Higgs Bosons in the Next-to-Minimal Supersymmetric Model

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Outline

- **Brief Review of Fine-Tuning and Little Hierarchy Problems and Proposed Solutions**
- **Brief Review of the NMSSM**
- **NMHDECAY**
- **Evasion of Fine-Tuning and Little Hierarchy Problems In the NMSSM**
- **NMSSM LHC and Tevatron Phenomenology**

The Fine-Tuning and (Big or Little) Hierarchy Problems

SM problems:

- No explanation for the huge hierarchy of $m_{h_{\rm SM}} \ll M_{\rm P}$, as required for perturbativity of $W_L W_L \to W_L W_L$, ... If the scale of new physics is Λ, then

$$\delta m_h^2\big|_{top} \sim -\frac{N_c |\lambda_t|^2}{8\pi^2}\Lambda^2 \tag{1}$$

  and in the absence of new physics communicating to the Higgs sector before $M_{\rm P}$, $\lambda \sim M_{\rm P}$ leads to huge fine-tuning.

- No explanation for negative m^2 in Higgs potential needed for EWSB.

- Gauge coupling unification does not take place.

MSSM successes:

- Gauge coupling unification works very well (though not perfectly).

- Evolution from GUT scale to m_Z can naturally produce $m_{H_u}^2 < 0$ and, hence, EWSB.

- Dark matter.

- Low-Scale ($\lesssim \mathrm{TeV}$) Supersymmetry could in principle solve the naturalness / hierarchy problem.

$\boxed{\text{BUT}}$ there are significant problems for the MSSM

$\boxed{\text{MSSM problems:}}$

- The CP-conserving MSSM is being pushed into parameter regions characterized by substantial fine tuning and a "little" hierarchy problem (i.e. large stop masses) in order to have a heavy enough Higgs boson for consistency with LEP limits.

- A strong phase transition for baryogenesis is hard to arrange when the Higgs is heavy and the stops are heavy.

- No really attractive explanation for the μ parameter has emerged.

- **One can marginally escape all but the last of these problems if significant Higgs sector CP violation is introduced through SUSY loops.**

What are the alternatives to the MSSM?:

- **We can ignore the naturalness and hierarchy issues and accept the huge fine-tuning of "Split Supersymmetry" (Arkani-Hamed etal).**

- **We can "temporarily" solve the hierarchy problem up to $\mathbf{\Lambda \sim 10}$ TeV using Little Higgs models (Arkani-Hamed etal).**
  - **After $\mathbf{\Lambda \sim 10}$ TeV new strong interactions must enter.**
  - **Is there really consistency with precision electroweak?**
  - **A recent paper (Casas etal) argues that fine tuning in the little Higgs models is comparable to that of the SM and larger than in the MSSM.**

- **Large Extra Dimensions? (Dimopoulos, ...)**

  This remains a possibility, but could we really be so "lucky" (or unlucky, given that all physics would end at a scale of order a TeV).

- **Higgsless Models? (Terning etal)**

– Not only do we need extra dimensions, RS warping, and so forth, but we also need special ($v \to \infty$) boundary conditions on the TeV brane.
– Lots of special arrangements regarding fermions are needed for consistency with precision electroweak.

- **The NMSSM?**

  – We will show that the CP-conserving NMSSM can solve all these problems.
  We will show that the NMSSM can have a very low-level of fine-tuning, small little hierarchy, good electroweak baryogenesis,...
  Thus, **is it not time to adopt the NMSSM as the baseline supersymmetric model?**
  – The NMSSM phenomenology is considerably richer than that of the MSSM in many important ways. The focus here is on **Higgs physics**.

There has been a huge amount of work on the NMSSM. The new contributions discussed here clarify just how completely the fine-tuning and little hierarchy problems can be resolved and what the preferred scenarios imply regarding phenomenology at colliders (especially Tevatron and LHC).

A bibliography of the important NMSSM references appears below and will be appropriately cited in what follows.

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The NMSSM

- The Next to Minimal Supersymmetric Standard Model (NMSSM [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13]) provides a very elegant solution to the μ problem of the MSSM via the introduction of a singlet superfield $\widehat{S}$.

  For the simplest possible scale invariant form of the superpotential, the scalar component of $\widehat{S}$ acquires naturally a vacuum expectation value of the order of the SUSY breaking scale, giving rise to a value of μ of order the electroweak scale.

- The NMSSM is actually the simplest supersymmetric extension of the standard model in which the electroweak scale originates from the SUSY breaking scale only.

- The NMSSM preserves all the successes of the MSSM (gauge coupling unification, RGE EWSB, dark matter, . . .).

Hence, the phenomenology of the NMSSM deserves to be studied at least as fully and precisely as that of the MSSM.

Its particle content differs from the MSSM by the addition of one CP-even and one CP-odd state in the neutral Higgs sector (assuming CP conservation), and one additional neutralino. Thus, the physics of the Higgs bosons – masses, couplings and branching ratios [1, 7, 8, 9, 10, 11, 12, 13] can differ significantly from the MSSM.

I will be following the conventions of Ellwanger, Hugonie, JFG [14]. The NMSSM parameters are as follows.

a) Apart from the usual quark and lepton Yukawa couplings, the scale invariant superpotential is

$$\lambda \ \widehat{S}\widehat{H_u}\widehat{H_d} + \frac{\kappa}{3} \ \widehat{S}^3 \tag{2}$$

depending on two dimensionless couplings λ, κ beyond the MSSM. (Hatted capital letters denote superfields, and unhatted capital letters will denote their scalar components).

b) The associated trilinear soft terms are

$$\lambda A_\lambda S H_u H_d + \frac{\kappa}{3} A_\kappa S^3 \,. \tag{3}$$

c) The final two input parameters (at tree-level) are

$$\tan\beta = \langle H_u \rangle / \langle H_d \rangle \ , \ \mu_{\rm eff} = \lambda \langle S \rangle \ . \tag{4}$$

These, along with M_Z, can be viewed as determining the three SUSY breaking masses squared for H_u, H_d and S through the three minimization equations of the scalar potential.

Thus, as compared to three independent parameters in the Higgs sector of the MSSM (often chosen as μ, $\tan\beta$ and M_A, before m_Z is input), the Higgs sector of the NMSSM is described by the six parameters

$$\lambda \ , \ \kappa \ , \ A_\lambda \ , \ A_\kappa, \ \tan\beta \ , \ \mu_{\rm eff} \ . \tag{5}$$

We will choose sign conventions for the fields such that λ and $\tan\beta$ are positive, while κ, A_λ, A_κ and $\mu_{\rm eff}$ should be allowed to have either sign.

In addition, values for the gaugino masses and of the soft terms related to the squarks and sleptons that contribute to the radiative corrections in the Higgs sector and to the Higgs decay widths must be input.

NMHDECAY

We (Ellwanger, Hugonie, JFG [14]) have developed the NMSSM analogue of HDECAY. We provide two forms of the NMHDECAY program:

- **NMHDECAY_SLHA.f** — for study of one parameter point in the SLHA conventions for particle labeling etc. familiar to experimentalists;
- **NMHDECAY_SCAN.f** — designed for general phenomenological work including scanning over ranges of NMSSM parameters.

The programs, and associated data files, can be downloaded from the two web pages:

`http://www.th.u-psud.fr/NMHDECAY/nmhdecay.html`

`http://higgs.ucdavis.edu/nmhdecay/nmhdecay.html`

The web pages provide simplified descriptions of the programs and instructions on how to use them. The programs will be updated to include additional features and refinements in subsequent versions. We welcome comments with regard to improvements that users would find helpful.

