Progress in Heavy Flavor Theory

Thomas Becher

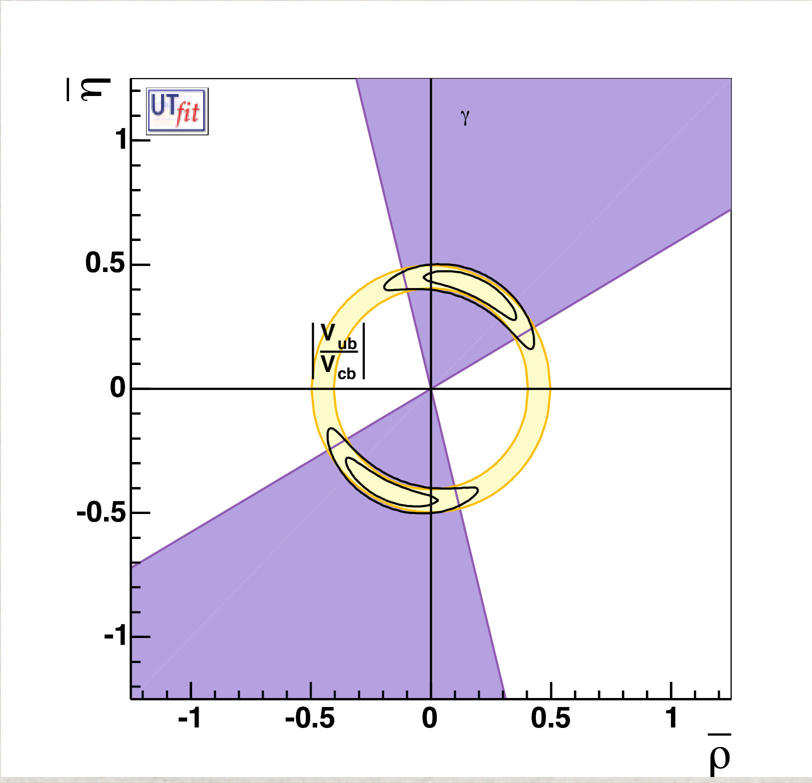
Fermilab

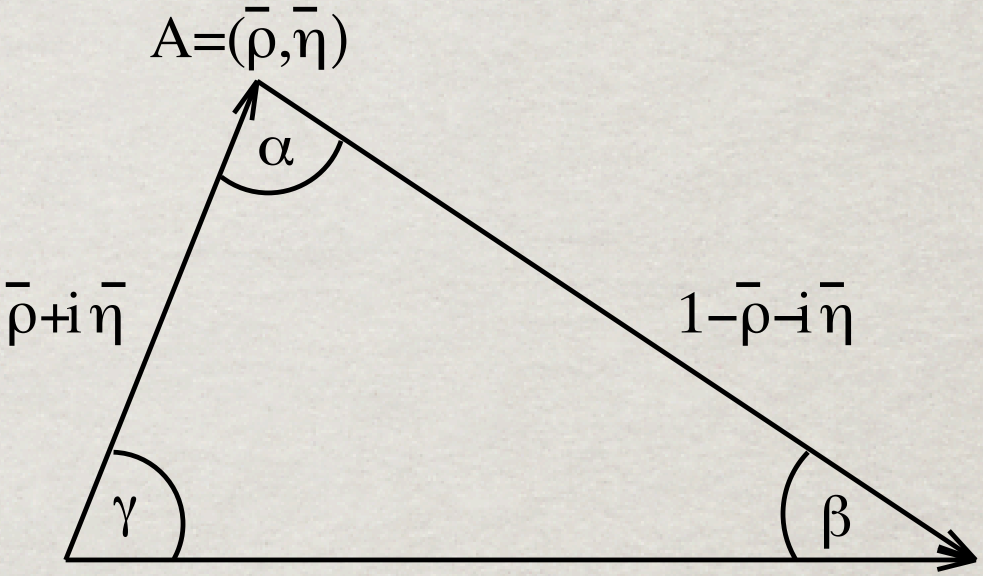
Funkadelic, "standing on the verge of getting it on"

- Flavor physics is especially appropriate topic at this conference:
  - have been *on the verge of discovery* of New Physics for a while...

- New physics effects in flavor sector are (almost) guaranteed
  - Potentially large signals in FCNC interactions which are suppressed in SM.
    - Flavor physics probes very high scales
  - NP Models typically contain large numbers of flavor changing interactions and CP phases.
  - Even if there are no new flavor changing interactions, they are induced by "misalignment" of SM fields.
    - Minimal Flavor Violation

Tree-Level Triangle





- $V_{ub} \sim \bar{\rho} - i\bar{\eta}$ from tree level processes only
  - $|V_{ub}|$ from semi-leptonic $b \to u\,\ell\nu$
  - Angle γ from $B^{\pm} \to D^{(*)}K^{\pm}$

Strategy

- To identify effects of new physics:
  - Use tree-level determination of CKM,
  - calculate loop processes, search for (pattern of) deviations
    - MFV, NMFV, ... ?
- Limitation: experimental precision and ability to calculate hadronic effects
  - → this talk.

Outline

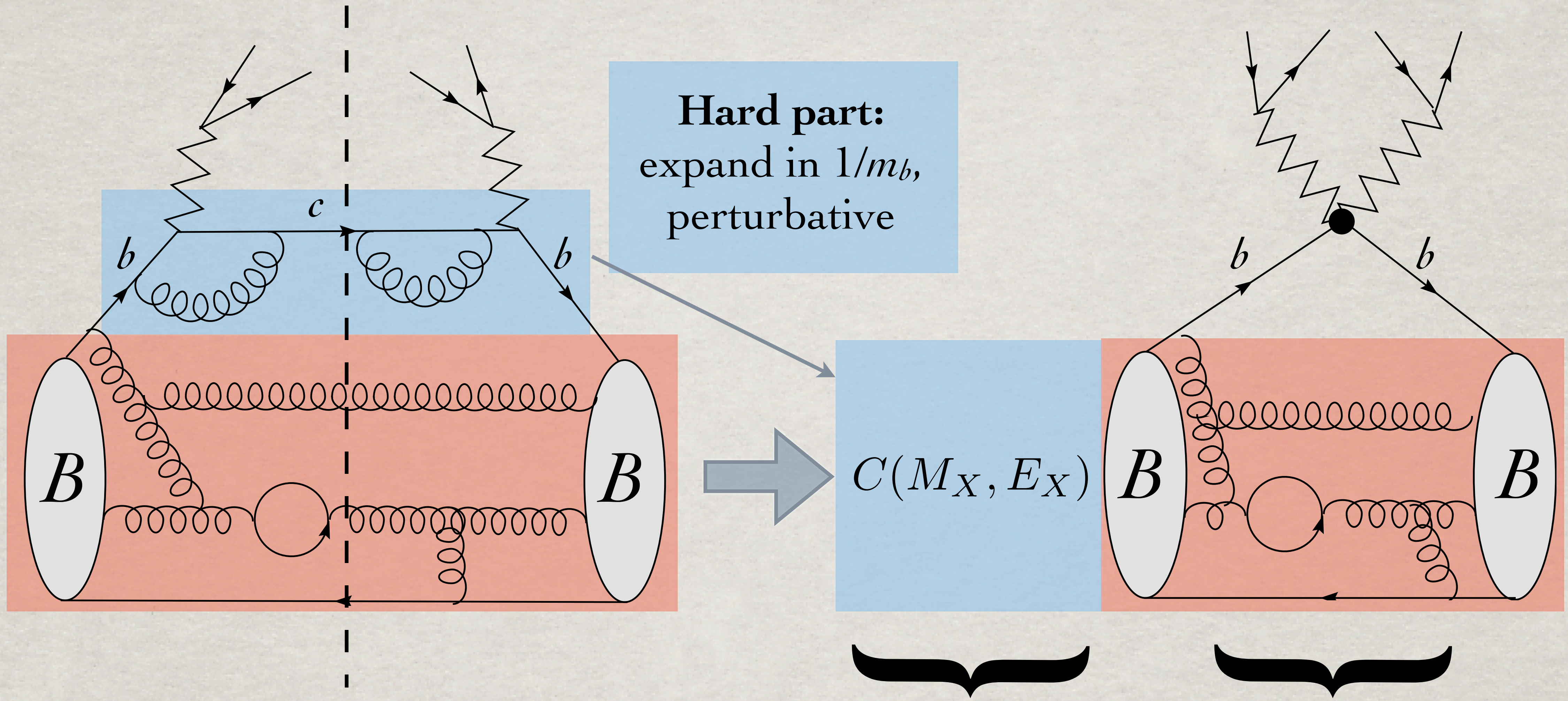
- (Not entirely) inclusive B-decays
  - Dealing with experimental cuts
  - With SCET from all orders to two loops
  - Towards $\bar{B} \to X_s \gamma$ at NNLO
- Exclusive B-decays
  - With SCET from one-loop to tree level
  - Hadronic input from $\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}$

