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Measuring the spin of the Higgs boson in Higgs-strahlung

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Work performed in collaboration with B. Eberle and P. M. Zerwas

1. Introduction

Higgs properties

After the Higgs discovery we must make sure it is the Higgs boson of the Standard Model and Spontaneous Electroweak Symmetry Breaking. We must measure the following properties:

(i) J^{PC} Quantum numbers

- Behaviour of $Z \to ZH$ at threshold
- Angular correlations in Higgs decays, eg. $H \to f\bar{f}$
- Yang's theorem

(ii) The HVV and Hff couplings

In particular $Ht\bar{t}$:

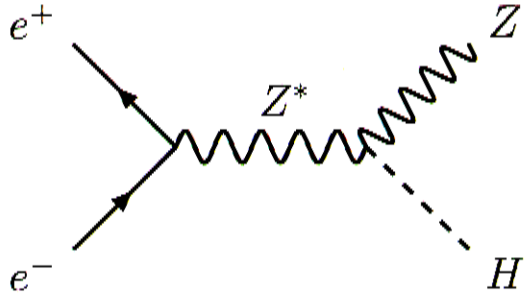
- For light higgs bosons: radiation of Higgs bosons off top quarks $t \to Ht$
- indirectly by measuring $H\gamma\gamma$ and Hgg couplings, which are mediated by virtual top quark loops

(ii) The triple and quartic Higgs self-couplings

Can use to reconstruct the Higgs potential itself.

Today I will discuss how to definitively confirm the spinless nature of the Higgs boson.

2. Higgs-strahlung



Standard Model:

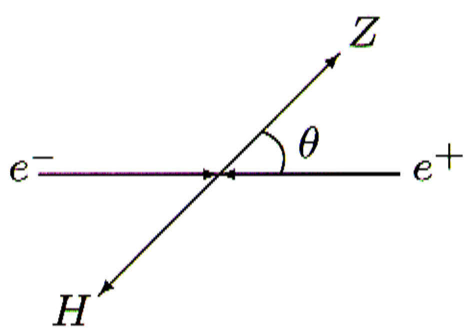
$$\sigma[e^+e^- \to ZH] = \frac{G_F^2 M_Z^4}{96\pi s}\,(a_e^2 + v_e^2)\,\beta\,\frac{\beta^2 + 12M_Z^2/s}{(1 - M_Z^2/s)^2}$$

with $\beta = 2|\vec{p_Z}|/\sqrt{s}$

Characteristics:

- Threshold behaviour $\sim \beta$
- Scaling behaviour $\sim 1/s$ asympotically

Angular dependence:



$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = \frac{3}{4}\frac{\beta^2\sin^2\theta + 8M_Z^2/s}{\beta^2 + 12M_Z^2/s}$$

- Isotropic at threshold
- asymptotically $\to \frac{3}{8}\sin^2\theta$ as equivalence theorem

Angular dependence in a general model

- Angular dependence constrained by properties of angular momentum operator

Matrix Element:

$$M = J_\mu^{e^+e^-\to Z^*} \frac{1}{s-M_Z^2} J^\mu_{Z^*\to ZH}$$

(Lepton current picks out transverse components of ZH current)

- Generally we can write ZH current as

$$J_m = \mathcal{D}^{S*}_{m\lambda}(\phi,\theta,-\phi)\Gamma_m^{\lambda_Z\lambda_H}$$

with $\lambda = \lambda_Z - \lambda_H$,

$$\mathcal{D}^S_{m\lambda}(\alpha,\beta,\gamma) = e^{-im\alpha} d^S_{m\lambda}(\theta)\, e^{-i\lambda\gamma}$$

$\Gamma_m^{\lambda_Z\lambda_H}$ are form factors which depend on the model

- Then we have:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = \frac{3}{4}\frac{4M_Z^2/s}{\beta^2+12M_Z^2/s}\left[\sin^2\theta(|\Gamma^{00}|^2+2|\Gamma^{11}|^2)\right.$$
$$\left.+(1+\cos^2\theta)\left(|\Gamma^{01}|^2+|\Gamma^{10}|^2+|\Gamma^{12}|^2\right)\right]$$

where in the Standard Model,

$$\Gamma^{00} = E_Z/M_Z$$
$$\Gamma^{10} = 1$$
$$\Gamma^{01} = \Gamma^{11} = \Gamma^{12} = 0$$