NMHDECAY performs the following tasks:

1. It computes the masses and couplings of all physical states in the Higgs, chargino and neutralino sectors.

   Error messages are produced if a Higgs or squark mass squared is negative.

2. It computes the branching ratios into two particle final states (including charginos and neutralinos — decays to squarks and sleptons will be implemented in a later release) of all Higgs particles.

3. It checks whether the Higgs masses and couplings violate any bounds from negative Higgs searches at LEP, including many quite unconventional channels that are relevant for the NMSSM Higgs sector.

   It also checks the bound on the invisible Z width (possibly violated for light neutralinos).

   In addition, NMHDECAY checks the bounds on the lightest chargino and on neutralino pair production.

   Corresponding warnings are produced in case any of these phenomenological constraints are violated.

4. It checks whether the running Yukawa couplings encounter a Landau singularity below the GUT scale.

   A warning is produced if this happens.

5. Finally, NMHDECAY checks whether the physical minimum (with all vevs non-zero) of the scalar potential is deeper than the local unphysical minima with vanishing $\langle H_u \rangle$ or $\langle H_d \rangle$.

   If this is not the case, a warning is produced.

Thus, by processing a possible NMSSM parameter choice through NMHDECAY, we can be certain of the associated Higgs phenomenology and of the fact that the parameter choice does not violate LEP and other experimental limits.

Fine Tuning

w. Radovan Dermisek

The MSSM

Sample discussions of the issues appear in the papers cited in [16]. A typical and useful discussion for the MSSM is that given by Kane and King.

We have repeated the MSSM analysis allowing substantial freedom for soft parameters (that might in principle have led to the possibility of smaller fine-tuning than found in the above references).

The basic fine-tuning measure is

$$F = \mathrm{Max_a} \left| \frac{d \log m_Z}{d \log a} \right| \tag{6}$$

where the parameters a are the GUT scale soft-SUSY-breaking parameters and the μ parameter.

The essence of the fine-tuning problem is revealed by looking at how the soft-SUSY-breaking parameters at the GUT scale enter into the m_Z-scale

relation

$$\frac{1}{2}m_Z^2 = -\mu^2 + \frac{m_{H_d}^2 - t_\beta^2 m_{H_u}^2}{t_\beta^2 - 1} \,. \tag{7}$$

RGE's are used to related the above quantities to GUT scale parameters. One finds huge coefficients in front of the GUT scale soft-SUSY parameters, especially $M_3(GUT)$, $\mu(GUT)$, $m_{H_u}^2(GUT)$ and $m_{H_d}^2(GUT)$.

In our approach:

- We choose m_Z-scale values for all the squark soft masses squared, the gaugino masses, $M_{1,2,3}(m_Z)$, $A_t(m_Z)$ and $A_b(m_Z)$ (with no requirement of universality at the GUT scale).

- We also choose m_Z-scale values for $\tan\beta$, μ and m_A.

  These uniquely determine the soft-SUSY-breaking parameter $B_\mu(m_Z)$ (which must be non-zero for $m_A^2 > 0$.)

- The vevs $h_u \equiv \langle H_u \rangle$ and $h_d \equiv \langle H_d \rangle$ at scale m_Z are fixed by $\tan\beta = h_u/h_d$ and $m_Z^2 = \overline{g}^2(h_u^2 + h_d^2)$ (where $\overline{g}^2 = g^2 + g'^2$).

- Finally, $m_{H_u}^2(m_Z)$ and $m_{H_d}^2(m_Z)$ are determined from the two potential minimization conditions.

- We then evolve all parameters to the MSSM GUT scale (including μ and B_μ).

- Next, we shift each of the GUT-scale parameters in turn, evolve back down to scale m_Z, and reminimize the Higgs potential using the shifted values of μ, B_μ, $m_{H_u}^2$ and $m_{H_d}^2$.

  This gives new values for h_u and h_d from which we compute a new value for m_Z (and $\tan\beta$).

Results will be presented for $\tan\beta(m_Z) = 10$, $M_{1,2,3}(m_Z) = 100$, 200, $300~\mathrm{GeV}$.

- We scan randomly over $A_t(m_Z)$, $A_b(m_Z)$ and 3rd generation squark and slepton soft masses-squared above $(200~\mathrm{GeV})^2$, as well as over $|\mu(m_Z)| \geq 100~\mathrm{GeV}$, $\mathrm{sign}(\mu) = \pm$ and over $m_A > 100~\mathrm{GeV}$.

- In the left plot of Fig. 1, we plot F as a function of the mean stop mass $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$, which enters into the computation of the radiative correction to the SM-like light Higgs mass m_h.

