Incorporating systematic uncertainties in CL limits

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  - → Using search for $X \to t\bar{t}$ as an example
- Incorporating systematic uncertainties
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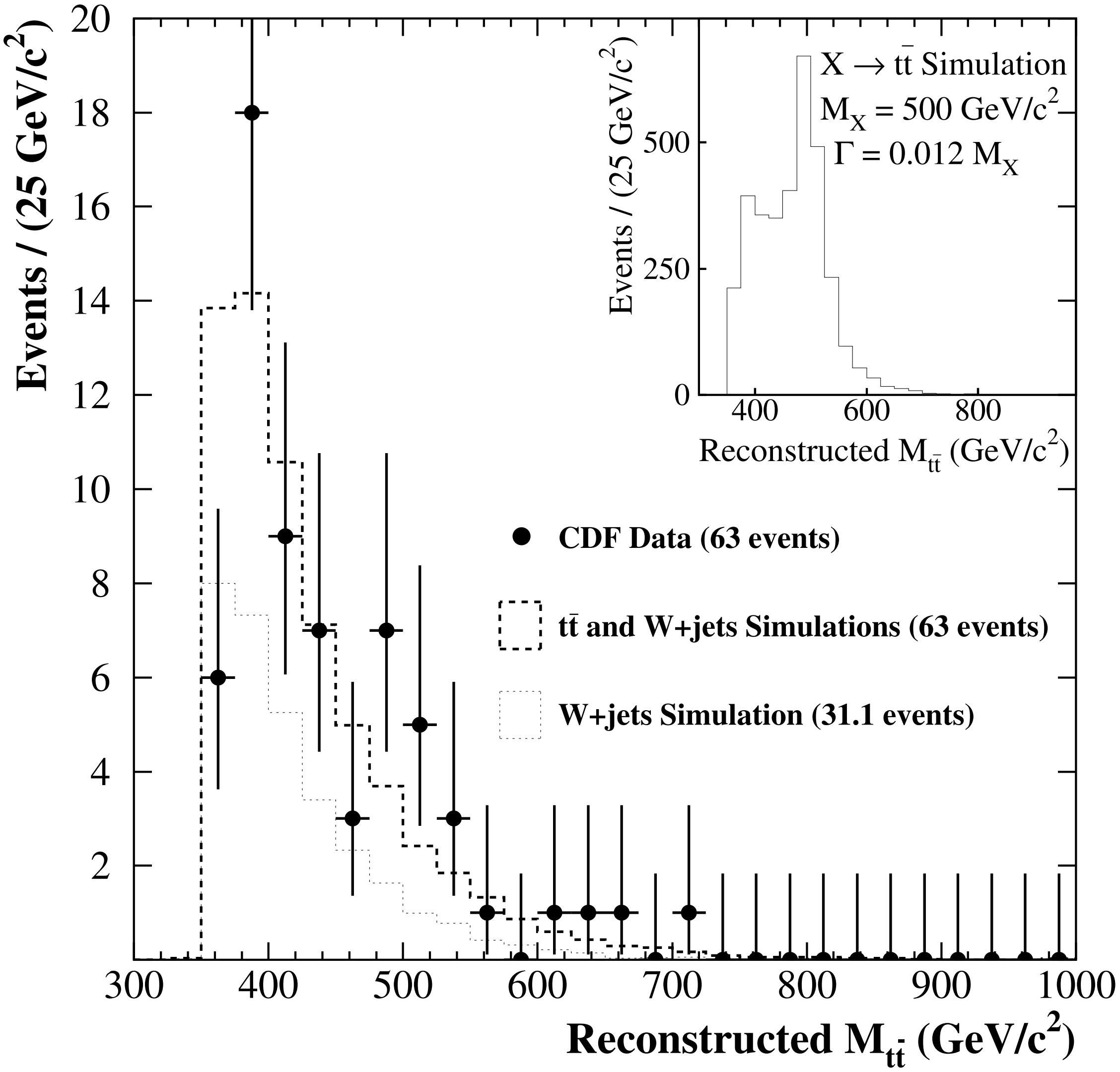
Introduction

- In searching for new particles, we can do simple counting experiments using a set of selection criteria optimized for the given signal, and set limits based solely on the *numbers* of observed and expected events.

