

Extra Dimensions

+

The Linear Collider

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Why Build the NLC?

• Zero'th Order Reason: we are greedy physicists. We want to be certain that we cover the physics of the TeV scale from every possible angle.

• "But why not wait till the LHC finds SUSY so we can set the scale for next machine? etc etc etc"

... Because we should try our best to ignore theoretical prejudices in the experimental investigation of the TeV scale.

• This is OK, because theoretical prejudices are often wrong or miss the point.

• Example: fine-tuning of electroweak symmetry breaking.

"We're bound to see almost all superpartners, at least for 3rd generation, at LHC, otherwise EWSB is too fine-tuned".

• May be true, but lots of things are "accidental" fine-tunings in nature...

- Ancient example: why does the moon nearly perfectly eclipse the sun?

$$\frac{R_{\text{moon}}}{R_{\text{earth-moon}}} \bigg/ \frac{R_{\text{sun}}}{R_{\text{earth-sun}}} = 1$$

Big accidental fine-tuning!

- More closely related example from nuclear physics. All the mass scales in nucl. phys.

are set by $\sim m_{\pi} \sim 100's \text{ MeV}$. Why is Deuteron

binding energy $\sim 2 \text{ MeV}$? Because of

an accidental cancellation between

kinetic + potential energies to $\sim 1\%$.

• Many more examples like this ... but perhaps more importantly, our notions of naturalness don't have a wonderful track record:

e.g. "all dimensionless couplings should be $\mathcal{O}(1)$..." then why is only the top heavy?

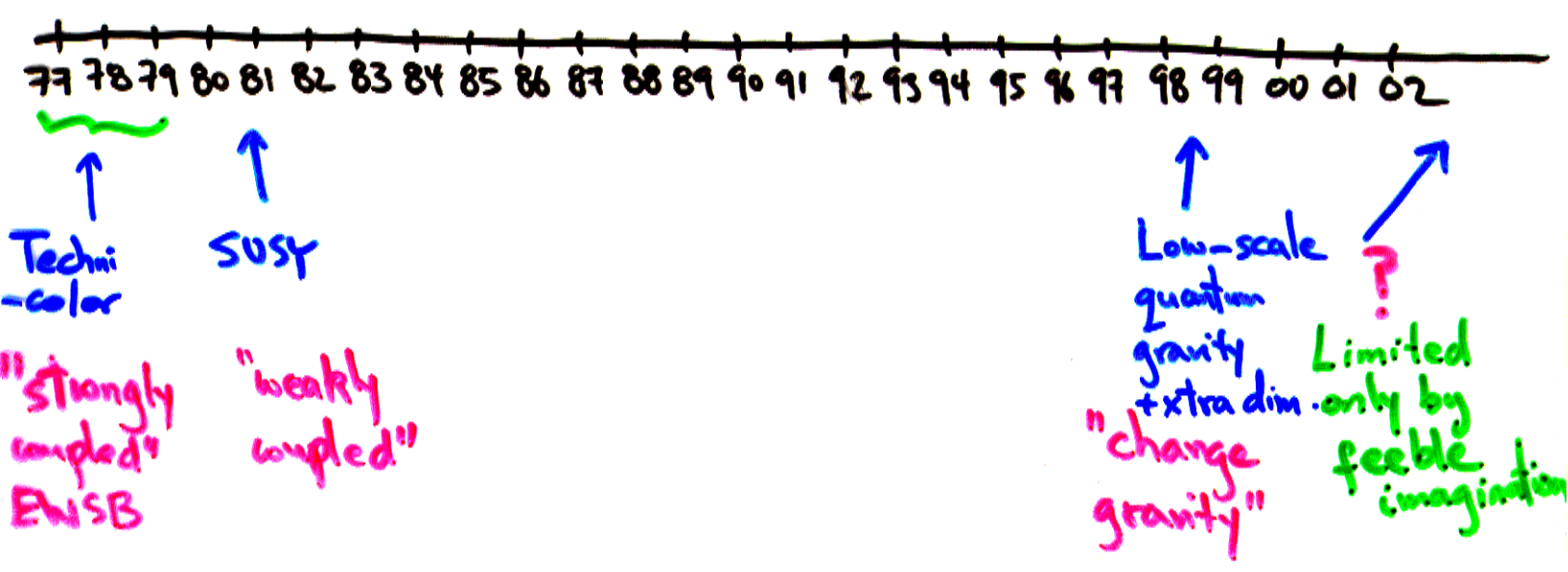
more famously, "all dimensionful parameters are scaled by the UV cutoff.."

$$m_H^2 \sim M_{pl}^2 \quad \text{Hierarchy Problem}$$

$$\Lambda \sim M_{pl}^4 \quad \text{Cosm. Const. Problem}$$

• Another theoretical prejudice: "we pretty much know the sweep of possibilities for TeV scale physics. We should try to figure out which one of these models is right!"

• The set of qualitatively different possibilities for TeV scale physics is quite time dependent, however.



• Finally: a little exercise in model-building, with the aim of refuting this attitude:

"Sure precision measurements + pristine environment of NLC is wonderful.

But its a luxury. Don't we mainly care about determining particle quantum #'s, decay patterns, branching ratios etc.?"

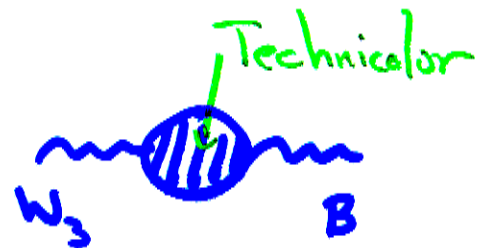
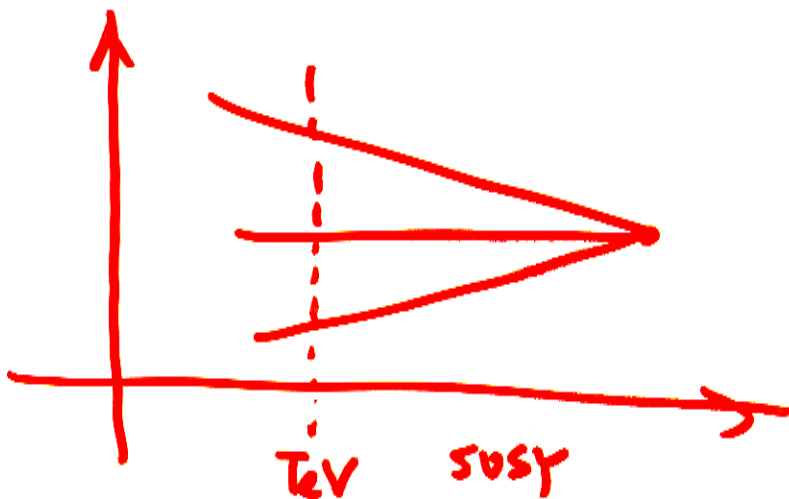
Precision measurements
are not a luxury,
they are crucial for
determining what is really
going on, and pointing us
in the right direction.

e.g. precision electroweak

measurements did much more

than confirm the SM to

1% accuracy:



Reveal amazing connection
between SUSY + Unification!

Make trouble
for strong
EWSB!

But you could get even more misled.

e.g. suppose you see

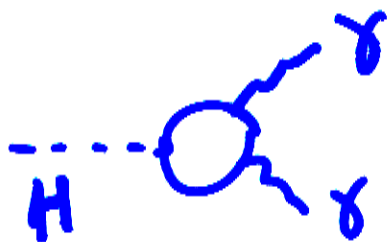


What have you seen?

Could be

or

Could be



A little fun:

Suppose that after combined effort of the Tevatron + LHC, we have determined the existence of the following scalar particles, and decay modes:

$$\phi_1 \rightarrow l + \text{missing energy}$$

$$\phi_2 \rightarrow \tau + \text{missing energy}$$

WHAT HAVE WE SEEN?

... 12,000 papers on hep-ph
would say you've seen

SUSY

$$\Phi_1 \sim \tilde{l}, \tilde{e}^c$$

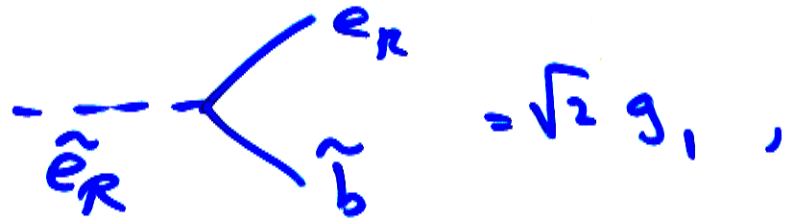
$$\tilde{l} \rightarrow l \tilde{b}$$

$$\Phi_2 \sim \tilde{q}, \tilde{u}^c, \tilde{d}^c$$

$$\tilde{q} \rightarrow q \tilde{b}$$

Somehow gluinos, winos were a little
to heavy to make. But we got everything
else!

• So even though we haven't measured e.g.



you might be tempted to think

that you've seen a solid hint

for SUSY.

• You would be dead wrong.

Here's the actual model.

Add a γ_R to the SM,
and strongly gauge the

(anom.) U(1) symmetry where

$$\begin{array}{cccccc} Q & U^c & D^c & L & E^c & \gamma_R \\ +1 & +1 & +1 & +1 & +1 & -1 \end{array} .$$

The U(1) is broken by GS mech,
say near $\sim \text{TeV}$. This gives
strong attractive force between
($Q U^c D^c L E^c$) and γ_R .

.... So you make bound states

$$(Q \nu_R) \sim \tilde{Q}$$

quantum #'s of squark!

$$(U^c \nu_R) \sim \tilde{U}^c$$

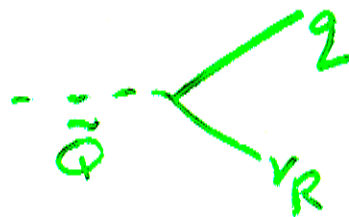
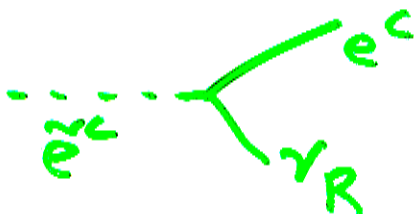
$$(D^c \nu_R) \sim \tilde{D}^c$$

$$(L \nu_R) \sim \tilde{L}^c$$

slepton!

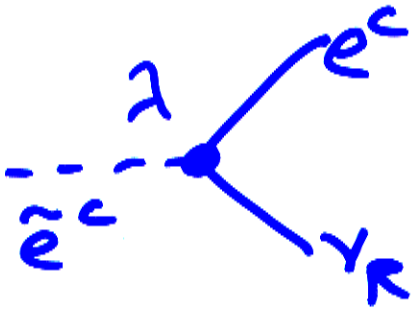
$$(E^c \nu_R) \sim \tilde{E}^c$$

• Furthermore they decay into their constituents, e.g.