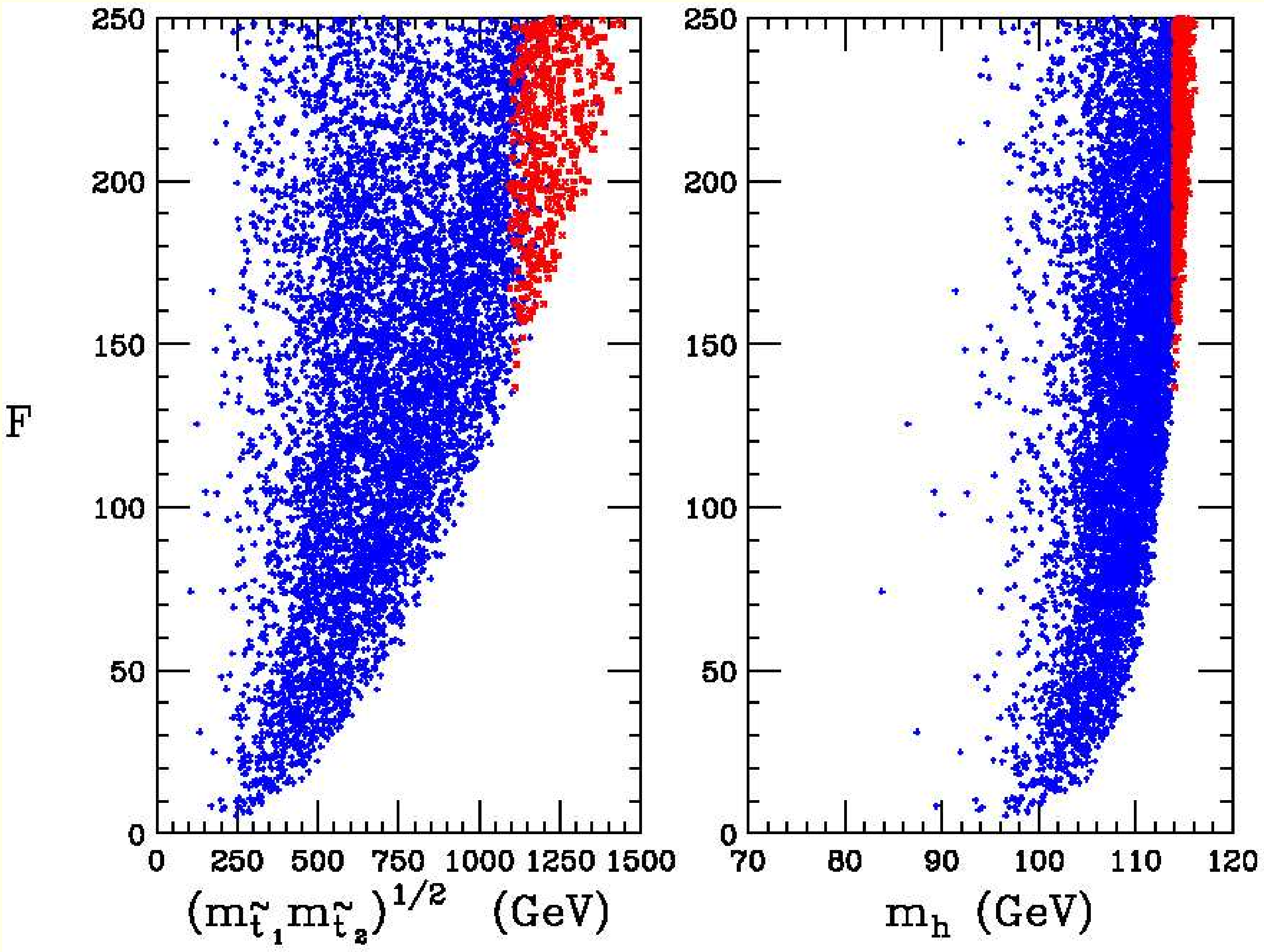

Figure 1: Left: the fine-tuning measure F in the MSSM is plotted vs. $\sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$, without regard to LEP constraints on m_h. Right: F is plotted vs. m_h for all scanned points. Points plotted as $+$'s ($\times$'s) have $m_h < 114$ GeV ($m_h \geq 114$ GeV) and are excluded (allowed) by LEP data.

- **Very modest values of F (of order $F \sim 5$) are possible for $m_h < 114~\mathrm{GeV}$ but the smallest F value found for $m_h \geq 114~\mathrm{GeV}$ is of order $F \sim 140$.**

- **The very rapid increase of the smallest achievable F with m_h is illustrated in the right plot of Fig. 1.**

  This is the essence of the current fine-tuning problem for the CP-conserving MSSM.

- **Also, to achieve $m_h > 114~\mathrm{GeV}$, $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \geq 1.1~\mathrm{TeV}$ is required, an indicator of the little hierarchy problem.**

  If (as in the talk by Wagner) one chooses small $m_{\widetilde{t}_1}$ for a strong phase transition for baryogenesis, this means that $m_{\widetilde{t}_2}$ must be very large for $m_h > 114~\mathrm{GeV}$.

The NMSSM

We now contrast this to the NMSSM situation. Here, the computation of finetuning for m_Z^2 is much more complicated.

Earlier discussions of fine-tuning in the NMSSM have appeared in refs. [2] and [3], but we claim they missed the most interesting part of parameter space with the smallest finetuning.

We start with

$$\begin{aligned} V &= \lambda^2(h_u^2 s^2 + h_d^2 s^2 + h_u^2 h_d^2) + \kappa^2 s^4 - 2\lambda\kappa h_u h_d s^2 - 2\lambda A_\lambda\, h_u h_d s \\ &\quad + \frac{2}{3}\kappa A_\kappa s^3 + m_{H_u}^2 h_u^2 + m_{H_d}^2 h_d^2 + m_S^2 s^2 + \frac{1}{4}\overline{g}^2 (h_u^2 - h_d^2)^2 \,. \end{aligned} \tag{8}$$

In the above, h_u and h_d are the vevs of the up and down type Higgs fields and s is the vev of the singlet Higgs field. (We have defined $\overline{g}^2 \equiv \frac{1}{2}(g_2^2 + g'^2)$ so that $m_Z^2 = \overline{g}^2(h_u^2 + h_d^2)$.)

Once λ, A_λ, κ, A_κ, s and $\tan\beta = h_u/h_d$ have been chosen (with $h_u^2 + h_d^2 = v^2$), one must then solve the minimization equations

$$\frac{\partial V}{\partial h_u} = 0, \quad \frac{\partial V}{\partial h_d} = 0, \quad \frac{\partial V}{\partial s} = 0 \tag{9}$$

for the soft masses squared $m^2_{H_u}$, $m^2_{H_d}$ and m^2_S and explore combinations thereof for reexpressing the minimization conditions. One finds

$$m^2_{Hu} = \frac{1}{2h_u}\left(g^2h_d^2h_u - g^2h_u^3 - 2h_d^2h_u\lambda^2 + 2A_\lambda h_d\lambda s + 2h_d\kappa\lambda s^2 - 2h_u\lambda^2 s^2\right) \tag{10}$$

$$m^2_{H_d} = \frac{1}{2h_d}\left(g^2h_dh_u^2 - g^2h_d^3 - 2h_dh_u^2\lambda^2 + 2A_\lambda h_u\lambda s + 2h_u\kappa\lambda s^2 - 2h_d\lambda^2 s^2\right) \tag{11}$$

$$m^2_S = \frac{1}{s}\left(\lambda A_\lambda h_dh_u + 2h_dh_u\kappa\lambda s - h_d^2\lambda^2 s - h_u^2\lambda^2 s - \kappa A_\kappa s^2 - 2\kappa^2 s^3\right) \tag{12}$$

Defining $\mu_{\rm eff} = \lambda s$, it is easy to eliminate terms linear in s to find that

$$\frac{1}{2}m_Z^2 = -\mu^2_{\rm eff} + \frac{m^2_{H_d} - \tan^2\beta m^2_{H_u}}{\tan^2\beta - 1}\,. \tag{13}$$

However, $\mu_{\rm eff}$ is not a fundamental parameter in this case. Taking

$(\kappa\lambda/\tan\beta-\lambda^2)$(11) $-(\kappa\lambda\tan\beta-\lambda^2)$ (10), we obtain a second equation

$$\begin{aligned}
&\kappa\lambda\left(\frac{1}{\tan\beta}m^2_{H_d}-m^2_{H_u}\tan\beta\right)-\lambda^2\left(m^2_{H_d}-m^2_{H_u}\right)\\
&=\frac{1}{2}m_Z^2\frac{\tan^2\beta-1}{\tan^2\beta+1}\left[\kappa\lambda\left(\frac{1}{\tan\beta}+\tan\beta\right)-2\lambda^2+\frac{2}{g^2}\lambda^4\right]\\
&\qquad+\mu_{eff}A_\lambda\lambda^2\left(\frac{1}{\tan\beta}-\tan\beta\right)
\end{aligned}\tag{14}$$

Let's make it simpler by defining

$$a = -\frac{1}{2}\frac{\tan^2\beta-1}{\tan^2\beta+1}\left[\kappa\lambda\left(\frac{1}{\tan\beta}+\tan\beta\right)-2\lambda^2+\frac{2}{g^2}\lambda^4\right]\tag{15}$$

$$b = \frac{1}{\tan\beta}k\lambda\left(m^2_{Hd}-m^2_{Hu}\tan^2\beta\right)-\lambda^2\left(m^2_{Hd}-m^2_{Hu}\right)\tag{16}$$

$$c = A_\lambda\lambda^2\left(\frac{1}{\tan\beta}-\tan\beta\right)\tag{17}$$

so that it is simply

$$aM_Z^2+b=c\mu_{eff}.\tag{18}$$

Squaring this equation and plugging in μ_{eff} from Eq. (13) we can eliminate μ_{eff} completely, and we obtain a quadratic equation for M_Z^2 with coefficients given in terms of soft susy breaking parameters:

$$AM_Z^4 + BM_Z^2 + C = 0, \tag{19}$$

where

$$\begin{aligned} A &= a^2 &(20)\\ B &= 2ab + c^2/2 &(21)\\ C &= b^2 + c^2\frac{m_{Hd}^2 - m_{Hu}^2\tan^2\beta}{1-\tan^2\beta}. &(22) \end{aligned}$$

This is the equivalent formula to that in the case of the MSSM. A, B, and C can be expressed in terms of SSB parameters at the GUT scale; the only difference is that it is a quadratic equation. Therefore there are two solutions:

$$m_Z^2 = \frac{1}{2A}\left(-B \pm \sqrt{B^2 - 4AC}\right). \tag{23}$$

Only one applies for any given set of parameter choices.

