On modifying gravity as an alternative to dark matter

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Roadmap

• The problem with gravity.
• Preaching to the converted: Dark Matter.
• An alternative approach: Modified Gravity.
• How to build a theory.
• Isn’t this dark matter?
• Conclusions.
NGC 3198

Keplerian: $v_c \propto \frac{1}{R^{1/2}}$

Begeman 1987
\( \Delta^2 = \frac{k^3 P(k)}{(2\pi^2)} \)
Density Perturbations

\[ \delta_B = \frac{\rho_B - \bar{\rho}_B}{\bar{\rho}_B} \quad \delta_\gamma = \frac{\rho_\gamma - \bar{\rho}_\gamma}{\bar{\rho}_\gamma} \]

Three Regimes

- Tight coupling
  \[ \ddot{\delta}_B + 2\frac{\dot{a}}{a}\dot{\delta}_B - c_s^2 k^2 \delta_B = S(\dot{\Phi}, \ddot{\Phi}) \]
- Recombination
  \[ \delta_B \propto e^{-(k_s t)} \]
- Free Streaming
  \[ \delta_B \propto a \]

\[ \nabla^2 \Phi = 4\pi G(\rho_B + \rho_\gamma) \]
Preaching to the converted: Dark Matter

\[ \nabla^2 \Phi = 8\pi G (\rho + \rho_C) \]

If \( \rho_C \propto 1/r^2 \)

then \( v^2 = \frac{GM(R)}{R} \propto \frac{R}{R} \rightarrow \text{constant} \)

and \[ \ddot{\delta}_B + 2\frac{\dot{a}}{a} \dot{\delta}_B - c_s^2 k^2 \delta_B \neq 0 \]

through recombination
Simple and “predictive”

- Cross sections of order the weak scale, masses between Gev and Tev.
- Gravity may not be aligned with light.
- Globular clusters look different from Dwarf Galaxies.
The Bullet Cluster

Clowe et al, 2006
Birkhoff's theorem: solution unique

$N = 10^5$ Stars

Globular Cluster

Dwarf Galaxy (same external gravity)
Alternative solution: modified law of inertia...

\[ \mu \left( \frac{a}{a_0} \right) a = \frac{GM}{r^2} \]

\[ \mu(x) = \begin{cases} 
1 & x \gg 1 \\
x & x \ll 1 
\end{cases} \]

\[ \frac{v^4}{a_0} \frac{1}{r^2} = \frac{GM}{r^2} \]

\[ v^4 = a_0 GM \rightarrow \text{constant} \]

Milgrom, 83
... is a modified theory of gravity

\[ \mu \vec{a} = -\vec{\nabla} \Phi_N \quad \nabla^2 \Phi_N = 4\pi G\rho \]

Rewrite as:

\[ \vec{a} = -\vec{\nabla} \Phi \quad \vec{\nabla} \cdot \left[ f\left( \frac{\nabla \Phi}{a_0} \right) \vec{\nabla} \Phi \right] = 4\pi G\rho \]

We need a relativistic theory (a generally covariant action) if we want true predictions.
Hints of relativity

\[ a_0 \sim cH_0 \sim 10^{-8}\text{cm/s}^2 \]

All in the metric: \[ g_{00} \sim (1 + 2\Phi) \]
Simplest approach: modify the Einstein-Hilbert action

\[
\frac{1}{16\pi G} \int d^4x \sqrt{-g} R
\]

\[
\frac{1}{16\pi G} \int d^4x \sqrt{-g} F(R, R_{\mu\nu}, R_{\mu\nu\alpha\beta}, C_{\mu,\nu\alpha\beta}, \ldots)
\]

- Higher derivatives in the metric.
- Instabilities, ghosts (although not inevitable)
- New degrees of freedom (frozen modes become dynamical) - new fields.
Keep Einstein Hilbert action

Dynamics (geodesic equation)

\[ a^\mu + \Gamma^\mu_{\alpha \beta} u^\alpha u^\beta = 0 \]

Gravity (Einstein equations)

\[ G_{\mu \nu} = 8\pi G T_{\mu \nu} \]

Common metric
“Bimetric” theories

Use two different metrics

“Physical” in Geodesic equations

\[ g_{\mu\nu} = e^{-2\phi}(\tilde{g}_{\mu\nu} + A_\mu A_\nu) - e^{2\phi} A_\mu A_\nu \]

“Geometric” metric in Einstein equations

\[ \tilde{g}_{\alpha\beta} A^\alpha A^\beta = -1 \]

Bekenstein, 04

Time-like (“aether”)

