, and

$$W^q(s,\mu^2) = \frac{\alpha_s(s)C_F}{4\pi}\left(\log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right)$$

$$W^g(s,\mu^2) = \frac{\alpha_s(s)C_A}{4\pi}\left(\log^2\frac{s}{\mu^2} - \frac{4}{C_A}\beta_0^{\mathrm{QCD}}\log\frac{s}{\mu^2}\right)$$

Solution solves generalized IR evolution (or RG) equation with IR singular anomalous dimensions:

$$\left(\frac{\partial}{\partial t} + \beta^{\mathrm{QCD}}\frac{\partial}{\partial g_s} + n_g\left(\Gamma_g(t) - \frac{1}{2}\frac{\alpha_s}{\pi}\beta_0^{\mathrm{QCD}}\right)\right.$$
$$\left. + n_q\left(\Gamma_q(t) + \frac{1}{2}\gamma_{q\bar{q}}\right)\right)\mathcal{M}(p_1,...,p_n,g_s,\mu) = 0$$

with

$$\Gamma_q(t) \equiv \frac{C_F\alpha_s}{4\pi}t \;\;;\;\; \Gamma_g(t) \equiv \frac{C_A\alpha_s}{4\pi}t$$

## Subleading EW Sudakov Logarithms

Consider massive(M) boson at rest: $k^\nu = (M, 0, 0, 0)$
Polarization vector is lin.comb. of:

$$e_1 \equiv (0,1,0,0) \quad , \quad e_1 \equiv (0,0,1,0) \quad , \quad e_3 \equiv (0,0,0,1)$$

Boost along 3-axis, pol. vect. still satisfy:

$$e_\nu k^\nu = 0 \quad , \quad e^2 = -1$$

$\longrightarrow$ 2 transverse $(e_1, e_2)$ and 1 longitudinal:

$$e_L^\nu(k) = (k/M, 0, 0, E_k/M) = k^\nu/M + \mathcal{O}(M/E_k)$$

Transverse d.o.f. correspond to massless theory!
Use massless virtual QCD results at high energies:
Additional complications: mixing & broken sym.

$$\begin{aligned} W_\nu^\pm &= \frac{1}{\sqrt{2}}\left(W_\nu^1 \pm i W_\nu^2\right) \\ Z_\nu &= \cos\theta_\mathrm{W} W_\nu^3 + \sin\theta_\mathrm{W} B_\nu \\ A_\nu &= -\sin\theta_\mathrm{W} W_\nu^3 + \cos\theta_\mathrm{W} B_\nu \end{aligned}$$

Strategy:
Calculate in terms of massless fields for $\mu \geq M$
(where $M_Z \sim M_W \sim M_{\text{Higgs}} \sim M$)
Then solve IR evolution equation:

$$\left(\frac{\partial}{\partial t} + \beta\frac{\partial}{\partial g} + \beta'\frac{\partial}{\partial g'} + \sum_{i=1}^{n_g} \Gamma_g^i(t) - n_{\text{W}}\frac{1}{2}\frac{\alpha}{\pi}\beta_0 - n_{\text{B}}\frac{1}{2}\frac{\alpha'}{\pi}\beta'_0 + \sum_{k=1}^{n_f}\left(\Gamma_f^k(t) + \frac{1}{2}\gamma_{q\overline{q}}^k\right)\right)\mathcal{M}^{\perp}(p_1, ..., p_n, g, g', \mu) = 0$$

$$\begin{aligned}
\text{with } \beta(g(\overline{\mu}^2)) &= \frac{\partial g(\overline{\mu}^2)}{\partial \log \overline{\mu}^2} \approx -\beta_0 \frac{g^3(\overline{\mu}^2)}{8\pi^2} \\
\beta'(g'(\overline{\mu}^2)) &= \frac{\partial g'(\overline{\mu}^2)}{\partial \log \overline{\mu}^2} \approx -\beta'_0 \frac{g'^3(\overline{\mu}^2)}{8\pi^2} \\
\text{and} \qquad \beta_0 &= \frac{11}{12}C_A - \frac{1}{3}n_{gen} - \frac{1}{24}n_h \\
\beta'_0 &= -\frac{5}{9}n_{gen} - \frac{1}{24}n_h \\
\Gamma_{f,g}^i(t) &= \left(\frac{\alpha}{4\pi}T_i(T_i+1) + \frac{\alpha'}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)t \\
\gamma_{q\overline{q}}^i &= -3\left(\frac{\alpha}{4\pi}T_i(T_i+1) + \frac{\alpha'}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)
\end{aligned}$$