Inclusive Decays

$\bar{B} \to X_c \ell \nu\,,\; \bar{B} \to X_u \ell \nu\,,\; \bar{B} \to X_s \gamma$

Methods

	exp. cuts	method	hadr.input
$\bar{B} \to X_c \ell \nu$	loose $E_l > 1$GeV	OPE, HQET	$\frac{\mu_\pi^2}{m_b^2}, \frac{\mu_G^2}{m_b^2}, \ldots$
$\bar{B} \to X_s \gamma$	intermediate $E_\gamma > E_0 \approx 2$GeV	MSOPE, SCET	$\frac{\mu_\pi^2}{\Delta^2}, \frac{\mu_G^2}{m_b^2}, \ldots$
$\bar{B} \to X_u \ell \nu$	severe, $M_X < M_D$	factorization, SCET	$S(\omega), \frac{S_i(\omega)}{m_b}, \ldots$



$\Gamma \propto \mathrm{Im}\,\langle B|J^\dagger J|B\rangle$

Wilson coeff.

operator matrix element

- Fully inclusive B-decay can be calculated using the OPE in an expansion in $1/m_b$
- Nonperturbative input: matrix elements of local operators

V_{cb} DETERMINATION

$$\Gamma(B \to X_c \ell \nu) = \frac{G_F m_b^5}{192\pi^3} |V_{cb}^2| \left[c^{(0)} \left(1 - \frac{\mu_\pi^2}{m_b^2} \right) + c^{(2)} \frac{\mu_G^2}{m_b^2} + \mathcal{O}\left(\frac{1}{m_b^3} \right) \right]$$

- Coefficients $c_i \equiv c_i(\alpha_s, \frac{m_c}{m_b})$ are evaluated in perturbation theory
- Expansion in $1/(m_b - m_c) \approx 1/m_b$
- With predictions and measurements of moments of decay spectra:

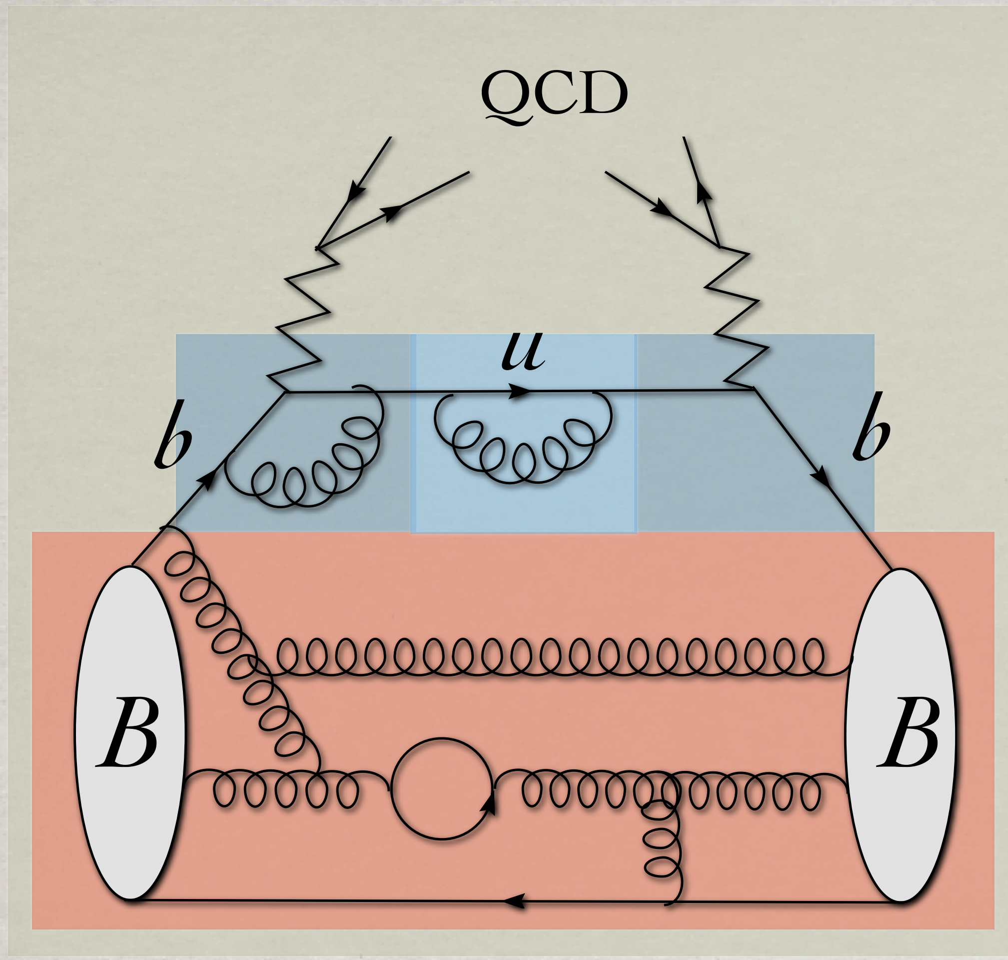
$$|V_{cb}| = (42.0 \pm 0.7) \times 10^{-3} \qquad \bar{m}_b(\bar{m}_b) = (4.20 \pm 0.04)\text{GeV}$$