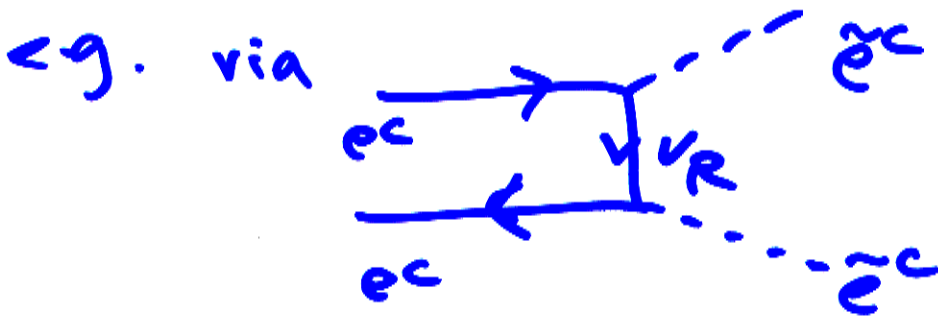


you would call ν_R a very light "bino".

... We would know this isn't SUSY by e.g. measuring



at the NLC, since it's very big in this model.



By proving $\lambda \neq \sqrt{2} g_2$, we see it is not SUSY.

Ideally, we should DISREGARD

theoretical prejudices in preparing to confront TeV physics.

• What we want from our colliders :

(1) Find new particles ϕ + their quantum #'s.

(2) Determine interactions so that we can find $\mathcal{L} = \mathcal{L}_{SM}(\psi_{SM}) + \mathcal{L}'(\psi_{SM}, \phi)$

• Once we know \mathcal{L} , we can figure out what the model is.

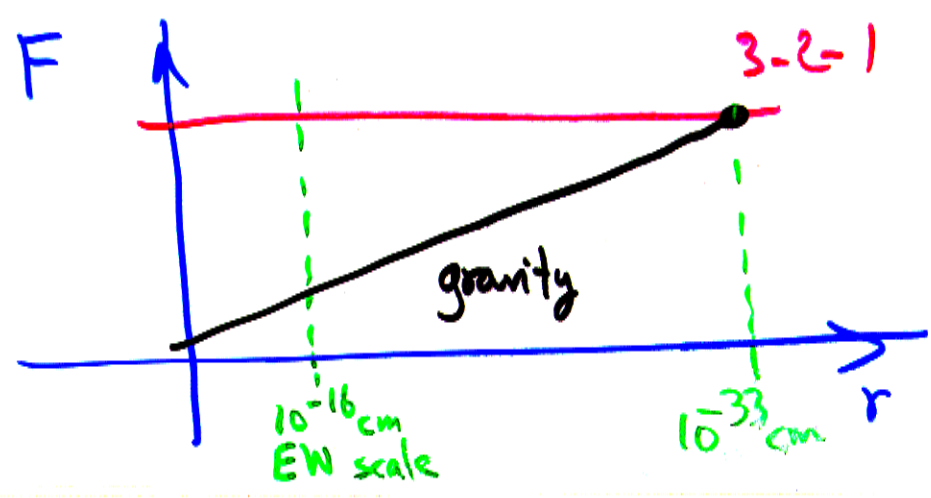
• We'll need all we can get to make sure we can do this ; LHC + NLC will play complementary roles.

. For the rest of this talk
I will discuss these issues
in the context of theories
with extra dimensions +
accessible quantum gravity.

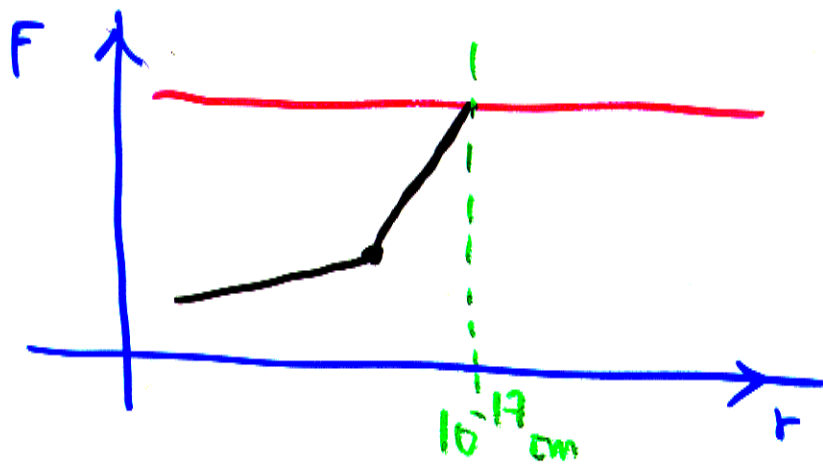
300 Years after Newton,
we don't understand the
answer to a simple

question: Why is gravity weak?

Usual picture: gravity comes
from tiny distances $\sim 10^{-33}$ cm

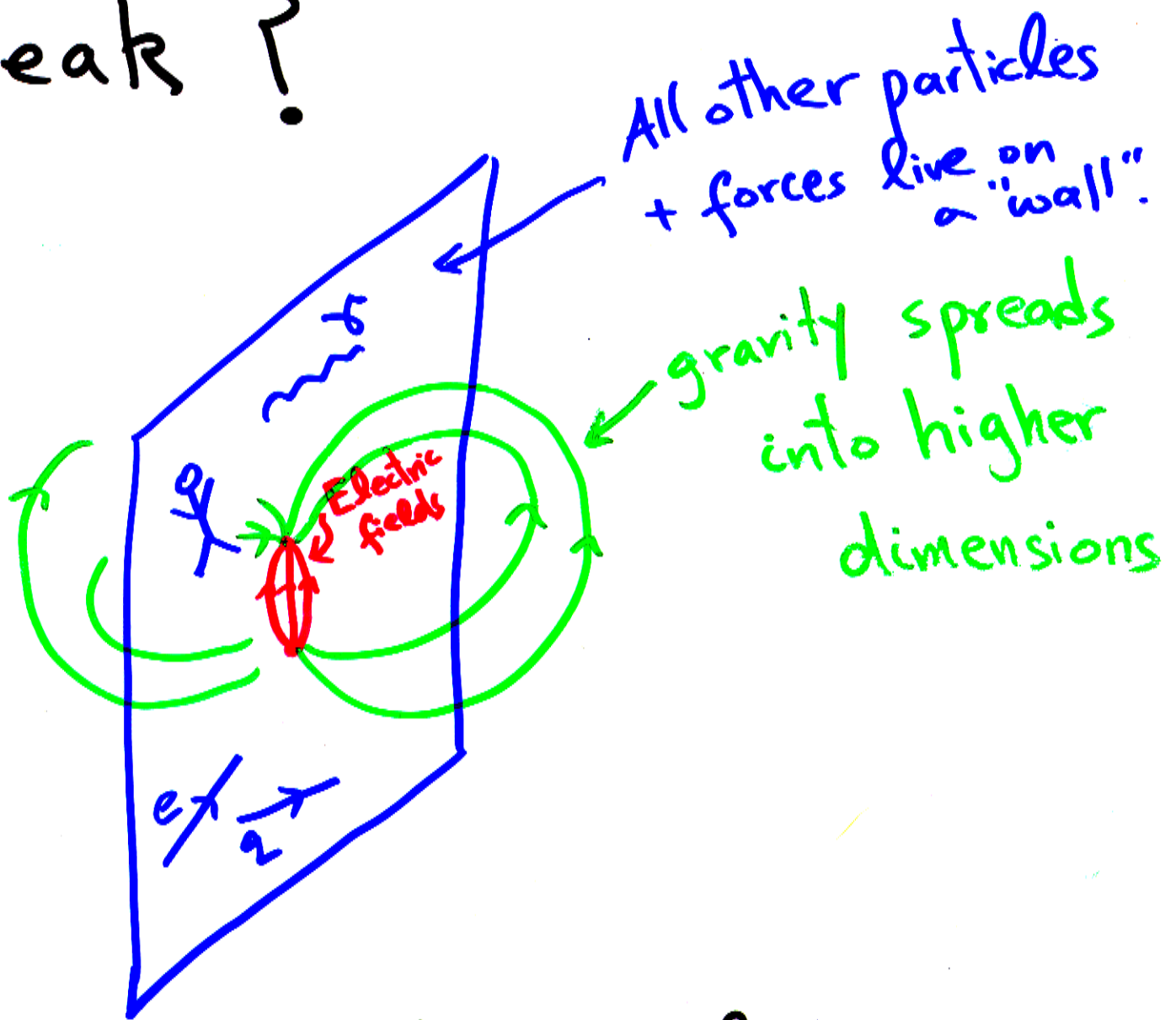


But can we instead have
gravity catch up with the
other forces near 10^{-17} cm,
so that the scales of
gravity + mass generation
are the same?



YES.
We can
imagine this
because our
exp. knowl. is poor

But then why is gravity weak?



Gravity Dilutes itself in Large New Dimensions of Space

Since the fundamental
scale of gravity is now 10^{-17} cm,

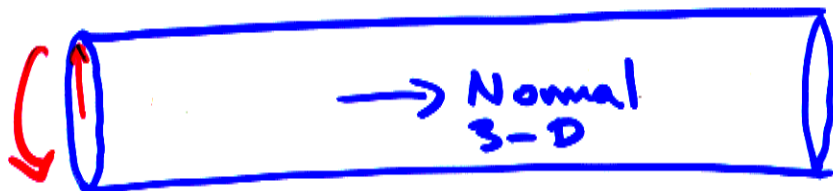
and accelerators (such as
the LHC) probe $\sim 10^{-17}$ cm,

Quantum Gravity / String Theory
will be probed at accelerators!
(on 5-10 yr timescale).

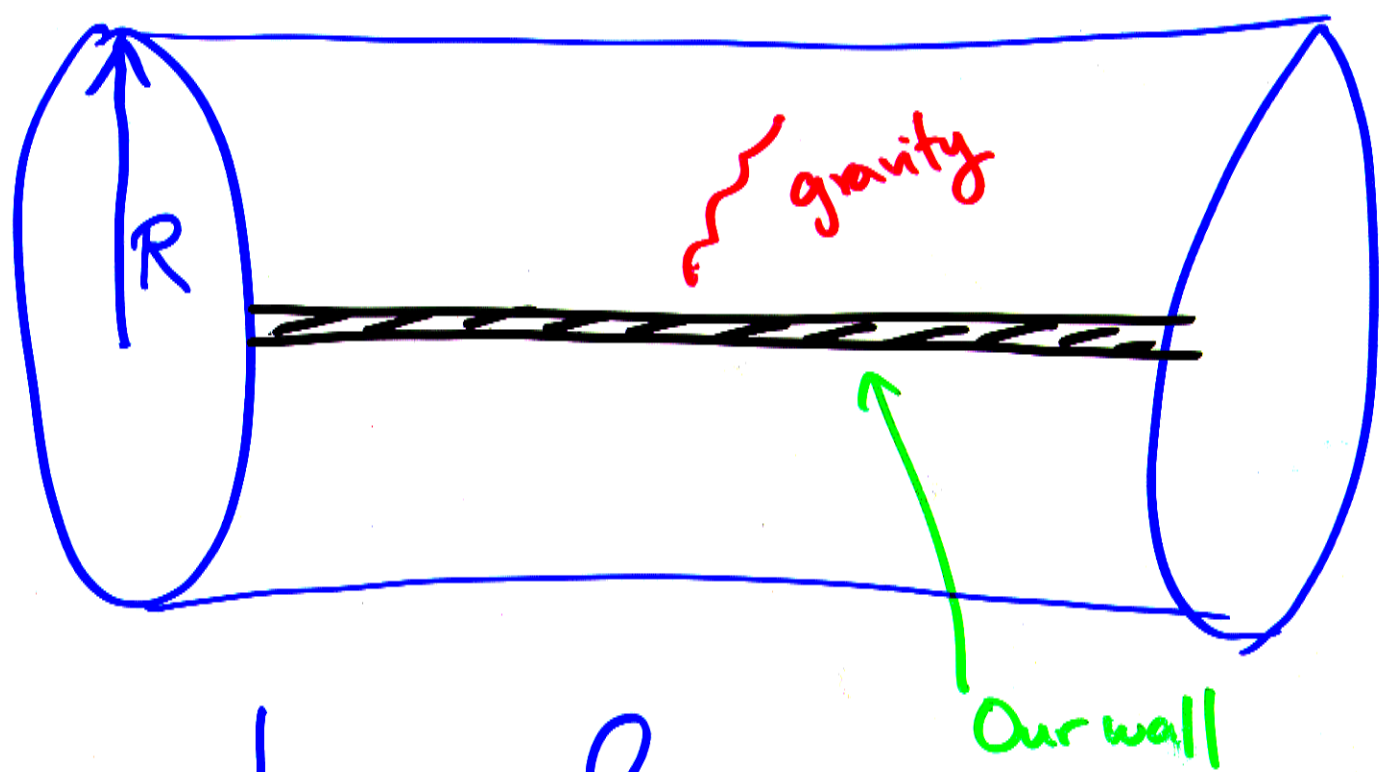
People have thought about extra dimensions for a long time, string theory predicts **7** extra spatial dimensions.

