EXTRA-DIMENSIONAL EWSB
AND FERMION MASSES

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– Research done in collaboration with David E. Kaplan *

Outline:

1. Hierarchy
2. Flavor
3. Experimental Constraints
4. Outlook

* JHEP 0006 020,2000, and more to appear soon!
The Hierarchy Problem

- As we all know, the hierarchy between $M_W$ and $M_P$ is puzzling
  - *Why* is the weak scale so much smaller than the Planck scale?
  - *How* is the hierarchy maintained in the face of radiative corrections?

- The ADD solution: There is NO Hierarchy
  - Gravity seems weak because it is “diluted” in $n$ extra dimensions

$$M_P^2 \sim R^n M_*^{n+2}$$
The Flavor Puzzle

- Our goal is to see if extra dimensions can also shed some light on flavor:
  - Large Hierarchy of Masses
    \[ M_u \ll M_c \ll M_t \]
    \[ M_d \ll M_s \ll M_b \]
  - Shouldn’t we expect all the quarks to have masses of order \( M_t \)?
  - Puzzling Pattern of Mixings
    \[ V_{us} \sim 1/5 \]
    \[ V_{cb}, V_{ub} \ll 1 \]
    \[ V_{us} \times V_{cb} \sim V_{ub} \]

- Can we solve the flavor puzzle and the hierarchy problem at the same time?

- Maybe a hybrid model with flavor from one extra dimension and gravity in even more? [J. Lykken & S. Nandi]
The Arkani-Hamed & Schmaltz Model

- Fermion zero-modes localized in an extra dimension
- $SU(2)$ singlets and doublets at different locales
- Higgs (VEV) distributed evenly in the bulk
- Gauge fields in the bulk
- Small overlap of the wave functions
  → small 4d Yukawa couplings
- Some fine-tuning for $M_t$
- Empty Matrices $\rightarrow \epsilon_K$?
Localizing Fermions

- 5d fermion $\Psi$ coupled to scalar $\Phi$

$$\bar{\Psi}(i\partial - \gamma^5\partial_5 + f\Phi)\Psi$$

- Notice a massless $\Psi$ is a 4-component field

- A non-trivial $\langle \Phi \rangle$ that crosses zero somewhere localizes a zero mode about that point:

- We can split fermions by adding masses:
Compactification

• In order for the theory to look 4d at low energies, we want to compactify the extra dimension

• For example, on a circle: $S^1$

• ...but this causes other problems:

  – Generally, 5d theories are not chiral
  – Each SM fermion has a mirror partner
  – The VEV $\langle \Phi \rangle$ is unstable

• We address these problems with orbifold boundary conditions
Orbifold

• For example, $S^1/\mathbb{Z}_2$:

$$\Psi(x, x_5) = \gamma_5 \Psi(x, -x_5)$$
$$\Psi(x, L + x_5) = \gamma_5 \Psi(x, L - x_5)$$
$$\Phi(x^\mu, -y) = -\Phi(x^\mu, y)$$
$$\Phi(x^\mu, L/2 + y) = -\Phi(x^\mu, -L/2 + y)$$

• This mods out the zero mass mirror partner:

• A bulk scalar potential $(\Phi^2 - u^2)^2$ clashes with the boundary conditions [H. Georgi, A. Grant, G. Hailu]

• Fermion mass terms forbidden by the orbifold
Odd Mass Term

- We can build the AS model with an orbifold by adding an “odd” mass:

\[ (x) = (x) + F \]

- For example, the odd mass can arise from another bulk scalar coupled to the fermions.