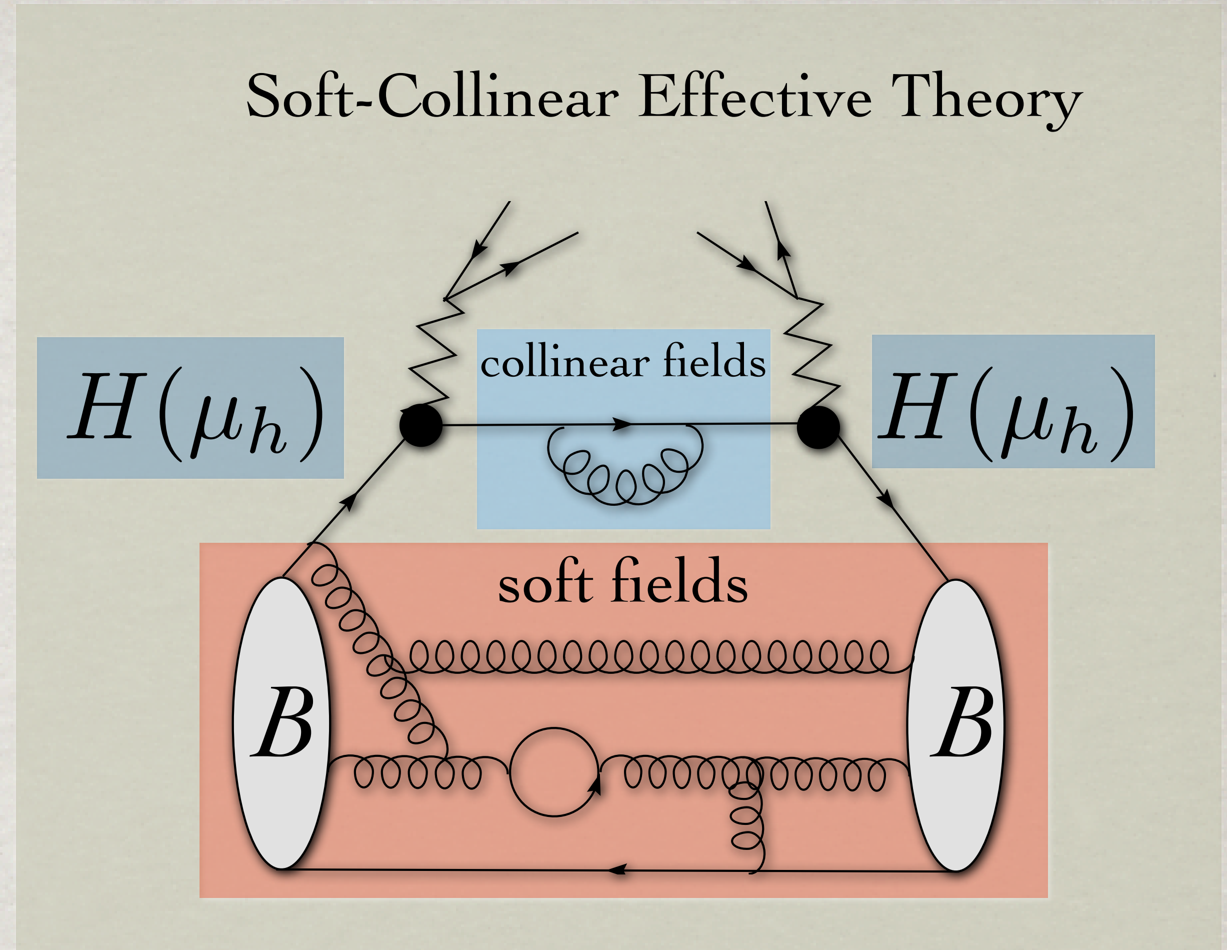
$$\mu_\pi^2 = (0.40 \pm 0.04)\text{GeV}^2 \qquad \bar{m}_c(\bar{m}_c) = (1.24 \pm 0.07)\text{GeV}$$

Bauer et. al. '04; Gambino et al. '04, Buchmüller and Flächer '05

Inclusive V_{ub}

- To discriminate against the huge $b \to c$ background, cuts need to be imposed which enforce $M_X < M_D$.
- Decay products with large energy but small invariant mass: OPE breaks down!
- Still possible to expand in $1/E_X$. ⇒Soft-Collinear Effective Theory
- Three scales: hard $\mu_h \sim m_b$, soft $\mu_0 \sim \Lambda$
  - Intermediate, jet-scale $\mu_i \sim \sqrt{\Lambda m_b}$



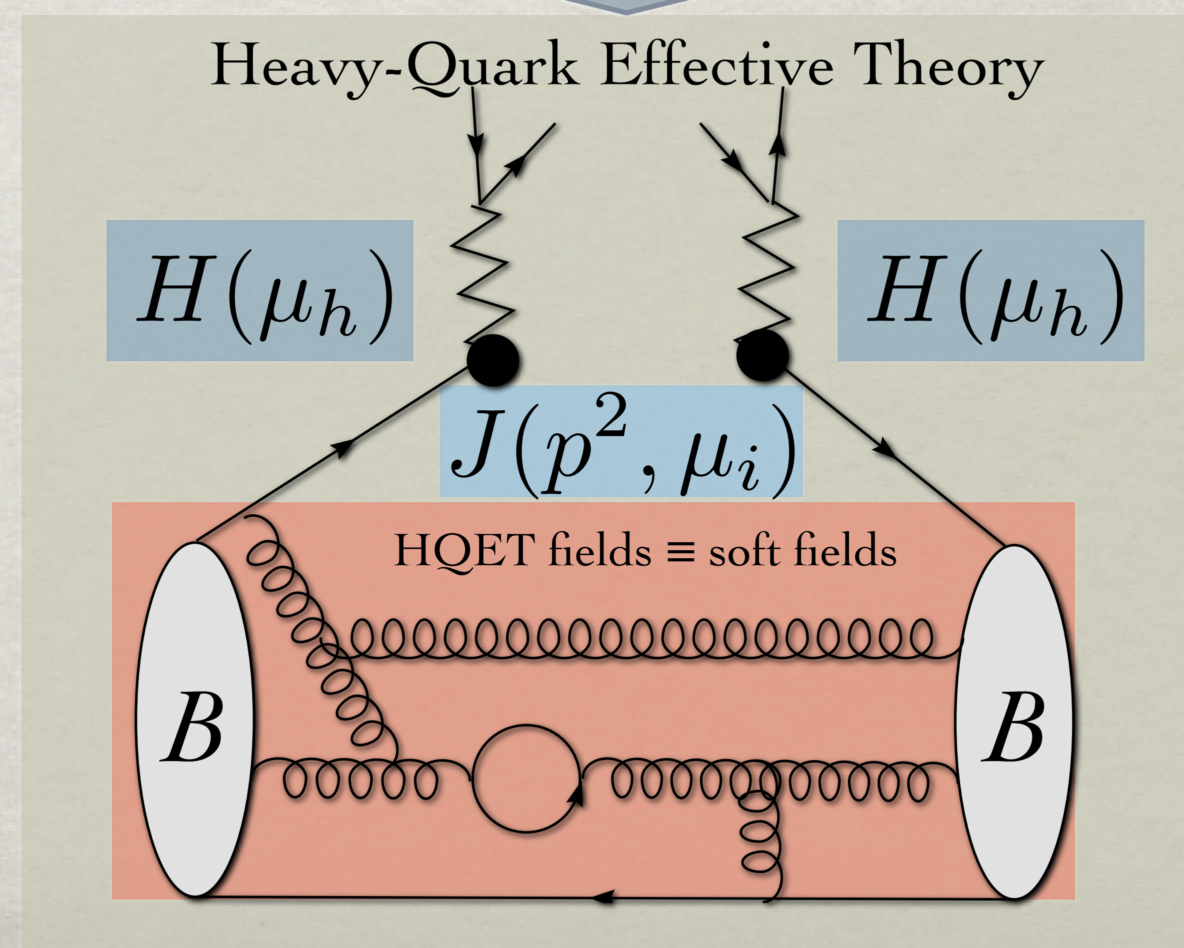


Factorization theorem

$$\Gamma \sim H^2 J \otimes S$$

hard jet soft

shape function



Korchemsky, Sterman '94

Scale Separation

$$\Gamma \sim H^2(\mu_h) U(\mu_h, \mu_i) J(\mu_i) \otimes U(\mu_i, \mu_0) \otimes S(\mu_0)$$

