Applications of Schottky Spectroscopy at the Storage Ring ESR of GSI

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Galena, IL
September 20, 2005
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Single Particle Signal

\[ j(x, y, t) = Q e \sum_{i=-\infty}^{\infty} \delta(x - x_n) \delta(y - y_n) \delta(t - nT - t_0) \]

\[ U(t) = \frac{S(x, y, t)}{2} \ast j(x, y, t) \]

Spectrum depends very much on the \( x \) and \( y \) dependence of \( S \)!
Pick-up Response Models

- Longitudinal Pick-up: $S(s, y, \Omega) = S^L(\Omega)$
- Horizontal Pick-up: $S(s, y, \Omega) = xS'_x(\Omega)$
- Vertical Pick-up: $S(s, y, \Omega) = yS'_y(\Omega)$
Voltage Spectral Density

**U(t):** Voltage from Schottky probe with Fourier transform \( U(\Omega) \)

First definition of voltage spectral density of a stationary process

\[
\langle U(\Omega)U(\Omega') \rangle = C_U(\Omega)\delta(\Omega + \Omega')
\]

or

Define \( C_U(\Omega) \) as the Fourier transform of the autocorrelation function

\[
R_U(\tau) = \langle U(t)U(t+\tau) \rangle
\]

Both definitions are equivalent (Wiener-Khinchine)

True expectation value can only be estimated from single measurements
Longitudinal Schottky Spectral Density

- Longitudinal harmonics
- good for heavy ions
- good for small rings
- good for cool beams

\[ C_U(m\omega) = \frac{(Z_L Q e)^2 \omega}{8\pi |m\eta|} \left| S^L(m\omega) \right|^2 \psi \left( \frac{\delta p}{p} \right) \]

Momentum distribution function!
Transverse Schottky Spectral Density

\[ C_U \left( \left[ m \pm Q_{x,y} \right] \omega \right) = \frac{(Z_L Qe)^2 \omega}{32\pi |m\eta|} \left\langle \beta_{x,y} E_{x,y} \right\rangle S'_{x,y} (m\omega)^2 \Psi \left( \frac{\delta p}{p} \right) \]

Average square of betatron amplitude at given \( \frac{\delta p}{p} \)
Power Considerations

- Total power in one longitudinal harmonic: \( P \propto NQ^2 \)
- Total power in one betatron sideband: \( P \propto NQ^2<J> \)
- We are not dealing here with signal suppression, which is \( \propto N(\delta p/p)^{-2} \)
Schottky Probe with Off-line Analysis
How to get to the ultimate frequency resolution

relative frequency difference in a multi-species beam: \[ \frac{\delta f}{f} = \eta \frac{\delta (\beta \gamma)}{\beta \gamma} - \alpha_p \left[ \frac{\delta (m/q)}{m/q} - \frac{\delta B}{B} \right] \]

Properties of one-dimensional ordered beams in the ESR

- Sudden drop of \( \delta f / f \) as \( N \) decreases below \( \approx 1000 \)
- Typical fwhm frequency width \( 4 \times 10^{-7} \)

Frequency resolution and record length: \[ \delta f = \frac{2.56}{t_{record}} \]
$^{207}$TI$^{81+}$ Decay Spectra with Isomer

$\delta_m = 1.35$ MeV

6.4 s after injection

640 ms
Schottky Mass Spectrometry: Resolution

Frequency separation:
\[
\frac{\delta f}{f} = -\alpha_p \frac{\delta (B \rho)}{B \rho}
\]

Available Resolution:
\[
\Delta f = \frac{2.56}{t_{record}}
\]

Available Mass Resolution:
\[
\frac{\delta (m/q)}{m/q} \approx \frac{2.56}{|\alpha_p| f t_{record}}
\]
Typical Fragment Spectrum
Vertical Schottky Spectra

- Left vertical sideband
- Spurious long harmonic
- Right vertical sideband

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Decay of one Ion out of Two

$t=150$ s

$140^\text{Pr}^{58+}$ (H-like)  $140^\text{Ce}^{58+}$ (bare)

$3.388$ MeV

$12$ averages per trace
distance between trace

$t=0$ s

$\delta f/f$

Particle Counting in Schottky Spectrum
A taper is a window in the time domain: \( x_n \ast h_{nm} \ast x_n \)

Use orthogonal taper families:

\[
\sum_n h_{nl} \cdot h_{nm} = \delta_{lm}
\]

yielding uncorrelated weighted spectral averages

\[
S_j = \frac{1}{K} \sum_{k=0}^{K-1} S_{kj}
\]

with

\[
S_{kj}(f) = t_{\text{record}} \sum_{l=0}^{N-1} x_n \cdot h_{nk} \exp\left(\frac{2\pi i ft_l}{N} \right)
\]
DPSS Multitapers

Chose the appropriate taper family

Solution: Discrete Prolate Spheroidal Sequences

Prolate Spheroidal Functions are solutions to

\[
\int_{-F}^{F} \frac{\sin(N \pi (x - x'))}{\sin(\pi (x - x'))} h_k^F (x') \, dx' = \lambda_k \ h_k^F (x)
\]

The DPSS are the discrete version of these and satisfy the energy concentration criterion