But they were always thought to be rolled up to tiny size

extra dim,
 $\sim 10^{-33}$ cm
big.



The size of the dimensions we are talking about are **enormous** by contrast



R can be as large as 1 millimeter.

~~*~~ [Not any more ... more like 2 mm]

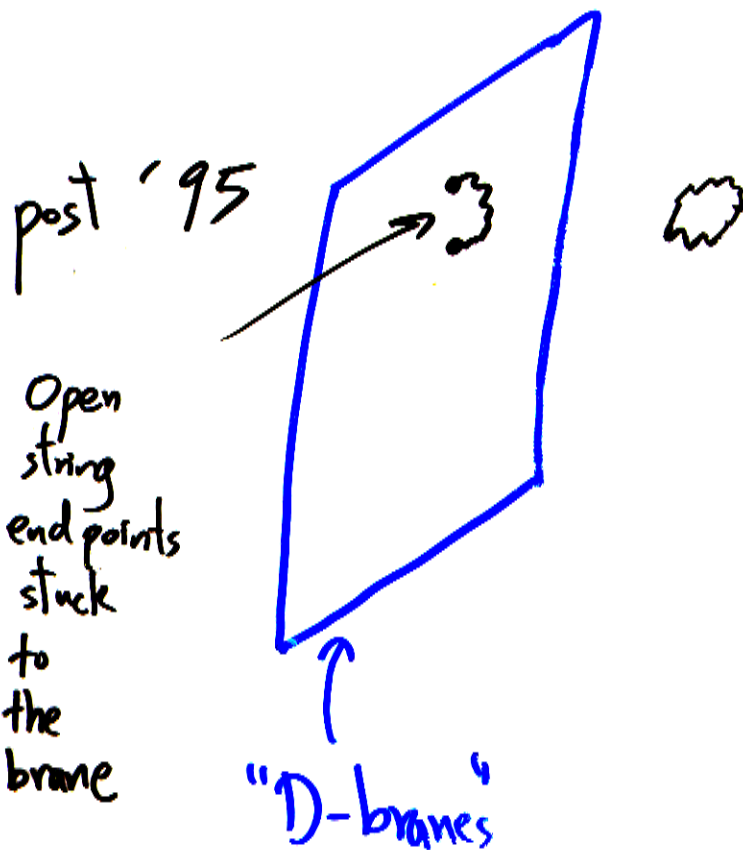
Can be realized in string theory.