- However, often one can increase the sensitivity for particle searches by exploiting differences in kinematic distributions between signal and background.
- To do this we construct a likelihood function which expresses the probability of observing N events with a distribution $\{x_k\}$.
- This can be a *binned* or *unbinned* likelihood function. For the $X \to t\bar{t}$ search we use the former.
- In the $X \to t\bar{t}$ search our variable x is $M_{t\bar{t}}$, the invariant mass of the $t\bar{t}$ system.

- Note: this analysis, including many studies on likelihood procedures and the "best" way to treat systematics under various scenarios, is written up in the PhD thesis of Josh Cassada, "A search for new particles decaying to $t\bar{t}$ pairs", U. of Rochester, Feb 2000.

The CDF $M_{t\bar{t}}$ spectrum



- Sample is $W + 4$-jets with b-tags and with non-tagged events (with tighter 4th jet requirements).
- Event is fit to $t\bar{t}$ hypothesis with top mass constrained to $175 GeV/c^2$.
- Configuration selected with the lowest χ^2 to form the invariant mass of the fitted b, $\bar{b}$, j_1, j_2, ℓ and ν.

The binned Likelihood procedure (statistical only)

- The distribution in x of N data points can be written in terms of the signal distribution, n_s, and the distributions of the j background sources n_b^j (typically normalized to unity):

$$\frac{dN}{dx} = \alpha n_s + \sum_j \beta_j n_b^j$$

- The likelihood function L is defined as:

$$L = \prod_i \mathcal{P}_i \qquad \text{where} \quad \mathcal{P}_i = \frac{\mu_i^{N_i} e^{-\mu_i}}{N_i!}$$