Again! New degrees of freedom (new fields)
A theory (TeVeS)

\[ S = S_T + S_V + S_S + S_{\text{matt}} \]

\[ S_T = \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} R(\tilde{g}) \quad S_{\text{matt}} = \int d^4x \sqrt{-g} \mathcal{L}(g, \text{matter fields}) \]

\[ S_V = -\frac{1}{32\pi G} \int d^4x \sqrt{-\tilde{g}} [K_B F^{\alpha\beta} F_{\alpha\beta} - 2\lambda (A^\mu A_\nu + 1)] \]

Lagrange multipliers

\[ S_S = -\frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} \{\mu [(\tilde{g}^{\mu\nu} - A^\mu A^\nu) \phi,_{\mu} \phi,_{\nu}] + V(\mu; \mu_0, \ell_B)\} \]
Einstein-Hilbert, one metric and Aether field

- Rewrite action solely in terms of physical metric and vector field

\[ S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G_N} + \mathcal{L}(g, A) \right] + S_M \]

Extra degrees of freedom (the Aether)

Zlosnik et al
How do we modify gravity?

\[ G_{\alpha\beta} = 8\pi G T^{\text{matter}}_{\alpha\beta} + 8\pi G T^A_{\alpha\beta} \]

Take non-relativistic, weak field limit:

\[ g_{00} = -(1 + 2\Phi) \]
\[ A = (1 + \alpha_0, \bar{\alpha}) \]

\[ g_{\mu\nu} A^\mu A^\nu = -1 \rightarrow \Phi = -\alpha_0 \]

\[ G_{\alpha\beta}^{\text{mod}}(\Phi) \equiv G_{\alpha\beta}(\Phi) - 8\pi G T^A_{\alpha\beta}(\Phi) = 8\pi G T^{\text{matter}}_{\alpha\beta} + 8\pi G T^A_{\alpha\beta}(\bar{\alpha}) \]

Modified gravity

Extra d.o.f.
Extra degrees of freedom ...

Is modified gravity simply a more contrived form of Dark Matter?
Cosmology of Bekenstein’s theory

\[ H^2 = \frac{8\pi G_{\text{eff}}}{3} (\rho + \rho_\phi) \]

BBN:

\[ \Omega_\phi < 10^{-2} \]

Not Dark Matter

Skordis et al, 2006
Perturbations: definitions

Metric

\[ g_{00} = -a^2(1 + 2\Phi) \quad g_{ij} = a^2(1 - 2\Psi)\delta_{ij} \]

Scalar Field

\[ \phi = \phi + \varphi \]

Vector Field

\[ A = (1 + \alpha_0, \alpha) \]

\[ \Phi = -\alpha_0 \]

Extra d.o.f
Perturbations

Approximation: \[ \ddot{\alpha} + \frac{4}{\tau} \dot{\alpha} + \frac{2(1 + \epsilon)}{\tau^2} \alpha = S(\Phi, \Psi) \]

\( \alpha \propto \tau^p \)

with \[ p_{\pm} = -\frac{3}{2} \pm \frac{1}{2} \sqrt{9 - 8(1 + \epsilon)} \]

where \[ \epsilon \simeq -\frac{12 \ln(a/5 \times 10^{-5})}{\mu_0 K} \]

Dodelson and Liguori, 2006

\[ \nabla^2 \Phi = 4\pi G \delta \rho_B + 4\pi G \delta \rho_\alpha \]

\[ \nabla^2 (\Phi - \Psi) = 8\pi G \sigma_\alpha \]
Growth of $\alpha$ and $\Psi - \Phi$.
Need $m_\nu = 2.2$ eV to fit peaks ...

... but not necessary to fit large scale structure

Skordis et al

$\Omega_\phi \approx 10^{-3}$
A smoking gun for modified gravity?

Caldwell et al

\[ \Phi - \Psi \sim \omega \rho_{DE} \]

Cross correlate ISW and galaxies

\[ \Phi - \Psi \neq 0 \]

Cross correlate weak lensing and galaxies

Bertschinger

\[
C_{\ell}(\ell+1) = 3 \times 10^8
\]

Multipole \( \ell \)

\[
E_{\alpha}
\]

\( k \) (h/Mpc)
Main point:

Extra degrees of freedom in modified theories of gravity can play a significant role. This does not mean that they make a significant contribution to the overall energy density...
... and therefore:

Specific configurations (such as the "bullet cluster" or the discrepancy between globular clusters and dwarf galaxies) must be looked at in detail to see if extra degrees of freedom play a role.

PS: There may be a smoking gun ...