Simulations of dynamical friction including spatially-varying magnetic fields

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Motivation

• Parameters for RHIC II cooler are unprecedented
  – friction forces must be understood to within a factor of ~2

• There’s a need for high-fidelity simulations
  – We are using the VORPAL code
    • http://www-beams.colorado.edu/vorpal/

• Goals of the simulations
  – Resolve differences in theory, asymptotics, parametric models
    • Understand magnetization in limit of small Coulomb logarithm
  – Quantify the effect of magnetic field errors

• Numerical approach:
  – use $O(N^2)$ algorithm from astrophysical dynamics community
    • 4th-order predictor-corrector with aggressive variation of time step
    • accurately resolves close binary collisions
4th-Order Predictor/Corrector Hermite Algorithm

- Algorithm developed and used extensively by galactic dynamics community

- Predictor step:
  \[ v_{p,j} = \frac{1}{2} (t - t_j) \dot{a}_j + (t - t_j) a_j + v_j \]
  \[ x_{p,j} = \frac{1}{6} (t - t_j) \dot{a}_j + \frac{1}{2} (t - t_j) a_j + (t - t_j) v_j + x_j \]

- where

\[ a_i = \frac{q_i}{m_i} v_i \times B + \frac{q_i}{4\pi\varepsilon_0 m_i} \sum_j \frac{q_j r_{ij}}{(r_{ij}^2 + r_c^2)^{3/2}} \]
\[ \dot{a}_i = \frac{q_i}{m_i} a_i \times B + \frac{q_i}{4\pi\varepsilon_0 m_i} \sum_j q_j \left[ \frac{v_{ij}}{(r_{ij}^2 + r_c^2)^{3/2}} + \frac{3(v_{ij} \cdot r_{ij}) r_{ij}}{(r_{ij}^2 + r_c^2)^{5/2}} \right] \]

\[ r_{ij} = x_{p,j} - x_{p,i} \]
\[ v_{ij} = v_{p,j} - v_{p,i} \]
\[ r_c \geq 0 \quad \text{“cloud” radius} \]
Hermite Algorithm – including a Magnetic Field

- The corrector step:
  \[
  x_i(t_i + \Delta t_i) = x_{p,i} + \frac{1}{24} \Delta t_i^4 a^{(2)}_{0,i} + \frac{1}{120} \Delta t_i^5 a^{(3)}_{0,i}
  \]
  \[
  v_i(t_i + \Delta t_i) = v_{p,i} + \frac{1}{6} \Delta t_i^3 a^{(2)}_{0,i} + \frac{1}{24} \Delta t_i^4 a^{(3)}_{0,i}
  \]
  - where \(a^{(2)}_{0,i}\) and \(a^{(3)}_{0,i}\) are linear functions of \(a(t)\) and \(\dot{a}(t)\) evaluated at times \(t_i\) and \(t_i + \Delta t_i\)

- Adding B-field breaks 4\(^{th}\)-order scaling, unless
  - Lab-frame B is purely longitudinal, constant in time
  - \(v \times B\) force is evaluated again at the predicted positions
  - magnetic term in velocity correction (far right term above):
    - is split into self-field \(\mathbf{a}_{\text{self-field},i}\) & magnetic \(\mathbf{a}_{\text{magnetic},i}\) terms
    - the coefficient in front of \(\mathbf{a}_{\text{self-field},i}\) is changed from \(1/24\) to \(5/72\)
Initial Study of Magnetic Field Errors – Motivation

• The effect of magnetic field errors in a solenoid on the dynamical velocity drag (i.e. friction) of an ion in an electron cooler is not well understood
  – The parametric model of Parkhomchuk treats field errors as an effective transverse rms velocity of the electron Larmor circles
    • Contribution appears in same place as $V_{e,\text{rms},\parallel}$
    • In the absence of an explicit model, field errors have been treated as an effective increase in $V_{e,\text{rms},\parallel}$

• Our primary interest is the cooler for RHIC II
  – We consider the CELSIUS ring here, to take advantage of recent experiments
  – We consider two very different models for the errors
Magnetic field errors – “Model 1”