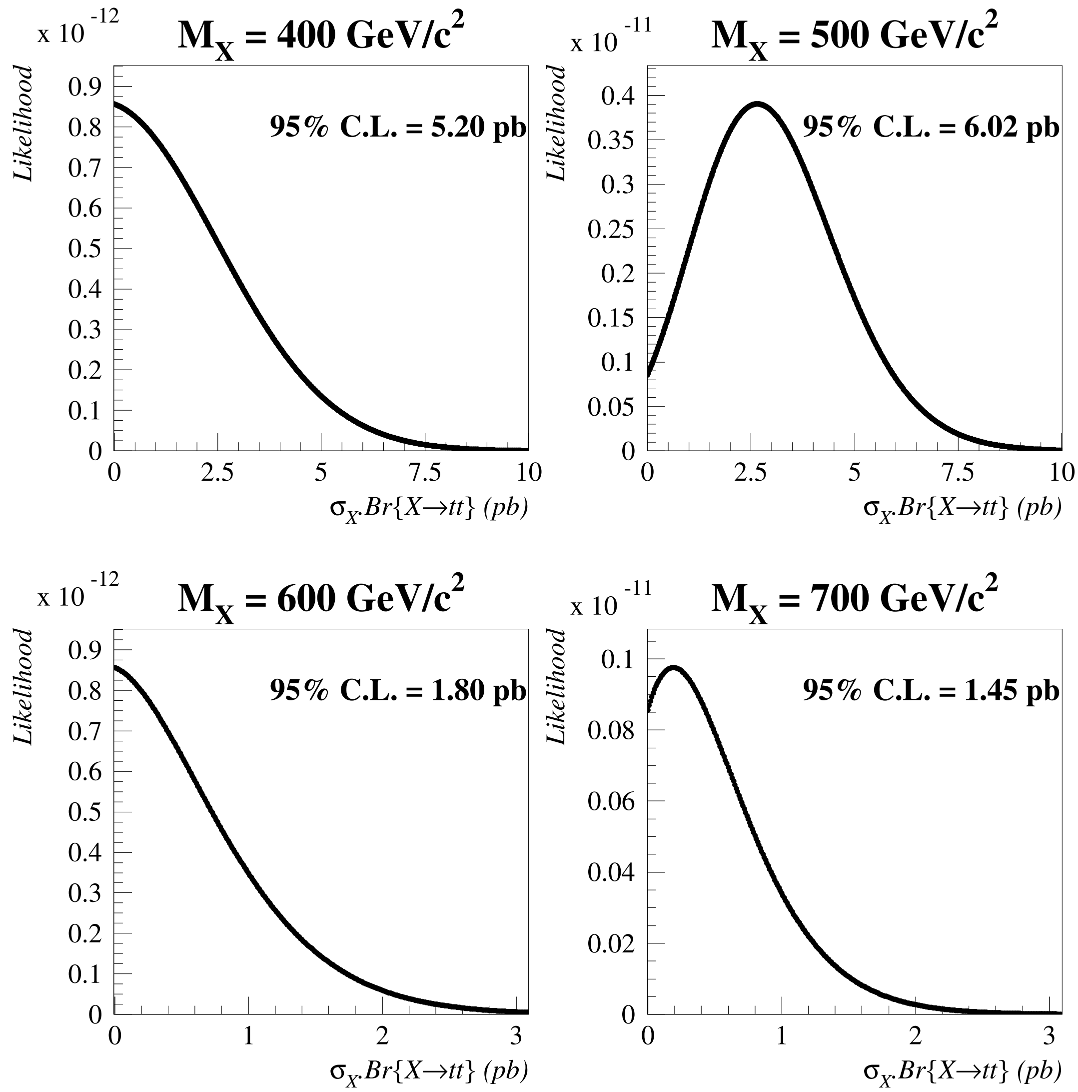
is the poisson probability for N_i data events and μ_i expected events in bin i.

$$\mu_i = \alpha n_s(i) + \sum_j \beta_j n_b^j(i)$$

- Depending on the analysis, gaussian background constraints (of the form $e^{(\beta_j - N_b^j)^2/\sigma(N_b^j)^2}$) are applied as multipliers to the likelihood function.
- The fitted signal is determined by minimising $-\ln L$ for α $(= \epsilon_s \sigma_s \int \mathcal{L} dt)$ and the β_j.
- The 95% CL upper limit on the signal cross-section, σ_{95}, is determined by mapping out L as a function of σ_s and integrating L wrt σ_s up to $\sigma_s = \sigma_{95}$ such that 95% of the area under the curve lies to the left of σ_{95}.

The binned Likelihood procedure for $M_{t\bar{t}}$

- $\frac{dN}{dM_{t\bar{t}}} = \alpha n_s + \beta n_{t\bar{t}} + \gamma n_{W+jets}$
- In searching for a narrow $t\bar{t}$ resonance we did NOT constrain the backgrounds so as to avoid making any assumptions about the standard model $t\bar{t}$ production cross-section.



Checking the statistical procedure

- Our 95% CL limits have used an inherently Bayesian procedure.
- We check our procedure using a frequentist definition:
  Suppose that the measurement of σ_X yields the value σ_{fit} and one wishes to establish the 95% CL limit, σ_{95} on σ_X. σ_{95} is that value such that if the true mean of σ_X were σ_{95} and the measurement were repeated a large number of times, then values of less than or equal to σ_{fit} will be observed 5% of the time.
- We check this using pseudo-experiments :
  - → Generate PE by poisson fluctuating the expected numbers of signal and background ($\bar{N}_s$ and $\bar{N}_b$, and then choosing these events according to the corresponding template shapes.
  - → Likelihood fit gives N_s, N_b and N_s^{95} as per our procedure above.
  - → 100 additional PE's are then generated by poisson fluctuating N_s^{95} and N_b.
  - → We then determine the fraction of these 100 PE's which fit to a number of signal events less than or equal to N_s.
  - → Repeat 200 times for each study.
  - → We find on average this fraction is indeed about 5%.
- In practice we could do this with our measured σ_{fit} by using PE's to find the value of σ_{95}. We don't because integrating the likelihood function is much easier and aesthetically appealing.