Polarized Cross-sections

For longitudinally polarized e^+e^-:

$$\frac{d\sigma}{d\cos\theta} \sim \sin^2\theta(|\Gamma^{00}|^2 + 2|\Gamma^{11}|^2)$$
$$+(1+\cos^2\theta)\left(|\Gamma^{01}|^2 + |\Gamma^{10}|^2 + |\Gamma^{12}|^2\right)$$

For transversly polarized e^+e^-:

$$\frac{d\sigma}{d\cos\theta d\phi} \sim (v_e^2 + a_e^2)[\sin^2\theta\left(|\Gamma^{00}|^2 + 2|\Gamma^{11}|^2\right)$$
$$+(1+\cos^2\theta)\left(|\Gamma^{01}|^2 + |\Gamma^{10}|^2 + |\Gamma^{12}|^2\right)]$$
$$-(v_e^2 - a_e^2)\cos 2\phi \sin^2\theta$$
$$\times\left(|\Gamma^{00}|^2 + 2|\Gamma^{11}|^2 - |\Gamma^{01}|^2 - |\Gamma^{10}|^2 - |\Gamma^{12}|^2\right)$$

3. Form factors for Higgs bosons of Spin S

- Split into two cases for even/odd normality

Normality $n = (-1)^S P$

e.g. scalars and vectors have normality $n = 1$
pseudoscalars and axial-vectors have normality $n = -1$

- $H \rightleftharpoons \gamma\gamma \Rightarrow \mathcal{J}_H \neq 1$ by Yang's Theorem

ie. $H \to \gamma\gamma$ at the LHC or LC,
or $\gamma\gamma \to H$ at a γ collider

- Rotational invariance $\Rightarrow \Gamma^{\lambda_Z \lambda_H}_{-m} = \Gamma^{\lambda_Z \lambda_H}_{m} \equiv \Gamma^{\lambda_Z \lambda_H}$

Odd Normality: $n = -1$

$$\text{Parity} \Rightarrow \Gamma^{\lambda_Z \lambda_H}_{m} = n_Z \, n_H \, \Gamma^{-\lambda_Z \, -\lambda_H}_{m}$$

$$\Rightarrow \Gamma^{00}_{m} = 0 \text{ for } n_H = -1$$

- Observance of Γ^{00} rules out normality $n = -1$.

eg. in Higgs-strahlung with $Z \to f\bar{f}$

For $n = 1$

$$\frac{d\sigma}{d\cos\theta\, d\cos\theta_*\, d\phi_*} \sim (\Gamma^{01}\Gamma^{11} - \Gamma^{00}\Gamma^{10}) \cos\phi_* \sin 2\theta_* \sin 2\theta$$

$$+2|\Gamma^{00}|^2 \sin^2\theta \sin^2\theta_*$$

$$+|\Gamma^{10}|^2 \sin^2\theta \sin^2\theta_* \cos 2\phi_*$$

$$+(|\Gamma^{10}|^2 + |\Gamma^{12})(1+\cos^2\theta)(1+\cos^2\theta_*)$$

$$+2(|\Gamma^{01}|^2 + |\Gamma^{11}|^2) \sin^2\theta(1+\cos^2\theta_*)$$

$$+4\,\frac{4\, v_f\, a_f\;\, v_e\, a_e}{(v_f^2+a_f^2)\,(v_e^2+a_e^2)}((\Gamma^{00}\Gamma^{10}+\Gamma^{01}\Gamma^{11}) \sin\theta \sin\theta_* \cos\phi_*$$

$$-(|\Gamma^{10}|^2 - |\Gamma^{12}|^2) \cos\theta \cos\theta_*)$$

For $n = -1$

$$\frac{d\sigma}{d\cos\theta\, d\cos\theta_*\, d\phi_*} \sim \Gamma^{01}\Gamma^{11} \cos\phi_* \sin 2\theta_* \sin 2\theta$$

$$-|\Gamma^{10}|^2 \sin^2\theta \sin^2\theta_* \cos 2\phi_*$$

$$+(|\Gamma^{10}|^2 + |\Gamma^{12})(1+\cos^2\theta)(1+\cos^2\theta_*)$$

$$+2(|\Gamma^{01}|^2 + |\Gamma^{11}|^2) \sin^2\theta(1+\cos^2\theta_*)$$

$$+4\,\frac{4\, v_f\, a_f\;\, v_e\, a_e}{(v_f^2+a_f^2)\,(v_e^2+a_e^2)}(\Gamma^{01}\Gamma^{11} \sin\theta \sin\theta_* \cos\phi_*$$

$$-(|\Gamma^{10}|^2 - |\Gamma^{12}) \cos\theta \cos\theta_*)$$

(θ_* and ϕ_* are Z decay angles)