- A sum of sinusoidal terms (lab frame)

\[
B_x = \sum_i b_i \frac{k_{x,i}}{k_{z,i}} \exp(k_{x,i}x)\exp(k_{y,i}y)\sin(k_{z,i}z + \varphi_{z,i})
\]

\[
B_y = \sum_i b_i \frac{k_{y,i}}{k_{z,i}} \exp(k_{x,i}x)\exp(k_{y,i}y)\sin(k_{z,i}z + \varphi_{z,i})
\]

\[
B_z = B_0 + \sum_i b_i \exp(k_{x,i}x)\exp(k_{y,i}y)\cos(k_{z,i}z + \varphi_{z,i})
\]

\[
k_{z,i}^2 = k_{x,i}^2 + k_{y,i}^2 \quad \lambda_i = 2\pi/k_{z,i}
\]

- a more general form of the equations is allowed
- we assume \( b_i << B_0 \) for all \( i \)
- appropriate choices for \( b_i, \lambda_i \), etc. are not yet clear
- here, we consider a single component
Magnetic field errors – “Model 2”

- A sum of piece-wise constant “tilts” (lab frame)

\[
B_x = \sum_i b_{x,i} H(z - z_{x,i}) H(z_{x,i+1} - z)
\]

\[
B_y = \sum_i b_{y,i} H(z - z_{y,i}) H(z_{y,i+1} - z)
\]

\[
B_z = B_0
\]

- \( H(x) \) is the unit Heaviside function
- we assume \( b_{ix}, b_{iy} \ll B_0 \) for all \( i \)

- small abuse of Maxwell’s eqn.’s at discontinuities
- parameters taken from design report
  - amplitude of tilts (highly variable) is \(~1.e-03\)
  - length of segments (highly variable) is \(~20\) cm
Fields are Lorentz-transformed to beam frame

- VORPAL cooling sim.’s are in the beam frame

\[
B_z' = B_z(x', y', \gamma \beta c t')
\]

\[
B_x' = \gamma B_x(x', y', \gamma \beta c t') \quad B_y' = \gamma B_y(x', y', \gamma \beta c t')
\]

\[
E_x' = -\beta c B_y' \quad E_y' = \beta c B_x' \quad E_z' = 0
\]

- \(E\) fields are dominant for “relativistic” coolers
  - because electrons are non-relativistic in the beam frame
  - not strictly true for CELSIUS, for which \(\beta \sim 0.3\)
Basic Parameter set with 5 variations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_0$</td>
<td>solenoid field</td>
<td>0.1</td>
<td>T</td>
</tr>
<tr>
<td>$L_{\text{sol}}$</td>
<td>solenoid length</td>
<td>2.5</td>
<td>m</td>
</tr>
<tr>
<td>$\beta$</td>
<td>proton bunch velocity / c</td>
<td>0.308</td>
<td></td>
</tr>
<tr>
<td>$\tau_{\text{lab}}$</td>
<td>interaction time (lab frame)</td>
<td>2.7x10^{-8}</td>
<td>s</td>
</tr>
<tr>
<td>$\tau_{\text{beam}}$</td>
<td>interact. time (beam frame)</td>
<td>2.6x10^{-8}</td>
<td>s</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>largest time step</td>
<td>2.6x10^{-12}</td>
<td>s</td>
</tr>
<tr>
<td>$dt_{\text{min}}$</td>
<td>smallest time step</td>
<td>8.0x10^{-14}</td>
<td>s</td>
</tr>
<tr>
<td>$\omega_{\text{pe}}$</td>
<td>e- plasma frequency</td>
<td>4.1x10^8</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\Omega_{\text{L}}$</td>
<td>e- Larmor frequency</td>
<td>1.8x10^{10}</td>
<td>rad/s</td>
</tr>
<tr>
<td>$r_{\text{L}}$</td>
<td>e- Larmor (gyro-) radius</td>
<td>7.9x10^{-6}</td>
<td>m</td>
</tr>
<tr>
<td>$L_{x,y,z}$</td>
<td>sim. domain dimensions</td>
<td>6.0x10^{-4}</td>
<td>m</td>
</tr>
<tr>
<td>$n_e$</td>
<td>e- number density</td>
<td>5.4x10^{13}</td>
<td>m^{-3}</td>
</tr>
<tr>
<td>$N_e$</td>
<td># of simulated e-’s</td>
<td>1.2x10^{3}</td>
<td></td>
</tr>
<tr>
<td>$\Delta e_{\perp}$</td>
<td>transverse rms e- velocity</td>
<td>1.4x10^{-5}</td>
<td>m/s</td>
</tr>
<tr>
<td>$\Delta e_{</td>
<td></td>
<td>}$</td>
<td>long. rms e- velocity</td>
</tr>
<tr>
<td>$\Delta e_{\text{eff,</td>
<td></td>
<td>}}$</td>
<td>effective long. rms e- vel.</td>
</tr>
</tbody>
</table>

