Neutrino “Theory”

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New Physics at the Electroweak Scale and New Signatures at Hadron Colliders

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Outline

1. What We Have Learned About Neutrinos;
2. What We Know We Don’t Know;
3. Neutrino Masses As Physics Beyond the Standard Model;
4. Ideas for Neutrino Masses, and Consequences;
5. Comments On Lepton Mixing;
6. Conclusions.
NEUTRINOS
HAVE MASS
[albeit very tiny ones...]

We don’t know why that is, but we have a “gut feeling” it means something important.

Are neutrinos fundamentally different?

Are neutrino masses generated by a distinct dynamical mechanism?
How Did We Find Out: Flavor Oscillations!

Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the neutrino energy $E_\nu$ and the baseline $L$.

- $\nu_\mu \to \nu_\tau$ and $\bar{\nu}_\mu \to \bar{\nu}_\tau$ — atmospheric experiments [“indisputable”];
- $\nu_e \to \nu_{\mu,\tau}$ — solar experiments [“indisputable”];
- $\bar{\nu}_e \to \bar{\nu}_{\text{other}}$ — reactor neutrinos [“indisputable”];
- $\nu_\mu \to \nu_{\text{other}}$ from accelerator experiments [“really strong”].

The simplest and only satisfactory explanation of all this data is that neutrinos have distinct masses, and mix. →

\[ P_{osc} = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right) \]
Solar

$\nu_e \rightarrow \nu_{\text{active}}$

+ KamLAND

$\bar{\nu}_e \leftrightarrow \bar{\nu}_e$

$\nu_e$ oscillation parameters compatible with $\bar{\nu}_e$: Sensible to assume CPT: $P_{ee} = P_{\bar{e}\bar{e}}$

$\Delta m^2 = \left(8^{+0.4}_{-0.5} \times 10^{-5}\right) \text{ eV}^2 \quad (1\sigma)$

$\tan^2 \theta_\odot = 0.45^{+0.05}_{-0.05}$

[Gonzalez-Garcia, PASI 2006]
K2K 2004: spectral distortion

MINOS 2006: spectral distortion

Confirmation of ATM oscillations

[Gonzalez-Garcia, PASI 2006]
Phenomenological Understanding of Neutrino Masses & Mixing

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

Definition of neutrino mass eigenstates (who are \(\nu_1, \nu_2, \nu_3?\)):

- \(m_1^2 < m_2^2\)
  \[\Delta m_{13}^2 < 0 - \text{Inverted Mass Hierarchy}\]

- \(m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|\)
  \[\Delta m_{13}^2 > 0 - \text{Normal Mass Hierarchy}\]

\[
\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} \equiv \sin \theta_{13} e^{-i\delta}
\]

January 11, 2007
3σ ranges:

\[ 7 \leq \frac{\Delta m_{21}^2}{10^{-5}\text{eV}^2} \leq 9.1 \]

\[ 1.9 \leq \frac{\Delta m_{32}^2}{10^{-3}\text{eV}^2} \leq 3.25 \]

\[ 0.34 \leq \tan^2 \theta_{12} \leq 0.62 \]

\[ 0.49 \leq \tan^2 \theta_{23} \leq 2.2 \]

\[ \sin^2 \theta_{13} \leq 0.045 \]

\[ -\pi \leq \delta \leq \pi \]

[Gonzalez-Garcia, PASI 2006]
What We Know We Don’t Know (1)

- What is the \( \nu_e \) component of \( \nu_3 \)? (\( \theta_{13} \neq 0 \)?)

- Is CP-invariance violated in neutrino oscillations? (\( \delta \neq 0, \pi \)?)

- Is \( \nu_3 \) mostly \( \nu_\mu \) or \( \nu_\tau \)? (\( \theta_{23} > \pi/4 \), \( \theta_{23} < \pi/4 \), or \( \theta_{23} = \pi/4 \)?)

- What is the neutrino mass hierarchy? (\( \Delta m_{13}^2 > 0 \)?)

