Precise Predictions for MSSM Higgs boson production at $e^+e^-$ colliders

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1. Introduction:

Higgs sector of the MSSM:

MSSM: Enlarged Higgs sector:

Two Higgs doublets

\[ H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \]

\[ H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \]

Higgs potential:

\[ V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \]

Physical states: \( h^0, H^0, A^0, H^\pm \)

Input parameters:

\[ \tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta) \]

Diagonalization of tree-level mass matrix

\[ \Rightarrow m_h, m_H, \text{ mixing angle } \alpha \]
Tree-level result for $m_h, m_H$:

$$m_{H,h}^2 = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2\cos^2 2\beta} \right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

Large radiative corrections:

Yukawa couplings: $\frac{e^{m_t}}{2M_Ws_w}, \frac{e^{m_t^2}}{M_Ws_w}, \ldots$

$\Rightarrow$ Dominant one-loop corrections: $G_{\mu}m_t^4 \ln \left( \frac{m_t^2}{m_{t1}m_{t2}} \right)$

$\tilde{t}$ sector:

$$M_t^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_{\tilde{t}}^2 + DT_1 & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_{\tilde{t}}^2 + DT_2 \end{pmatrix}$$

$X_t = A_t - \mu \cot \beta$; large mixing possible

Input parameters: $M_{\tilde{t}_L}, M_{\tilde{t}_R}, X_t$

often used: $M_{SUSY} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$

$\Rightarrow$ Physical parameters: $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$
At LC with $\sqrt{s} = 500 – 1000$ GeV:

→ Need prediction for Higgs production cross sections and branching ratios at 1% level

**Electroweak corrections at LEP ↔ LC:**

**At LEP1:**

Z-boson resonance ⇒ description in terms of universal corrections ($\Delta \alpha$, $\Delta \rho$, ...), effective couplings (propagator-type corrections)

⇒ Born-type structure

Genuine box contributions negligible
(Partially still true at LEP2 energies)

At LC with $\sqrt{s} = 500 – 1000$ GeV:

Propagator-type corrections no longer dominant

⇒ potentially large box corrections, Sudakov-logs, ...

**Finite width effects:**

→ need to include non-resonant contributions

→ $2 \rightarrow 4$, $2 \rightarrow 6$, ... processes
Remaining theoretical uncertainties in MSSM Higgs calculations:

→ From unknown higher-order corrections

Present estimate: $\Delta m_h^{\text{theo}} \approx 3$ GeV

→ From uncertainties in input parameters

$m_t, \ldots, M_A, \tan \beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \ldots$

Present situation:

$$\Delta m_t \approx 5 \text{ GeV} \Rightarrow \Delta m_h^{\text{theo}} \approx 5 \text{ GeV}$$

At LC:

$$\Delta m_t = 0.2 \text{ GeV} \Rightarrow \Delta m_h^{\text{theo}} \approx 0.2 \text{ GeV}$$

$$\Delta m_h^{\text{exp}} = 0.05 \text{ GeV}$$

⇒ High-precision measurement of $m_t$ important!
2. Higgs production at $e^+e^-$ colliders:

SM like: $e^+e^- \rightarrow Zh$

Complementary: $e^+e^- \rightarrow hA$

- Complete Feynman-diagrammatic one-loop result in on-shell scheme
  $\rightarrow$ box corrections included
  $\rightarrow$ full momentum dependence included
  [V. Driesen, W. Hollik, J. Rosiek '95]

- Dominant two-loop contributions $\mathcal{O}(G_\mu \alpha_s m_t^4)$
  incorporation via propagator corrections
  [S.H., W. Hollik, J. Rosiek, G. Weiglein '00] $\rightarrow$ FF
Diagrams for $e^+e^- \rightarrow hZ$:

\[ \begin{align*}
\text{Diagram a:} & & \text{Diagram b:} \\
\text{Diagram c:} & & \text{Diagram d:} \\
\text{Diagram e:} & & \text{Diagram f:} \\
\text{Diagram g:} & & \text{Diagram h: boxes} \\
\text{Diagram i:} & & \text{Diagram j:} \\
\end{align*} \]
Leading two-loop corrections:

$O(\alpha\alpha_s)$ contribution from $t - \bar{t}$-sector
(Yukawa contribution, $p^2 = 0$)

[S.H., W. Hollik, G. Weiglein '98]

1. $m_h$, $\sigma(e^+e^- \rightarrow hZ, hA)$, $\text{BR}(h \rightarrow ff)$
2. Complete result for $m_h$, $\sigma(e^+e^- \rightarrow hZ, hA)$, $\text{BR}(h \rightarrow ff)$
3. Included in: FeynHiggs, FeynHiggsXS

S. Heinemeyer, The Future of Higgs Physics, Fermilab, 05/03/01
Alternative approach:

**Renormalization group \( \alpha_{\text{eff}} \) approximation:**

Higgs mass matrix: (in \( \phi_1 - \phi_2 \) basis)
incl. dominant higher-order corrections:

\[
M_{\text{Higgs}}^2 = \begin{pmatrix}
m_{\phi_1}^2 & m_{\phi_1^2}^2 - \Sigma_{\phi_1}(0) & m_{\phi_2^2}^2 - \Sigma_{\phi_1\phi_2}(0) \\
m_{\phi_1^2}^2 - \Sigma_{\phi_1\phi_2}(0) & m_{\phi_2}^2 - \Sigma_{\phi_2}(0)
\end{pmatrix}
\]

\[
\downarrow \quad \text{Diag. } \alpha_{\text{eff}}
\]

\[
\begin{pmatrix}
M_{\text{H}}^2 & 0 \\
0 & M_{h}^2
\end{pmatrix}
\]

