The Naturalness Principle, Measures of Fine-Tuning

& Expectations for a Linear Collider

Greg Anderson
Northwestern University

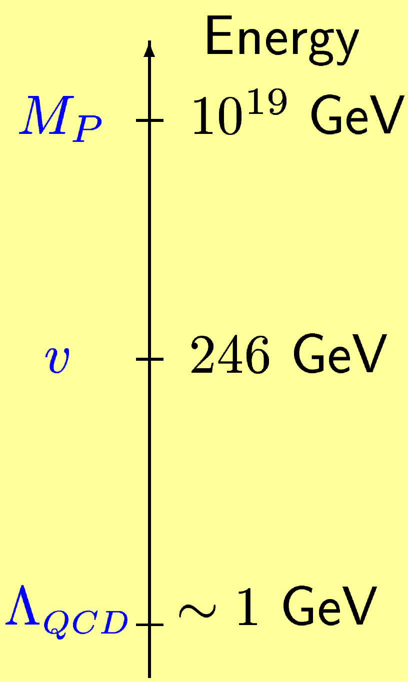
October, 2000

Outline

1. The Naturalness Principle
   - Fundamental Scales
   - Naturalness and hierarchy problems
   - Why scrutinize fine-tuning measures?
2. Naturalness Measures
   - Early attempts
   - The failure of the sensitivity parameter
   - The naturalness measure
   - Technical considerations
3. Naturalness and Superpartner Masses
   - Expectations for Sparticle Masses
   - Conclusions

Fundamental Energy Scales

We know of ≥ 3 fundamental scales in Nature.



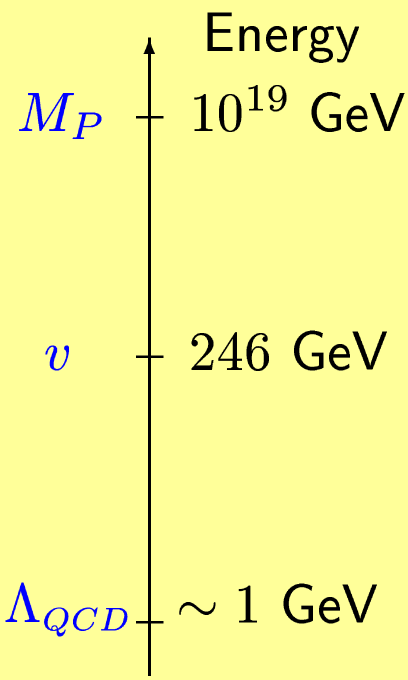
$$F_G = G_N \frac{M_1 M_2}{r^2}$$
$$G_N \sim 1/M_{Pl}^2$$

$$G_F \sim 1/v^2$$
$$M_W \sim g v$$

$$m_p \sim \Lambda_{QCD}$$
$$m_\pi^2 \sim \lambda v \Lambda_{QCD}$$

All of the masses of observed particles are related to the two fundamental scales v and Λ_{QCD} by dimensionless couplings.

Hierarchy & Naturalness Problems



Hierarchy Problem:
why is $v, \Lambda \ll M_P$

Naturalness Problem:
If we impose
$v, \Lambda \ll M_P$ is it stable?

Λ/M_P is both understood and natural.

$$1/g^2(\mu) \quad = \quad 1/g^2(M_P) + \frac{b}{(4\pi)^2} \ln M_P/\mu$$

$$\Lambda_{QCD} \quad \sim \quad M_P e^{-\frac{(4\pi)^2}{b g^2(M_P)}}$$

Symmetry: (broken) scale invariance

Much ado about nothing?

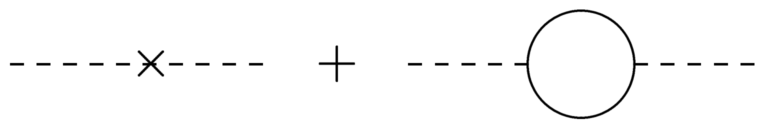
Why take the critique of naturalness measures and putative naturalness measures so seriously?

- ▷ Naturalness is the original and principle motivation for weak scale supersymmetry.
- ▷ The use of fine-tuning arguments to draw conclusions about the tenable mass scales associated with physics beyond the standard model is pervasive.
- ▷ In the absence of any direct evidence of an accessible scale beyond 246 GeV naturalness arguments can and should play an important role in shaping our expectations of what we may realistically expect to see at proposed future colliders.

The Naturalness Problem in the SM

Fundamental scalars are not naturally light.

- Assume the SM is valid up to some scale Λ.
- Compute the Higgs mass (or vev).



$$m_H^2 = m_{0H}^2 + g^2\Lambda^2 \quad \text{or}$$

$$v^2 = v_0^2 + g^2\Lambda^2$$

For $\Lambda \sim M_P$, $g \sim 1$

$$(246\ \text{GeV})^2 = -(\sim 10^{19}\ \text{GeV})^2 + g^2(10^{19}\text{GeV})^2$$

The tree-level and one-loop contributions need to cancel to on part in 10^{34}.