⇒ All of the above can be addressed in neutrino oscillation experiments if we get lucky, that is if \( \theta_{13} \) is large enough

- What is the smallest neutrino mass?
What We Know We Don’t Know (2): Are Neutrinos Majorana Fermions?

The neutrino is the only neutral elementary fermion. There is a left-handed one and a right-handed one.

as far as we can tell (experiments) . . .
the left-handed has lepton number $L = +1$, while the right-handed one has $L = -1$:

$$(\nu_\ell)_L + X \rightarrow \ell^- + X', \text{ while}$$

$$(\nu_\ell)_R + X \rightarrow \ell^+ + X', \text{ so we call } (\nu_\ell)_R \equiv \bar{\nu}_\ell$$

However:
If the neutrino is its own antiparticle (Majorana fermion), then the lepton number conservation law must not be exact $\rightarrow$ look for $L$-violation.
Search for the Violation of Lepton Number (or \( B - L \))

In order to make significant theoretical progress, we need to decide whether the neutrinos are Dirac or Majorana fermions.

**Best Bet:** search for Neutrinoless Double-Beta decay: \[ Z \rightarrow (Z + 2)e^-e^- \]

(neutrino exchange picture: \( 2n \rightarrow 2p + 2e^- + \bar{\nu}_e + \bar{\nu}_e \rightarrow 2p + 2e^- \))

Helicity Suppressed Amplitude \( \propto \frac{m_{ee}}{E} \)

Observable: \( m_{ee} \equiv \sum_i U_{ei}^2 m_i \)
What We Know We Don’t Know (3) – The LSND Anomaly

The LSND experiment looks for $\bar{\nu}_e$ coming from

- $\pi^+ \rightarrow \mu^+\nu_\mu$ decay in flight;
- $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$ decay at rest;

produced some 30 meters away from the detector region.

It observes a statistically significant excess of $\bar{\nu}_e$-candidates. The excess can be explained if there is a very small probability that a $\bar{\nu}_\mu$ interacts as a $\bar{\nu}_e$, $P_{\mu e} = (0.26 \pm 0.08)\%$.

However: the LSND anomaly (or any other consequence associated with its resolution) is yet to be observed in another experimental setup.
strong evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

If oscillations $\Rightarrow \Delta m^2 \sim 1 \text{ eV}^2$

$\times$ does not fit into 3 $\nu$ picture;

$\times$ 2 + 2 scheme ruled out (solar, atm);

$\circ$? 3 + 1 scheme disfavored (sbl searches);

$\times$ 3 $\nu$'s CPTV ruled out (KamLAND, atm);

$\times$ $\mu \rightarrow e\nu_e\bar{\nu}_e$ ruled out (KARMEN, TWIST);

$\circ$ 3 + 1 + 1 scheme works (finely tuned?);

$\circ$ 4 $\nu$'s CPTV

$\circ$ “heavy” decaying sterile neutrinos;

$\circ$? 3 $\nu$s and Lorentz-invariance violation;

$\circ$ something completely different.
LSND – Challenge Summarized

The LSND effect is very small ($P_{\mu e} \sim 0.3\%$). No other experiment has achieved better sensitivity to $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at these energies . . .

. . . except for the Karmen experiment → $L$ (or $L/E$) dependency necessary!

This implies that a small effect at LSND translates “always” into a much larger effects elsewhere (atmospheric/solar neutrinos, short baseline experiments, etc).

We have, so far, failed to find a consensus, “feel good” solution to the LSND anomay.

Does this mean it is wrong? of course NOT!\(^a\)

\(^a\)Some have “experimental complaints” about LSND. None seem to be entirely convincing, and qualify, at best, as well-justified suspicion that something is afoot...
**LSND Anomaly to be resolved by the MiniBooNE experiment:**

Ongoing experiment at Fermilab designed to definitively test the LSND anomaly with different beam and systematics.