Systematic uncertainties : statement of the problem

- Given that we only understand a given feature of our analysis (e.g. jet E_T scale, ISR/FSR,....) to some estimated precision, how do we incorporate this uncertainty in 95% CL limits ?
- A simple minded approach might simply vary each source of uncertainty by its estimated error and observe the effect on the 95% CL limit.
- However, varying a source by 1σ probably isn't relevant for a 95% CL limit.
- A more accepted procedure is to smear the likelihood function $L(x)$ at all values of x by a gaussian of width equal to the systematic effect at x, to obtain a new likelihood function which reflects the effect of systematic uncertainties.

Systematic uncertainties

- Two types of systematic uncertainty must be taken into account in the likelihood procedure:

  → Those that affect the template shapes.

  → Those that affect the signal acceptance.

- Most of time taken for a shape analysis is consumed by understanding and incorporating the systematic effects !

- The method for incorporating systematic uncertainties often depends on the analysis, but it generally involves “smearing” the likelihood function with the systematic effects.

- Generally, the likelihood function is smeared with a constant term due to shape systematics, and a fractional term (in σ_s) due to acceptance.

- For the top search c.1991 this smearing was done using a simple MC program. It is now more common to write the smeared likelihood as:

$$L'(\sigma) = \int_0^\infty L(x) \times \frac{1}{\sqrt{2\pi}\Delta(x)} e^{-\frac{\sigma-x)^2}{2\Delta(x)^2}} dx$$

- The methods for determining $\Delta(x)$ vary somewhat from analysis to analysis. Here we demonstrate the method used for the $X \to t\bar{t}$ search, which was based on previous methods and slightly improved upon.

Determining the systematics in the $X \to t\bar{t}$ search

- Sources of systematics include; Jet E_T scale, M_{top}, ISR/FSR, b-tagging bias, structure functions, QCD background shape, luminosity, other acceptance effects (lepton ID etc.).
- Determining shape effects : instead of using a constant term (most previous analyses use as the constant term the effect at the 95% CL limit).

$\rightarrow$ Generate many pseudo-experiments (PE's) containing the expected amount of background and σ pb of signal.

$\rightarrow$ Fit each PE to the "nominal" templates. Call the fitted (max L) value σ_{fit}.

$\rightarrow$ Fit each PE to templates which reflect a 1σ shift in the given systematic. Call the fitted value σ'_{fit}.

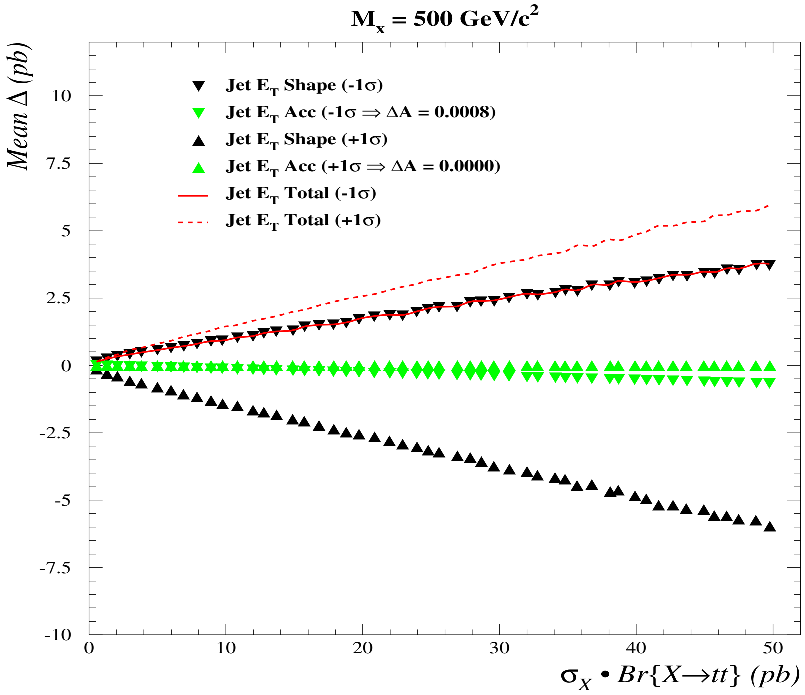
$\rightarrow$ Determine the mean of σ_{fit} ($\bar{\sigma}_{fit}$) and $\bar{\sigma}_{fit} - \sigma_{fit}$ ($\bar{\Delta}$).