This picture can be naturally realized in string theory

pre '95

~~~~~  
"open strings"  
contains particles  
like photons,  
electrons

⊙  
"closed strings"  
contains  
graviton.



# Old picture



←  $10^{-33}$  cm →



←  $10^{-33}$  cm →

too small  
to ever see

01

# New picture

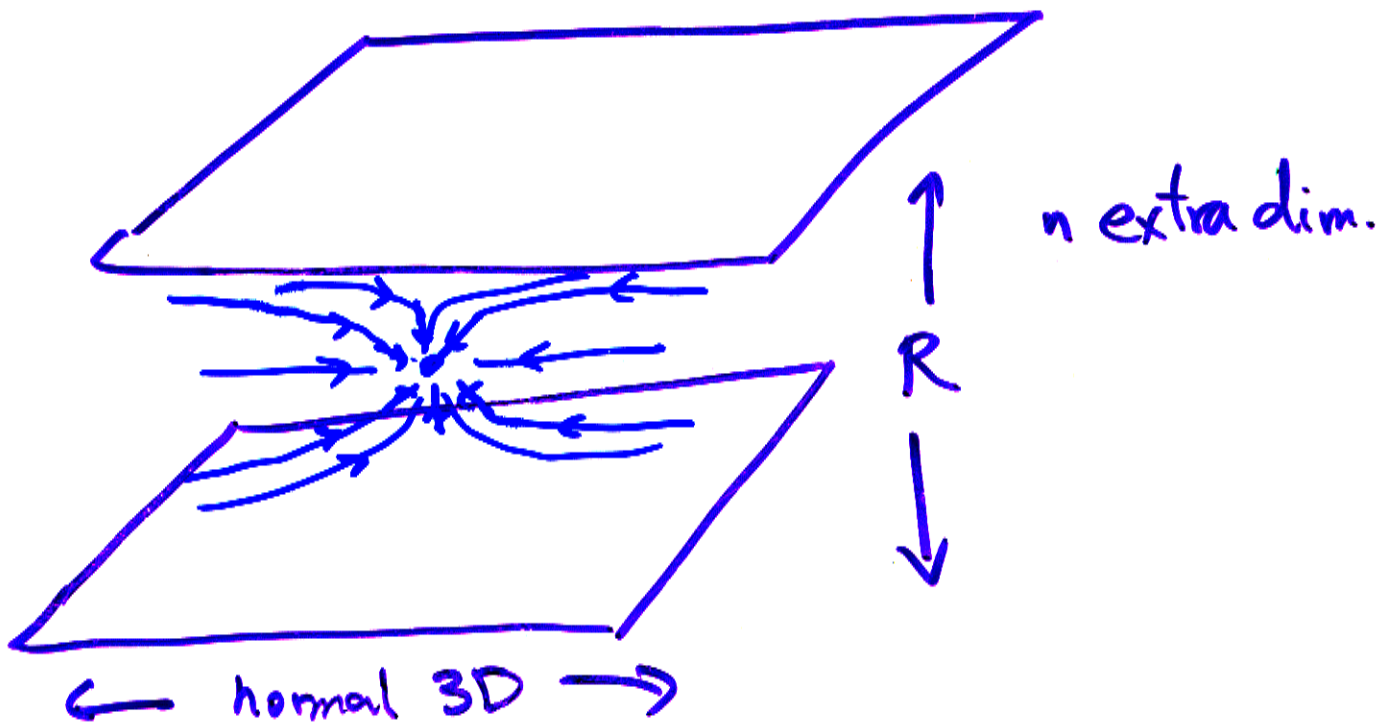


←  $10^{-17}$  cm →



Could be seen  
in 5-10  
years, at  
accelerators

- Extra dimensions can make gravity weak:



$$g(\vec{r}) = \frac{G_{N(4+n)} m}{r^{n+2}} \quad (r \ll R)$$

$$= \boxed{\frac{G_{N(4+n)}}{R^n}} \frac{m}{r^2} \quad (r \gg R)$$

↑  
 $G_{N4}$

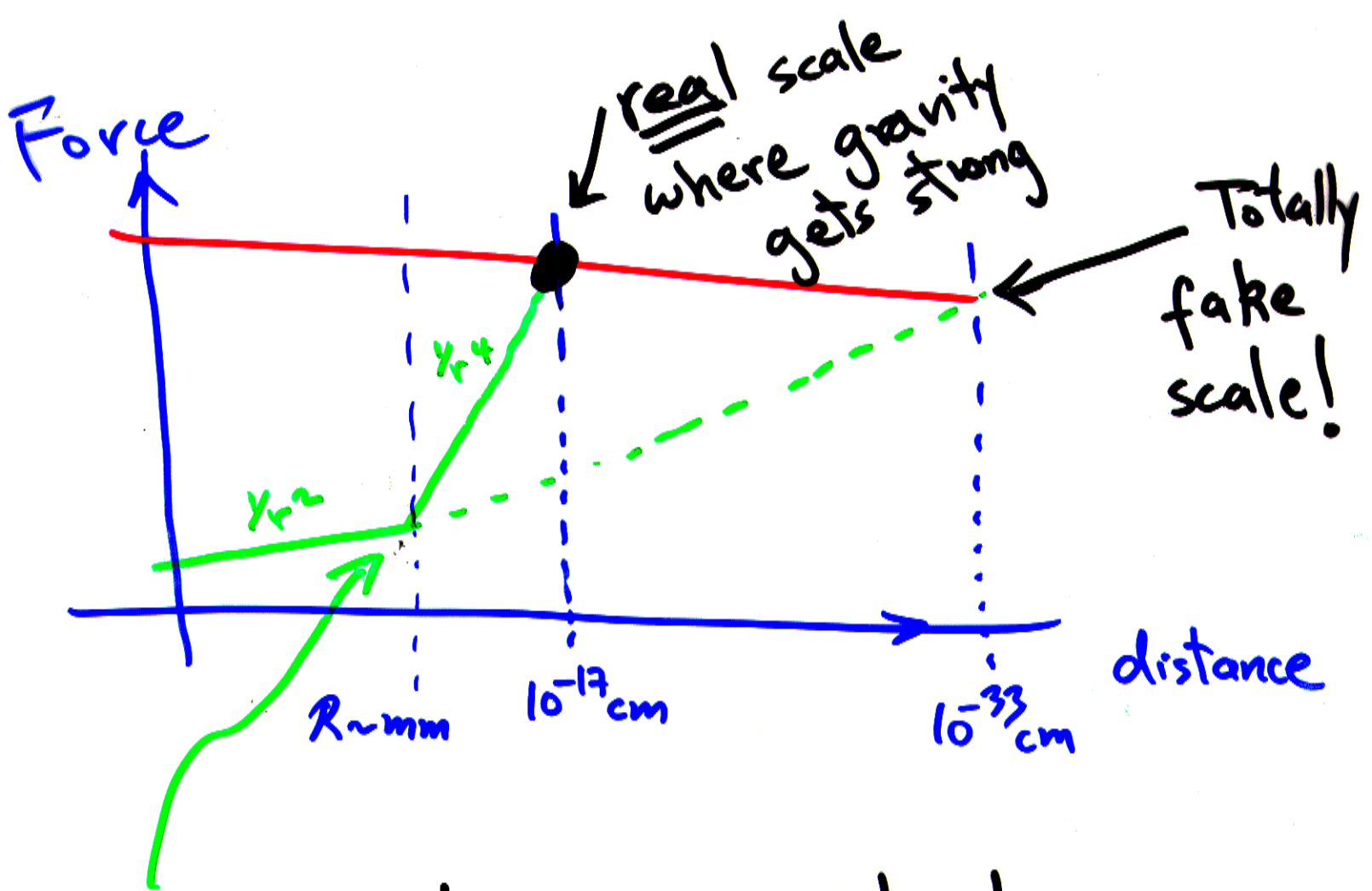
So, we have seen that

$$G_{N_4} = \frac{G_{N(4+n)}}{R^n}$$

If we want  $G_{N(4+n)} = \frac{1}{(\text{TeV})^{2+n}}$

then

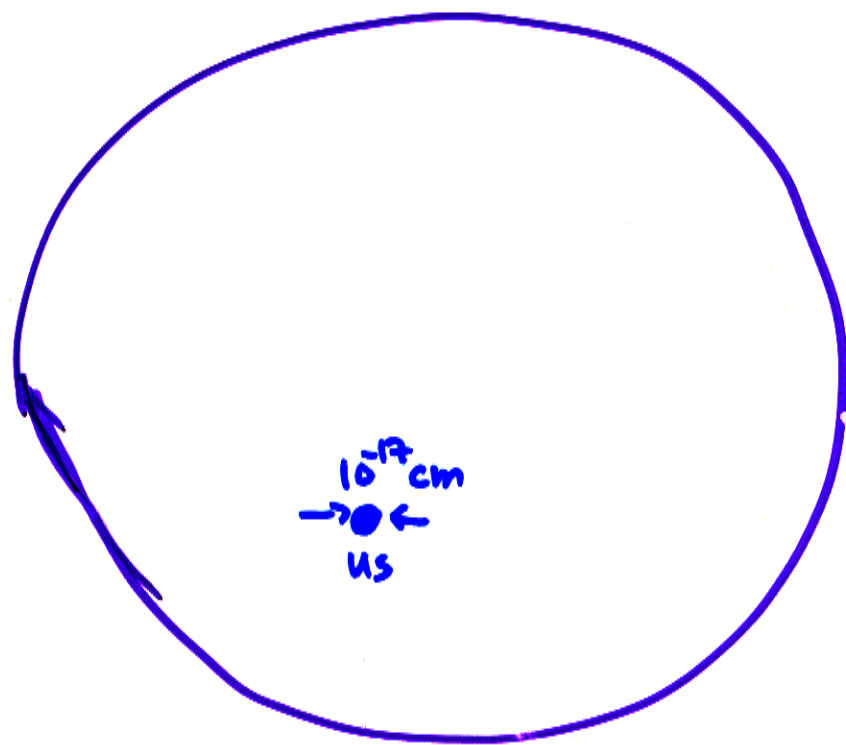
$$R \sim 10^{\frac{30}{n} - 17} \text{ cm} = \begin{cases} \sim 10^{13} \text{ cm} & n=1 \text{ X} \\ \sim \text{mm} & n=2 \\ \vdots \\ \sim 10 \text{ fm} & n=6 \end{cases}$$



• This deviation from Newtonian Gravity near  $\sim \text{mm}$  can be tested by table-top experiments!

• If this picture is realized in nature, it would represent a natural continuation of the Copernican tradition in physics.

← R →



$$\frac{V_{us}}{V_{\text{whole universe}}} \sim 10^{-32} !$$

# Is it alive?

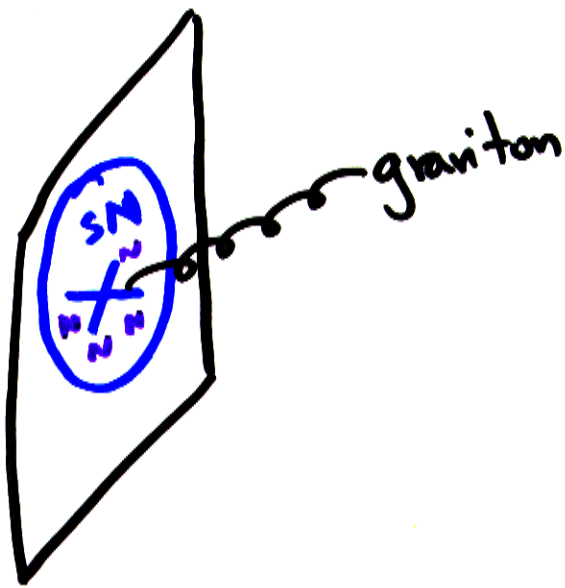
- This theory represents a drastic modification of our usual picture for the universe.

Can it be consistent with everything we already know about the world experimentally?

- Remarkably, the theory survives all lab, astrophysical + cosmological constraints, although  $n=2$  extra dimensions are tightly constrained. (Theory is safer with more dimensions!)



- Strongest constraints come from "evaporating" gravitons into the extra dimensions, in hot astrophysical systems + the early universe.
- Best limits from SN1987A



"Nucleon-nucleon gravistahlung"

$$\sigma \sim G_{N(4+n)} E^n \sim \frac{E^n}{M_{Pl(4+n)}^{2+n}}$$

$$n=2: M_{Pl6} \gtrsim 50 \text{ TeV.}$$

$$n > 2 \quad M_{Pl(4+n)} \sim \text{TeV OK.}$$

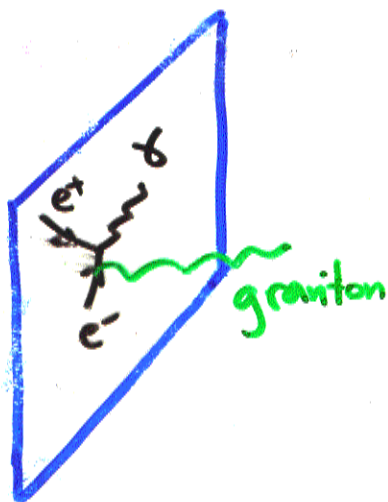
# • (Broad) Experimental Predictions:

(1) Deviations from Newtonian Gravity at sub-mm distances, can be tested by currently running table-top experiments.

Adelberger et al.  
Kapitulnik et al.  
Price et al.

(2) Quantum gravity at colliders.

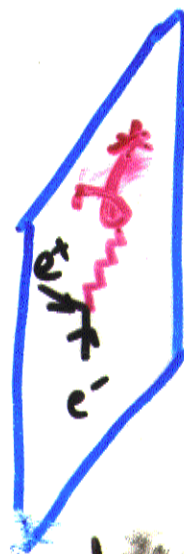