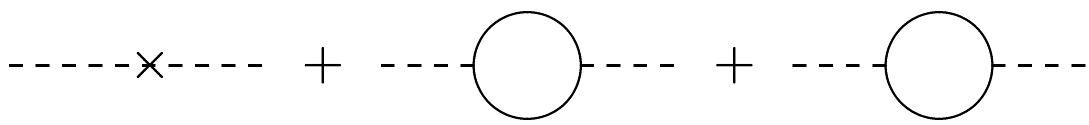
3 Solutions to the Naturalness Problem

Physics beyond the Standard Model can be classified according to their solution to the naturalness and or hierarchy problem.

1. Strong Dynamics
   - Scalars are not fundamental.
   - Origin of v is similar to the origin of Λ_{QCD}
2. Extra Dimensions
   - M_P is not what we think it is.
   - $M_P = M_F(M_F R)^{n/2}$
3. Supersymmetry
   - Scalars are fundamental
   - Supersymmetry Cancels Quadratic Divergences

Broken SUSY & Naturalness

- Supersymmetry must be broken.
- What happens to weak-scale naturalness?



$$\Delta m^2 = \tilde{m}^2 - m^2$$

$$v^2 = v_0^2 + g^2 \Delta m^2$$

For arbitrarily large superpartner masses, our understanding of weak-scale naturalness is lost.

Early Attempts to Quantify Naturalness

"For every complex problem, there is an answer that is short, simple and wrong." – H. L. Mencken

Early attempts to quantify naturalness provided a prescription which, while easy to impliment, is ultimately unsuccessful.

The sensitivity parameter:

Ellis, Enqvist, Nanopoulos, & Zwirner; Barbieri & Giudice

- y: "Observable" aka computed parameter.
- x: "Fundamental" aka Lagrangian parameter.

$$\frac{\delta y}{y} = c\frac{\delta x}{x}, \qquad c = \left|\frac{x}{y}\frac{\partial y}{\partial x}\right|$$

$$c \not\gg 1$$

Wilson as quoted by Susskind

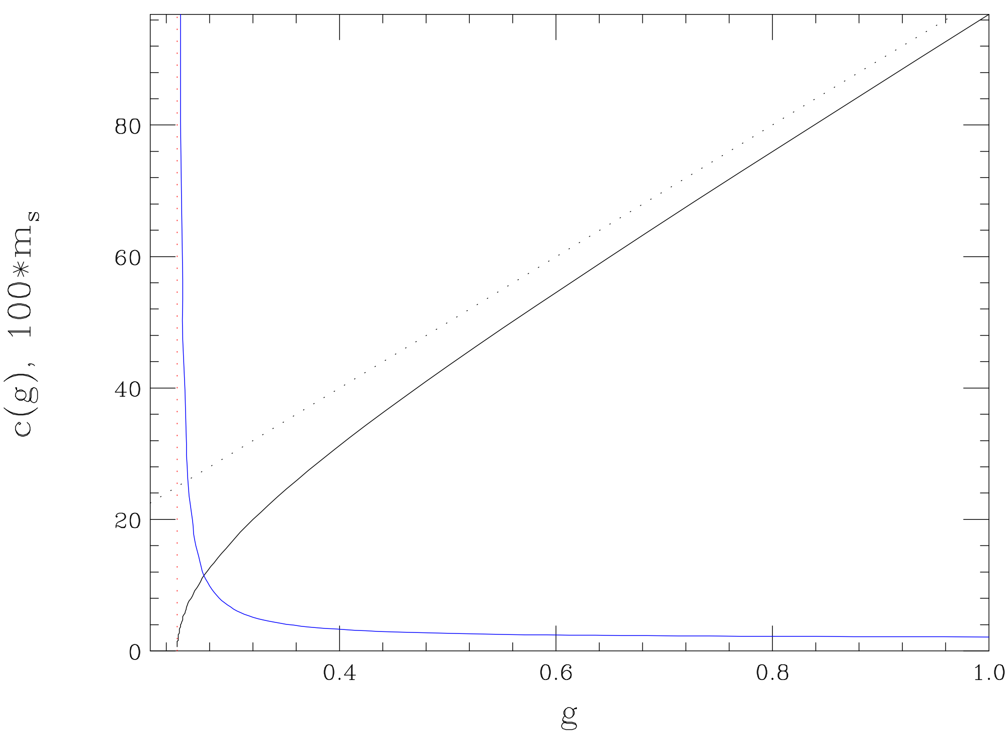
Observable properties of a system should not depend sensitively on variations in the fundamental parameters

Sensitivity and Simple Quadratic Divergence

For the simple quadratic divergence

$$m_S^2 = -m_0^2 + g^n \Lambda^2$$

$$c(g) = \frac{g}{m_s^2}\frac{\partial m_s^2}{\partial g} = n\left(\frac{g^n\Lambda^2}{g^n\Lambda^2 - m_0^2}\right) \to n$$



Sensitivity for a Simple Quadratic Divergence

The Failure of Sensitivity

Sensitivity as a criterion?

$$c(x) = \left| \frac{x}{y} \frac{\partial y}{\partial x} \right|, \qquad c \not\gg 1$$