$\rightarrow$ The functional form of $\bar{\Delta}$ versus $\bar{\sigma}_{fit}$ gives an estimate of the width of the gaussian by which L needs to smeared by at a given σ.

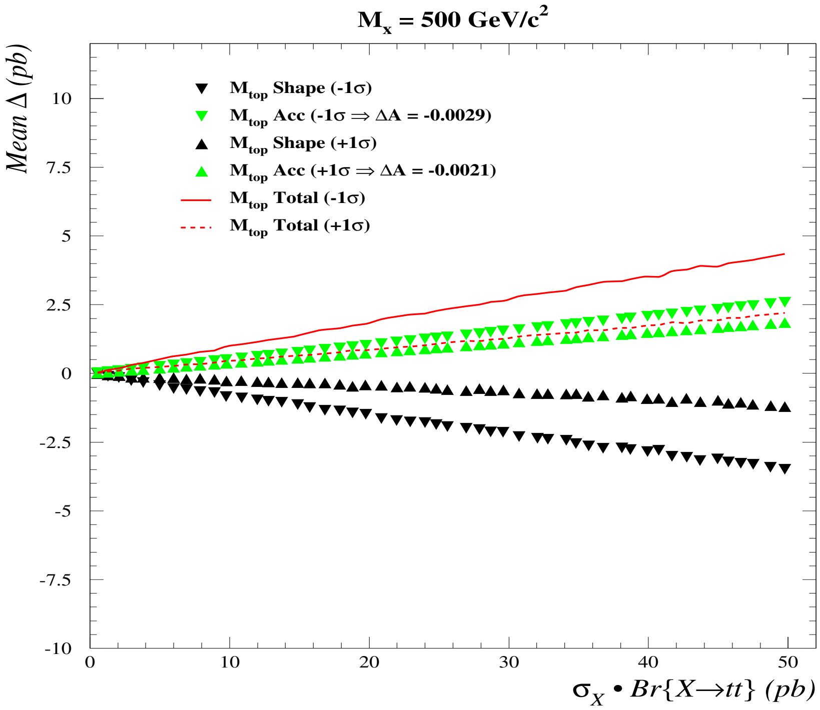
  - The acceptance systematics contribute $\frac{\Delta_{acc}}{\sigma_{fit}} = \frac{\Delta A}{A}$
  - For a given systematic source, the acceptance and shape effects are added in quadrature, unless they are clearly correlated, in which case they are added linearly.