LEP,  
Tevatron,  
LHC,  
NLC



gravitational radiation into the bulk



graviton exchange

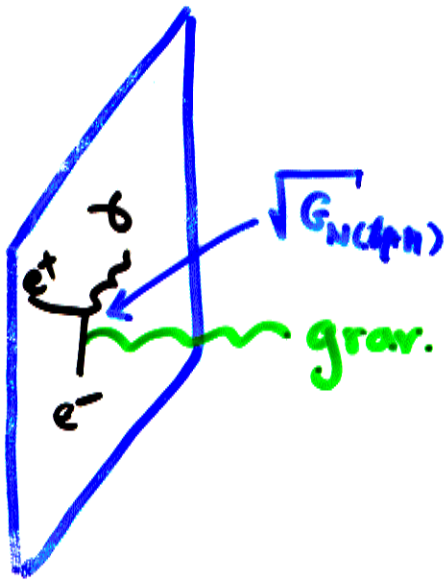


production of string excitations.

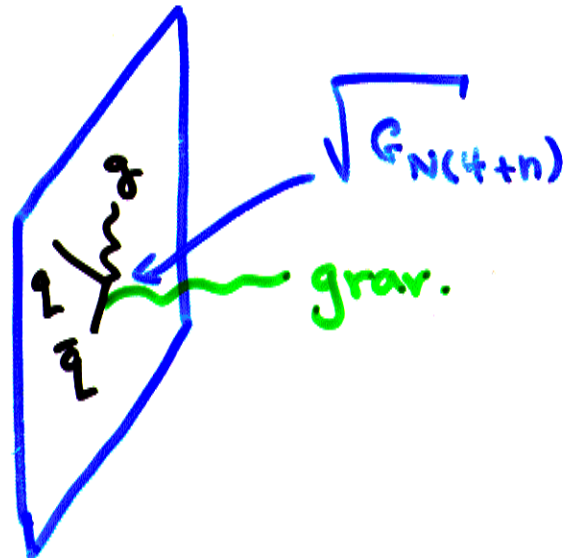
# Gravitational Radiation into the Bulk.

(Giudice, Rattazzi, Wells ; Mirabelli, Perelstein, Peskin)

(Han, Lykken, Zhang)



$$e^+e^- \rightarrow \gamma + \text{missing } E$$



$$pp \rightarrow \text{jet} + \text{missing } E$$

Signals depend on only 2 parameters;

$$n = 2, 3, 4, \dots \quad \text{and} \quad G_{N(4+n)}. \quad \sigma \sim G_{N(4+n)} E^n.$$

Writing  $G_{N(4+n)} = \frac{1}{M_*^{2+n}}$ , we get limits on  $M_*$  as a function of  $n$ .

# Limits

|                                                   | $M_*(n=2)$ | $M_*(n=4)$ | $M_*(n=6)$ TeV |
|---------------------------------------------------|------------|------------|----------------|
| 1 TeV NLC,<br>$\mathcal{L} = 100 \text{ fb}^{-1}$ | 7.7        | 4.5        | 3.1            |

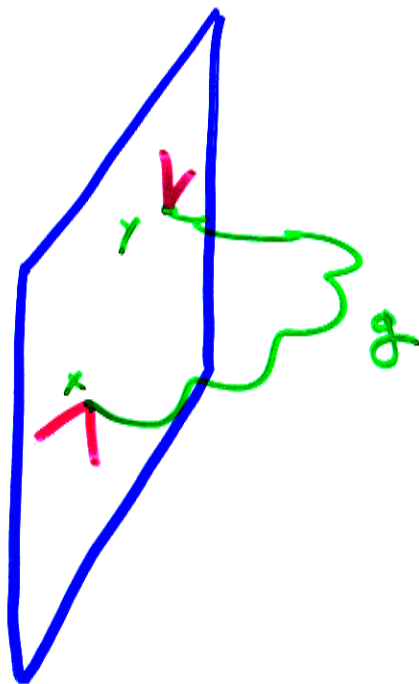
|                                            |      |     |     |
|--------------------------------------------|------|-----|-----|
| LHC<br>$\mathcal{L} > 100 \text{ fb}^{-1}$ | 12.5 | 7.5 | 6.0 |
|--------------------------------------------|------|-----|-----|

LHC can do better because of higher

energy,  $\sigma \sim \frac{E^n}{M_*^{2+n}}$ .

|                                               |     |   |     |
|-----------------------------------------------|-----|---|-----|
| Run II<br>$\mathcal{L} > 300 \text{ pb}^{-1}$ | 1.4 | 1 | 0.9 |
|-----------------------------------------------|-----|---|-----|

# Graviton exchange (Hewett, ...)



$$S = \int d^4x d^4y T^{\mu\nu}(x) \frac{G_{N(4+n)}}{|x-y|^{2+n}} T_{\mu\nu}(y)$$

$n=0, 1$  non-local operator ( $\sim \frac{d^4y}{y^2}$  or  $\frac{d^4y}{y^3}$ )

$n=2, 3, \dots$  local operator! Has UV divergence (Coulomb singularity) that must be cutoff.

Parametrize  $\frac{\lambda}{M_H^4} T_{\mu\nu} T^{\mu\nu}$ . Note: couples everything to everything!

# Limits

NLC

$$\begin{aligned} e^+e^- &\rightarrow ff \\ e^+e^- &\rightarrow e^+e^- \\ e^+e^- &\rightarrow e^+e^- \\ e^+e^- &\rightarrow \gamma\gamma \\ e^+e^- &\rightarrow WW/\tau\tau \end{aligned}$$

$M_H$  (TeV)

6.5  
6.2  
6.0  
3.2  
5.5

$\gamma\gamma$  Collider  
(100 fb<sup>-1</sup>)

