Combining CDF and D0 Physics Results

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Outline

- *W* mass (common systematic error method).
- Top quark mass (correlated error method).
- Leptoquark production limit (counting experiments).

Combined W mass (Common Systematic Error Method)

- Assumes that all measurements have the same common error.

  First mass: $m_1 \pm \sigma_1$

  Second mass: $m_2 \pm \sigma_2$

  Common error: σ_C

- Average and error:

$$m_{\text{ave}} = \frac{\dfrac{m_1}{\sigma_1^2 - \sigma_C^2} + \dfrac{m_2}{\sigma_2^2 - \sigma_C^2}}{\dfrac{1}{\sigma_1^2 - \sigma_C^2} + \dfrac{1}{\sigma_2^2 - \sigma_C^2}}$$

$$\sigma_{\text{ave}}^2 = \frac{1}{\dfrac{1}{\sigma_1^2 - \sigma_C^2} + \dfrac{1}{\sigma_2^2 - \sigma_C^2}} + \sigma_C^2$$

Combined W mass (cont.)

- Latest Tevatron *W* mass results:
  - CDF: $m_W = 80.433 \pm 0.079$ GeV
  - D0: $m_W = 80.482 \pm 0.091$ GeV
  - Common Error: 0.025 GeV
- Tevatron combined W mass:
  - $m_W = 80.454 \pm 0.062$ GeV (common error method)
  - $m_W = 80.454 \pm 0.060$ GeV (ignoring correlations)

Correlated Error Method

- Assume that measurements are correlated.
  - First mass: $m_1 \pm \sigma_1$
  - Second mass: $m_2 \pm \sigma_2$
  - Correlation Coefficient: $\rho = \langle m_1 m_2 \rangle / \sigma_1 \sigma_2$
- Minimize χ^2:

$$\chi^2 = \begin{pmatrix} \Delta m_1 & \Delta m_2 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}^{-1} \begin{pmatrix} \Delta m_1 \\ \Delta m_2 \end{pmatrix}$$

$$m_{\text{ave}} = \frac{m_1(\sigma_2^2 - \rho\sigma_1\sigma_2) + m_2(\sigma_1^2 - \rho\sigma_1\sigma_2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

$$\sigma_{\text{ave}}^2 = \frac{\sigma_1^2\sigma_2^2(1-\rho^2)}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$$

Connection with Common Error Method

- Define "correlated error" $\sigma_C^2 = \rho\sigma_1\sigma_2$
- Then previous equations reduce to:

$$m_{\text{ave}} = \frac{\dfrac{m_1}{\sigma_1^2 - \sigma_C^2} + \dfrac{m_2}{\sigma_2^2 - \sigma_C^2}}{\dfrac{1}{\sigma_1^2 - \sigma_C^2} + \dfrac{1}{\sigma_2^2 - \sigma_C^2}}$$

$$\sigma_{\text{ave}}^2 = \frac{1}{\dfrac{1}{\sigma_1^2 - \sigma_C^2} + \dfrac{1}{\sigma_2^2 - \sigma_C^2}} + \sigma_C^2$$

Calculating Correlation

- Calculate correlation by summing over uncorrelated error categories.

$$\rho = \frac{\langle \Delta m_1 \Delta m_2 \rangle}{\sigma_1 \sigma_2} = \frac{\sum_{ij} \langle \Delta m_{1i} \Delta m_{2j} \rangle}{\sigma_1 \sigma_2}$$

$$= \frac{\sum_{i} \langle \Delta m_{1i} \Delta m_{2i} \rangle}{\sigma_1 \sigma_2} = \frac{\sum_{i} \rho_i \sigma_{1i} \sigma_{2i}}{\sigma_1 \sigma_2}$$

$$\sigma_C^2 = \sum_i \rho_i \sigma_{1i} \sigma_{2i}$$

- Usually one assumes $\rho_i = 0$ or 1.