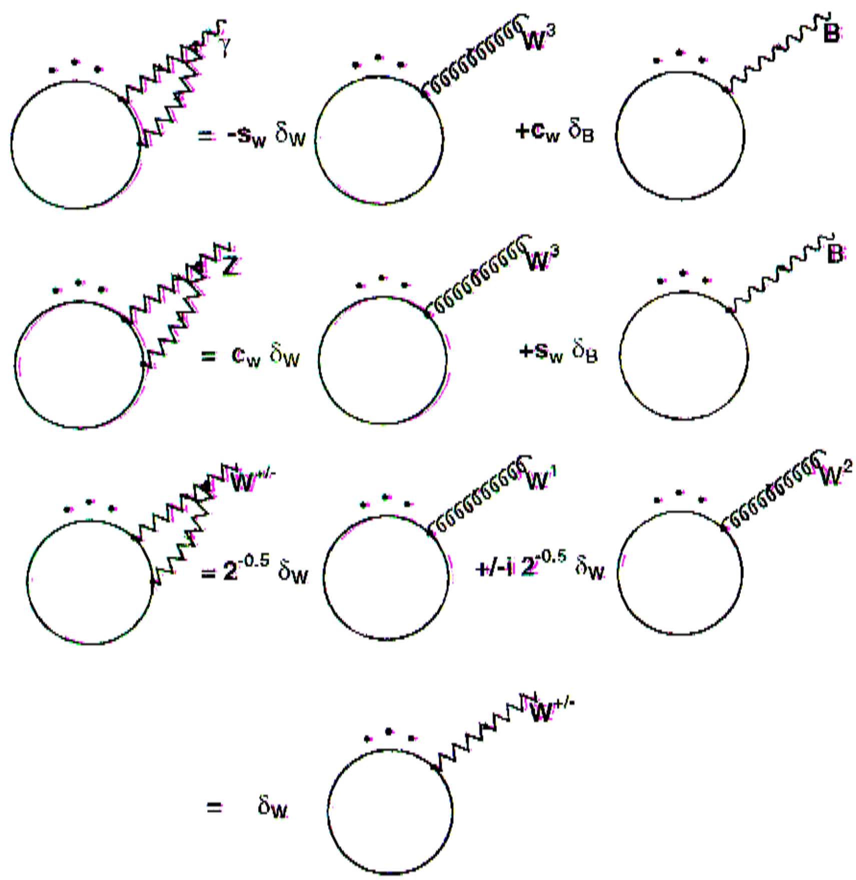
Solution:

$$\mathcal{M}^{\perp}(p_1,\ldots,p_n,g,g',\mu) = \mathcal{M}^{\perp}_{\text{Born}}(p_1,\ldots,p_n,g(s),g'(s))$$
$$\times \exp\left\{-\frac{1}{2}\sum_{i=1}^{n_g}\left(\frac{\alpha(s)}{4\pi}T_i(T_i+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)\log^2\frac{s}{\mu^2}\right.$$
$$+\left(n_{\text{W}}\frac{\alpha(s)}{2\pi}\beta_0+n_{\text{B}}\frac{\alpha'(s)}{2\pi}\beta_0'\right)\log\frac{s}{\mu^2}$$
$$-\frac{1}{2}\sum_{k=1}^{n_f}\left(\frac{\alpha(s)}{4\pi}T_k(T_k+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_k}{2}\right)^2\right)$$
$$\left.\times\left[\log^2\frac{s}{\mu^2}-3\log\frac{s}{\mu^2}\right]\right\}$$

In region $\mu \leq M$ masses cannot be neglected.

But: Only QED contributes!