$$\begin{aligned} \gamma\gamma &\rightarrow e^+e^-/t\bar{t}/jj \\ \gamma\gamma &\rightarrow \gamma\gamma \\ \gamma\gamma &\rightarrow WW \end{aligned}$$

4 $\sqrt{s}$   
4.5 $\sqrt{s}$   
11 $\sqrt{s}$

LHC

$$\begin{aligned} pp &\rightarrow e^+e^- \\ pp &\rightarrow t\bar{t} \\ pp &\rightarrow jj \\ pp &\rightarrow \gamma\gamma \end{aligned}$$

5.3  
6.0  
9.0  
5.4

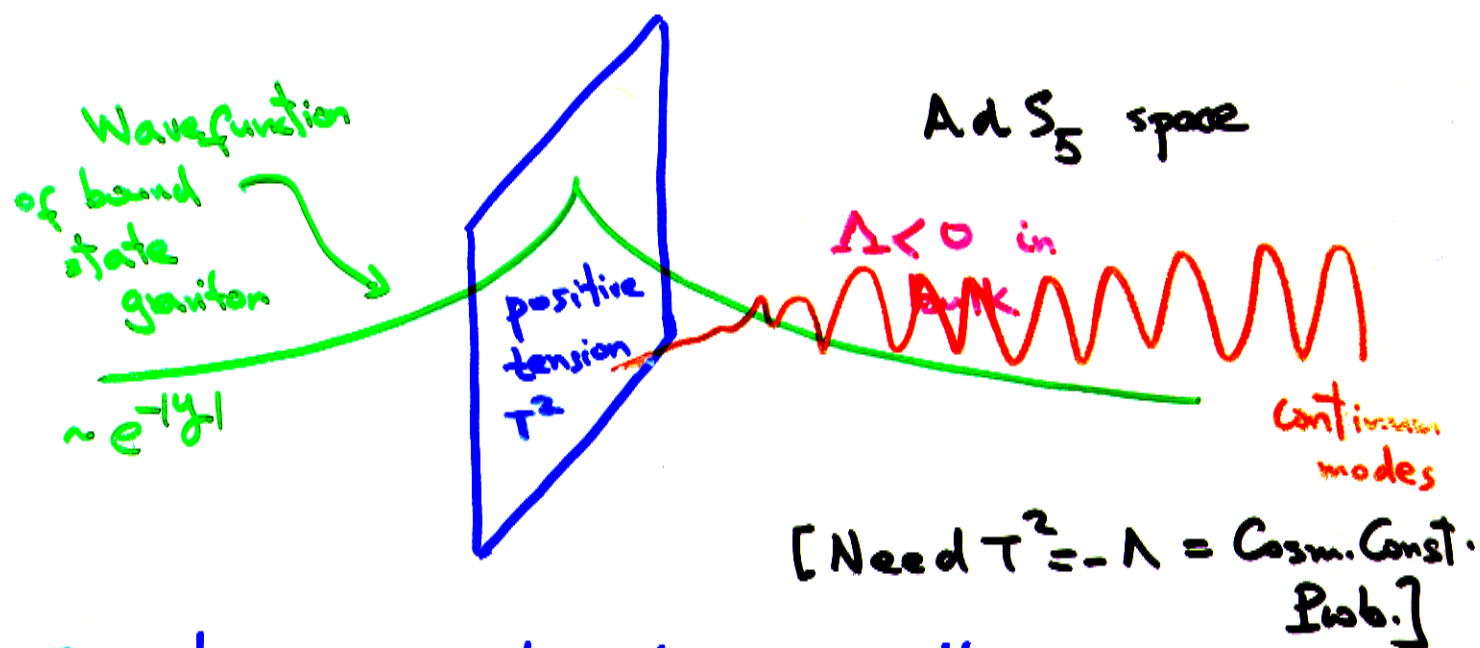
Run II

//

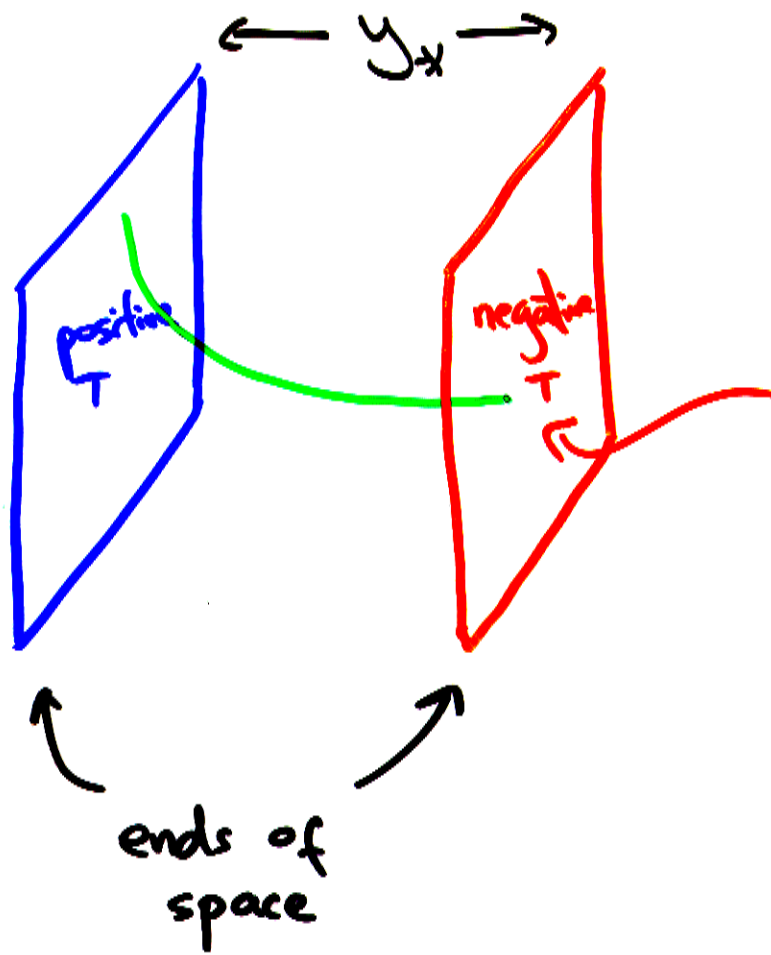
1.4  
1.0  
1.0  
1.4

# • Warped extra dimension (Randall+Sundrum, '99)

Idea is that gravity itself may be localized to a brane w/ 1 extra dimension.



• This traps gravity to a wall in an infinitely large 5<sup>th</sup> dimension! [But has no testable experimental consequences].



SM lives here.  
 Since our overlap with gravity wavefunction  $\sim e^{-\gamma}$ , gravity looks weak!

•  $\frac{M_*}{M_{pl}} \sim e^{-(y_x M_*)} \Rightarrow \text{need } (y_x M_*) \sim 30 \text{ for } M_* \sim \text{TeV}.$

• But the old continuum modes turn into discrete particle in a box modes with  $1/M_*$  couplings and  $M_*$  modes:  $kk$  graviton resonances.



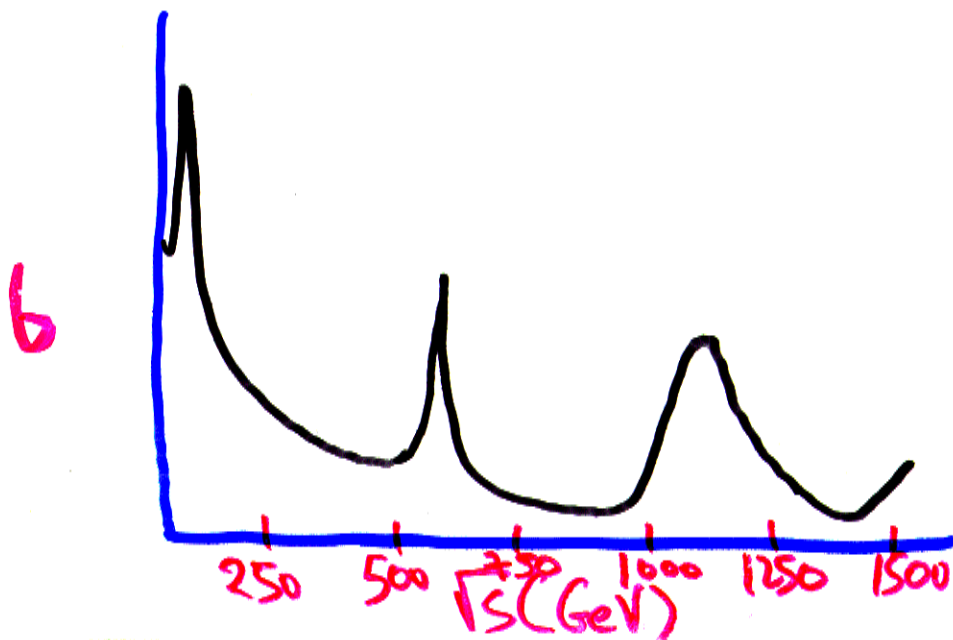
## (Broad) Experimental Predictions :

- No deviations from Newtonian gravity at sub-mm distances
- Discrete tower of KK graviton resonances at colliders.
- Quite different from large extra dimensions.

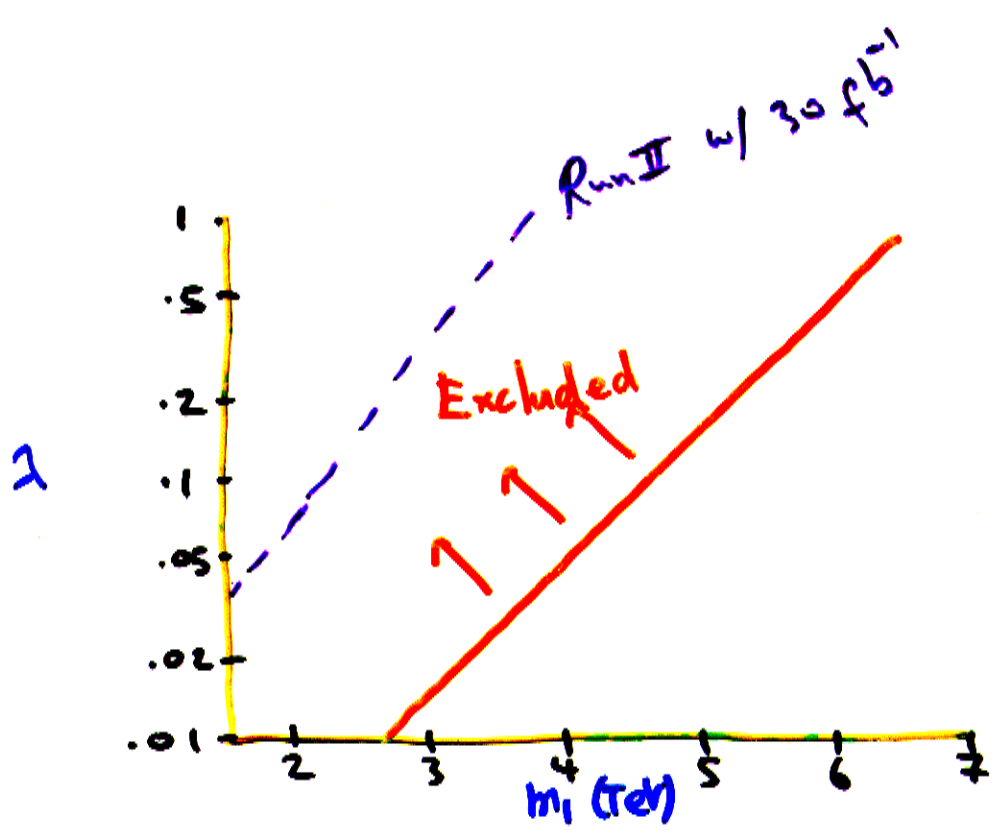
- Studied by Davoudiasl, Hewett + Rizzo:

With enough energy we can make the KK resonances !

Model controlled by 2 parameters ( $m_1, \Gamma_1$ )



# LHC Limits



• Summary for gravitons in the bulk:

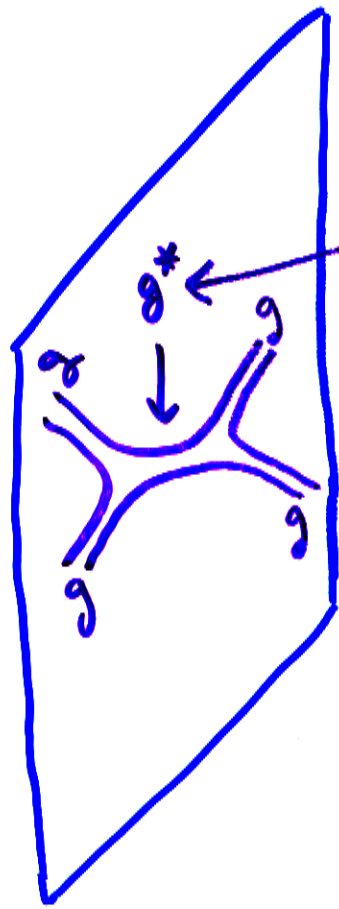
★ Gravitational Radiation / Resonant KK grav.  
production

LHC does "Better"

★ Virtual Graviton Exchange

LHC, 1 TeV NLC comparable

# • String excitations at Colliders



spin 2 "heavy gluon".



Strings  
are  
bigger!  
 $\sim 10^{-17}$  cm

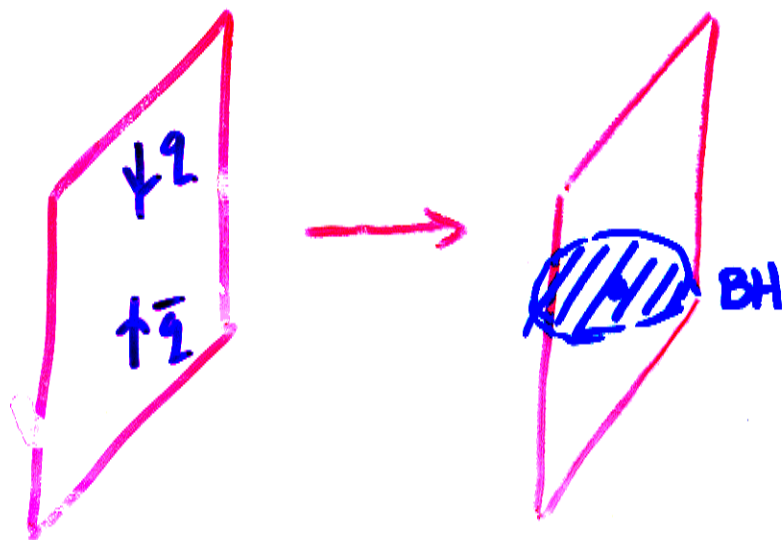
• Real study yet to be done for Tevatron, LHC.

Limits from  $e^+e^-$  collisions @  $\sqrt{s} = 1$  TeV

are  $M_s \gtrsim 10$  TeV [Peskin et al.].

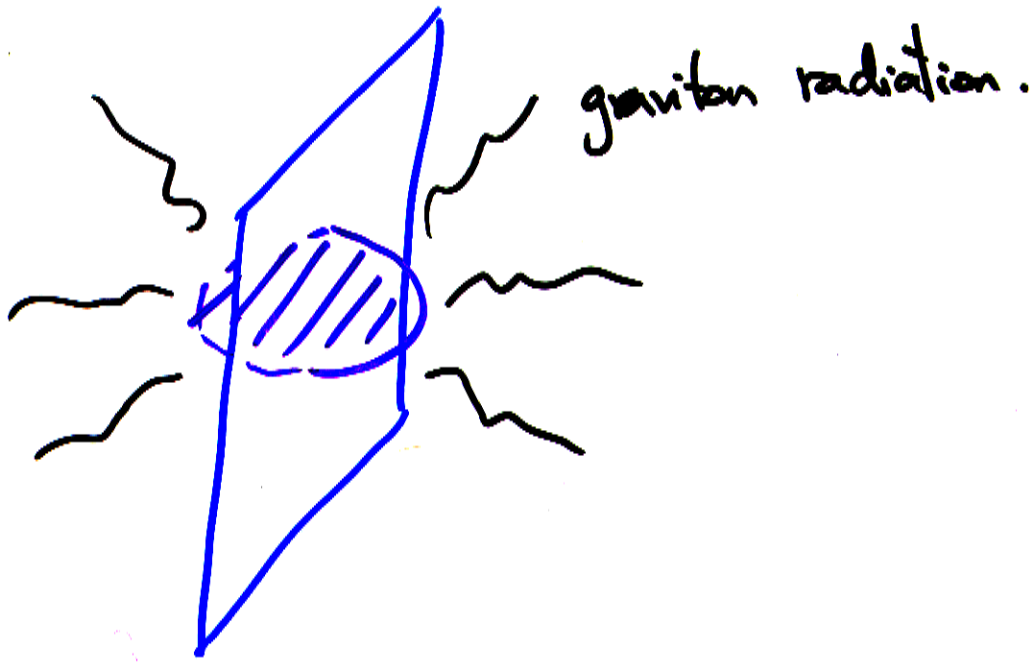
# Black-hole production at colliders

At high enough energies ( $\gtrsim M_{\text{Pl, fund.}}$ ), semi-classical black holes can be produced.



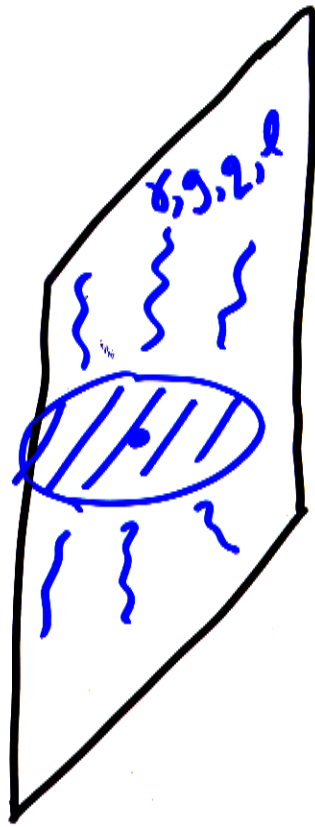
How does the BH decay?

. Naively: Hawking radiates into the bulk



Impossible to see this.

But...



BLACK HOLES RADIATE ON  