The sensitivity prescription fails miserably for the only hierarchy problem involving fundamental scales which we truly understand:

$$\begin{aligned} m_p &\sim \Lambda_{QCD} \sim M_P e^{-\frac{(4\pi)^2}{bg^2(M_P)}} \\ C(g) &= \left(\frac{4\pi}{b}\right) \frac{1}{\alpha_s(M_P)} \gtrsim 100 \end{aligned}$$

Sensitivity $\neq$ Naturalness

The Naturalness Measure

D. Castaño
& G.A.

"Derive" from an assumed metric on parameter space encapsulating theoretical prejudice:

$$dP = f(x)\frac{\delta x}{x},$$

For a scale invariant distribution, $f(x)$ is constant. Relative likelihood that a physical parameter y lies within some percentage of y:

$$dP = f(x)\frac{\delta x}{x} = f(x)\rho(x)\frac{\delta y}{y},$$

where $\rho = \mid c^{-1} \mid$ is the relative probability of finding "y" in some particular scale-invariant interval.

$$\frac{\rho}{\langle\rho\rangle} \quad \text{where} \quad \langle\rho\rangle = \frac{\int \frac{dx}{x} f\rho}{\int \frac{dx}{x} f(x)}$$

Define a parameter which is large in the case of fine-tuning and order one otherwise.

$$\gamma = \frac{\langle\rho\rangle}{\rho} \equiv \frac{c}{\bar{c}}, \qquad \gamma \not\gg 1$$

where

$$1/\bar{c} \equiv \langle\rho\rangle = \frac{\int \frac{dx}{x} f(x) c^{-1}(x)}{\int \frac{dx}{x} f(x)}$$

NB $\int c^{-1}$ ensures $\bar{c}$ is dominated by the most natural, $i.e.$ smallest, values of c. [1]

Observable properties of a system should not be unusually sensitive to minute variations in the fundamental parameters.

D. Castaño
& G.A.

[1] A more careful treatment gives: $\gamma = MAX(\langle\rho\rangle)/\rho$, or $\bar{c}^{-1} = MAX(\langle\rho\rangle, 1)$.