\( \Rightarrow M_h, M_H, \) mixing angle \( \alpha_{\text{eff}} \) at higher order

\( \alpha_{\text{eff}} \) contains dominant higher order corrections

\( \rightarrow \) production \( \rightarrow \sin(\beta - \alpha_{\text{eff}}), \cos(\beta - \alpha_{\text{eff}}) \)

\( \rightarrow \) decay

\( \Rightarrow \) Deviations from other approaches:

\(-\) different accuracies for \( \Sigma_s(0) \)
\(-\) momentum dependence
\(-\) vertex, box corrections
3. Numerical results

Comparison:

FD (1-loop, 2-loop) $\leftrightarrow$ RG-$\alpha_{\text{eff}}$ (2-loop)

[S.H., W. Hollik, J. Rosiek, G. Weiglein '01]

FD:

- Fortran code
  - including all FD corrections ($\rightarrow$ FeynHiggsXS)
    [V. Driesen, S.H., J. Rosiek '95/'01]

RG-$\alpha_{\text{eff}}$:

- Fortran code subhpole
  [M. Carena, M. Quiros, C. Wagner '95]

- $m_t(M_S)$ corrections included
  [M. Carena, H. Haber, S.H., W. Hollik, C. Wagner, G. Weiglein '00]

- On-shell $\overline{\text{MS}}$ transition ($M_{\text{SUSY}}, Xt$) included
  [M. Carena, H. Haber, S.H., W. Hollik, C. Wagner, G. Weiglein '00]

- $\rightarrow \alpha_{\text{eff}}$ approximation
$e^+e^- \rightarrow hZ, hA, \sqrt{s} = 500$ GeV:
$M_{\tilde{q}} = 1$ TeV, $M_{\tilde{t}} = 300$ GeV, $M_2 = \mu = 200$ GeV

$X_t = 2M_{\tilde{q}}, M_A = 200$ GeV

solid: FD 2L (with box), dot-dashed: FD 2L (no box)
dashed: FD 1L, dotted: RG 2L

**Diff. FD 2L – RG-α_{eff} result:** $\sigma_{Zh} \approx 10–15\%$
$\sigma_{Ah} \approx 25\%$

**Effect of 2L corrections:** $\sim 20\%$

**Effects of box contributions:** 5–10\% larger effects in differential cross sections
$e^+e^- \rightarrow hZ, hA, \sqrt{s} = 500 \text{ GeV}$

$M_{\tilde{q}} = 1 \text{ TeV}, M_{\tilde{t}} = 300 \text{ GeV}, M_2 = \mu = 200 \text{ GeV}$

$X_t = 0, \tan \beta = 5, 50$:

solid: FD 2L (with box), dot-dashed: FD 2L (no box)
dashed: FD 1L, dotted: RG 2L

$\Rightarrow$ Sizable deviations between FD and RG-$\alpha_{\text{eff}}$
result for large $M_A$
$e^+e^- \rightarrow hZ, hA$ as a function of $\sqrt{s}$:

$M_{\tilde{q}} = 1$ TeV, $M_{\tilde{\ell}} = 300$ GeV, $M_2 = \mu = 200$ GeV

$X_t = 0$, $M_A = 200$ GeV

solid: FD 2L (with box) , dot-dashed: FD 2L (no box)
dashed: FD 1L , dotted: RG 2L

$\Rightarrow$ Relative difference between FD and RG result grows with increasing $\sqrt{s}$
Code currently used for the LEP2 analysis: HZHA
[P. Janot '96-'00]

\[ e^+e^- \rightarrow hZ, \text{ comparison with HZHA: } M_{\tilde{q}} = 1 \text{ TeV}, \]
\[ M_{\tilde{t}} = 300 \text{ GeV, } M_2 = \mu = 200 \text{ GeV} \]
\[ X_t = 2M_{\tilde{q}}, \ M_A = 100, 200 \text{ GeV} \]
solid: FD 2L (with box), dashed: FD 2L (no box)
dotted: HZHA (FeynHiggs), dot-dashed: HZHA (subhpole)

\[ 
\Rightarrow \text{sizable deviations for large } M_A
\]
Outlook:

- Further comparison with HZHA
  → full effect of FD calculation

- Implementation of *FeynHiggsXS*
  into existing codes?
  → Pythia
  → HZHA

- leading two-loop (process specific) vertex corrections?
  ⇒ needed for 1% accuracy
4. Conclusions

• Calculation for Higgs production:
  \[ e^+ e^- \rightarrow hZ, hA \]

**Higgs propagator correction** combined with complete one-loop on-shell result
(→ vertices, boxes)

⇒ *FeynHiggsXS*

• Comparison of FD and \( \text{RG}^\alpha_{\text{eff}} \):
  
  \( \sigma_{Zh} : @ \sqrt{s} = 500 \text{ GeV} \)
  
  box corrections up to 7%
  two-loop corrections \( \sim 10\% \)
  FD larger than RG up to 15%

  \( \sigma_{Ah} : @ \sqrt{s} = 500 \text{ GeV} \)
  
  box contributions up to 10%
  two-loop corrections \( \sim 20\% \)
  FD larger than RG up to 25%

• **Improvement** factor of 10 needed in theoretical prediction to match the experimental precision