General Analysis

Write down most general current:

$$J^\mu = T^{\mu\alpha\beta_1\beta_2\ldots\beta_s}\,\epsilon^*_{Z\alpha}(p_Z,\lambda_Z)\,\epsilon^*_{H\beta_2\ldots\beta_s}(p_H,\lambda_H)$$

- Polarization tensor $\epsilon_{H\beta_2\ldots\beta_s}$ symmetric and traceless

$\Rightarrow T^{\mu\alpha\beta_1\beta_2\ldots\beta_s}$ must be:

  - symmetric in $\beta_i \leftrightarrow \beta_j$
  - cannot contain $g^{\beta_i\beta_j}$

- Only transverse component of J^μ contributes

$$J^\mu \equiv (g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})J_\nu + \frac{q^\mu q^\nu}{q^2}J_\nu$$

Longitudinal component removed by conserved Lepton current:

$$q^\mu \mathcal{L}_\mu = 0$$

$\Rightarrow$ Effective current is transverse:

$$q_\mu T^{\mu\alpha\beta_1\beta_2\ldots\beta_s} = 0$$

Spin 0

$$T^{\mu\alpha} = a_1(g^{\mu\alpha} - q^\mu q^\alpha/s) + a_2(k^\mu - q^\mu(M_Z^2 - M_H^2)/s)q^\alpha$$

where $q = p_Z + p_H$, $k = p_Z - p_H$

Each momentum contracted with a polarization vector gives either:

- $p_i \cdot \epsilon_i(p_i, \lambda_i)$=0, $i = Z, H$
- $p_i \cdot \epsilon_j(p_j, \lambda_j = \pm 1)$=0, $i, j = Z, H$
- $p_i \cdot \epsilon_j(p_j, \lambda_j = 0) = \frac{s}{2M_j}\beta$, $i, j = Z, H$, $i \neq j$

Leads to form factors:

$$\Gamma^{00} = \frac{1}{m_Z}(a_1 E_Z + \frac{1}{2} a_2\, s^{3/2}\, \beta^2)$$

$$\Gamma^{10} = a_1$$

- For Spin 0 predict cross section at threshold rises linearly with β (one power of β from the phase space).

- Will show that almost all other spin terms lead to higher powers of β (as did a_2)

Spin 1

General tensor:

$$T^{\mu\alpha\beta} = b_1\, g^{\alpha\beta}(k^\mu - (M_Z^2 - M_H^2)\, q^\mu/s)$$
$$+b_2\,(q^\alpha g^{\beta\mu} - q^\beta g^{\alpha\mu})$$
$$+b_3\, q^\alpha q^\beta (k^\mu - (M_Z^2 - M_H^2)\, q^\mu/s)/s$$
$$+b_4\,(q^\alpha g^{\beta\mu} + q^\beta g^{\alpha\mu} - 2\, q^\alpha q^\beta q^\mu/s)$$

Form factors:

$$\Gamma^{00} = [b_1\,(s - M_Z^2 - M_H^2) - b_2\, s + \frac{1}{2}\, b_3\, s\, \beta^2$$
$$+b_4\,(M_Z^2 - M_H^2)]\frac{\sqrt{s}}{2\, M_Z M_H}\beta$$
$$\Gamma^{11} = b_1\, \sqrt{s}\, \beta$$
$$\Gamma^{10} = (-b_2 + b_4)\frac{s}{2\, M_H}\, \beta$$
$$\Gamma^{01} = (b_2 + b_4)\frac{s}{2\, M_Z}\, \beta$$

⇒ Spin one Higgs displays at least a β^3 dependence at threshold

Spin 2

General tensor:

$$T^{\mu\alpha\beta_1\beta_2} = c_1 \left(g^{\alpha\beta_1}(g^{\mu\beta_2} - q^\mu q^{\beta_2}/s) + g^{\alpha\beta_2}(g^{\mu\beta_1} - q^\mu q^{\beta_1}/s)\right)$$
$$+c_2 \left(g^{\mu\alpha} - q^\mu q^\alpha/q^2\right) q^{\beta_1} q^{\beta_2}$$
$$+c_3 \left(g^{\mu\beta_1} q^{\beta_2} + g^{\mu\beta_2} q^{\beta_1} - 2 q^\mu q^{\beta_1} q^{\beta_2}/s\right) q^\alpha$$
$$+c_4 \left(g^{\alpha\beta_1} q^{\beta_2} + g^{\alpha\beta_2} q^{\beta_1}\right)(k^\mu - q^\mu (M_Z^2 - M_H^2)/s)$$
$$+c_5 \left(k^\mu - q^\mu (M_Z^2 - M_H^2)/s\right) q^\alpha q^{\beta_1} q^{\beta_2}$$