QCD $\longrightarrow$ SCET $\longrightarrow$ HQET

- Resummation of (Sudakov) logs using two-step matching and RG evolution in effective theory.
- New: general solution of evolution equations in momentum instead of moment space. Lange, Neubert '03
- Shape function develops radiative tail. Bauer and Manohar '03 Bosch et al. '04
- New: NLO resummation. Involves 3-loop cusp anomalous dimension! Moch, Vermaseren, Vogt '04 Two-loop anomalous dimensions for J and S. Kochemsky, Marchesini '93 Neubert '04, Gardi '04, TB and Neubert '05

Power corrections

$$\Gamma \sim H^2 J \otimes S + \frac{1}{m_b} \sum_i H_i J_i \otimes S_i + \ldots$$

Korchemsky, Sterman '94

Lee & Stewart '04
Bosch, Neubert, Paz '04
Beneke et al. '04

- Factorization of power corrections using SCET!
- Same shape function $S(\omega)$ enters $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_u \ell \nu$ at leading power.
- Different combination of the subleading shape functions in the two decays.

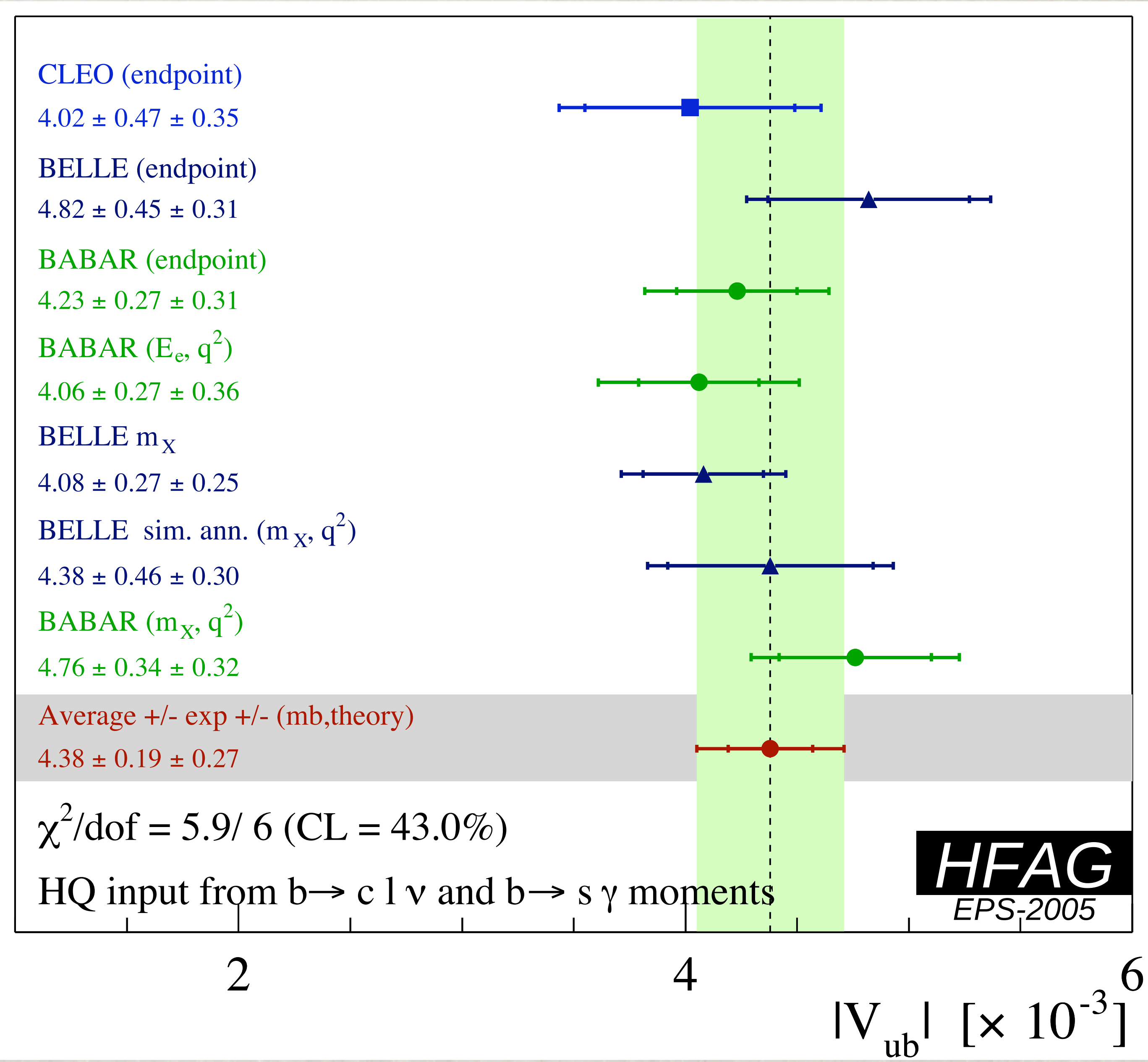
State of the art

- Lange, Neubert and Paz '05: theoretical expressions which incorporate all known contributions to differential decay rate
  - NLO result near end-point. Leading shape function from $\bar{B} \to X_s \gamma$, models for subleading shape functions.
  - reproduces 1-loop OPE result when integrated.
- Other possibility: shape function independent relations between $\bar{B} \to X_u \ell \nu$ and $\bar{B} \to X_s \gamma$ decay spectra.

Neubert '93
Leibovich et al. ; Neubert '00
Lange et al. '05

Inclusive $V_{UB}=(4.38\pm0.29)\times10^{-3}$

error budget	
Stat	2.2%
Syst.	2.5%
b→c	1.9%
b→u 7.6 %	2.2%
SF	4.7%
sub SF	3.5%
Total	7.6%



$\bar{B} \to X_s\gamma$

- FCNC process, stringent constraint on New Physics.
- Current experimental uncertainties match theoretical uncertainty in the prediction of the (cut) rate. E.g. Belle '04

$$\mathrm{Br}(E_\gamma > E_0 = 1.8\mathrm{GeV}) = (3.38 \pm 0.30 \pm 0.29) \times 10^{-4}$$

- Lower values of cut energy E_0 . (BaBar has E_0=1.9 GeV.)

- NNLO calculation of the rate is underway.
  - Reduce perturbative uncertainty
  - Reduce dependence on the choice of scheme for m_c. (NLO: 10% shift between pole and $\overline{MS}$ mass.)
- Most ambitious flavor physics calculation!
  - ☑ 2- and 3-loop matching at $\mu=M_W$.
    Misiak & Steinhauser '04
  - ☐ 3- and 4-loop anomalous dimensions.
    3-loop: Gorban, Haisch, Misiak '04,'05
  - ☐ 2- and 3-loop matrix elements
    n_f-parts: Bieri et al. '03. Q_7: Blokland et al. '05
- Cut on the photon energy? For what value of E_0 is OPE for the cut rate valid?