Convergence Criteria

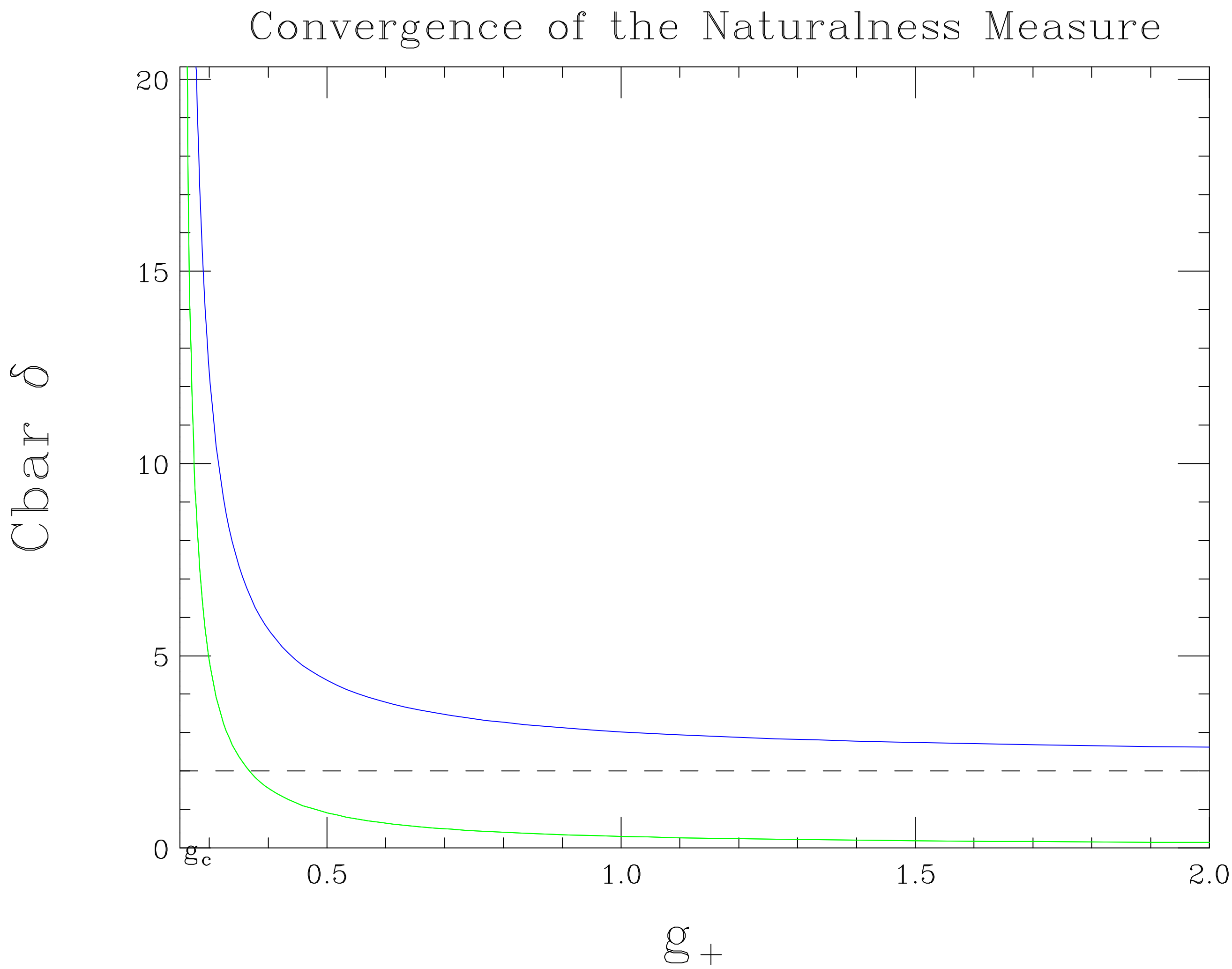
Naturalness: $\gamma \not\gg 1$,

$$\gamma = c/\bar{c}, \qquad \bar{c}^{-1} \equiv \frac{\int \frac{dx}{x} f(x) c^{-1}(x)}{\int \frac{dx}{x} f(x)}$$

Formally $\gamma \overset{x_+ \to x_-}{\longrightarrow} 1$. What constitutes a sufficient domain $\{x_-, x_+\}$?

$$\begin{aligned} \delta_\pm &\equiv \left| \frac{x_\pm}{\bar{c}} \frac{d\bar{c}}{dx_\pm} \right| = \left| \pm \hat{f}(x_\pm) \left(1 - \gamma_\pm^{-1}\right) \right| \\ \hat{f} &\equiv f(x) / \int \frac{dx}{x} f(x) \end{aligned}$$

Criteria: $\delta_- \leq \epsilon$ or $\delta_+ \leq \epsilon$ $\quad \epsilon = \mathcal{O}(1)$.



For the scalar quadratic divergence:

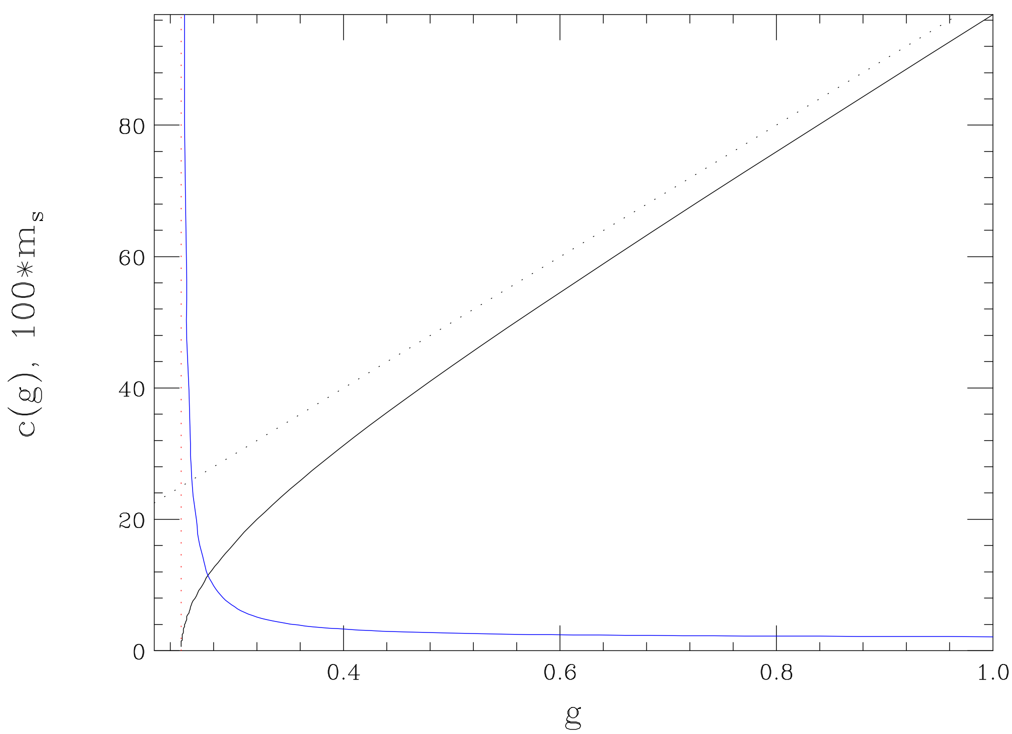
$$\bar{c} \to \frac{2}{1-\delta_+} \quad \text{for small} \quad \delta_+$$

Sensitivity and Simple Quadratic Divergence

The sensitivity parameter can be re-scaled by a global constant to provides an accurate measure of fine tuning in one and only one case:

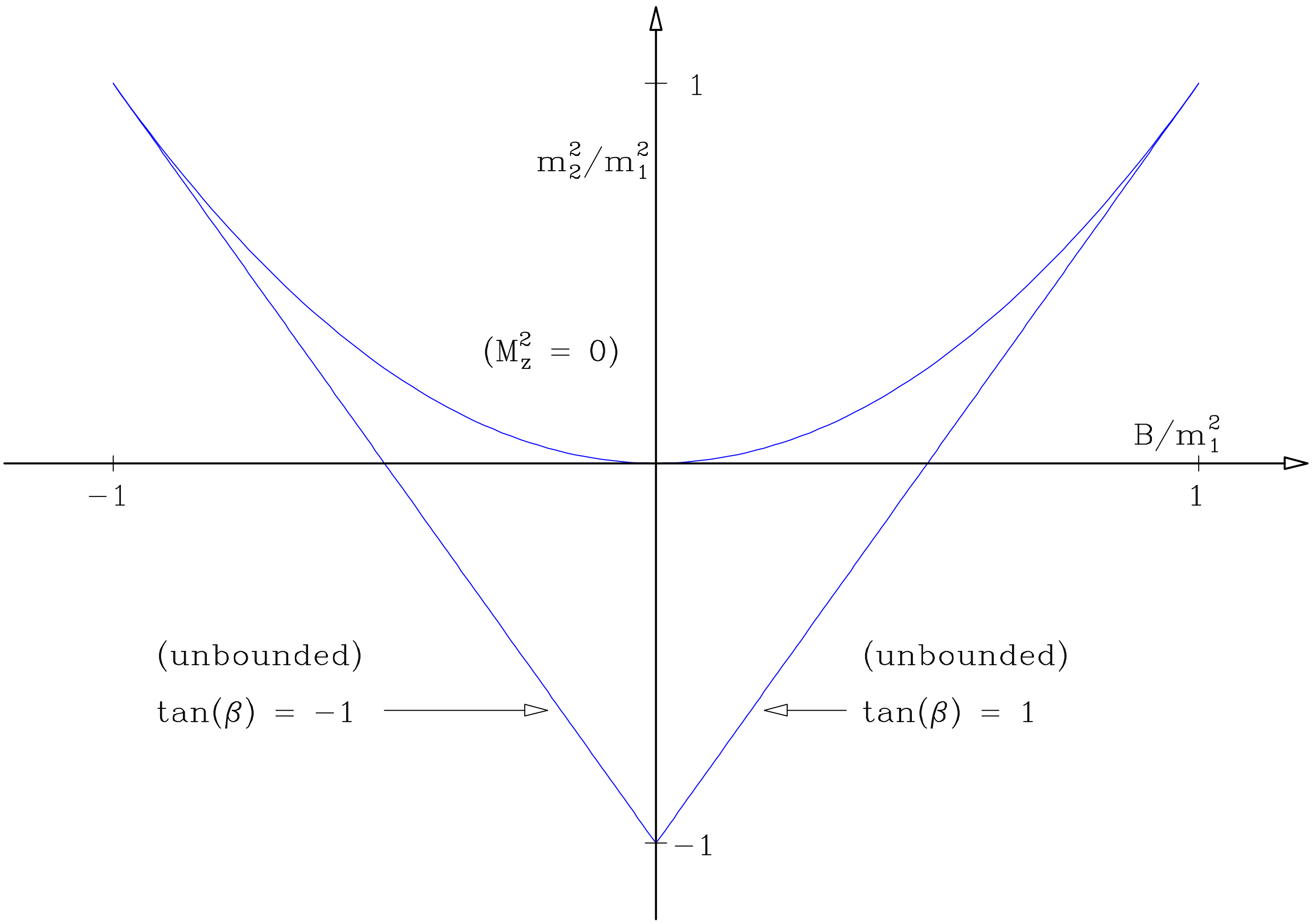
$$m_S^2 = -m_0^2 + g^n \Lambda^2$$

$$c(g) = \frac{g}{m_s^2}\frac{\partial m_s^2}{\partial g} = n\left(\frac{g^n\Lambda^2}{g^n\Lambda^2 - m_0^2}\right) \to n$$