**LSND:** $\mu^+ \rightarrow \bar{\nu}_\mu \rightarrow \bar{\nu}_e$

**MiniBooNE:** $\pi^+ \rightarrow \nu_\mu \rightarrow \nu_e$

[See Wasko’s and Monroe’s talks]
The best solution (my opinion) to the LSND anomaly we have been able to concoct is 3+2 neutrino oscillations (or 3+3, 3+4, etc).

While a good fit can be obtained, it seems to be “taylor made.” Why haven’t LSND effects been observed in disappearance experiments?

**There are many left-over theoretical complaints.**

- What are these sterile neutrinos? [LEP data tell us there are only three light neutrinos that couple to the $Z$-boson...]
- Why are they so light? Sterile neutrinos are “theoretically expected” to be very heavy...
- Can we say anything about the expected sterile–active neutrino mixing? Can LSND oscillations be predicted?
- ...
Who Cares About Neutrino Masses: Only* “Palpable” Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

* There is only a handful of questions our model for fundamental physics cannot explain properly. These are, in order of “palpability” (my opinion!):

- What is the physics behind electroweak symmetry breaking? (Higgs or not in SM).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM – is this “particle physics?”).
What I Mean By the Standard Model

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group \((SU(3)_c \times SU(2)_L \times U(1)_Y)\);
- Particle Content (fermions: \(Q, u, d, L, e\), scalars: \(H\)).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done! (after several decades of hard experimental work…)

If you follow these rules, neutrinos have no mass. Something has to give.
What is the New Standard Model? \([\nu\text{SM}]\)

The short answer is – WE DON’T KNOW. Not enough available info!

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the \(\nu\text{SM}\) candidates can do. [are they falsifiable?, are they “simple”? , do they address other outstanding problems in physics?, etc]

We need more experimental input, and it looks like it may be coming in the near/intermediate future!
Options include:

- modify SM Higgs sector (e.g. Higgs triplet) and/or
- modify SM particle content (e.g. $SU(2)_L$ Triplet or Singlet) and/or
- modify SM gauge structure and/or
- supersymmetrize the SM and add R-parity violation and/or
- augment the number of space-time dimensions and/or
- etc
Candidate $\nu$SM

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu SM} \supset -\lambda_{ij} \frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$ 

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

after EWSB $\mathcal{L}_{\nu SM} \supset \frac{m_{ij}}{2} \nu^i \nu^j$; $m_{ij} = \lambda_{ij} \frac{v^2}{\Lambda}$.

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- $\nu$SM effective theory – not valid for energies above at most $\Lambda$.
- What is $\Lambda$? First naive guess is that $M$ is the Planck scale – does not work. Data require $\Lambda < 10^{15}$ GeV (anything to do with the GUT scale?)

What else is this “good for”? Depends on the ultraviolet completion!
Full disclosure:

All higher dimensional operators are completely negligible, except those that mediate proton decay, like:

$$\frac{\lambda_B}{M^2} QQQQ_L$$

The fact that the proton does not decay forces $M/\lambda_B$ to be much larger than the energy scale required to explain neutrino masses.

Why is that? We don’t know...
Massive Neutrinos and the Seesaw Mechanism

A simple\textsuperscript{a}, renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\text{old}} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where $N_i$ ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions. $\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the $N_i$ fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: \textbf{six neutrinos}.

\textsuperscript{a}Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.
To be determined from data: $\lambda$ and $M$.

The data can be summarized as follows: there is evidence for three neutrinos, mostly “active” (linear combinations of $\nu_e$, $\nu_\mu$, and $\nu_\tau$). At least two of them are massive and, if there are other neutrinos, they have to be “sterile.”