Transition between shape-function and OPE regime: MultiScaleOPE

Neubert '05

shape function $\quad H \times J$

$$\int_{-\bar{\Lambda}}^{\Delta} d\omega \, S(\omega) \, f(\omega)$$

$$\Delta = m_b - 2E_0 \approx 1\text{GeV}$$

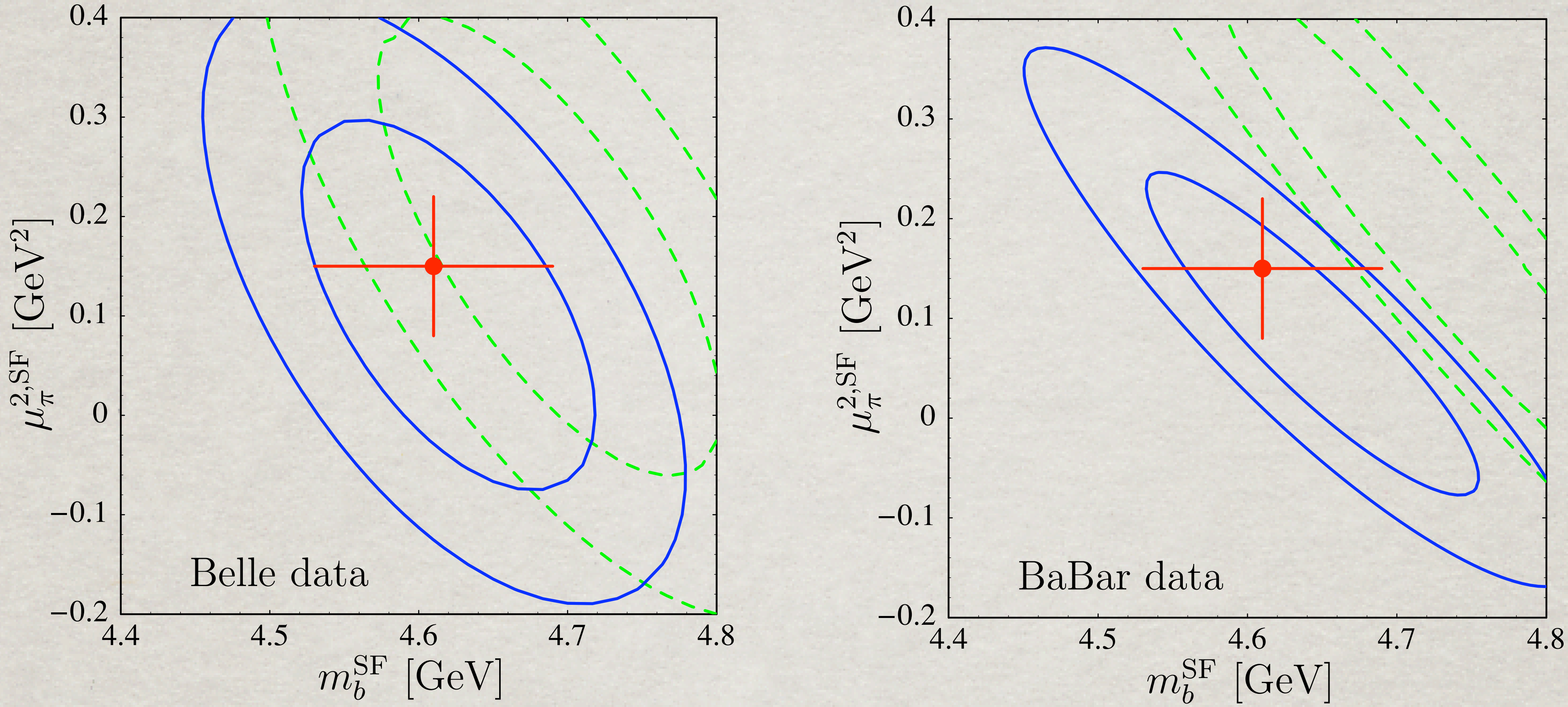
$$\bar{\Lambda} = m_B - m_b$$

- Integral over S can be evaluated in OPE if energy window Δ is large enough.
- OPE is expansion in Λ/Δ , $\alpha_s(\Delta)$!
- NLO: $\text{Br}(E_\gamma > 1.8\text{GeV}) = (3.38^{+0.31}_{-0.42}{}^{+0.32}_{-0.30}) \times 10^{-4}$
  pert. param.

- Perturbative error dominant.
- Parametric uncertainties can be reduced with precise E_γ moment measurements. NNLO predictions available Neubert '05, '06

$$\langle E_\gamma \rangle \sim m_b \qquad \langle E_\gamma^2 \rangle - \langle E_\gamma \rangle^2 \sim \mu_\pi^2$$

- New: 2-loop calculations of jet-function and partonic shape function. Cut-effects to NNLO TB and Neubert '05 & to appear

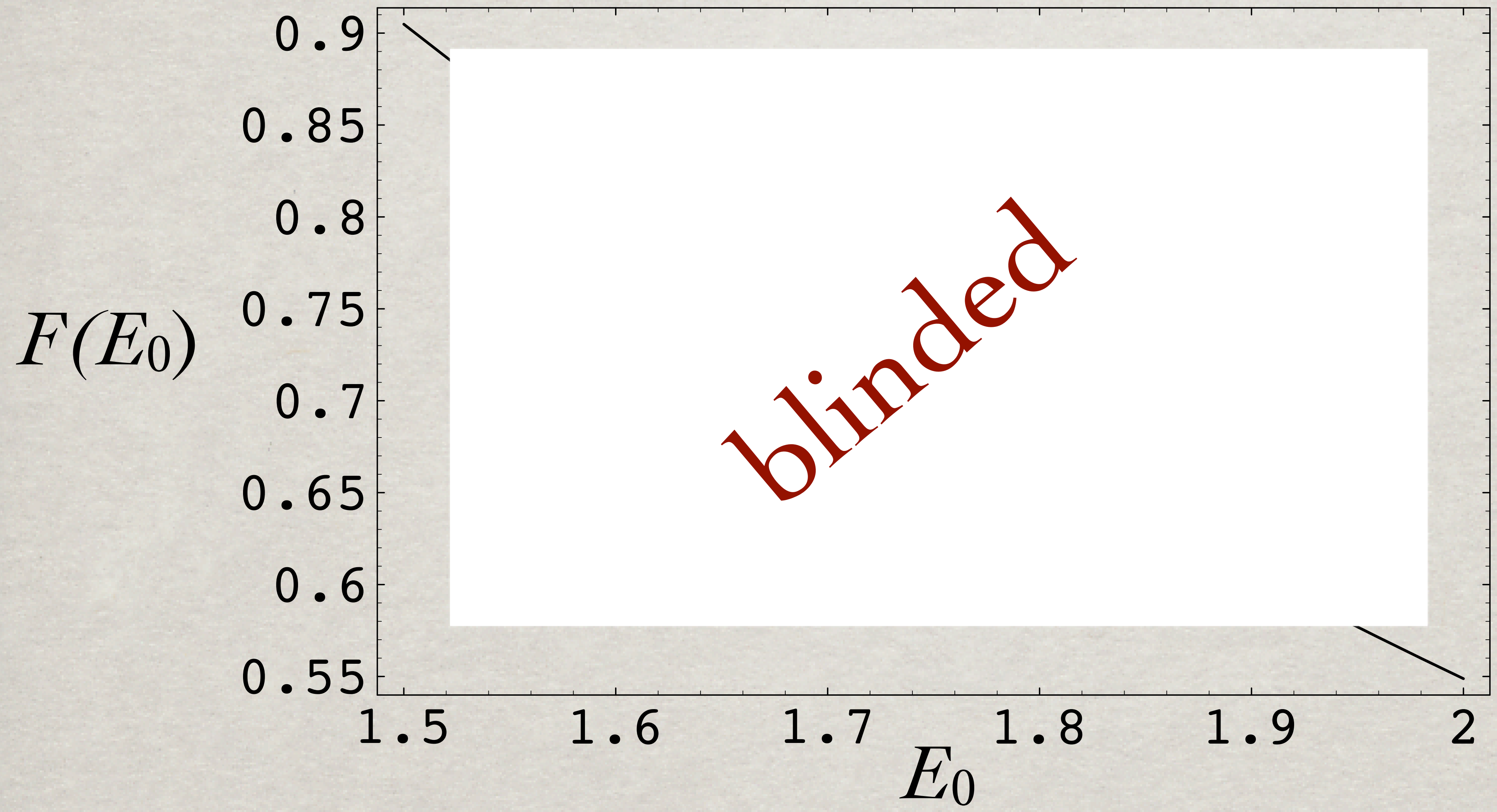


Neubert ‘05

- Consistency check:
  - Red: $\bar{B} \to X_c \ell \nu$ moments
  - NLO and NNLO $\bar{B} \to X_s \gamma$ moments

68% and 90% c.l. contours

Event Fraction at NNLO



- Fairly large NNLO correction!
- Reduced scale dependence.