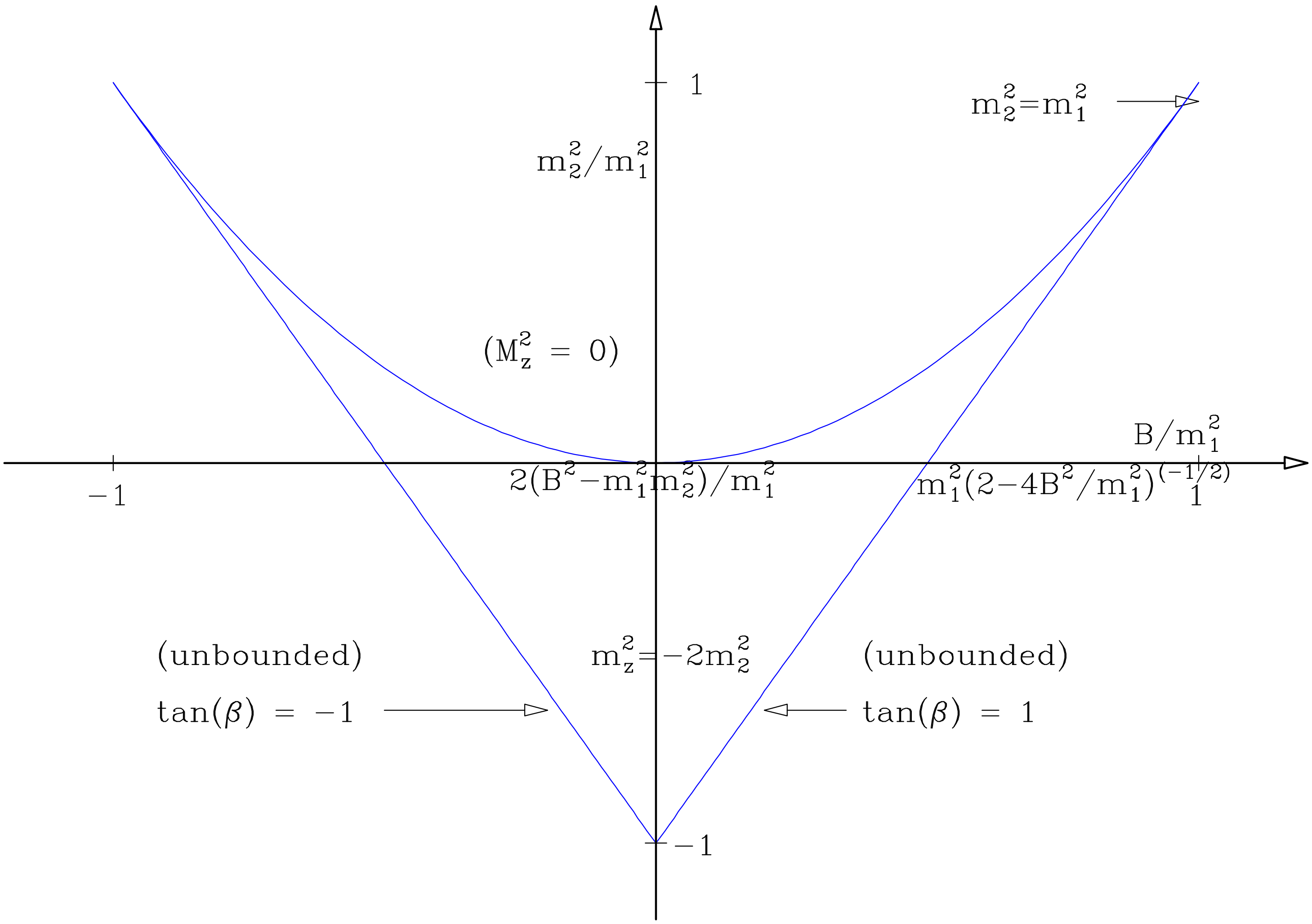
Sensitivity for a Simple Quadratic Divergence