This provides very little information concerning the magnitude of $M_i$ (assume $M_1 \sim M_2 \sim M_3$)

Theoretically, there is prejudice in favor of very large $M$: $M \gg v$. Popular examples include $M \sim M_{\text{GUT}}$ (GUT scale), or $M \sim 1 \text{ TeV}$ (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest $M_i$ to be around $10^{10}$ GeV.

we can impose very, very few experimental constraints on $M$
What We Know About $M$:

- $M = 0$: the six neutrinos “fuse” into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all $M_i$ vanish. Small $M_i$ values are ’tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i} \quad [m = 1/\Lambda \Rightarrow \Lambda = M/\mu^2]$. This the seesaw mechanism. Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though $L$-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).
High-energy seesaw has no other observable consequences, except, perhaps,

**Baryogenesis via Leptogenesis**

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the observed *baryon asymmetry* of the Universe can be obtained from a baryon–antibaryon symmetric initial condition plus well understood dynamics. [Baryogenesis]

This isn’t just for aesthetic reasons. If the early Universe undergoes a period of *inflation*, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

It turns out the seesaw mechanism contains all necessary ingredients to explain the baryon asymmetry of the Universe as long as the right-handed neutrinos are heavy enough – $M > 10^8$ GeV (with some exceptions that I won’t have time to mention).
Low-Energy Seesaw [AdG PRD72,033005])

Lets peek in the other end of the $M$ spectrum. What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$; $
\Rightarrow$

- No standard thermal leptogenesis – right-handed neutrinos way too light;

- No obvious connection with other energy scales (EWSB, GUTs, etc);

- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;

- sterile–active mixing can be predicted – hypothesis is falsifiable!

- Small values of $M$ are natural (in the ‘tHooft sense). In fact, theoretically, no value of $M$ should be discriminated against!
On very small Yukawa couplings

We would like to believe that Yukawa couplings should naturally be of order one.

Nature, on the other hand, seems to have a funny way of showing this. Of all known fermions, only one (1) has a “natural” Yukawa coupling – the top quark!

Regardless there are several very different ways of obtaining “naturally” very small Yukawa couplings. They require more new physics.
Also effects in $0\nu\beta\beta$, tritium beta-decay, supernova neutrino oscillations, NEEDS non-standard cosmology.
Neutrinos Masses And Colliders: Non-Anomalous, Gauged $U(1)_{\nu}$

And it could turn out that neutrino masses are deeply connected to physics at the electroweak symmetry breaking scale:

Add to the SM a new, non-anomalous $U(1)_{\nu}$ under which both SM fermions and the right-handed neutrinos transform. Charges are heavily constrained by anomaly cancellations and the fact that quarks and charged leptons have relatively large masses.

One can choose $U(1)_{\nu}$ charges so that all neutrino masses are forbidden by gauge invariance. This way, neutrino masses are only generated after $U(1)_{\nu}$ is spontaneously broken, and only through higher dimensional operators, suppressed by a new ultraviolet scale $\Lambda$.

Neutrino masses might be small because they are a consequence of very high dimensional operators: $m_\nu \propto (\frac{\varphi}{\Lambda})^{|p|}$, where $p$ is an integer exponent.

---

Supplementary Note:

Assume $U(1)_{\nu}$ is spontaneous broken when SM singlet scalar $\Phi$ gets a vev, $\langle \Phi \rangle \equiv \varphi$. 

---

After $U(1)_{\nu}$ breaking → see-saw Lagrangian plus “left–left” neutrino mass:

$$\mathcal{L} \supset \sum_{ik} \epsilon^{p_{ik}} \overline{L}_i (\lambda^\nu)^{ik} \tilde{n}_k \tilde{H} + \sum_{ij} \epsilon^{q_{ij}} \overline{L}_i \frac{(h^L)^{ij}}{\Lambda} L_j HH + \sum_{kk'} \epsilon^{r_{kk'}} \Lambda \tilde{n}_k (h^R)^{kk'} \tilde{n}_{k'},$$

$\lambda^\nu$ – neutrino Yukawa coupling, $h^L$ – “left–left” coupling), and $h^R$ – “right–right” Majorana mass term). $i, j = 1, 2, 3, k, k' = 1 \ldots N$. Only allowed for integer values of $p, q,$ and $r$.