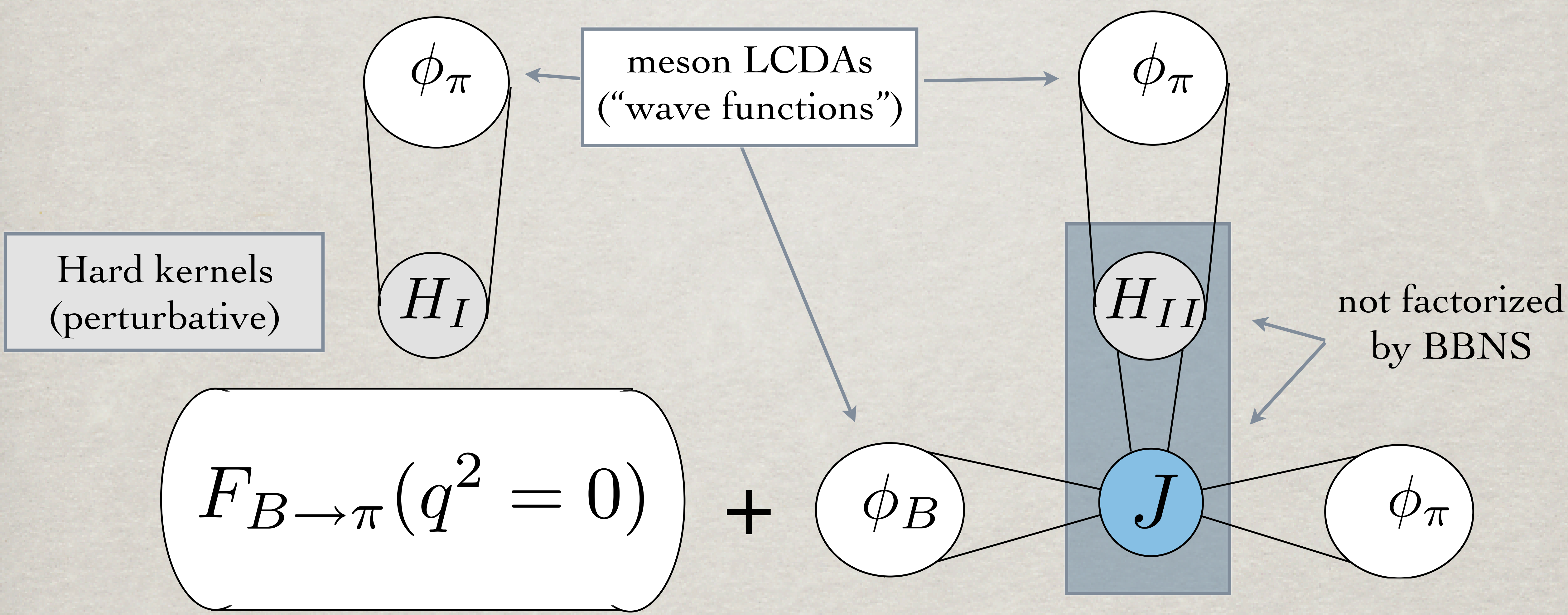
Exclusive Decays

Overview

- QCD factorization vs. SCET
  - Comparison of SCET analysis of $B \to M_1 M_2$, of Bauer et *al*. to QCD factorization results of Beneke et *al*.
- Hadronic input from $B \to \pi l \nu$
  - Formfactor constraints + Exp. + Lattice $\to F_+(0)$, λ_B

Factorization Theorem for $B \to \pi\pi$

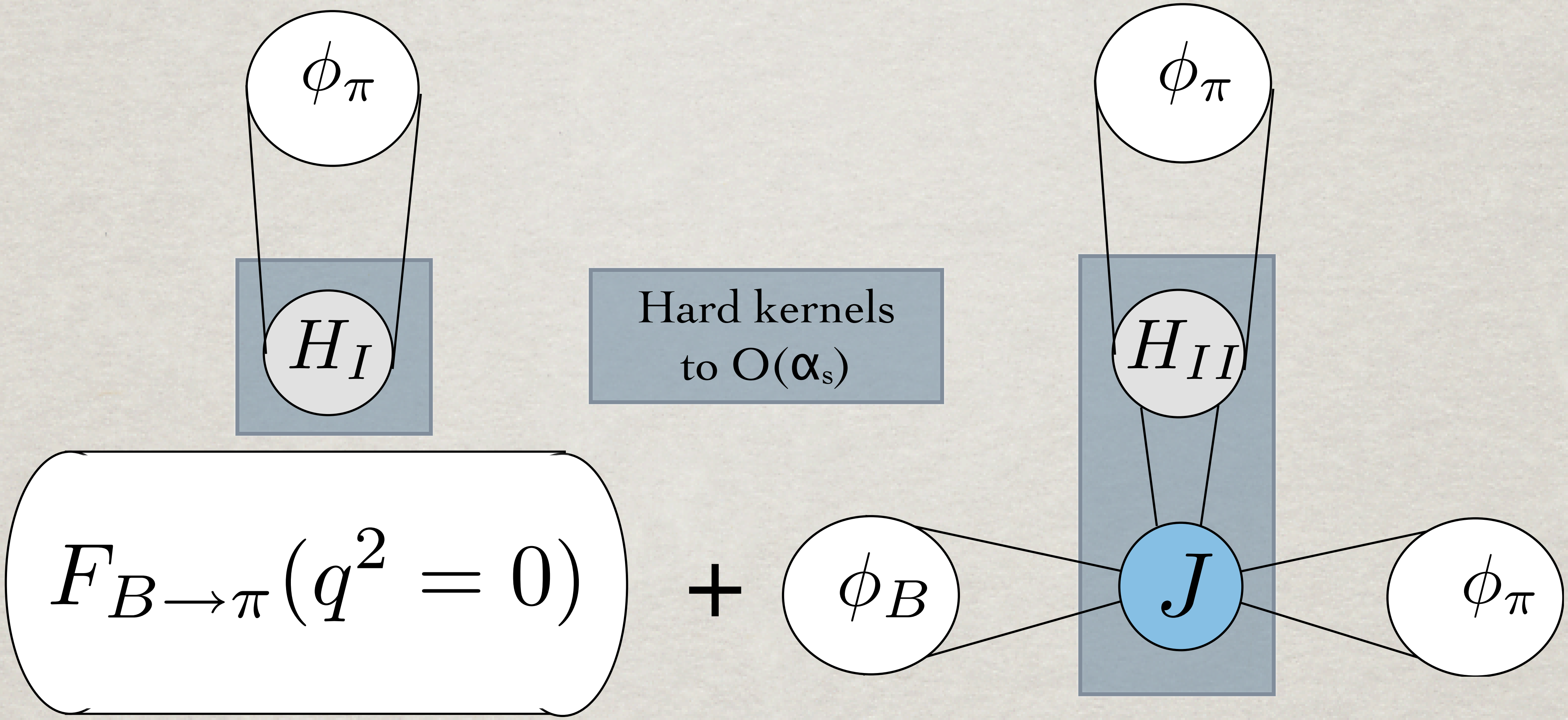
Beneke, Buchalla, Neubert, Sachrajda '99



Jet-function
(interm. scale)

Same Factorization theorem in SCET!