Examples of systematics vs σ

- Jet E_T at $M_X = 500\,GeV/c^2$

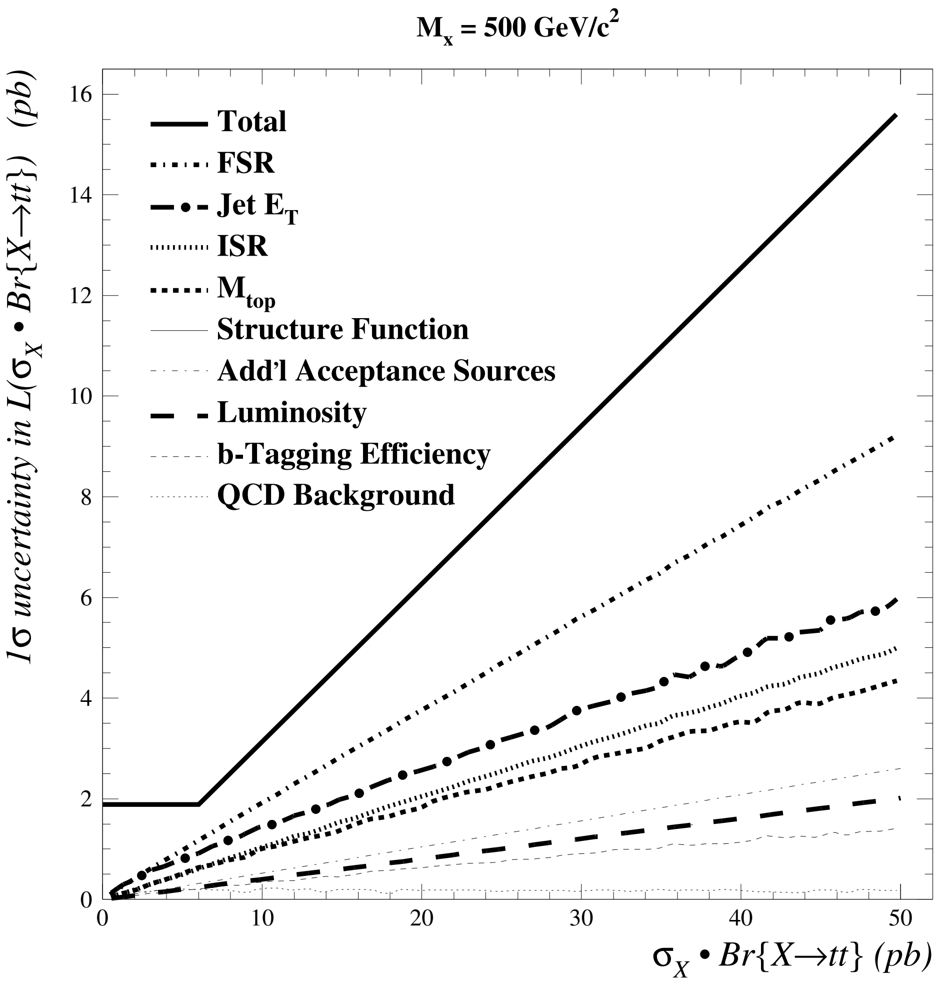


- M_{top} at $M_X = 500\,GeV/c^2$



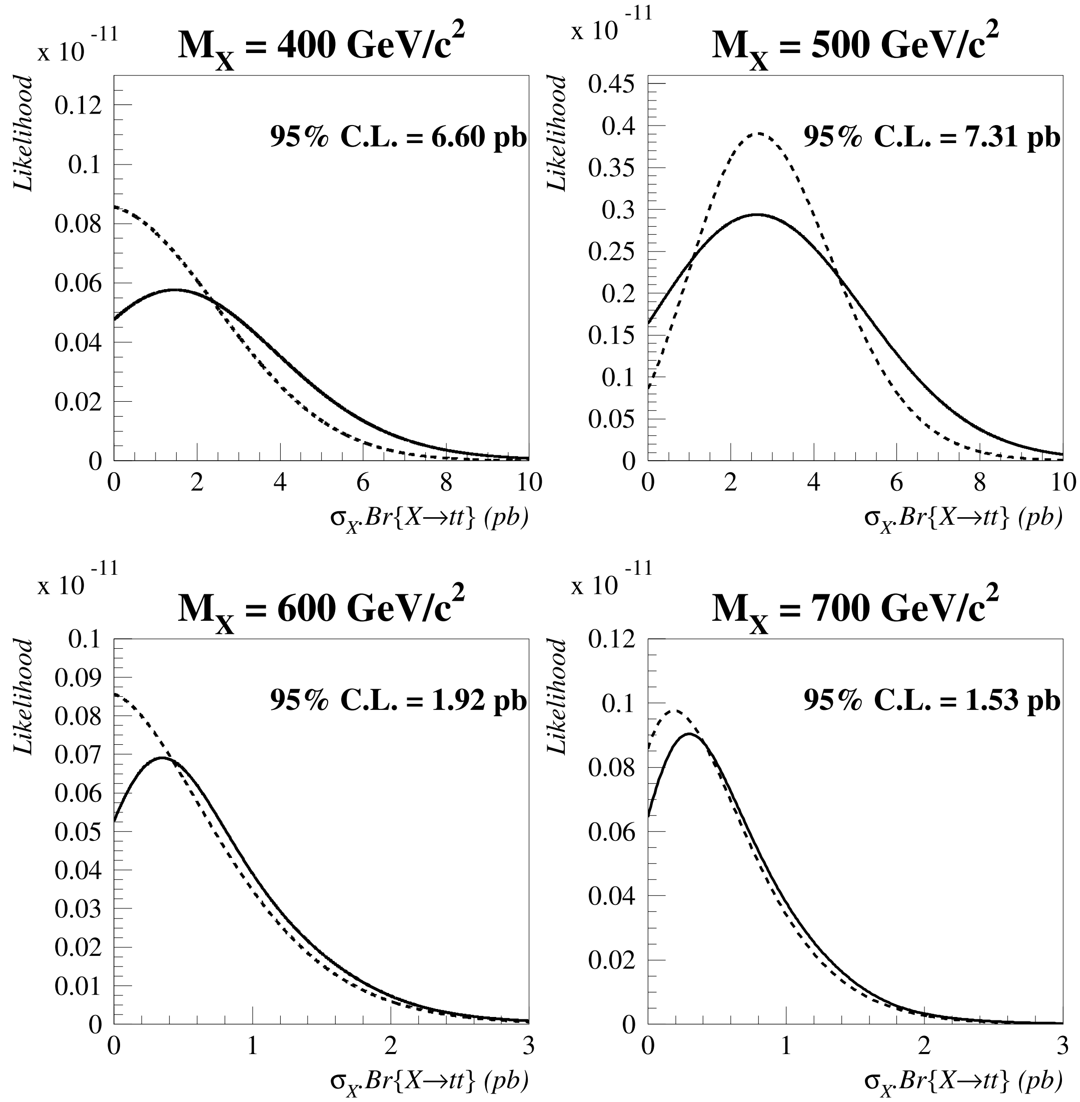
The function used to smear L

- Systematic uncertainty is given as a percentage of $\sigma_X \cdot BR(X \to t\bar{t})$ for $\sigma_X \cdot BR(X \to t\bar{t}) > 95\%$ limit including only statistical uncertainties.
- Systematic uncertainty is flat for $\sigma_X \cdot BR(X \to t\bar{t}) < 95\%$ value (to avoid problems at origin).