Form factors:

$$\Gamma^{00} = \frac{\sqrt{2/3}}{M_Z M_H^2} \left(c_1 E_H (s - M_Z^2 - M_H^2)\right.$$
$$-\frac{1}{4} s^2 \beta^2 \left[c_2 E_Z - 2 c_3 E_H + 2 c_4 (s - M_Z^2 - M_H^2)/\sqrt{s}\right]$$
$$\left.-\frac{1}{8} c_5 s^{7/2} \beta^4\right)$$

$$\Gamma^{01} = -\frac{1}{2\sqrt{2} M_Z M_H} (2 c_1 (s - M_Z^2 - M_H^2) + c_3 s^2 \beta^2)$$

$$\Gamma^{10} = \sqrt{2/3}(c_1 + \frac{1}{4 M_H^2} c_2 s^2 \beta^2)$$

$$\Gamma^{11} = \frac{\sqrt{2}}{M_H}(c_1 E_H - \frac{1}{2} c_4 s^{3/2} \beta^2)$$

$$\Gamma^{12} = 2 c_1$$

Problem: c_1 presents terms with no β dependence

Mimics Spin 0: cross-section will rise like β at threshold

Distinguishing Spin 0 from Spin 2

- Observation of β rise at threshold $\Rightarrow c_1 \neq 0$
- Thus if $S = 2$ then $\Gamma^{12} \neq 0$
- Consider Z decaying via $Z \to f\bar{f}$ and examine angular correlations to measure $|\Gamma^{12}|$

$$\frac{d\sigma}{d\cos\theta\, d\cos\theta_*} \sim$$

$$(1+\cos^2\theta)(1+\cos^2\theta_*)(|\Gamma^{10}|^2 + |\Gamma^{12}|^2)$$

$$+4\frac{4\, v_e\, a_e\, a_f\, v_f}{(v_e^2+a_e^2)(a_f^2+v_f^2)} \cos\theta\cos\theta_*(|\Gamma^{10}|^2 - |\Gamma^{12}|^2) + \ldots$$

- Depends only on $S = 2$, $c_1 \neq 0$ hypothesis and well-known properties of the Z boson
- Can also measure Γ^{12} from azimuthal correlation between H and Z decay planes

Spin $S > 2$

For $S > 2$ have no new tensor structures and the same number of independent coefficients

- Can write $T^{\mu\alpha\beta_1\beta_2\beta_3}$ in terms of the spin 2 case

$$T_S^{\mu\alpha\beta_1\beta_2\beta_3} = \sum_{i<j} T_2^{\mu\alpha\beta_i\beta_j} q^{\beta_1} \ldots q^{\beta_{i-1}} q^{\beta_{i+1}} \ldots q^{\beta_{j-1}} q^{\beta_{j+1}} \ldots q^{\beta_S}$$

Only have extra momenta which contract with the Higgs polarization tensor

- Have only higher powers of β for higher spins

For $S \geq 2$ leading threshold behaviour:

$$\sigma \sim \beta\, \beta^{2\,[S-2]} = \beta,\ \beta^3,\ \beta^5, \ldots$$

Normality $n = -1$

Now,

$$\Gamma_m^{\lambda_Z \lambda_H} = -\Gamma_m^{-\lambda_Z - \lambda_H} \Rightarrow \Gamma_m^{00} = 0$$

Spin 0

General tensor:

$$T^{\mu\alpha} = a_1 \, \epsilon^{\mu\alpha\rho\sigma} q^\rho k^\sigma$$

Form factors:

$$\Gamma^{00} = 0$$

$$\Gamma^{10} = a_1 \, s \, \beta$$

Observe β^3 behaviour at threshold

Spin 1

General tensor:

$$T^{\mu\alpha\beta} = b_1\,\epsilon^{\mu\alpha\beta\rho}q^\rho + b_2\,(\epsilon^{\mu\alpha\beta\rho}k^\rho - \epsilon^{\alpha\beta\rho\sigma}q^\rho k^\sigma q^\mu/s)$$
$$+b_3\,\epsilon^{\alpha\beta\rho\sigma}(k^\mu - q^\mu(M_Z^2 - M_H^2)/s)\,q^\rho k^\sigma$$