Above solution is matching condition at $\mu = M$.



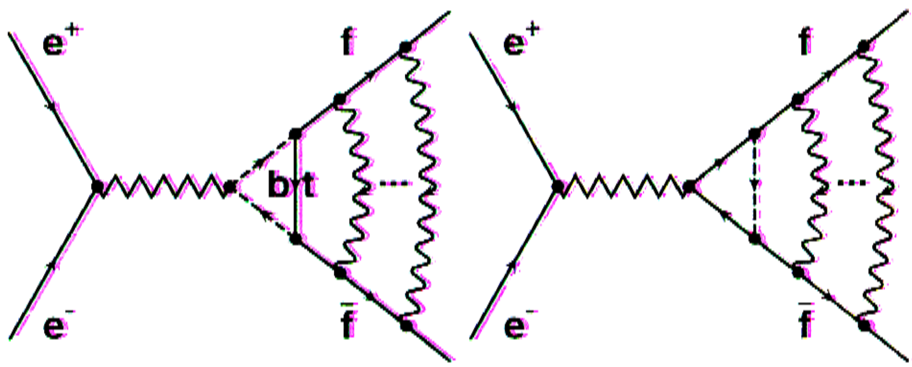

Only corrections to external $W^{\pm}$ factorize with respect to the physical amplitude!

The general solution is thus:

$$\mathcal{M}^{\perp}(p_1,\dots,p_n,g,g',\mu) = \mathcal{M}^{\perp}_{\text{Born}}(p_1,\dots,p_n,g(s),g'(s))$$
$$\times \exp\left\{-\frac{1}{2}\sum_{i=1}^{n_g}\left(\frac{\alpha(s)}{4\pi}T_i(T_i+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)\log^2\frac{s}{M}\right.$$
$$+\left(n_{\text{W}}\frac{\alpha(s)}{2\pi}\beta_0+n_{\text{B}}\frac{\alpha'(s)}{2\pi}\beta_0'\right)\log\frac{s}{M^2}$$
$$-\frac{1}{2}\sum_{k=1}^{n_f}\left(\frac{\alpha(s)}{4\pi}T_k(T_k+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_k}{2}\right)^2\right)$$
$$\left.\times\left[\log^2\frac{s}{M^2}-3\log\frac{s}{M^2}\right]\right\}\exp\left[-\frac{1}{2}\sum_{i=1}^{n_\gamma}w_i^{\gamma}(M^2,m_j^2)\right.$$
$$-\frac{1}{2}\sum_{i=1}^{n_f}\left(w_i^f(s,\mu^2)-w_i^f(s,M^2)\right)$$
$$\left.-\frac{1}{2}\sum_{i=1}^{n_g}\left(w_i^{\text{W}}(s,\mu^2)-w_i^{\text{W}}(s,M^2)\right)\right]$$

For external $Z$ and $\gamma$ we have mixing!

In addition we have top-yukawa corrections for $b_L$, $t_L$ and $t_R$ external lines!




Resummation to all orders by using the Gribov theorem possible, since coupling inside the inner loop gives only sub-sub-leading corrections:

$$\mathcal{M}(p_1, \ldots, p_n; \mu^2) = \mathcal{M}_{1\text{loop}}(p_1, \ldots, p_n) \times \exp\left(-\frac{1}{2}\sum_{l=1}^{n} W_l(s, \mu^2)\right)$$

We find for left handed $t, b$ at 1 loop:

$$\mathcal{M}^{DY}_{\text{1loop}}(p_1, ..., p_4) = \mathcal{M}^{DY}_{\text{Born}}(p_1, ..., p_4) \times \left\{1 - \frac{g^2}{16\pi^2}\frac{1}{4}\frac{m_t^2}{M^2}\delta_{f,t_L/b_L} \log\frac{s}{M^2}\right\}$$

and for right handed $t$ at 1 loop:

$$\mathcal{M}^{DY}_{\text{1loop}}(p_1, ..., p_4) = \mathcal{M}^{DY}_{\text{Born}}(p_1, ..., p_4) \times \left\{1 - \frac{g^2}{16\pi^2}\frac{1}{2}\frac{m_t^2}{M^2}\delta_{f,t_R} \log\frac{s}{M^2}\right\}$$