Small fine tuning is typically achieved when $4AC \ll B^2$ and derivatives of m_Z^2 with respect to a GUT scale parameter tend to cancel between the $-B$ and $+\sqrt{B^2 - 4AC}$ ($-\sqrt{B^2 - 4AC}$) for $B > 0$ (for $B < 0$).

To explore fine tuning, we proceed as follows.

- At scale m_Z, we fix $\tan\beta$ and scan over values of $\lambda \leq 0.5$ ($\lambda \lesssim 0.7$ is required for perturbativity up to the GUT scale), $|\kappa| \leq 0.3$, $\mathrm{sign}(\kappa) = \pm$ and $100~\mathrm{GeV} \leq |\mu_{\mathrm{eff}}| \leq 1.5~\mathrm{TeV}$, $\mathrm{sign}(\mu_{\mathrm{eff}}) = \pm$.

- We choose m_Z-scale values for the soft-SUSY-breaking parameters A_λ, A_κ, $A_t = A_b$, M_1, M_2, M_3, m_Q^2, m_U^2, m_D^2, m_L^2, and m_E^2, all of which enter into the evolution equations.

- We process each such choice through NMHDECAY to check that the scenario satisfies all theoretical and available experimental constraints (ignoring constraints on $m_{\widetilde{t}_1}$).

- For accepted cases, we then evolve to determine the GUT-scale values of all the above parameters.

- The fine-tuning derivative for each parameter is determined by:

- shifting the GUT-scale value for that parameter by a small amount,
- evolving all parameters back down to m_Z,
- redetermining the potential minimum (which gives new values h'_u and h'_d and s')
- and finally computing a new value for m_Z^2 using $m_Z'^2 = \overline{g}^2(h_u'^2 + h_d'^2)$.

Results for $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV and randomly chosen values for the soft-SUSY-breaking parameters listed earlier are displayed in Fig. 2.

- We see that F as small as $F \sim 5.5$ can be achieved for $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 250 \div 400$ GeV.

- In the figure, the $+$ points have $m_{h_1} < 114$ GeV and escape LEP exclusion by virtue of the dominance of $h_1 \to a_1 a_1$ decays, a channel to which LEP is less sensitive as compared to the traditional $h_1 \to b\overline{b}$ decays.

- Points marked by $\times$ have $m_{h_1} > 114$ GeV and will escape LEP exclusion regardless of the dominant decay mode.

  For most of these latter points $h_1 \to b\overline{b}$ decays are dominant, even if somewhat suppressed; $h_1 \to a_1 a_1$ decays dominate for a few.

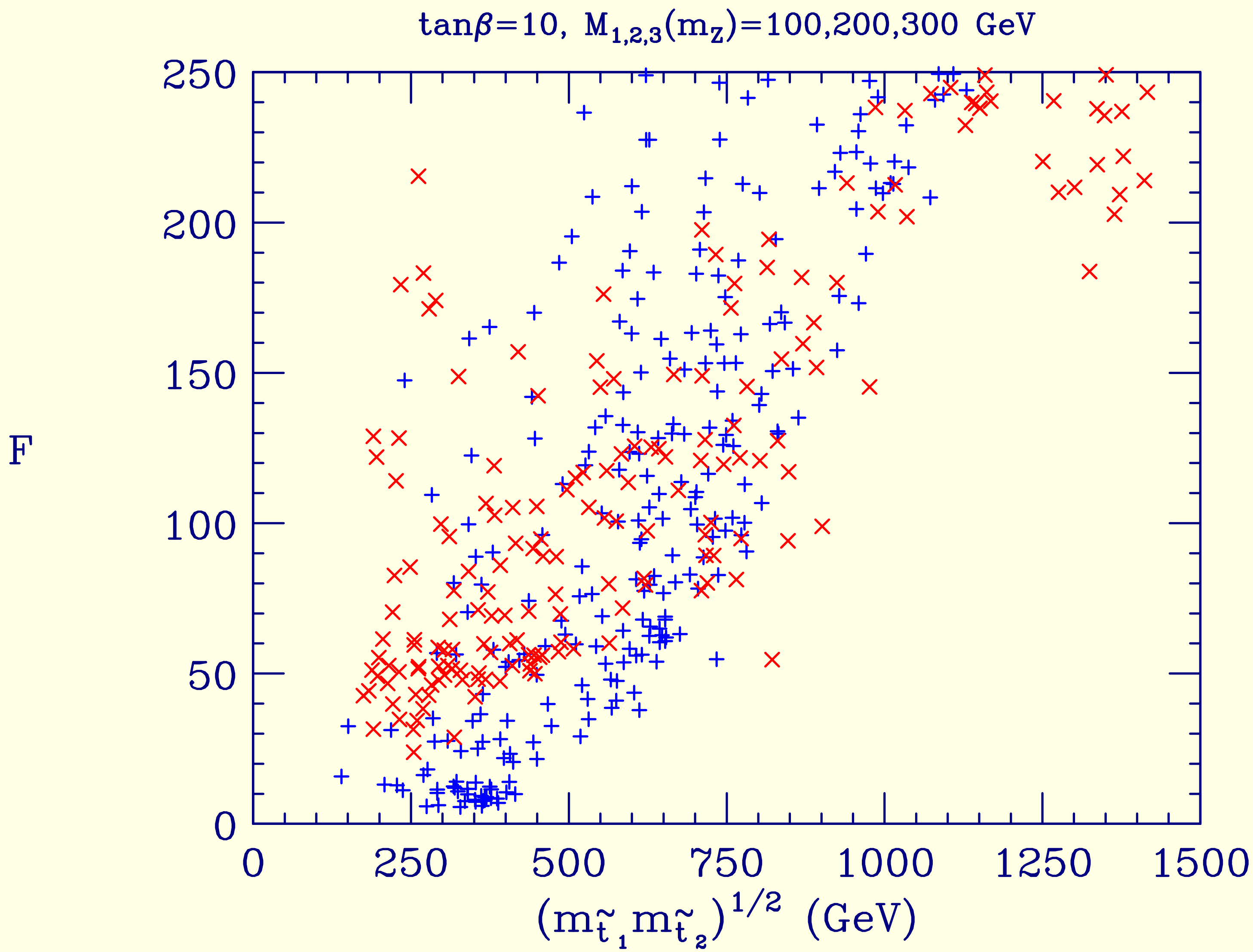

Figure 2: For the NMSSM, we plot the fine-tuning measure F vs. $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ for NMHDECAY-accepted scenarios with $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Points marked by '$+$' ('$\times$') escape LEP exclusion primarily due to dominance of $h_1 \to a_1 a_1$ decays (due to $m_{h_1} > 114$ GeV).