THE BRANE (Horowitz + Meyers).

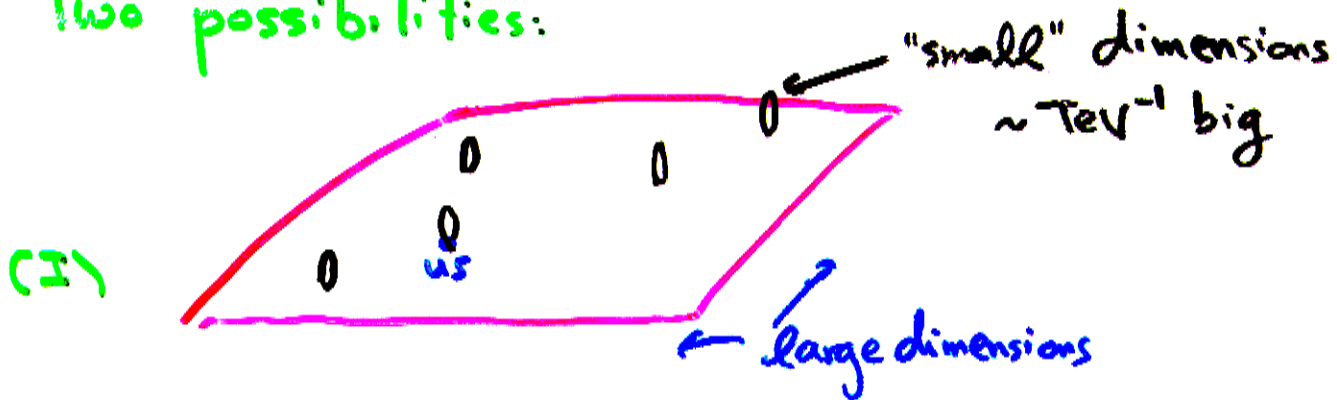
Signal:  $p\bar{p} \rightarrow \text{BH} \rightarrow$  Thermal Spectrum  
of SM particles  
with  $T \sim \text{TeV}$ .

No detailed studies have yet been done on  
this — in progress.

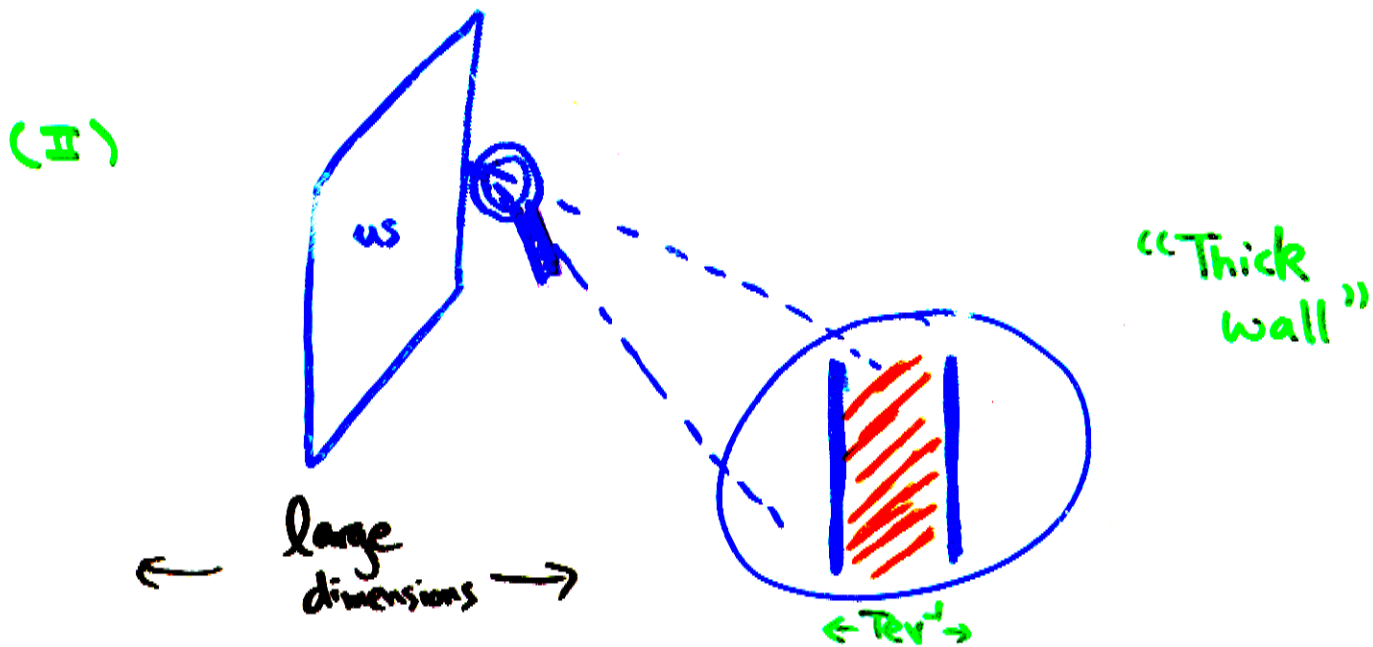


• In addition to these large dimensions, there may be "small" dimensions of size  $\sim \text{TeV}^{-1} \sim 10^{-17}$  cm where SM field can go:

Two possibilities:

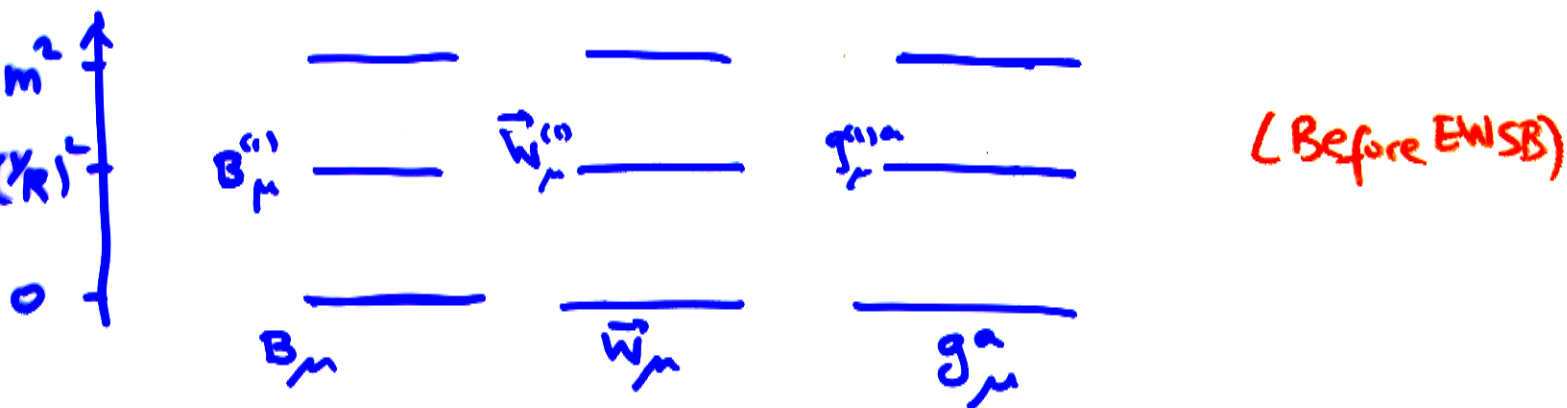


"Real  $\perp$  dimensions"



# • Kaluza-Klein Gauge Bosons

(Gauge fields propagating in some  $\sim \text{TeV}^{-1}$  size extra dimensions).



• Most conservatively,  $(1/R) \gtrsim 4 \text{ TeV}$  from precision electroweak constraints (sig. weaker with Fermions in the Bulk)

$\Rightarrow$  Can only produce first KK mode,

@ LHC. Looks like  $Z'$ . [How can you tell you actually have 2 degen. states  $\gamma, Z'$ ]

. Less conservatively, though, the only direct limit on  $1/R$  is from  $Z'$  searches, require  $1/R \gtrsim 800 \text{ GeV}$ .

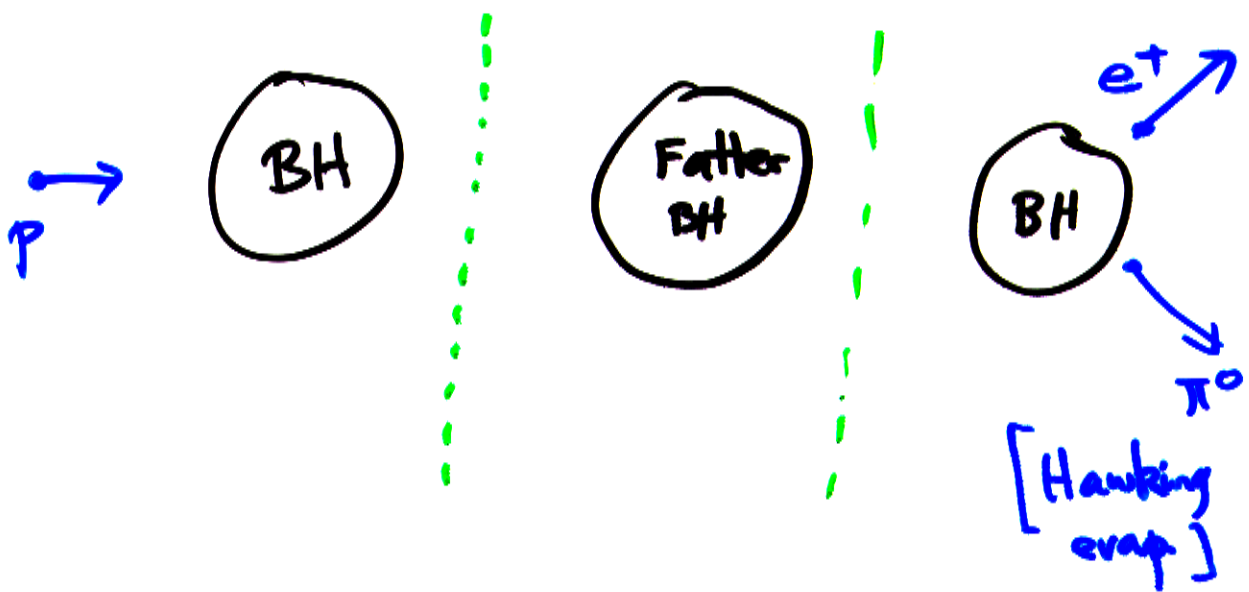
. In this case you produce whole tower of them @ LHC, and get to study the first one @ 1 TeV NLC, determining its couplings precisely and resolving the degeneracy.

. [Rizzo: even if  $1/R \gtrsim 4 \text{ TeV}$ , 1 TeV NLC may be able to resolve  $\gamma^{(1)}/Z^{(1)}$  degeneracy].

Quantum gravity + Proton Decay

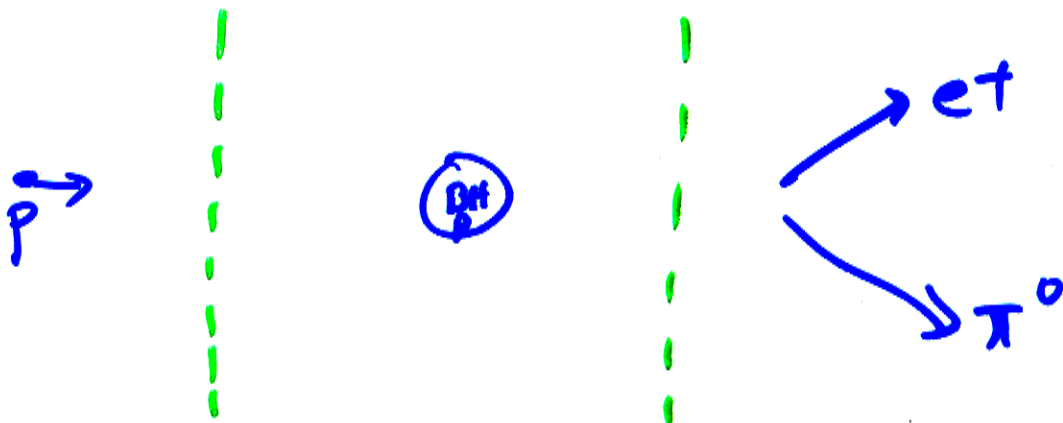
Usual lore: Black holes

violate all global symmetries



Lost Baryon #!

Even if you don't have a real  
BH, you get this from  
virtual BH



$$Amp \sim \frac{1}{M_{pl}^2} e^{-S_{BH}} \sim \frac{1}{M_{pl}^2} \text{ for } M_{pl}^{-1} \text{ size BH.}$$

For  $M_{pl} \sim 10^{19}$  GeV no problem.

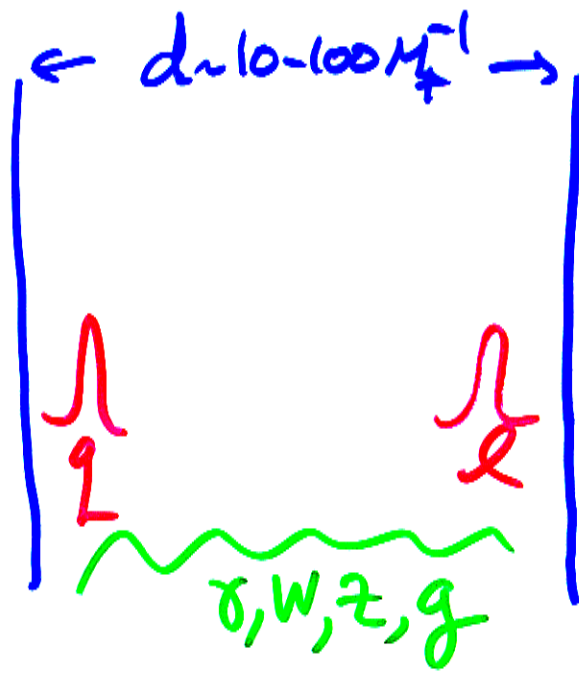
For  $M_{pl} \sim 10^3$  GeV huge disaster!

Naively kills idea (although  
it also kills SUSY GUT's ...)

... But of course there are many ways to solve this problem...

Most boring idea: as in SUSY, impose a discrete (gauge) symmetry....

... But extra dimensions allow us to understand proton stability without imposing symmetries but rather as a consequence of geography.



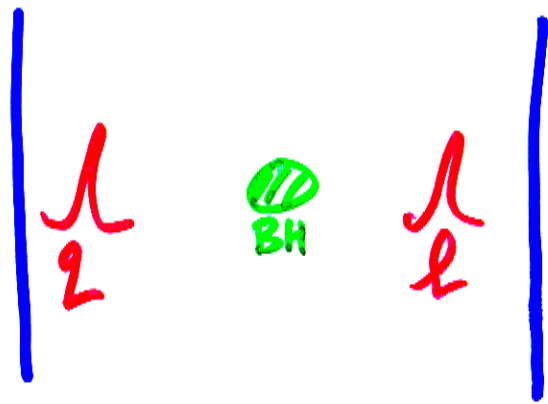
To decay proton, you need to

directly couple  $qql$ .

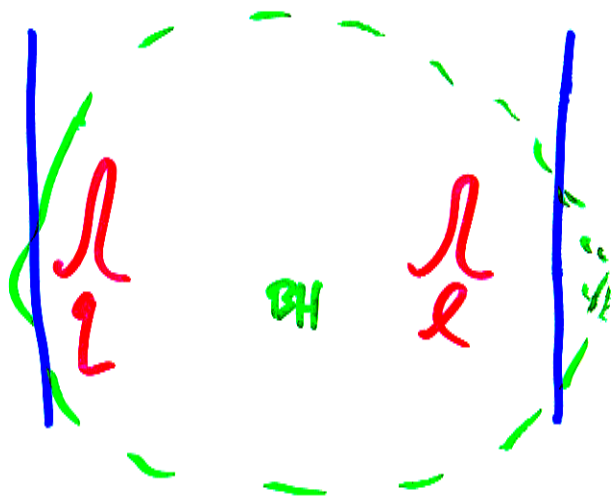
Exponentially suppressed!



# What about Black holes?



little BH  