$\longrightarrow$ subleading probability for fermions:

$$W_i^f(s, \mu^2) = \frac{g^2(s)}{16\pi^2}\left[\left(T_i(T_i+1) + \tan^2\theta_W \frac{Y_i^2}{4}\right) \times \left(\log^2\frac{s}{\mu^2} - 3\log\frac{s}{\mu^2}\right) + \left(\frac{1+\delta_{f,R}}{4}\frac{m_f^2}{M^2} + \delta_{f,L}\frac{m_{f'}^2}{4M^2}\right)\log\frac{s}{\mu^2}\right]$$

$\gamma$ semi-inclusively: Assume $\mu_{exp} \leq M$:

$$d\sigma^{\perp}(p_1,\ldots,p_n,g,g',\mu_{exp}) = d\sigma^{\perp}_{\text{elastic}}(p_1,\ldots,p_n,g(s),g'(s),\mu) \times \exp(w^{\gamma}_{\text{exp}}(s,m_i,\mu,\mu_{exp}))$$

and thus ( $\mu$-dependence cancels):

$$d\sigma^{\perp}(p_1,\ldots,p_n,g,g',\mu_{exp}) = d\sigma^{\perp}_{\text{Born}}(p_1,\ldots,p_n,g(s),g'(s)$$
$$\times \exp\left\{-\sum_{i=1}^{n_g}\left(\frac{\alpha(s)}{4\pi}T_i(T_i+1)+\frac{\alpha'(s)}{4\pi}\left(\frac{Y_i}{2}\right)^2\right)\log^2\frac{s}{M^2}\right.$$
$$+\left(n_{\text{W}}\frac{\alpha(s)}{\pi}\beta_0+n_{\text{B}}\frac{\alpha'(s)}{\pi}\beta_0'\right)\log\frac{s}{M^2}$$
$$\left.-\sum_{k=1}^{n_f}W_i^f\left(\frac{s}{M^2}\right)\right\}\exp\left[-\sum_{i=1}^{n_\gamma}w_i^{\gamma}(M^2,m_j^2)\right.$$
$$-\sum_{i=1}^{n_f}\left(w_i^f(s,\mu^2)-w_i^f(s,M^2)\right)$$
$$\left.-\sum_{i=1}^{n_g}\left(w_i^{\text{W}}(s,\mu^2)-w_i^{\text{W}}(s,M^2)\right)\right]$$
$$\times \exp(w^{\gamma}_{\text{exp}}(s,m_i,\mu,\mu_{exp}))$$

## Longitudinal degrees of freedom

Use Goldstone-boson equivalence theorem:

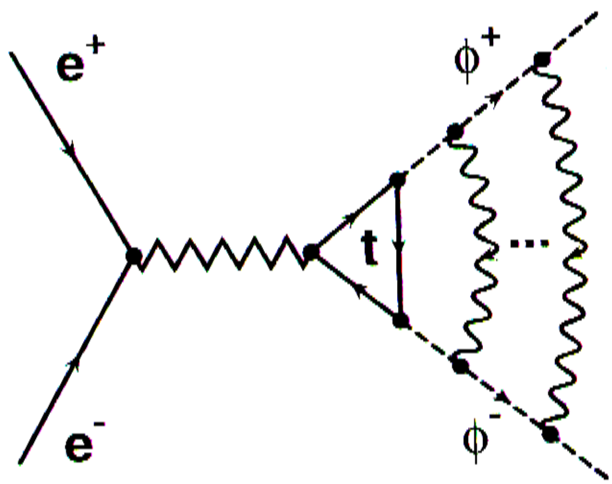
$$
\begin{aligned}
\mathcal{M}(W_L^{\pm}(k), \psi_{\text{phys}}) &= C_{\text{W}} \mathcal{M}(\phi^{\pm}(k), \psi_{\text{phys}}) + \mathcal{O}\left(\frac{M_{\text{W}}}{\sqrt{s}}\right) \\
\mathcal{M}(Z_L(k), \psi_{\text{phys}}) &= C_{\text{Z}} \mathcal{M}(\phi(k), \psi_{\text{phys}}) + \mathcal{O}\left(\frac{M_{\text{Z}}}{\sqrt{s}}\right)
\end{aligned}
$$