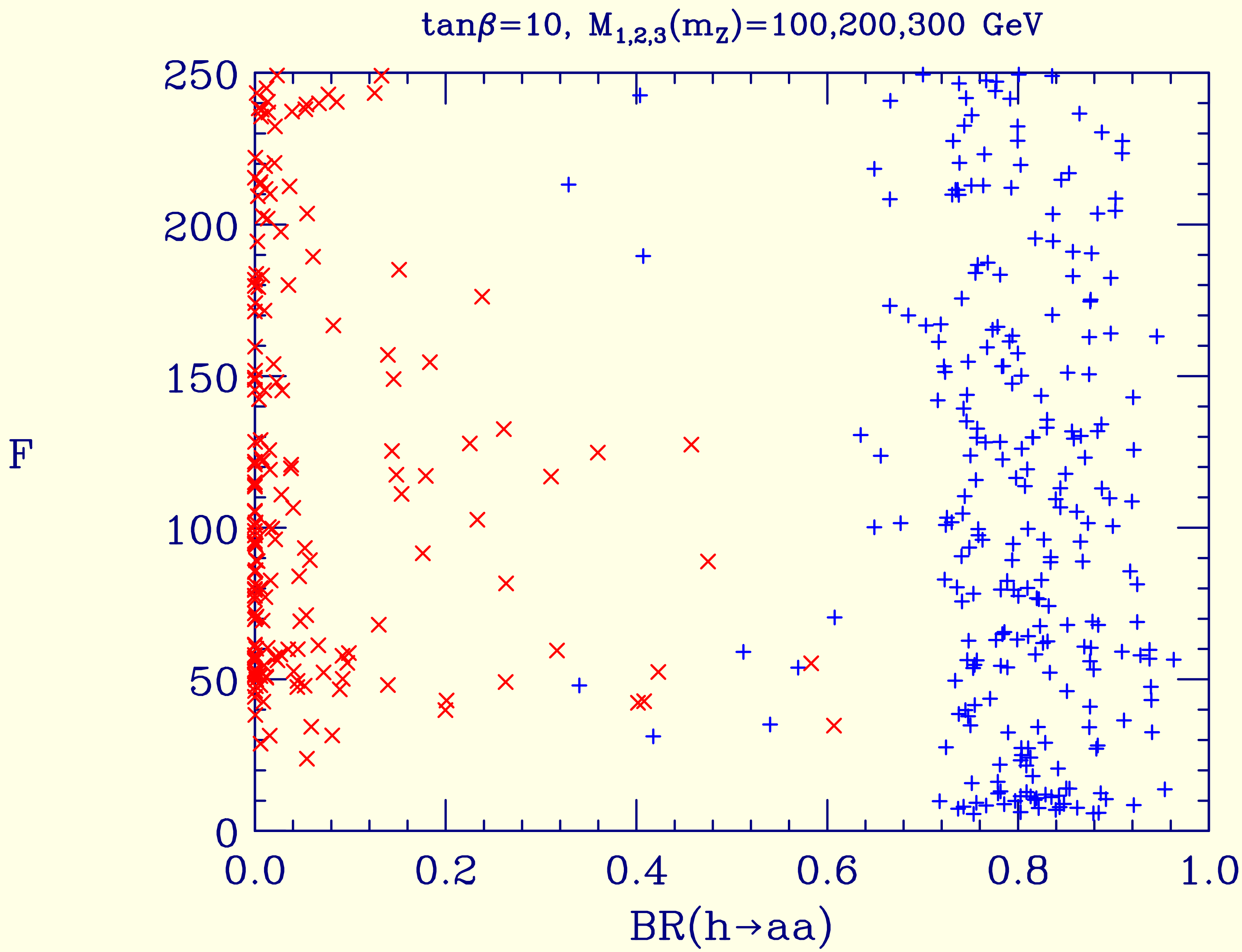

Figure 3: For the NMSSM, we plot the fine-tuning measure F vs. $BR(h_1 \to a_1 a_1)$ for NMHDECAY-accepted scenarios with $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Point notation as in Fig. 2.

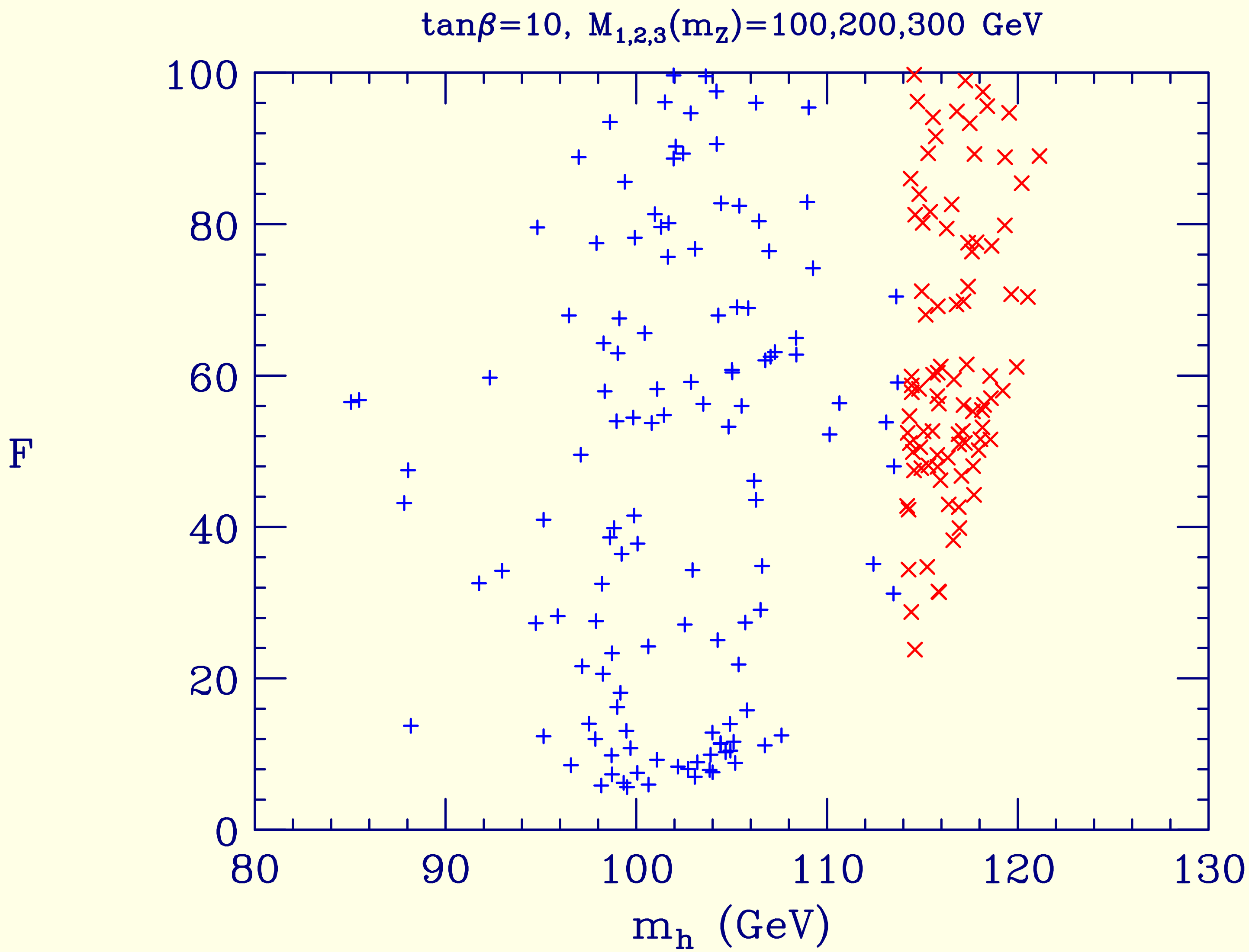

Figure 4: For the NMSSM, we plot the fine-tuning measure F vs. m_{h_1} for NMHDECAY-accepted scenarios with $\tan\beta = 10$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. Point notation as in Fig. 2.