Consequences for collider physics:

- Non-standard $Z'$ – branching ratios to different fermion species can be matched to neutrino mass structure!

- Enhanced Higgs sector.

Understanding Fermion Mixing

The other puzzling phenomenon uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

\[
V_{\text{MNS}} \sim \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix} \quad V_{\text{CKM}} \sim \begin{pmatrix} 1 & 0.2 & 0.001 \\ 0.2 & 1 & 0.01 \\ 0.001 & 0.01 & 1 \end{pmatrix}
\]

\[| (V_{\text{MNS}})_{e3} | < 0.2 \]

They certainly look VERY different, but which one would you label as “strange”??
Theoretical predictions:

The literature on this subject is very large. The most exciting driving force (my opinion) is the fact that one can make *bona fide* predictions:

\[ \Rightarrow U_{e3}, \text{CP-violation, mass-hierarchy unknown!} \]

Type-I seesaw GUT models

<table>
<thead>
<tr>
<th>Model</th>
<th>(\Delta m^2_{13} &gt; 0)</th>
<th>(\Delta m^2_{13} &lt; 0)</th>
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<tbody>
<tr>
<td>SO(10)</td>
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<tr>
<td>Goh, Mohapatra, Ng [40]</td>
<td>0.18</td>
<td>0.13</td>
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<tr>
<td>Orbifold SO(10)</td>
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<tr>
<td>Asaka, Buchmüller, Covi [41]</td>
<td>0.1</td>
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“typical” prediction of all*

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<td>SO(10)</td>
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<tr>
<td>Babu, Pati, Wilczek [42]</td>
<td>5.5 (\cdot) 10^{-4}</td>
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<td>Blazek, Raby, Tobe [43]</td>
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<td>Albright, Barr [45]</td>
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<td>Mackawa [46]</td>
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<td>Ross, Velasco-Sevilla [47]</td>
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<tr>
<td>Chen, Mahanthappa [48]</td>
<td>0.15</td>
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Type-II seesaw GUT models

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<tr>
<td>Bando, Obara [51]</td>
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<td>4 (\cdot) 10^{-4} .. 0.01</td>
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Inverted hierarchy requires*

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<td>King, Ross [57]</td>
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“more flavor structure”

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<td>Lebed, Martin [59]</td>
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<td>Bando, Kaneko, Obara, Tanimoto [60]</td>
<td>0.01 .. 0.05</td>
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<tr>
<td>Ibarra, Ross [61]</td>
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* Albright,hep-ph/0407155 (inverted hierarchy) \(> 0.006 > 1.6 \(\cdot\) 10^{-4}\)

More data needed to “sort things out.”

\[ \Delta m^2_{13} > 0 \]

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<td>Chen, Mahanthappa [48]</td>
<td>0.15</td>
<td>0.09</td>
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<td>Raby [49]</td>
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<tr>
<td>SO(10) + texture</td>
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<td>Buchmüller, Wyler [50]</td>
<td>0.1</td>
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<tr>
<td>Bando, Obara [51]</td>
<td>0.01 .. 0.06</td>
<td>4 (\cdot) 10^{-4} .. 0.01</td>
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Flavor symmetries

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<th>Model</th>
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<td>SO(10)</td>
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<td>Grimus, Lavoura [52, 53]</td>
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<td>Grimus, Lavoura [52]</td>
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<td>Babu, Ma, Valle [54]</td>
<td>0.14</td>
<td>0.08</td>
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<td>Kuchimanchi, Mohapatra [55]</td>
<td>0.08 .. 0.4</td>
<td>0.03 .. 0.5</td>
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<td>Ohlsson, Seidl [56]</td>
<td>0.07 .. 0.14</td>
<td>0.02 .. 0.08</td>
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<tr>
<td>King, Ross [57]</td>
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Textures