Form factors:

$$\Gamma^{00} = 0$$
$$\Gamma^{11} = \frac{1}{\sqrt{s}}(b_1\,s + b_2\,(M_Z^2 - M_H^2) + b_3\,s^2\beta^2)$$
$$\Gamma^{10} = \frac{1}{\sqrt{s}\,M_H}(b_1\,s\,E_H + b_2\,(E_H\,(M_Z^2 - M_H^2) + \frac{1}{2}\,s^{3/2}\beta^2))$$
$$\Gamma^{01} = \frac{1}{\sqrt{s}\,M_Z}(b_1\,s\,E_Z + b_2\,(E_Z\,(M_Z^2 - M_H^2) - \frac{1}{2}\,s^{3/2}\beta^2))$$

Again, have terms with no β dependence

Distinguish by measuring Γ^{00}

Also ruled out by observing $H \to \gamma\gamma$

Spin 2

General tensor:

$$T^{\mu\alpha\beta_1\beta_2} = (\epsilon^{\mu\alpha\beta_1\rho} q^{\beta_2} + \epsilon^{\mu\alpha\beta_2\rho} q^{\beta_1})(c_1 q^\rho + c_2 k^\rho)$$
$$+c_3\,(\epsilon^{\alpha\beta_1\rho\sigma} q^{\beta_2} + \epsilon^{\alpha\beta_2\rho\sigma} q^{\beta_1})\,(k^\mu - q^\mu(M_Z^2 - M_H^2)/s)\, q^\rho k^\sigma$$
$$+c_4\,\epsilon^{\mu\alpha\rho\sigma} q^\rho k^\sigma q^{\beta_1} q^{\beta_2}$$

Form factors:

$$\Gamma^{00} = 0$$

$$\Gamma^{11} = \frac{\sqrt{s}}{\sqrt{2}\,M_H} \beta\,(c_1\,s + c_2\,(M_Z^2 - M_H^2) + c_3\,s^2\beta^2)$$

$$\Gamma^{10} = \sqrt{2/3}\frac{\sqrt{s}}{M_H^2}\beta\,(c_1\,sE_H + c_2\,(E_H(M_Z^2 - M_H^2) + \frac{1}{2}s^{3/2}\beta^2)$$
$$+\frac{1}{4}c_4\,s^{5/2}\beta^2)$$

$$\Gamma^{01} = \frac{\sqrt{s}}{\sqrt{2}\,M_Z M_H}\beta\,(c_1\,sE_Z + c_2\,(E_Z(M_Z^2 - M_H^2) - \frac{1}{2}s^{3/2}\beta^2)$$

$$\Gamma^{12} = 0$$

Have β^3 rise at threshold

Spin $S > 2$

As with even normality, have no new tensor structures byond spin 2:

$$T_S^{\mu\alpha\beta_1\beta_2\beta_3} = \sum_{i<j} T_2^{\mu\alpha\beta_i\beta_j} q^{\beta_1} ... q^{\beta_{i-1}} q^{\beta_{i+1}} ... q^{\beta_{j-1}} q^{\beta_{j+1}} ... q^{\beta_S}$$

All higher spins contribute higher powers of β

For $S \geq 2$ leading threshold behaviour:

$$\sigma \sim \beta^3 \beta^{2[S-2]} = \beta^3,\, \beta^5, ...$$

Mixed Normality

- What happens if H is not a parity eigenstate?

Must add tensors $T^{\mu\alpha\beta_1\beta_2...\beta_S}$ for odd and even normality together.

$$|\mathcal{M}|^2 = |\mathcal{M}_+ + \mathcal{M}_-|^2 = |\mathcal{M}_+|^2 + |\mathcal{M}_-|^2 + 2\,\mathcal{R}e\,\mathcal{M}_+\,\mathcal{M}_-^*$$

- $|\mathcal{M}_\pm|^2$ terms give same β dependence as before
- Interference term always gives non-linear β dependence

Again, will observe non-linear rise in σ at threshold for all spins > 2.

- Use angular correlations to rule out spin 1 and 2.

4. Summary and Conclusions

- The dependence of $e^+e^- \to ZH$ on β at threshold provides a definitive confirmation of a spinless Higgs boson.

If cross-section grows like $\beta \Rightarrow$

Higgs has normality $n = 1$ and $S = 0$ or 2

OR

Higgs has normality $n = -1$ and $S = 1$

- Normality $n = -1$ can be ruled out by measuring non-zero Γ^{00}

- $S = 2$ can be ruled out be examining the angular dependence of the cross-section