Simple EWSB

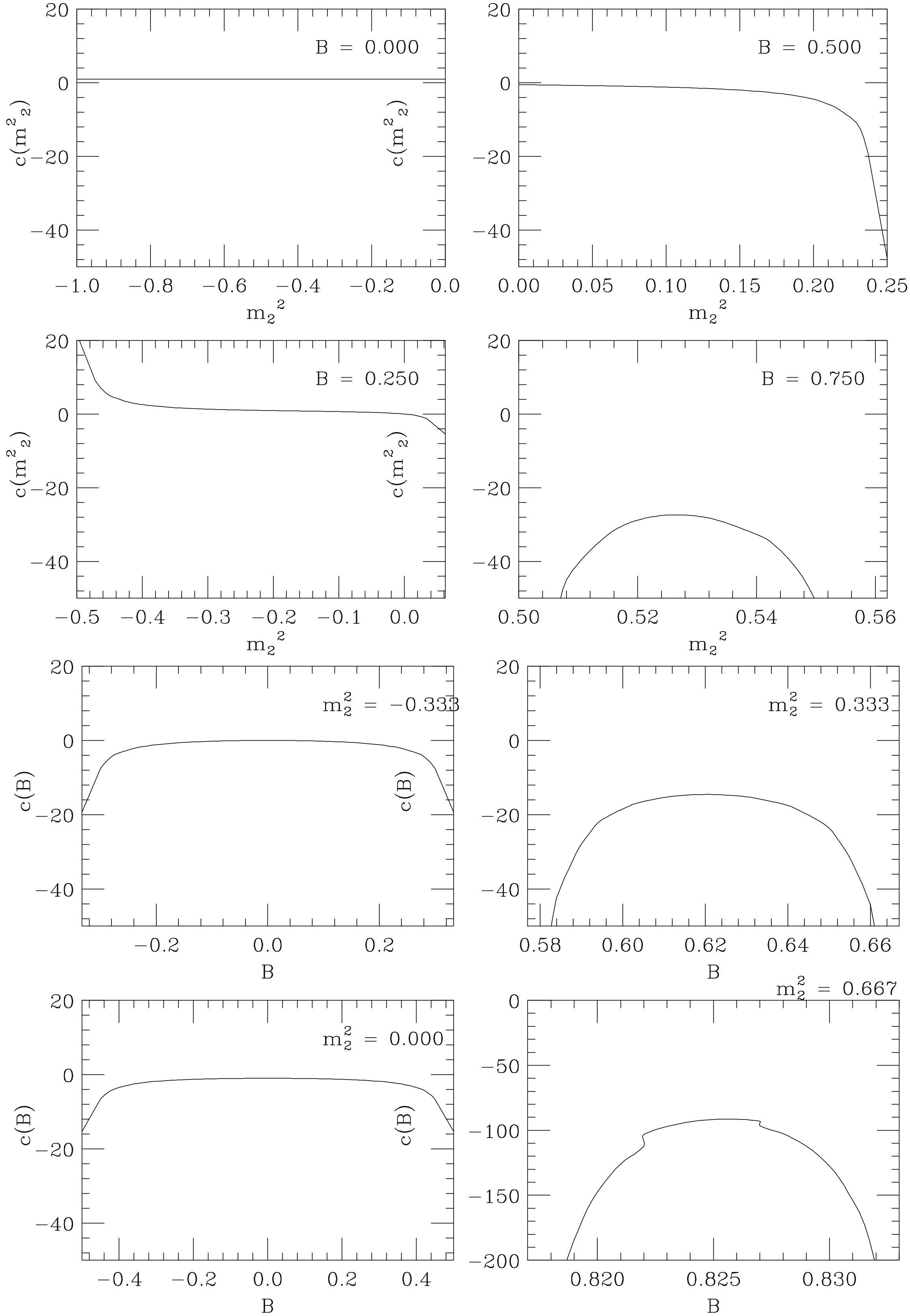


$$m_Z^2 = \frac{\left(m_1^4 - m_2^4\right)}{\sqrt{\left(m_1^2 + m_2^2\right)^2 - 4B^2}} - \left(m_1^2 + m_2^2\right)$$