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<tbody>
<tr>
<td>SO(10)</td>
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<tr>
<td>Honda, Kaneko, Tanimoto [58]</td>
<td>0.08 .. 0.20</td>
<td>0.03 .. 0.15</td>
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<tr>
<td>Lebed, Martin [59]</td>
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</tr>
<tr>
<td>Bando, Kaneko, Obara, Tanimoto [60]</td>
<td>0.01 .. 0.05</td>
<td>4 (\cdot) 10^{-4} .. 0.01</td>
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<tr>
<td>Ibarra, Ross [61]</td>
<td>0.2</td>
<td>0.15</td>
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</table>

Anarchy

<table>
<thead>
<tr>
<th>Reference</th>
<th>(\sin \theta_{13})</th>
<th>(\sin^2 2\theta_{13})</th>
</tr>
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<tbody>
<tr>
<td>de Gouvêa, Murayama [66]</td>
<td>&gt; 0.1</td>
<td>&gt; 0.04</td>
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</tbody>
</table>

Renormalization group enhancement

<table>
<thead>
<tr>
<th>Reference</th>
<th>(\sin \theta_{13})</th>
<th>(\sin^2 2\theta_{13})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mohapatra, Parida, Rajasekaran [67]</td>
<td>0.08 .. 0.1</td>
<td>0.03 .. 0.04</td>
</tr>
</tbody>
</table>

January 11, 2007

Table 1: Incomplete selection of predictions for \(\theta_{13}\). The numbers should be considered as order of magnitude statements.
pessimist – “We can’t compute what $|U_{e3}|$ is – must measure it!”

(same goes for the mass hierarchy, $\delta$)
How Do We Learn More?

In order to learn more, we need more information. Any new data and/or idea is welcome, including

- searches for charged lepton flavor violation ($\mu \to e\gamma$, etc);
- searches for lepton number violation (neutrinoless double beta decay, etc);
- precision measurements of the neutrino oscillation parameters;
- searches for fermion electric/magnetic dipole moments (electron edm, muon $g - 2$, etc);
- searches for new physics at the TeV scale – we need to understand the physics at the TeV scale before we can really understand the physics behind neutrino masses (is the low-energy SUSY?, etc).
CONCLUSIONS

The venerable Standard Model has finally sprung a leak – neutrinos are not massless!

1. we have a very successful parametrization of the neutrino sector, and we have identified what we know we don’t know.

2. neutrino masses are very small – we don’t know why, but we think it means something important.

3. lepton mixing is very different from quark mixing – we don’t know why, but we think it means something important.

4. we need a minimal $\nu$SM Lagrangian. In order to decide which one is “correct” (required in order to attack 2. and 3. above) we must uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).
5. We need more experimental input – and more seems to be on the way (this is a truly data driven field right now). We only started to figure out what is going on.

6. The fact that neutrinos have mass may be intimately connected to the fact that there are more baryons than antibaryons in the Universe. How do we test whether this is correct?

7. There is plenty of room for surprises, as neutrinos are very narrow but deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are “quantum interference devices” – potentially very very sensitive to whatever else may be out there (e.g., $\Lambda \simeq 10^{14}$ GeV).

8. Finally, we need to resolve the LSND anomaly. If MiniBooNE agrees with the LSND result, life will be much more interesting!
THIS 8-LETTER PARTICLE NAMED FOR ITS LACK OF CHARGE IS BEING STUDIED BY BEAMING IT 450 MILES IN .0025 SECONDS.
Back-up Slides:

(mostly stolen from earlier presentations)

more on LSND fits, on why is Dirac neutrino masses non-trivial new physics, and some details of the low-energy seesaw.
Karmen has a similar sensitivity to \( \bar{\nu}_\mu \to \bar{\nu}_e \), but a shorter baseline \((L = 18 \text{ m})\)