- The EWSB Higgs can be even or odd under the orbifold, and might have a non-trivial VEV as a result.
Flavor on an Orbifold I: Bulk Higgs

• Another flavor model does not invoke the odd mass but different \( \mathcal{O}(1) \) couplings to \( \Phi \)

• Zero modes go like \( 1/\cosh^{2f}[x_5/2] \)

• Doublets at \( x_5 = 0 \), singlets at \( x_5 = L/2 \):

• For \( M_*L \sim 20 \), the widths range from \( 1/5 \) to \( 3/2 \), and 5d Yukawas from \( 1/3 \) to \( 3 \)
Model II: Higgs on a Boundary

- Higgs (VEV) only at a boundary \((x_5 = 0)\)

- Yukawa suppression from overlap with “Higgs brane”

- For \(M_\ast L \sim 20\), this requires widths and 5d Yukawas between \(1/3\) and \(3\)

- No fine-tuning for \(M_t\)

- Automatically have: \(V_{us} \times V_{cb} \sim V_{ub}\)
Flavor-changing Neutral Currents

- The gauge boson zero modes couple universally to all of the fermions, but the higher modes do not:

  \[
  g_1, g_2
  \]

- After CKM rotation, this introduces flavor-violating interactions

  \[
  \left( \begin{array}{ccc} \bar{d} & \bar{s} & \bar{b} \end{array} \right) L_d^\dagger \left( \begin{array}{ccc} c_1^n & 0 & 0 \\ 0 & c_2^n & 0 \\ 0 & 0 & c_3^n \end{array} \right) L_d \left( \begin{array}{c} d \\ s \\ b \end{array} \right)
  \]

- The flavor-violating terms are proportional to rotation matrices (left- or right-handed) and \( \Delta c^n \)
$K^0 - \overline{K^0}$ Mixing

- For example, consider a contribution to $K^0 - \overline{K^0}$ mixing from KK gluons

[A. Delgado, A. Pomarol, M. Quiros]

\begin{equation}
\begin{aligned}
\text{s} & \quad \text{d} \\
\text{g}_n & \\
\text{d} & \quad \text{s}
\end{aligned}
\end{equation}

- In the SM these processes are weak loop effects

- The $\Delta S = 2$ effective Hamiltonian

\begin{equation}
H^{\Delta S=2} \sim \frac{\alpha_s}{M_c^2} \sum_{n=1}^{n^*} \frac{V^\dagger CV}{n^2}
\end{equation}

- KK mass scale $M_c = L^{-1}$

- The sum is cut off at $n^*$ (but results basically independent of $n^*$)
Limits on Models of Flavor

- By requiring the right order of magnitude for $\Delta m_K$ and $\epsilon_K$, we derive limits on $M_c$.

- In specific models, this is related to the fundamental scale $M_*$.

<table>
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<tr>
<th>Model</th>
<th>AS $\Delta m_K$</th>
<th>KT(I) $\Delta m_K$</th>
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<td>120 TeV</td>
<td>5 TeV</td>
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<tr>
<td></td>
<td>3000 TeV</td>
<td>80 TeV</td>
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| $L$   | $50/M_*$             | $10/M_*$                |

- One can obtain similar bounds from $\epsilon'_K/\epsilon_K$ (somewhat weaker than the $\epsilon_K$ ones).

- $M_* \sim (10^4 - 10^6) \times M_W$: Extra-dimensional flavor seems unlikely to coexist with a hierarchy solution.

- SUSY models?
What about the Higgs?

- If the Higgs field lives in the extra dimension, this may impact Higgs phenomenology

- Kaluza-Klein modes for the Higgs ($n$-Higgs doublet model) [J. Gunion]

- EWSB VEV could be “shared”:
  - Reduced $Z\cdot Z\cdot h$ ($W\cdot W\cdot h$) interaction strengths
  - Processes like $t\bar{t}h$ and $b\bar{b}h$ could become more important as discovery modes
  - Rare decays like $h \rightarrow \gamma\gamma$ could be enhanced (or suppressed)

- Some Higgs modes may have flavor-violating couplings like the gauge boson KK modes did
  - $h \rightarrow \tau\mu$? [T. Han]

- Some modes may couple strongly to *light* fermions
Outlook

- An extra dimension can provide interesting solutions to the flavor puzzle

- FCNC constraints are quite strong, and seem to disfavor a combined extra-dimensional solution of hierarchy and flavor

- Unusual Higgs phenomenology can result!