Additional Remarks

- For both classes of points, the h_1 has fairly SM-like couplings.

- The minimum F increases rapidly with m_{h_1} as seen in Fig. 4.

  The lowest F values are only achieved for $m_{h_1} \lesssim 105$.

  However, even for $m_{h_1} \geq 114~\mathrm{GeV}$, the lowest F value of $F \sim 24$ is far below that attainable for $m_h \geq 114~\mathrm{GeV}$ in the MSSM.

- A small value for $A_\kappa(m_Z)$ (typically of order a few GeV) appears to be essential to achieve small F.

  First, the naturally less fine-tuned values of $m_{h_1} < 114~\mathrm{GeV}$ can be allowed by virtue of m_{a_1} being small enough [as possible for small $A_\kappa(m_Z)$] that $h_1 \to a_1 a_1$ decays can dominate.

  Second, small F is frequently (nearly always) achieved for $m_{h_1} < 114~\mathrm{GeV}$ ($m_{h_1} \geq 114~\mathrm{GeV}$) via the cancellation mechanism noted earlier, where $4AC \ll B^2$, and this mechanism generally works mainly for small A_κ.

Indeed, there are many phenomenologically acceptable parameter choices with $m_{h_1} > 114~\mathrm{GeV}$ that have large A_κ, but these all also have very large F.

- For lower $\tan\beta$ values such as $\tan\beta = 3$, extremely large $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ is required for $m_h > 114~\mathrm{GeV}$ in the MSSM, leading to extremely large F.

  Results in the NMSSM for $\tan\beta = 3$ are plotted in Fig. 5 for $M_{1,2,3}(m_Z) = 100, 200, 300~\mathrm{GeV}$ and scanning as in the $\tan\beta = 10$ case.

  We see that $F \sim 15$ is achievable for $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}} \sim 300~\mathrm{GeV}$. No points with $m_{h_1} > 114~\mathrm{GeV}$ were found.

  All the plotted points escape LEP limits because of the dominance of the $h_1 \to a_1 a_1$ decay.

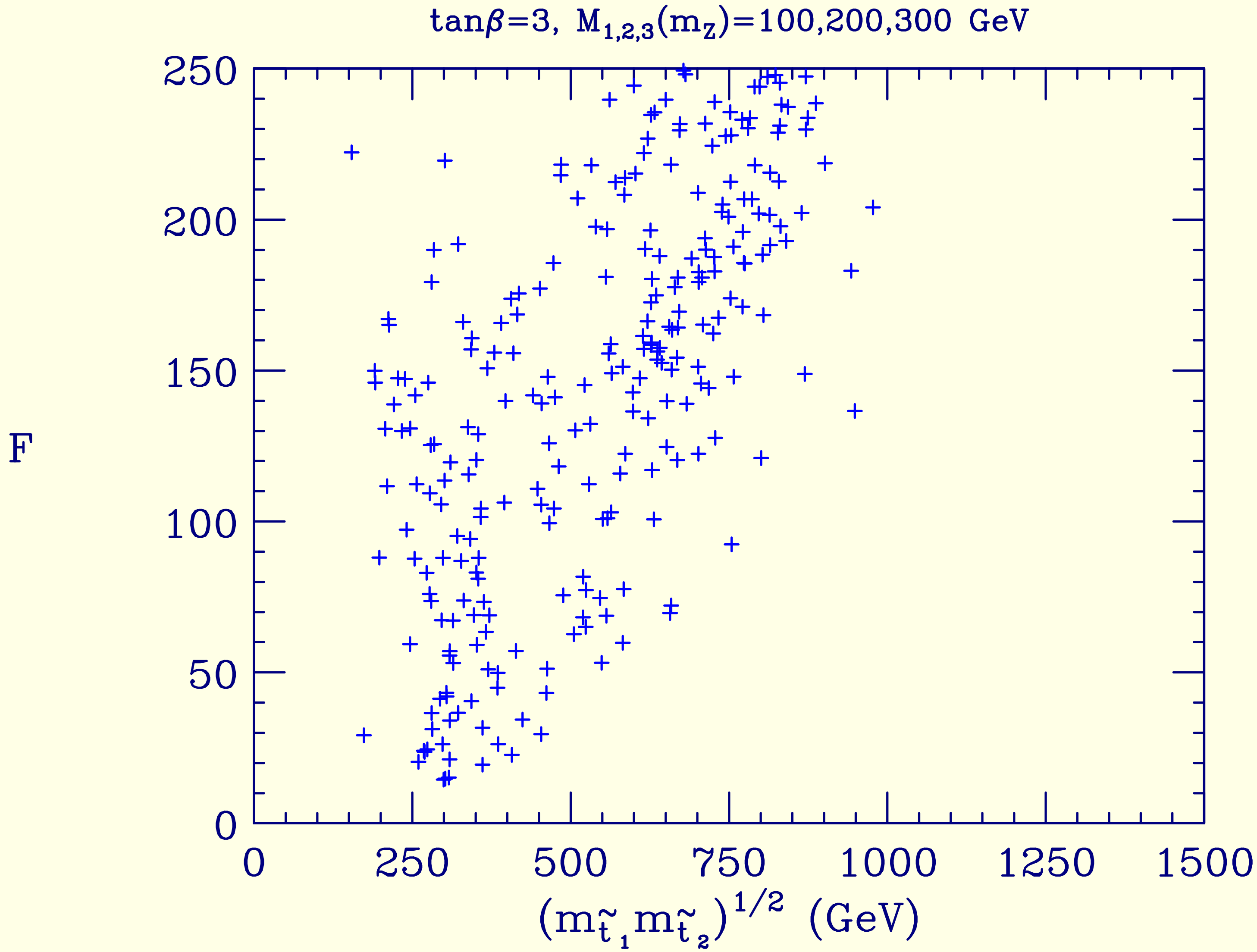

Figure 5: For the NMSSM, we plot the fine-tuning measure F vs. the mass of the lightest stop for NMHDECAY-accepted scenarios with $\tan\beta = 3$ and $M_{1,2,3}(m_Z) = 100, 200, 300$ GeV. There are no points with $m_{h_1} \geq 114$ GeV.

- **For very large $\tan\beta$ ($e.g.$ $\tan\beta \sim 50$), it is possible to obtain a light Higgs mass $> 114~\mathrm{GeV}$ with relatively small $\sqrt{m_{\widetilde{t}_1} m_{\widetilde{t}_2}}$ in the MSSM as well as in the NMSSM. We have not yet studied finetuning at very large $\tan\beta$ in either model.**

- **For $M_3(m_Z) \sim 700~\mathrm{GeV}$ (leading to unified GUT scale gaugino masses for $M_1 = 100~\mathrm{GeV}$ and $M_2 = 200~\mathrm{GeV}$) and $\tan\beta = 10$, the smallest F we find is of order $F \sim 40$.**

  This is starting to represent significant fine tuning and suggests that we should adopt smaller M_1 and M_2 at scale m_Z (but $M_2 \lesssim 120~\mathrm{GeV}$ leads to too light a chargino).