Other curves are failed searches for

\( \nu_\mu \) disappearance (CCFR),

\( \bar{\nu}_e \) disappearance (Bugey), etc

Remember: \( P_{\mu e} = \sin^2 2\theta \sin^2 \left[ 1.27 \left( \frac{\Delta m^2}{\text{eV}^2} \right) \left( \frac{L}{\text{m}} \right) \left( \frac{\text{MeV}}{E} \right) \right] \)
\[ (\Delta m^2)^{\text{sol}} (\Delta m^2)^{\text{sol}} (\Delta m^2)^{\text{atm}} (\Delta m^2)^{\text{atm}} (\Delta m^2)^{\text{LSND}} \]

\[ \Rightarrow 2+2 \text{ requires large sterile effects in either solar or atmospheric oscillations, not observed} \]
\begin{equation}
\sin^2 2\theta_{\mu e} \approx 4|U_{e4}|^2|U_{\mu 4}|^2,
\end{equation}

while for disappearance searches
\begin{equation}
\sin^2 2\theta_{\alpha\alpha} \approx 4|U_{\alpha 4}|^2(1 - |U_{\alpha 4}|^2).
\end{equation}

Nontrivial constraints from short-baseline disappearance searches!...
...but one can still speak of a “best fit” region.
3+1+1 Fits Introduce an Extra $\Delta m^2$ and Effective Mixing Angle.

Can only be better than 3+1 fit (decoupling)

The fit works by “splitting” the constraints imposed by short baseline data between the two frequencies, whose effect add up at LSND.

Is this “finely tuned”? In what sense?
[Courtesy of Michel Sorel]
Another νSM

Why don’t we just enhance the fermion sector of the theory?

One may argue that it is trivial and simpler to just add

\[ \mathcal{L}_{\text{Yukawa}} = -y_{i\alpha} L^i H N^\alpha + H.c., \]

and neutrinos get a mass like all other fermions: \( m_{i\alpha} = y_{i\alpha} v \)

- Data requires \( y < 10^{-12} \). Why so small?
- Neutrinos are Dirac fermions. \( B - L \) exactly conserved.
- νSM is a renormalizable theory.

This proposal, however, violates the rules of the SM (as I defined them)!
The operator \( \frac{M_N}{2} NN \), allowed by all gauge symmetries, is absent. In order to explain this, we are forced to add a symmetry to the νSM. The simplest candidate is a global \( U(1)_{B-L} \).

\( U(1)_{B-L} \) is upgraded from accidental to fundamental (global) symmetry.
Old Standard Model, Encore

The SM is a quantum field theory with the following defining characteristics:

• Gauge Group \( SU(3)_c \times SU(2)_L \times U(1)_Y \);

• Particle Content (fermions: \( Q, u, d, L, e \), scalars: \( H \)).

Once this is specified, the SM is unambiguously determined:

• Most General Renormalizable Lagrangian;

• Measure All Free Parameters, and You Are Done.

This model has accidental global symmetries. In particular, the anomaly free global symmetry is preserved: \( U(1)_{B-L} \).
New Standard Model, Dirac Neutrinos

The SM is a quantum field theory with the following defining characteristics:

- Gauge Group \( SU(3)_c \times SU(2)_L \times U(1)_Y \);
- Particle Content (fermions: \( Q, u, d, L, e, N \), scalars: \( H \));
- Global Symmetry \( U(1)_{B-L} \).

Once this is specified, the SM is unambiguously determined:

- Most General Renormalizable Lagrangian;
- Measure All Free Parameters, and You Are Done.

Naively not too different, but nonetheless qualitatively different → enhanced symmetry sector!
The key property of $\mathcal{L}_\nu$ is that it does not lead, after EWSB, to the most general active–sterile mass-matrix:

$$
\mathcal{M} = \begin{pmatrix}
0 & \mu^T \\
\mu & M
\end{pmatrix}, \quad \mu = \text{Dirac mass matrix}; \quad M = N_i \text{ Majorana mass matrix}.
$$