can't eat  
3q's + l.



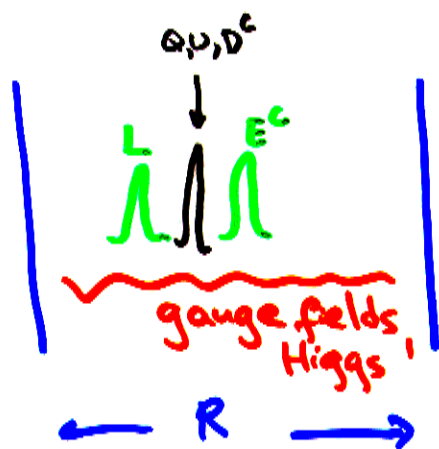
Bigger BH's can,  
but  $\Sigma_{BH}$   
 $\sim (M_{pl} d)^{2+n}$   
 $\sim 10^{2+n}$

$$\text{So, Amp} \sim \frac{1}{(\text{TeV})^2} e^{-10^{2+n}}$$

TOTALLY safe for any  $n$ .

- A scenario where you get qualitatively new information from the NLC:

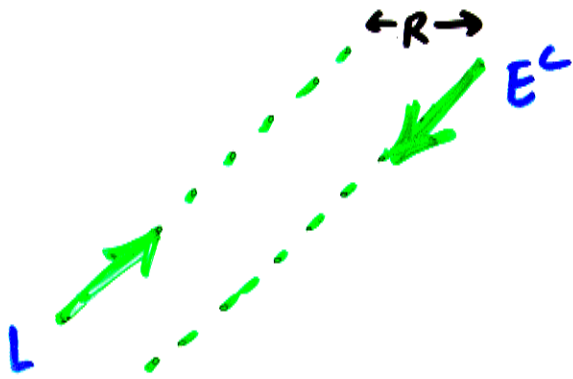
Split leptons in extra dimension.



- Small electron mass explained by tiny overlap of  $L, E^c$  wave functions (NAH + Schmaltz).
- LHC would discover KK gauge bosons, but would not be able to tell anything about  $L, E^c$  separation....

... But  $L/E^c$  separation could give

spectacular signal at NLC.

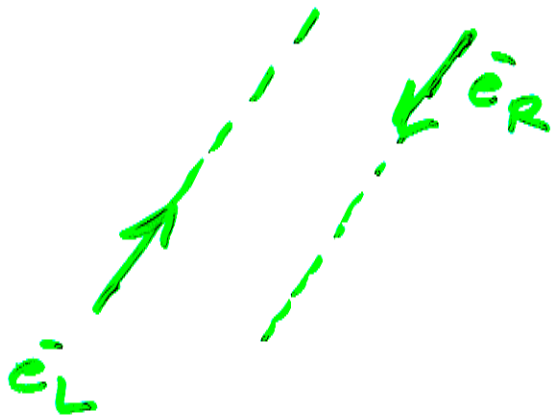


• At energies  $E \ll 1/R$ , normal scattering amplitude for  $L E^c$  scattering.

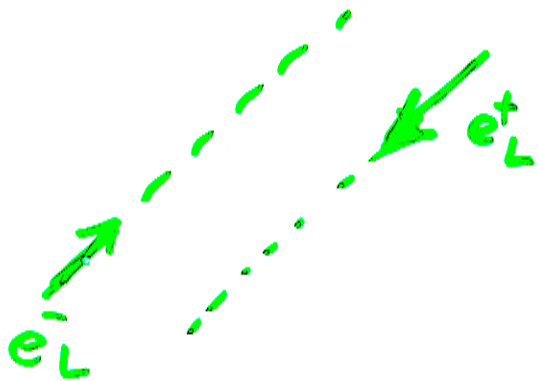
• But for energies  $E \gtrsim 1/R$ , they miss each other! Cross section vanishes exponentially

(NAT, Grossman, Schwalte). Note: this is not just the "new physics" contribution, the whole cross-section vanishes, (SM contribution is cancelled by KK modes).

e.g.  $\bar{e}_L \bar{e}_R \rightarrow \bar{e}_L \bar{e}_R$



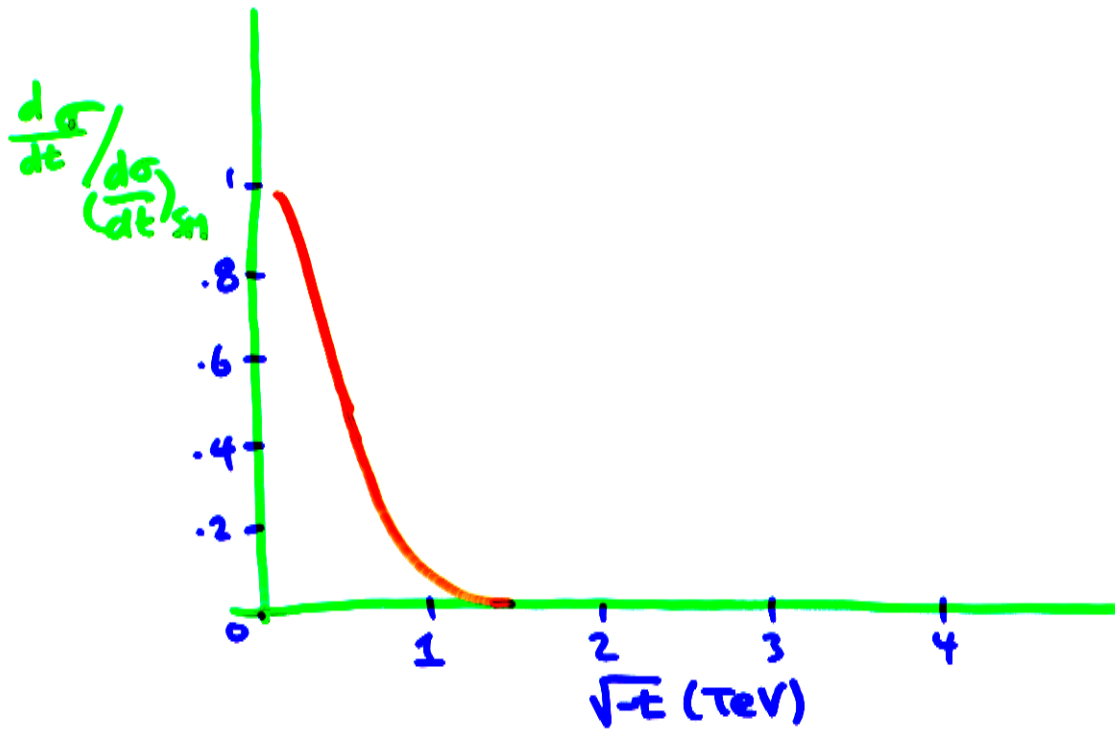
or  $\bar{e}_L e_L^+ \rightarrow \bar{e}_L e_L^+$



Cross-sections vanish exponentially.

[ All  $t$ -channel diagrams  
vanish exponentially as  $\sim e^{-\sqrt{-t}R}$  ]

$$\sigma(e_L^- e_L^+ \rightarrow e_L^- e_L^+)$$



$$\frac{1}{R} = 1 \text{ TeV}.$$

[Factor of  $\sim 10^3$  reduction @ 1.5 TeV NLC].

• Summarizing: For KK gauge bosons, LHC + NLC can complement each other very nicely. LHC has better reach for production, NLC can disentangle the states.

• NLC can further probe new physics in the lepton sector, totally inaccessible to LHC.

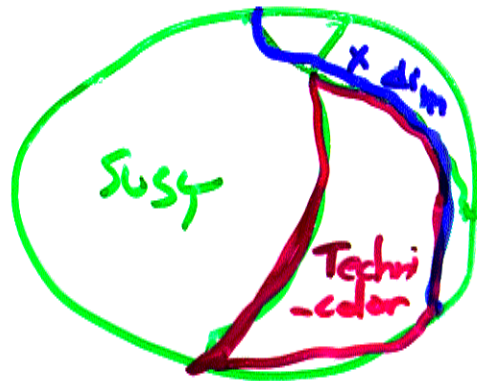
So, I hope to have convinced you that physics with accessible extra dimensions is theoretically well motivated + experimentally exciting.

LHC, LC would play complementary roles in disentangling the physics.

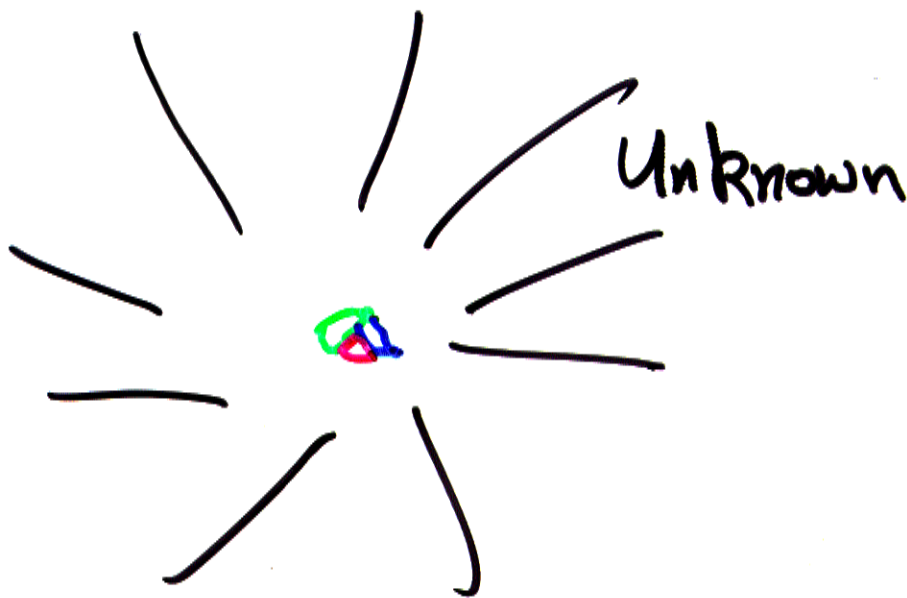
It also shows that, as much as we think we know about nature, what we don't know leaves room for possibilities we haven't dreamed of.



# Space of Possibilities for TeV scale physics



WRONG!



RIGHT!

Where is real world in this picture ???

- We need to be armed to the hilt to REALLY FIND OUT.
- We need both the LHC and the LC.