We consider 5 separate cases – 2 with field errors & 3 without

\[ V_{\text{ion,||}} = 10,000 \text{ m/s} \]
\[ V_{\text{ion,\perp}} = 10,000 \text{ m/s} \]

“cld” – $\Delta e_{||} = 3,000$ (no errors)
“wrm”– $\Delta e_{||} = 9,000$ (no errors)
“hot” – $\Delta e_{||} = 18,000$ (no errors)

“sin” – $\Delta e_{||} = 3,000$ (Model 1)
“err” – $\Delta e_{||} = 3,000$ (Model 2)
“cld” parameters – $\Delta_{\text{rms},||} = 3000$ (no field errors)
Diffusive dynamics can obscure friction/drag

- For a single pass through the cooler
  - Diffusive velocity kicks are larger than velocity drag
  - Consistent with theory
- For sufficiently large $\Delta e,||$
  - Numerical trick of e-/e+ pairs can suppress diffusion
  - Not valid for CELSIUS parameters
- Only remaining tactic is to generate 100’s of trajectories
  - Central Limit Theorem states that mean velocity drag is drawn from a Gaussian distribution, with rms reduced by $N_{\text{traj}}^{1/2}$ as compared to the rms spread of the original distribution
  - Hence, error bars are +/- 1 rms / $N_{\text{traj}}^{1/2}$
- Not practical to routinely generate 100’s or 1000’s of trajectories “by hand”
  - Run 8 trajectories simultaneously
  - Use “task farming” approach to automate many runs
“cld” parameters – $\Delta_{\text{rms},||} = 3000$ (no field errors)
Error models yield similar results –

- Longitudinal velocity drag is significantly reduced
  - in agreement with parametric increase of $\Delta_{\text{rms,||}}$
- Transverse velocity drag is less affected
  - NOT consistent with parametric increase of $\Delta_{\text{rms,||}}$
Wiggler approach to RHIC cooler – Motivation

• Why look for alternatives to solenoid design?
  – solenoid design & beam requirements are challenging
  – accelerator physics group of the RHIC electron cooling project is now considering a wiggler-based approach

• Advantages of a wiggler
  – provides focusing & suppresses recombination
    • Modest fields (~10 Gauss) effectively reduce recombination via ‘wiggle’ motion of electrons:
      \[
      \rho_w = \frac{\Omega_{gyro}}{v_{beam}} \sim 1.4 \times 10^{-3} \lambda^2_w [m] B_w [G] / \gamma
      \]
    – e- bunch is easier: less charge and un-magnetized

• What’s the effect of ‘wiggle’ motion on cooling?
  – increase minimum impact parameter of Coulomb logarithm: \[ \rho_{min} \geq \rho_w \] …needs to be simulated
Unmagnetized simulations for “wiggler” param.’s
Wiggler fields in the beam frame

- Tests in absence of wiggler field look promising
  - unmagnetized dynamical friction agrees with theory
  - numerical e-/e+ trick suppresses diffusion by 4x

- Wiggler field has following form (lab frame)

\[
\begin{align*}
B_x &= B_w \cosh(k_w x) \cos(k_w z) \\
B_y &= B_w \cosh(k_w y) \sin(k_w z) \\
B_z &= B_w \sinh(k_w y) \cos(k_w z) - B_w \sinh(k_w x) \sin(k_w z)
\end{align*}
\]

\[
k_w = \frac{2\pi}{z_w}
\]