$C_{\text{Z}}$, $C_{\text{W}}$ depend on wave-function renormalization constants and mass counterterms. $\longrightarrow$ Use on-shell scheme: Universal $\log \frac{s}{M^2}$ from vertex corrections

Effective theory: $\sim$ scalar QCD

$$
\begin{aligned}
\widetilde{W}^{\phi}(s, \mu^2) &= \frac{g^2(s)}{16\pi^2}\left[\left(T_{\phi}(T_{\phi}+1) + \tan^2\theta_{\text{W}} \frac{Y_{\phi}^2}{4}\right)\right. \\
&\qquad \left.\left(\log^2 \frac{s}{\mu^2} - 4 \log \frac{s}{\mu^2}\right)\right]
\end{aligned}
$$

In EW-theory, also top-Yukawa corrections:




Again we can use the non-Abelian Gribov-theorem:

$$\mathcal{M}(p_1, ..., p_n; \mu^2) = \mathcal{M}_{1\text{loop}}(p_1, ..., p_n) \times \exp\left(-\frac{1}{2}\sum_{l=1}^{n_\phi} \widetilde{W}_l^\phi(s, \mu^2)\right)$$

and thus for the full EW-corrections:

$$W^\phi(s,\mu^2) = \frac{g^2(s)}{16\pi^2}\left[\left(T_\phi(T_\phi+1) + \tan^2\theta_{\text{W}}\frac{Y_\phi^2}{4}\right)\left(\log^2\frac{s}{\mu^2} - 4\log\frac{s}{\mu^2}\right) + \frac{3\,m_t^2}{2\,M^2}\log\frac{s}{\mu^2}\right]$$

Thus, also subleading Sudakov logarithms exponentiate for longitudinal degrees of freedom:

$$d\sigma^{L}(p_1,\ldots,p_n,g,g',\mu_{exp}) = d\sigma^{\phi}_{\mathrm{Born}}(p_1,\ldots,p_n,g(s),g'(s))$$
$$\times \exp\left\{-\sum_{i=1}^{n_\phi} W_i^{\phi}\left(\frac{s}{M^2}\right)\right\} \exp\left[-\sum_{i=1}^{n_{\mathrm{W}}}\left(w_i^{\mathrm{W}}(s,\mu^2)\right.\right.$$
$$\left.\left.-w_i^{\mathrm{W}}(s,M^2)\right)\right] \exp(w^{\gamma}_{\mathrm{exp}}(s,m_i,\mu,\mu_{exp}))$$

In all cases we neglect $\log\frac{m_i^2}{M^2}$ terms.

Corrections are the same for $\phi^{\pm},\phi$ and H.

Comparison with explicit results:

Agreement with Denner & Pozzorini at one loop for generic EW 1 loop logarithmic corrections (transverse and longitudinal NLO)

Agreement with Beenakker et al. at one loop for $e^+ e^- \longrightarrow W^+ W^-$ transverse and longitudinal NLO

Agreement with explicit two loop DL-calculation $g \longrightarrow f + \overline{f}$
Melles, Beenakker et al., Hori et al.

Agreement with Kühn et al. to all orders for $e^+ e^- \longrightarrow f_l^+ f_l^-$ (universal terms)
But Yukawa corrections missing!

Disagreement with Ciafaloni/Comelli at DL for $Z' \longrightarrow f^+ f^-$ (2 loop level)

## Conclusions

NLO Sudakov logarithms in broken gauge theories are calculated to all orders using fields of the effective high energy theory and the gauge invariant IR evolution equation method.

They *factorize* and *exponentiate* (with reasonable experimental cuts, mixing).

$\log \frac{s}{M^2} \log \frac{t}{u}$ should be calculated at least at one loop.

Agreement at one loop up to NLO , to two loops for DL with calculation using physical fields!

The method is universal for all processes at a future collider (NLC, LHC).