BBNS, "QCD Factorization"



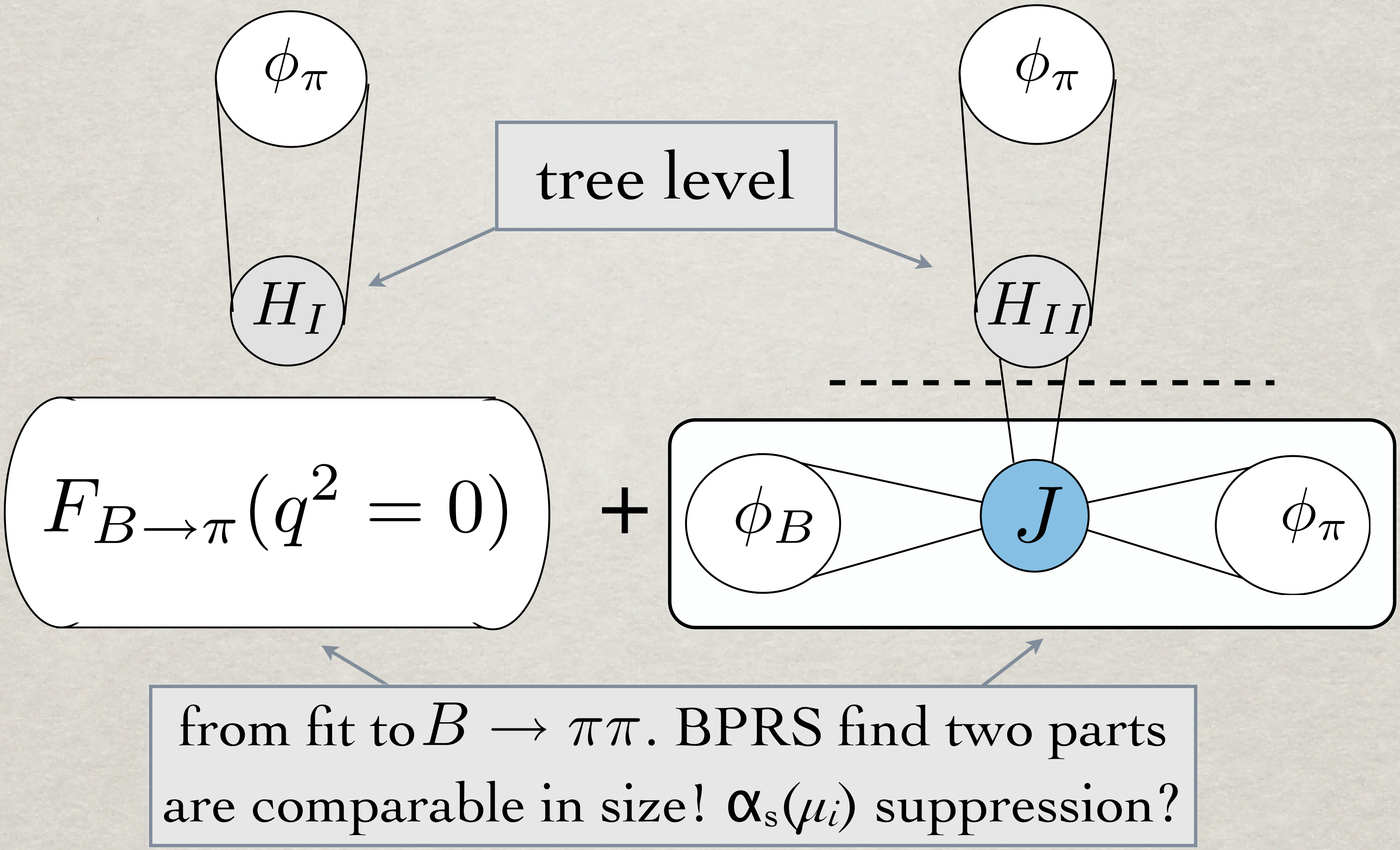
Jet-function ∝ $\alpha_s(\mu_i)$

LCDAs and $F(q^2=0)$ from light-cone sum rules

Estimate dominant power corrections.

BPRS, "SCET Approach"

Bauer, Pirjol, Stewart, Rothstein '04



Leave (charming) penguins unfactorized.

Neglect all power corrections.

Comparison

- "SCET approach":
  - ☑ Model independent; no dependence on light-cone sum rules.
  - ☐ might not be very precise: no power and no perturbative corrections. (BBNS find large power corrections.)
  - ○ More modest/less predictive. Penguins from fit, strong phases from fit, ...

$B \longrightarrow \pi \ell \nu$ TO THE HELP

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|\vec{p}_\pi|^3|V_{ub}|^2|F_+(q^2)|^2$$

- New measurements can be used to extract both $F_+(q^2 = 0)$ and, with help from lattice $H \sim \phi_B \otimes J \otimes \phi_\pi$
- Challenging! Have three- and five-bin measurement of partial decay rate.
  - Extrapolation to $q^2 = 0$! For H need first derivative of F_+ and F_0 at q^2=0.
  - F_0 from lattice.