$$\rho = \frac{\sum_{\text{correlated } i} \sigma_{1i} \sigma_{2i}}{\sigma_1 \sigma_2}$$

$$\sigma_C^2 = \sum_{\text{correlated } i} \sigma_{1i} \sigma_{2i}$$

Combining More Than Two Results

- Correlation coefficient $\rho \rightarrow$ correlation matrix ρ_{ij}.
- Minimize χ^2:

$$\chi^2 = \sum_{ij} w_{ij} \Delta m_i \Delta m_j$$

where

$$w_{ij} = \left(\rho_{ij} \sigma_i \sigma_j\right)^{-1} \qquad \text{(matrix inverse)}$$

$$m_{\text{ave}} = \frac{\sum_{ij} w_{ij} m_i}{\sum_{ij} w_{ij}}$$

$$\sigma^2_{\text{ave}} = \frac{1}{\sum_{ij} w_{ij}}$$

Separating Combined Error into Statistical and Systematic Components

- Define the statistical error by propagating errors on the equation for m_{ave}.

$$m_{\text{ave}} = \frac{\sum_{ij} w_{ij} m_i}{\sum_{ij} w_{ij}}$$

$$\sigma^2_{\text{stat}} = \frac{\sum_i \left(\sum_j w_{ij} \right)^2 \sigma^2_{\text{stat}i}}{\left(\sum_{ij} w_{ij} \right)^2}$$

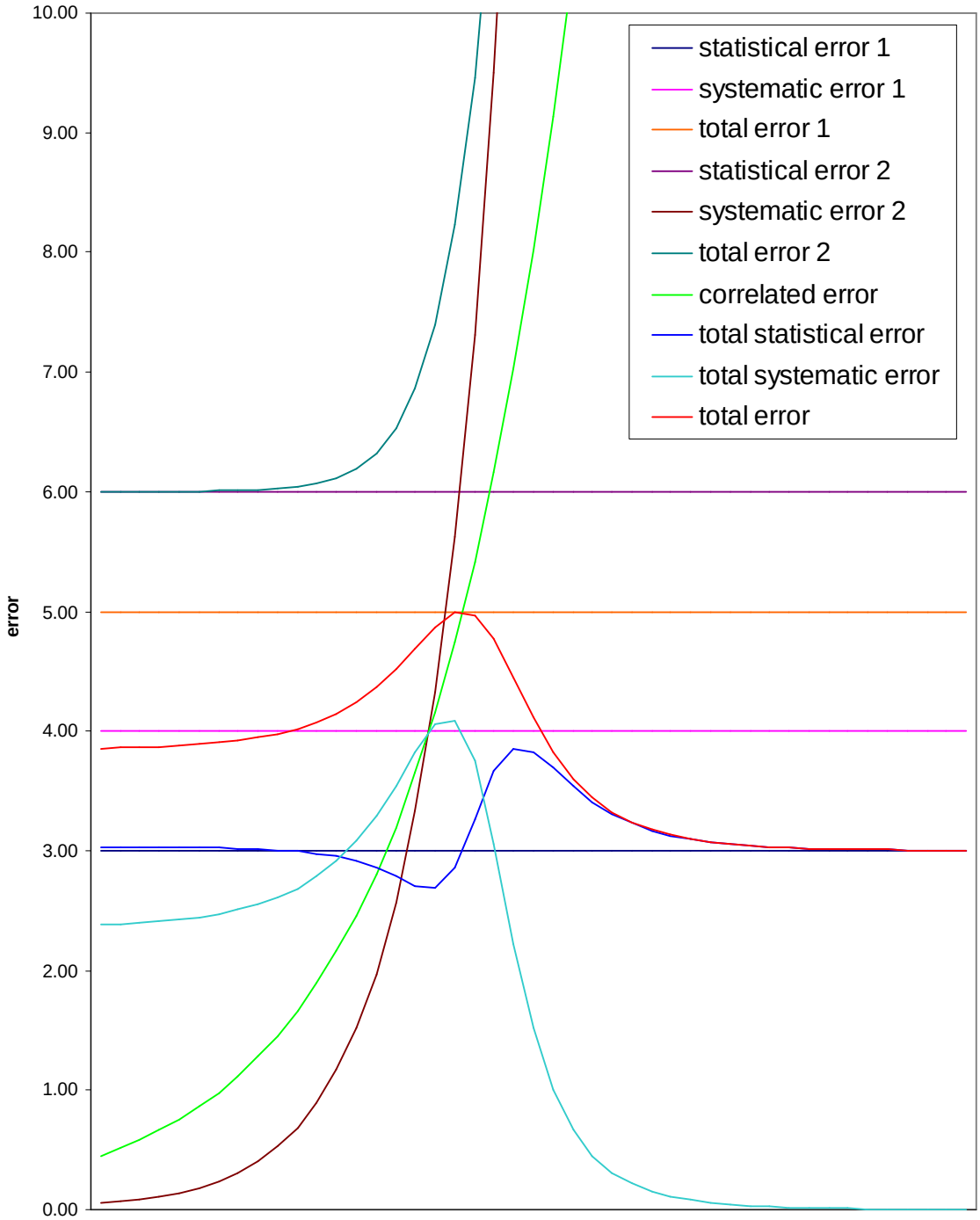
$$\sigma^2_{\text{syst}} = \sigma^2_{\text{ave}} - \sigma^2_{\text{stat}}$$