  Of course the corresponding MSSM F is huge for $M_3 = 700~\mathrm{GeV}$.

Hadron Collider Implications

- The importance of Higgs to Higgs decays was first realized at Snowmass 1996 (JFG, Haber, Moroi [19]) and was later elaborated on in a paper by Dobrescu and Matchev [25]. Detailed NMSSM scenarios were first studied in several papers by Ellwanger, Hugonie and JFG [26, 27].

  In the latter work, we found that all NMSSM parameter choices for which discovery of even one NMSSM Higgs boson is not possible at the LHC in the "standard modes"

  1) $gg \to h/a \to \gamma\gamma$;
  2) associated Wh/a or $t\bar{t}h/a$ production with $\gamma\gamma\ell^{\pm}$ in the final state;
  3) associated $t\bar{t}h/a$ production with $h/a \to b\bar{b}$;
  4) associated $b\bar{b}h/a$ production with $h/a \to \tau^+\tau^-$;
  5) $gg \to h \to ZZ^{(*)} \to$ 4 leptons;
  6) $gg \to h \to WW^{(*)} \to \ell^+\ell^-\nu\bar{\nu}$;
  7) $WW \to h \to \tau^+\tau^-$;
  8) $WW \to h \to WW^{(*)}$.
  9) $WW \to h \to invisible$.

are such that there is a SM-like Higgs h_H which decays to a pair of lighter Higgs, $h_L h_L$.

In general, the h_L decays to $b\bar{b}$ and $\tau^+\tau^-$ (if $m_{h_L} > 2m_b$) or to jj and $\tau^+\tau^-$ (if $2m_\tau < m_{h_L} < 2m_b$) or, as unfortunately still possible, to jj if $m_{h_L} < 2m_\tau$.

In the first two cases, a possibly viable LHC signal then comes [26, 27] from $WW \to h_H \to h_L h_L \to jj\tau^+\tau^-$ in the form of a bump in the $M_{jj\tau^+\tau^-}$ reconstructed mass distribution. It is not a wonderful signal, but it is a signal.

For most such cases, h_L is actually the lightest CP-odd scalar a_1 and h_H is the lightest or 2nd lightest CP-even scalar, h_1 or h_2.

Experimentalists should work hard to see if our crude estimates that there would be an observable signal at the LHC will survive reality.

- As regards the cases where $m_{a_1} < 2m_\tau \Rightarrow a_1 \to c\bar{c}, s\bar{s}, gg$, these can often evade LEP limits (but we are pushing the LEP people for improvements).

  It will be very difficult extract a signal in these cases where neither b nor

τ tagging is relevant. The only hope would be jet counting, but QCD backgrounds are probably enormous.

Since the $b\bar{b}$ coupling of these very light a_1's is not enhanced significantly (typically), there are no reliable exclusions coming from Υ or $B_{s,d}$ decays. We believe there is simply too much model dependence in the theory for such decays, although we would be happy to be persuaded otherwise.

- There are also cases in which $h_H = h_2$ and $h_L = h_1$, $m_{h_1} > 2m_b$, but yet $h_1 \to c\bar{c}, gg$ decays are completely dominant — parameters are chosen near a special region where the h_1 decouples from leptons and down-type quarks.

  (But, we have not found such cases to have small fine-tuning.)

  For these scenarios, it is very hard to imagine a technique for extracting a signal at a hadron collider.

- **Question:** Can the Tevatron be sensitive to the Higgs-to-Higgs decay scenarios?
  - We have started to look at the $gg \to h_1 \to a_1 a_1 \to 4\tau$ mode. (McElrath, Chertok, Conway, JFG), assuming $2m_\tau < m_{a_1} < 2m_b$.

- Assuming $m_{h_1} \sim 100~\mathrm{GeV}$, the a_1's will be highly boosted and the τ pairs emerging from each a_1 will tend to be pretty collinear.
- We find that the CDF $\mu+$jet trigger will be $> 50\%$ efficient in tagging the events.
- As a very first thing, we have looked at:
  - the mass peak reconstructed from the visible decay products (one of which is the trigger μ) of the two a_1's.
  - the mass peak of the visible tracks coming from each of the two a_1's.

  There are peaks. But, what are the backgrounds.
- Our procedure will be to pass the signal through Conway's simplified parameterized detector simulation program and see if the peaks survive after identifying 2τ-like events using an analogue of the current τ trigger (adjusted to account for the fact that there are two collinear τ's).
- Then, we will look at existing events from CDF to see how big the backgrounds are, and then refine to see if the predicted signal might possibly be seen with enough data.

- On the next page, I show some plots.