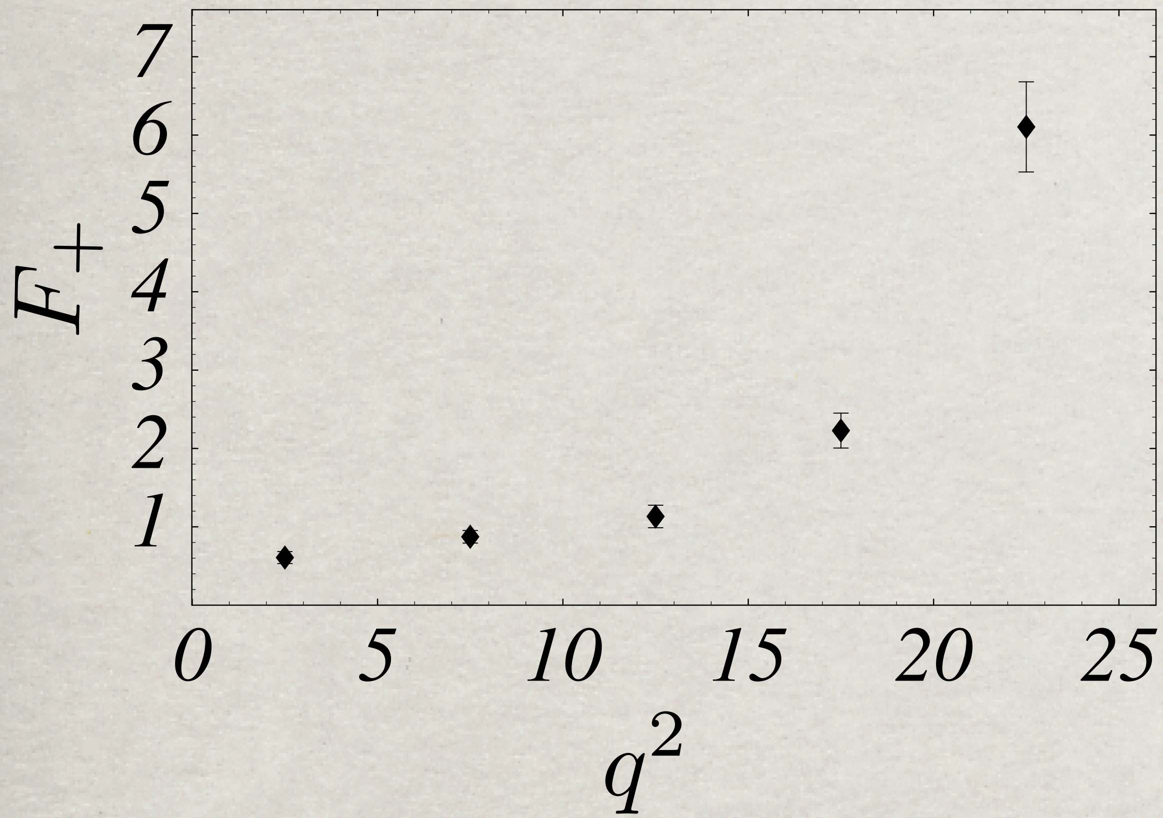
Form factor constraints

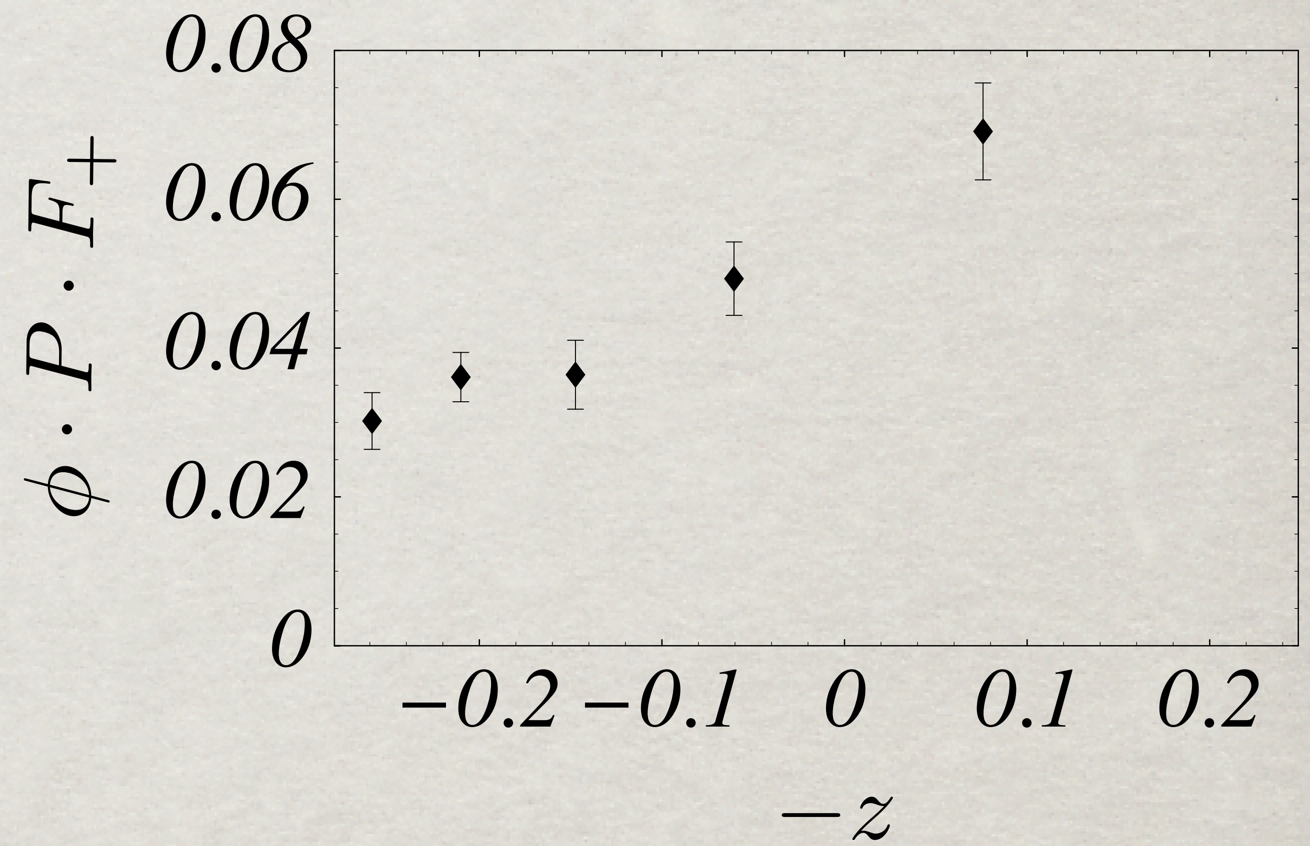
$$F_+(q^2) = \frac{1}{P(q^2)\phi(q^2)} \sum_{k=0}^{\infty} a_k \, [z(q^2)]^k \qquad A = \sum_{k=0}^{\infty} a_k^2$$

- Constrained series parameterization:
  - Map $q^2 \to z(q^2)$. Improved convergence of series $|z|_{\mathrm{max}} \approx 0.5$.
  - Bound $A < 1$ from unitarity.
  - Much stronger bound $A \sim \left(\frac{\Lambda}{m_b}\right)^3$ from heavy-quark power counting. TB, Hill '05

Illustration: Babar 5-bin Data

hep-ex/0506064





- Current experiments measure intersect and slope, but cannot yet resolve curvature.
- No need for model parameterizations.

Results

Exp. data and $F_+(16\text{GeV}^2) = 0.8 \pm 0.1$

$$|V_{ub}| = 3.7 \pm 0.2\,^{+0.6}_{-0.4} \pm 0.1 \qquad = (3.7 \pm 0.2) \times \frac{0.8}{F_+(16\,\text{GeV}^2)}\,,$$

$$F_+(0) = 0.25 \pm 0.04 \pm 0.03 \pm 0.01 \qquad = (0.25 \pm 0.04) \times \frac{F_+(16\,\text{GeV}^2)}{0.8}\,.$$

$$(m_B^2 - m_\pi^2)\frac{F_+'(0)}{F_+(0)} = 1.5 \pm 0.6 \pm 0.4\,,$$

A<1

A<0.1

!

- With A<0.1

Factorization test:

$$\frac{\Gamma(B^- \to \pi^-\pi^0)}{d\Gamma(\bar{B}^0 \to \pi^+\ell^-\bar{\nu})/dq^2|_{q^2=0}} = 0.76\,^{+0.22}_{-0.18} \pm 0.05\,\text{GeV}^2\,,$$

Naive fact: 0.62 ± 0.07 BBNS: $0.66^{+0.13}_{-0.08}$ BPRS: $1.27^{+0.22}_{-0.29}$

Summary

- Impressive progress in calculation of inclusive *B*-decays
  - NLO (even some NNLO) resummations
  - Factorization of power corrections
  - First 2-loop results for partial decay rates, more to come.
- Calculation of exclusive decays suffer from our lack of knowledge of input parameters.
  - Semileptonic decays can provide some of these.