- Lorentz transformation to beam frame
  - yields circularly-polarized EM wave
  - relatively strong, rapidly-oscillating external fields
    - not well-suited for Hermite algorithm; need operator splitting
Operator Splitting Approach

• Numerical technique used for ODE’s & PDE’s
• Consider Lorentz force equations

\[
\begin{align*}
\dot{x} &= v \\
\dot{v} &= \frac{q}{m} E_{\text{Coulomb}} + \frac{q}{m} (E_{\text{ext}} + v \times B_{\text{ext}})
\end{align*}
\]

• Robust 2\textsuperscript{nd}-order ‘Boris’ uses operator splitting

\[
\Delta x(\Delta t/2), \Delta v_E(\Delta t/2), \Delta v_B(\Delta t), \Delta v_E(\Delta t/2), \Delta x(\Delta t/2)
\]

• Add external \(E, B\) fields via operator splitting
  – Hermite algorithm: drift + coulomb fields
  – Boris ‘kick’: all external \(E, B\) fields

• Benchmark w/ pure Hermite alg. for constant \(B_{\parallel}\)
Operator splitting is implemented for ‘reduced’ model

- Use analytical two-body theory for ion/e- pairs
  - handle each pair separately in center-of-mass frame
  - calculate initial orbit parameters in relevant plane
  - advance dynamics for a fixed time step
    - electron’s new position and velocity are known
    - changes to ion position/velocity are small perturbations
  - total ion shift is sum of individual changes
- Initial algorithm is in place and partially tested
  - Speed and stability need to be improved
  - Initial comparisons with Hermite algorithm look good
  - Value of operator-splitting approach is verified
    - Hermite implementation in VORPAL will be modified so it can also be used with operator splitting
Semi-analytic ‘Reduced’ Model for Binary Collisions

- Must find the plane in which partial orbit occurs
  - necessary rotations (yaw, pitch, roll) are complete
  - transformations are messy, but straightforward
  - “initial” positions & velocities obtained in this plane
- Then standard orbital parameters are calculated

\[ \bar{x}^0 = (x, y, z) \]

\[ R\bar{x}^0 = \begin{pmatrix} |\bar{x}^0| \\ 0 \\ \pm \end{pmatrix} \]

\[ R^T(\Delta x) \]
Hermite & Reduced Model agree well for B=0

Hermite Algorithm

Reduced Model

Simulating dynamical friction...
Hermite algorithm & Reduced Model compare well for RHIC parameters w/ 5 Tesla B-field

- Agreement validates reduced model & operator splitting approach

Results for initial ion speeds: Vx=0.0 m/s; Vz=3.0E+05 m/s; 800 trajectories

<table>
<thead>
<tr>
<th></th>
<th>Hermite</th>
<th>Binary Collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;delta_Vz_ion&gt; [m/s]</td>
<td>-0.067</td>
<td>-0.066</td>
</tr>
<tr>
<td>dVz_rms/sqrt(# traj)</td>
<td>0.007</td>
<td>0.005</td>
</tr>
<tr>
<td>Time steps / TSPG</td>
<td>3978 / 70</td>
<td>576 / 10</td>
</tr>
<tr>
<td>Processor - min/run</td>
<td>145</td>
<td>25</td>
</tr>
</tbody>
</table>

Results for initial ion speeds: Vx=2.83E+05 m/s; Vz=1.0E+05 m/s; 800 trajectories

<table>
<thead>
<tr>
<th></th>
<th>Hermite</th>
<th>Binary Collision</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;delta_Vx_ion&gt; [m/s]</td>
<td>-0.033</td>
<td>-0.043</td>
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<tr>
<td>dVx_rms/sqrt(# traj)</td>
<td>0.008</td>
<td>0.003</td>
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<tr>
<td>&lt;delta_Vz_ion&gt; [m/s]</td>
<td>-0.068</td>
<td>-0.062</td>
</tr>
<tr>
<td>dVz_rms/sqrt(# traj)</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>Time steps / TSPG</td>
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<td>1017 / 20</td>
</tr>
<tr>
<td>Processor - min/run</td>
<td>144</td>
<td>44</td>
</tr>
</tbody>
</table>

* TSPG = Time steps per gyroperiod
Acknowledgements

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