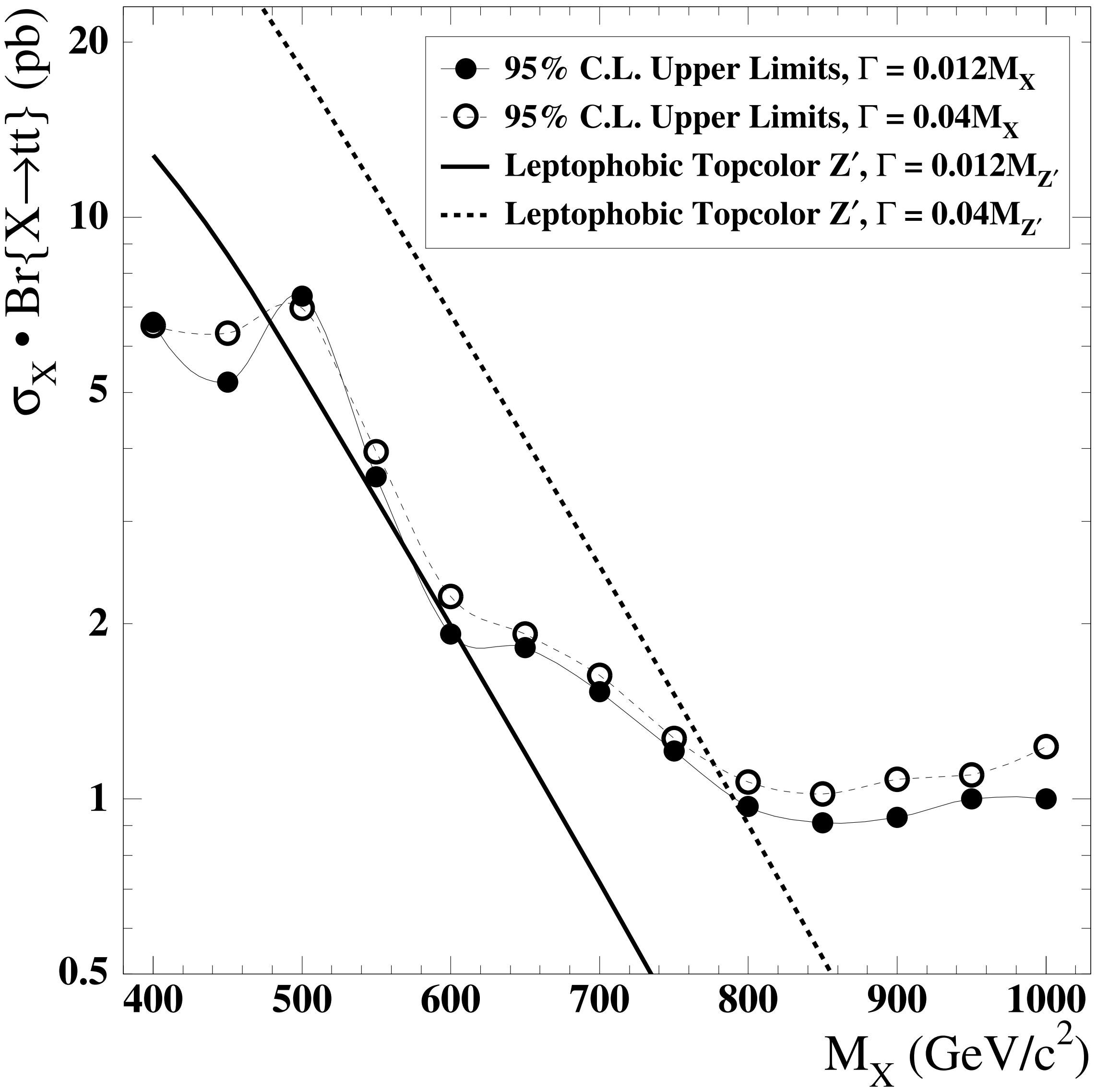
- Consistent with what CDF has done in the past, but more conservative (and correct as determined from PE's) at large values of $\sigma_X \cdot BR(X \to t\bar{t})$.

Smeared likelihood functions



The results

- The effect of systematics in this analysis is to increase the 95% CL limits by 10 to 20%.



Concluding remarks

- Systematic uncertainties are unavoidable in HEP analyses, and have a significant effect on CL limits.
- The incorporation of systematic effects into a likelihood procedure is typically the hardest part of any shape analysis, and no standard method exists to do this. It would be nice to settle on a general procedure/guidelines if possible
- We generally take very conservative approaches, a reflection of the inherent difficulty in treating systematics.
- In the example of $X \to t\bar{t}$ we have used an inherently Bayesian approach to obtain the statistical 95% CL limits, but a frequentist approach to determine the systematics. This is not atypical in CDF searches.