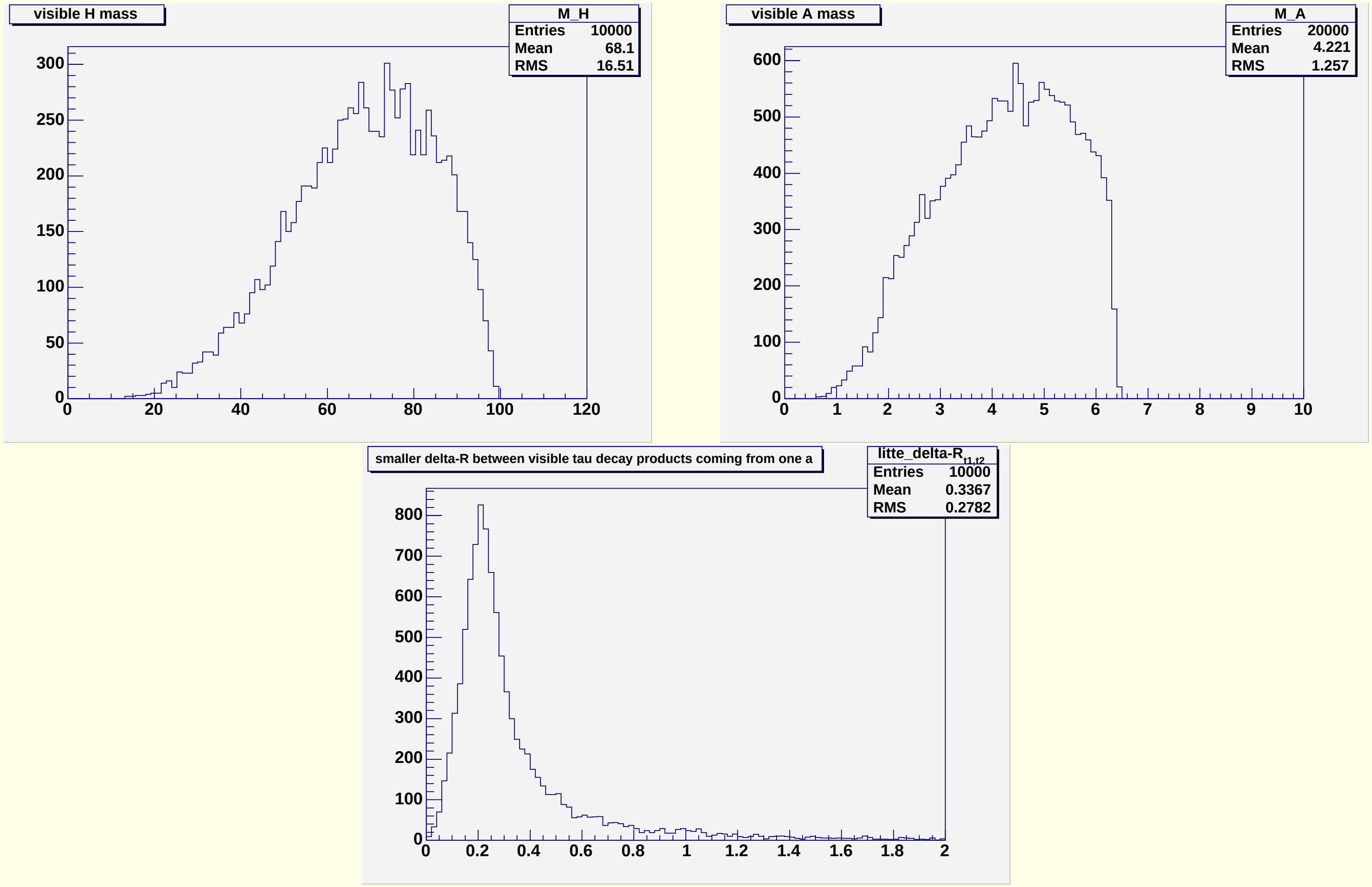

Figure 6: Left: m_{h_1} from visible decay products. Right: m_{a_1} from visible decay products. Bottom: angular separation ΔR between two τ's from same a_1.

- **However, there are not many events. One must first of all pay the price of $BR(\tau \to \mu\nu\bar{\nu}) \sim 0.17$ for the trigger.**

Cross Section Reality Check

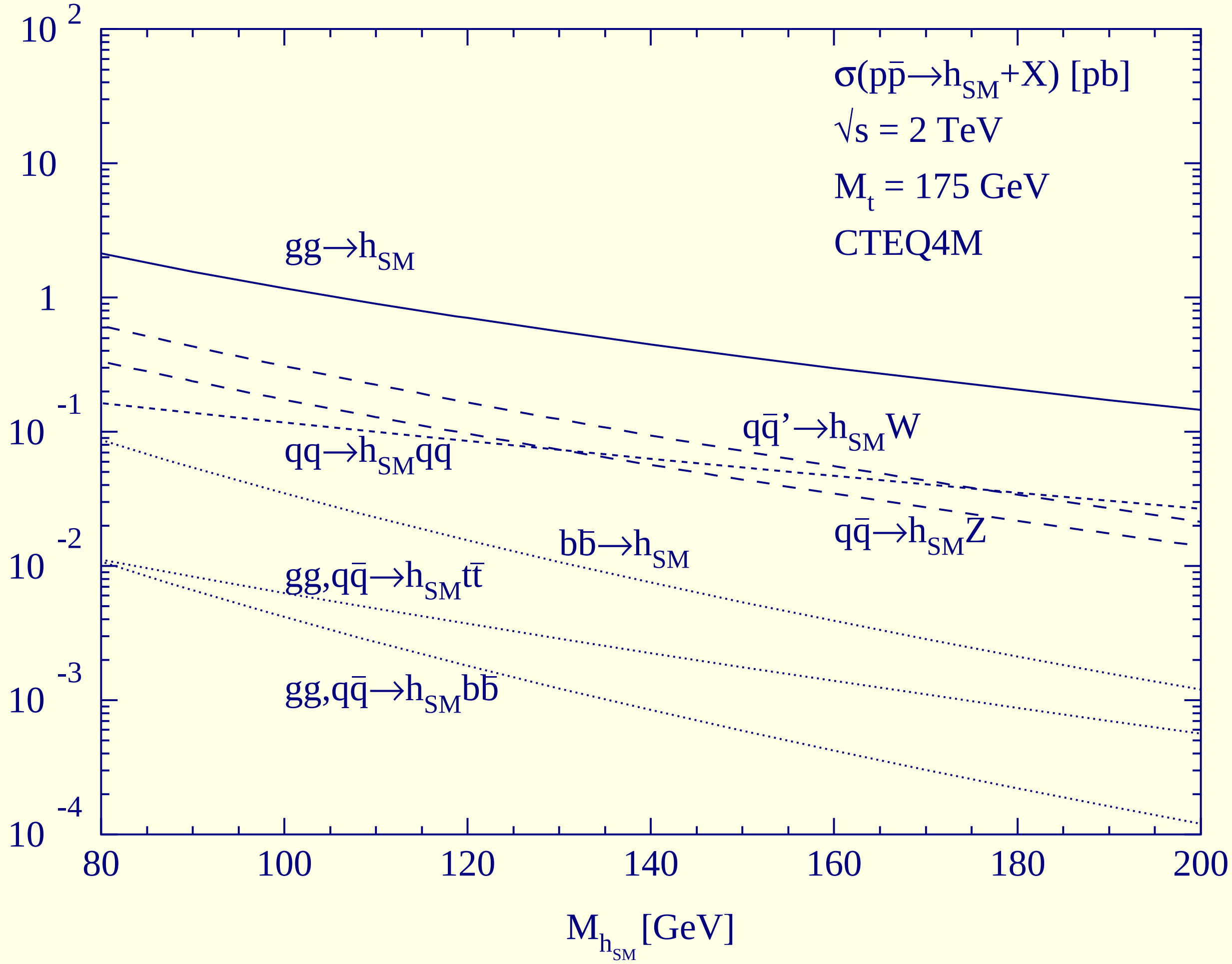

Figure 7: Various cross sections at the Tevatron for a SM Higgs boson. Note the small size of WW fusion at low m_h. Better is Wh associated production,

Conclusions

- The NMSSM is an attractive model, and the $h \to aa$ decay modes have significantly nice features with regard to finetuning and electroweak baryogenesis.
- If low fine-tuning is imposed for an acceptable model, we should expect:
  - a $m_{h_1} \sim 100~\mathrm{GeV}$ Higgs decaying via $h_1 \to a_1 a_1$
    Higgs detection will be quite challenging at a hadron collider (but not at the ILC using the missing mass $e^+e^- \to ZX$ method of looking for a peak in M_X or using $\gamma\gamma \to h_1 \to a_1 a_1$ signals as examined by Szleper and JFG [28]);
  - the very smallest F values are attained when:
    * h_2 and h_3 have "moderate" mass, i.e. in $300~\mathrm{GeV}$ to $700~\mathrm{GeV}$ mass range;
    * the a_1 mass is typically in the $5~\mathrm{GeV}$ to $20~\mathrm{GeV}$ range (but with a few exceptions) and the a_1 is always mainly singlet.
  - light stops;
  - a light gluino, and by implication a light wino and bino;
  - an LSP that is largely bino in the low fine-tuning cases — the singlino is heavy since s is large.

- **The modest mass and typically fairly SM-like couplings of the lightest Higgs boson imply that the Tevatron production rates are significant after accumulating a few fb^{-1}.**

  It will be a question of backgrounds.

  It is not impossible that the backgrounds will be better at the Tevatron than at the LHC.

  Detailed studies by the experimental groups at both the Tevatron and the LHC should receive significant priority.

- **It seems likely that other models in which the MSSM μ parameter is generated using additional scalar fields (such as the type of model that Cvetic will be discussing where the additional scalars can be charged under a new U(1)) can achieve small fine-tuning in a manner similar to the NMSSM.**

- **In general, very natural solutions to the fine-tuning and little hierarchy problems are possible in relatively simple extensions of the MSSM.**

  One does not have to employ more radical approaches or give up on small fine-tuning!

And now we take a commercial break.

CPNSH (CP-Violating and Non-Standard Higgs Models) Working Group