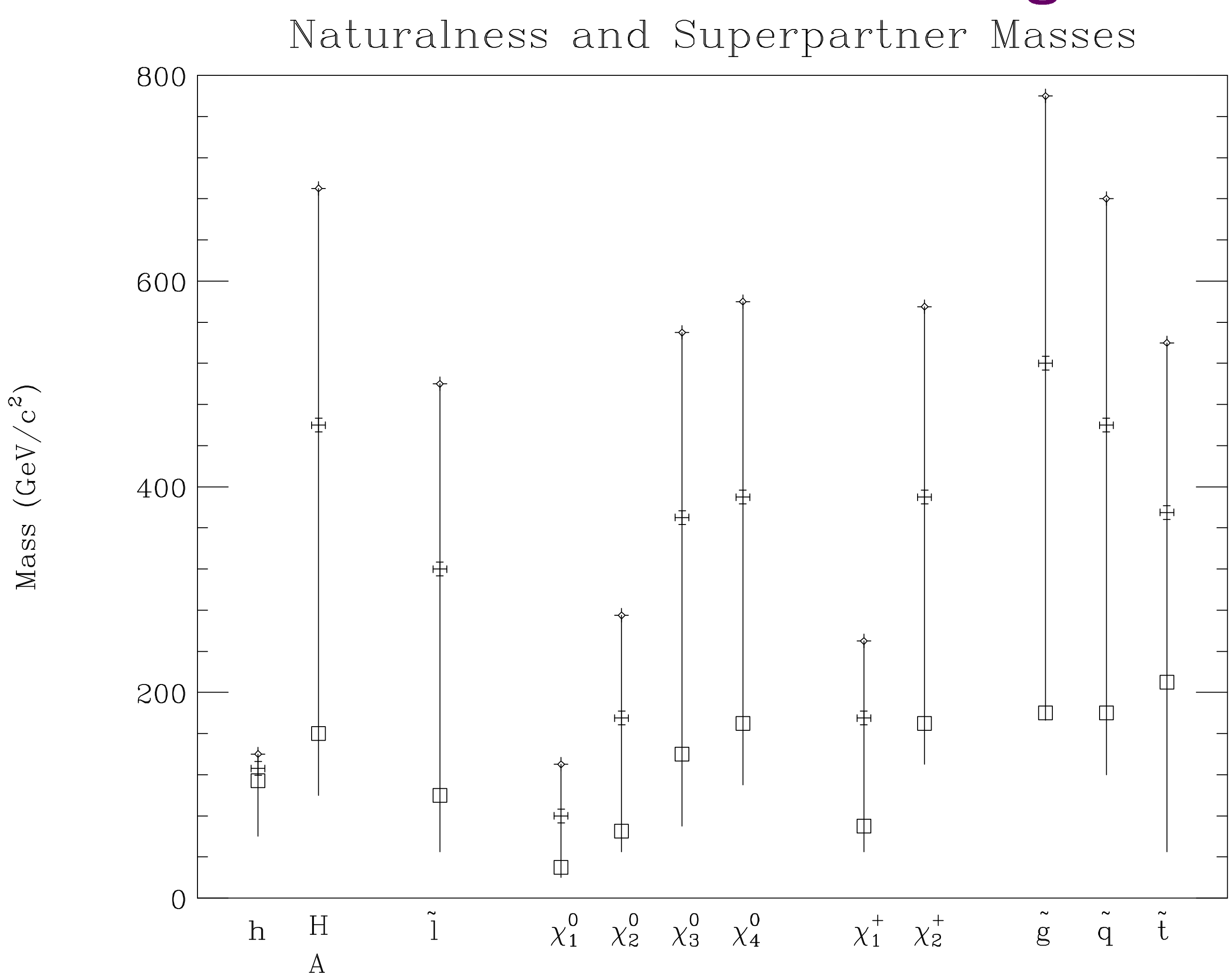
EWSB Regimes



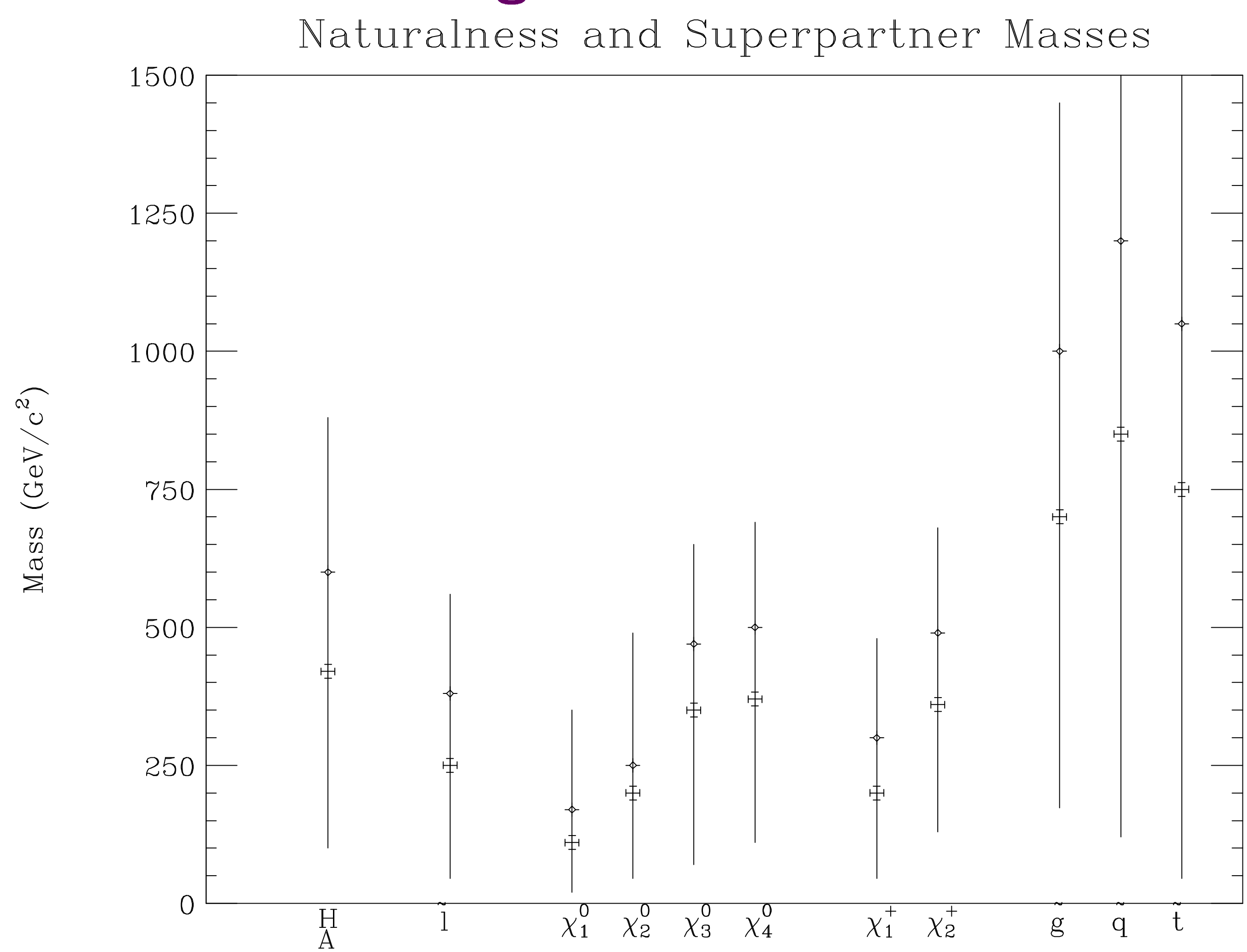
$$m_Z^2 \quad = \quad \frac{\left(m_1^4 - m_2^4\right)}{\sqrt{\left(m_1^2+m_2^2\right)^2 - 4B^2}} - \left(m_1^2 + m_2^2\right)$$


B = 0.000
B = 0.500
B = 0.250
B = 0.750
m2² = −0.333
m2² = 0.333
m2² = 0.000
m2² = 0.667
c(m2²)
m2²
c(B)
B

Numerical Results for Vanilla Sugra



Gauge Mediation



Conclusion

- Any claims about the expected values of sparticle masses based on unprincipled fine-tuning criteria should be treated with extreme skepticism. In most cases they should be disregarded.

- Contrary to claims in the literature, I find no evidence that third generation squarks in the multi-TeV region are compatible with the absence of fine-tuning.

- Supersymmetry (in whatever form) if relevant to the weak scale should provide a multitude of sparticles kinematically accessible at a 1 TeV NLC.

- Naturalness arguments do not guarantee that the entire spectrum of superpartners would be kinematically accessible at 1 TeV NLC.