Statistical and Systematic Error (cont.)

- This definition of statistical error is in general different than what one would get by averaging based on statistical errors alone.
- This definition remains sensible as $\rho \to 1$.
  - Combined statistical error remains less than the combined total error.
  - Combined statistical error dominates combined systematic error as $\rho \to 1$ (both approach zero, but systematic approaches zero faster).
- In general, it is not possible to consistently separate the systematic error into components.

High Correlation Limit

- If $\rho > \sigma_1/\sigma_2$ or $\rho > \sigma_2/\sigma_1$ we are in the high correlation region.
  - One result enters with negative weight into the average.
  - The average mass is not between the two measurements.
- For given σ_1 and σ_2, error is maximum at the boundary between high and low correlation regions.
  - Weaker experiment has zero weight.
  - Central value and error are the same as the stronger experiment.
- In the high correlation region, increasing correlated components of the systematic error of the weaker experiment actually makes the total error smaller.
- If $\rho = 1$, the total error is zero.


10.00
9.00
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error
statistical error 1
systematic error 1
total error 1
statistical error 2
systematic error 2
total error 2
correlated error
total statistical error
total systematic error
total error

Averaging CDF and D0 Top Quark Mass Results

- Five top quark mass results from both experiments.
  - CDF lepton + jets.
  - CDF dilepton.
  - CDF all jets.
  - D0 lepton + jets.
  - D0 dilepton.
- Average all five results together simultaneously using correlated error method.
  - Reference Fermilab-TM-2084

Summary of Top Quark Mass Results and Errors

- Results and systematic errors (100% correlation within each shaded color):

	CDF			D0	
	λ + jets	λλ	All had.	λ + jets	λλ
Mass (GeV)	175.9	167.4	186.0	173.3	168.4
Total Error	7.1	11.4	11.5	7.8	12.8
Statistical Error	4.8	10.3	10.0	5.6	12.3
Systematic Error	5.3	4.8	5.7	5.5	3.6
Jet Energy Scale	4.4	3.8	5.0	4.0	2.4
Signal Model	2.6	2.8	1.8	1.8	1.7
MC Generator	0.1	0.6	0.8	0.0	0.0
U noise/Mult. Int.	0.0	0.0	0.0	1.3	1.3
Background Model	1.3	0.3	1.7	2.5	1.0
Fitting Method	0.0	0.7	0.6	1.5	1.1

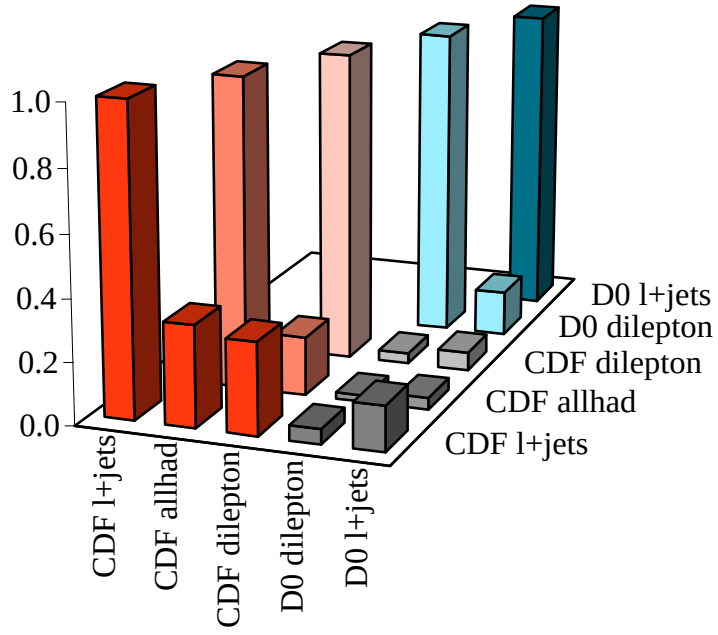
- Correlation matrix and weights:

		CDF			D0	
		λ + jets	λλ	All had.	λ + jets	λλ
CDF	λ + jets	1	0.296	0.327	0.146	0.048
	λλ	0.296	1	0.187	0.058	0.034
	All had.	0.327	0.187	1	0.038	0.021
D0	λ + jets	0.146	0.058	0.038	1	0.145
	λλ	0.048	0.034	0.021	0.145	1

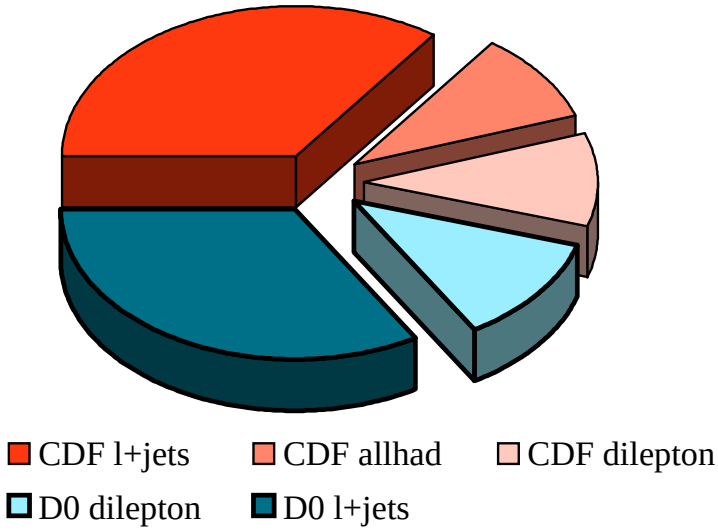
Weight ($1/\sigma^2$)	0.343	0.132	0.135	0.106	0.283
Weight (correlated)	0.351	0.096	0.101	0.115	0.337

Top Quark Correlation and Weights (cont.)

Correlation in top mass



Relative weight in top mass average



Top Quark Mass Combined Results

- Combined top mass
  - With correlation

    $m_t = 174.3 \pm 3.2$ (stat.) ± 4.0 (syst.) GeV/c^2

    $= 174.3 \pm 5.1$ GeV/c^2

    χ^2/DOF = 1.92/4
  - Without correlation

    $m_t = 174.4 \pm 4.2$ GeV/ c^2

Combined First Generation Leptoquark Limit

- Counting experiments:
  - Result (cross section or cross section limit) depends on four numbers:
    - Number of observed events n
    - Acceptance ε
    - Background b
    - Luminosity $\int L$

$$n = \sigma \varepsilon \int L + b$$

  - Sources of error:
    - Counting error (square root of n).
    - Systematic errors (σ_ε, σ_b, $\sigma_{\int L}$).
    - Correlations ($\rho_{\varepsilon b}$, $\rho_{b \int L}$).

Combining Counting Experiments

- Combine events, acceptance and background.

$$n = \sum_i n_i$$

$$\varepsilon \mathcal{L} = \sum_i \varepsilon_i \mathcal{L}_i$$

$$b = \sum_i b_i$$

- Propagate errors in sums using standard correlated propagation errors formula.
- Calculate cross section (limit) using combined events, acceptance, and background.
  - Detailed procedure described in hep-ex/9810015

Combined First Generation Leptoquark Limit

- Results:
  - CDF
    - 0 candidate events.
    - σ < 0.0993 pb.
    - M_{LQ} > 213.5 GeV/c^2.
  - D0
    - 0 candidate events.
    - σ < 0.0663 pb.
    - M_{LQ} > 225.8 GeV/c^2.
  - Combined
    - 0 candidate events.
    - σ < 0.0383 pb.
    - M_{LQ} > 242.4 GeV/c^2.

Summary

- Three results have been combined between CDF and D0.
  - W mass.
  - Top quark mass
  - First generation leptoquark limit.
- Practical considerations.
  - Agreeing on categories of systematic errors.
  - Handling high correlation regions.
    - Don’t overestimate systematic errors.
- Methods
  - Common systematic error method is special case of correlated error method for averaging type of combinations.
  - Combined counting method is simple to apply, but is not necessarily optimal.