In the limit $\mu \ll M$ (the seesaw limit),

$$
m_{\nu}^{\alpha\beta} = \sum_i \frac{\mu_\alpha^i \mu_\beta^i}{M_i} = \sum_i U_\alpha^i U_\beta^i m_i,
$$

where $U$ is the active neutrino mixing matrix (MNS matrix). In this case, it is easy to solve for $\mu$ in terms of active neutrino observables and $M$:\(^a\)

$$
\mu_\alpha^i = U_\alpha^i \sqrt{M_i m_i}
$$

---

\(^a\)This is a particularly simple solution. The most general solution does not lead to any qualitative changes to rest of the discussion.
Active–sterile mixing:

$$\langle \nu_\alpha | M_i \rangle \equiv \vartheta_{\alpha i} = \frac{\mu_{\alpha i}}{M_i} + O \left( \frac{\mu^2}{M^2} \right) = U_{\alpha i} \sqrt{\frac{m_i}{M_i}} + O \left( \frac{m}{M} \right),$$

such that, for example, $|U_{e4}|^2 = |U_{ej}|^2 \frac{m_j}{M_j}$, where $M_j$ is the lightest of the $M_i$.

$\left( \nu_4, \nu_5 \text{ and } \nu_6, \text{ are mass eigenstates with masses, respectively, } m_4 < m_5 < m_6. \text{ In the seesaw limit, } m_4 = \text{ lightest } M_i, m_5 = \text{ second lightest } M_i \text{ and } m_6 = \text{ heaviest } M_i, \text{ where } i = 1, 2, 3. \text{ The } i \text{ index refers to the position of } M_i \text{ in } \mathcal{M} \right)$
Other predictions: Tritium beta-decay

Heavy neutrinos participate in tritium $\beta$-decay. Their contribution can be parameterized by

$$m^2_\beta = \sum_{i=1}^{6} |U_{ei}|^2 m^2_i \simeq \sum_{i=1}^{3} |U_{ei}|^2 m^2_i + \sum_{i=1}^{3} |U_{ei}|^2 m_i M_i,$$

as long as $M_i$ is not too heavy (above tens of eV). For example, in the $3+2$ scenario of the previous slide, $m^2_\beta \simeq 0.7 \text{ eV}^2 \left( \frac{|U_{e1}|^2}{0.7} \right) \left( \frac{m_1}{0.1 \text{ eV}} \right) \left( \frac{M_1}{10 \text{ eV}} \right)$. 

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1}) \text{ eV}^2$. 
sensitivity of tritium beta decay to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]
Other predictions: **Neutrinoless Double-Beta Decay**

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \rightarrow (Z + 2)e^-e^-$. For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

$$m_{ee} = \left| \sum_{i=1}^{6} U_{ei}^2 m_i \right| \sim \left| \sum_{i=1}^{3} U_{ei}^2 m_i + \sum_{i=1}^{3} \vartheta_{ei}^2 M_i \right|.$$

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. The contribution of light and heavy neutrinos exactly cancels! This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \rightarrow m_{ee} \text{ is identically zero!}$$
(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

$M_{ee} = Q^2 \sum U_{ei}^2 \frac{m_i}{Q^2 + m_i^2}$

Region Required to explain Pulsar kicks and warm dark matter

$Q = 50$ MeV

$M_{ee}$: $\nu_{light}$

$M_{ee}$: $\nu_{light} + \nu_{heavy}$

$[\text{AdG, Jenkins, Vasudevan, hep-ph/0608147}]$
On Early Universe Cosmology / Astrophysics

A combination of the SM of particle physics plus the “concordance cosmological model” severely constrain light, sterile neutrinos with significant active-sterile mixing. Taken at face value, not only is the eV-seesaw ruled out, but so are all oscillation solutions to the LSND anomaly.

Hence, **eV-seesaw → nonstandard particle physics and cosmology.**

On the other hand...

- Right-handed neutrinos may make good warm dark matter particles.


- Sterile neutrinos are known to help out with r-process nucleosynthesis in supernovae, ...

- ...and may help explain the peculiar peculiar velocities of pulsars.