

Braneworld Gravity

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hep-th/0511266 + in progress with R. Bao, M. Carena, J. Lykken and M. Park



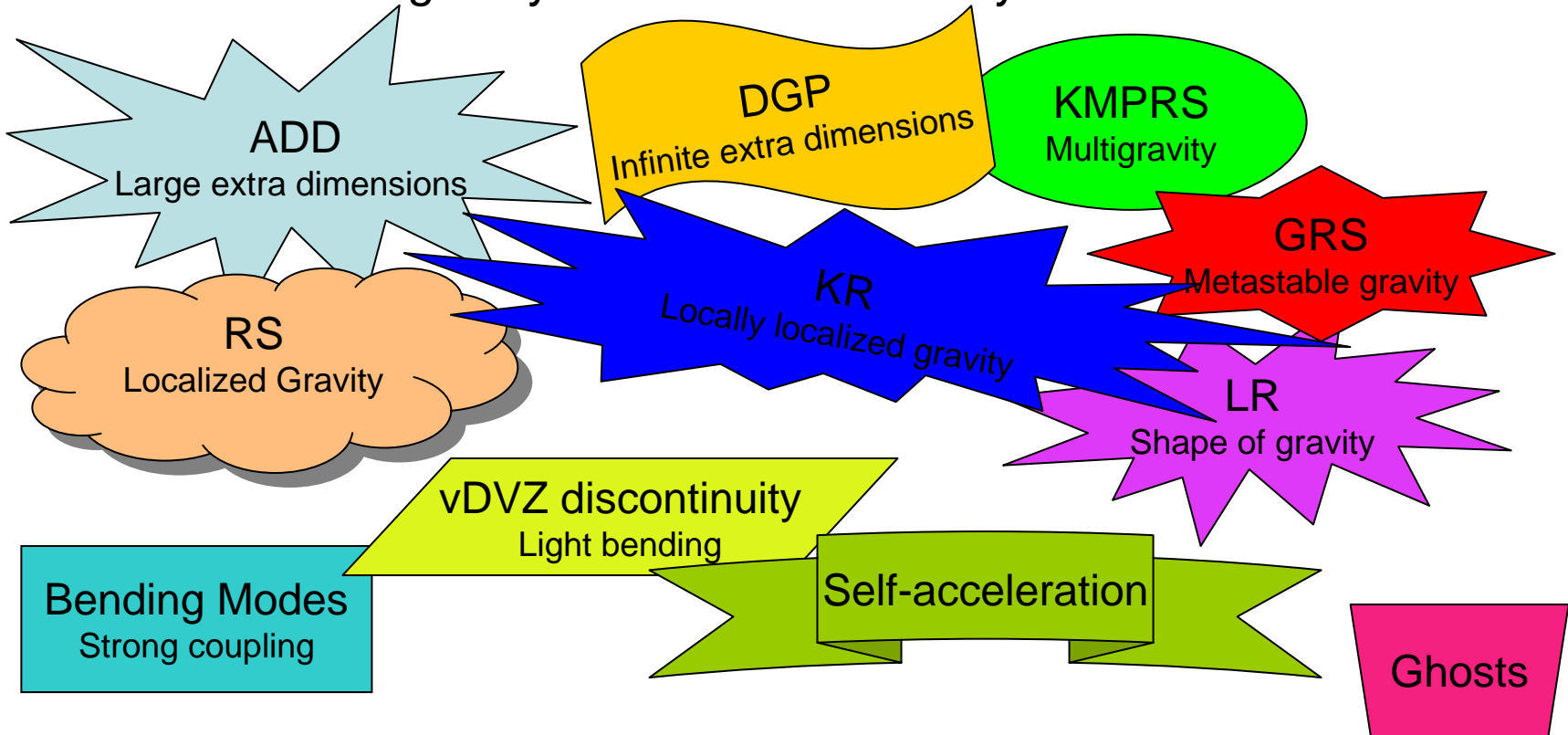
Why Braneworld Gravity?

- **New framework** to look at (and attempt to solve) long standing problems: **Hierarchy problem, Cosmological Constant problem(s), ...**
- Can have a wealth of **phenomenological implications**:
 - Graviton KK production in accelerators
 - Table-top (sub-millimeter) experiments
 - Solar System observations
 - [Cosmology](#)
 - Black hole physics



Braneworld Gravities

- Braneworld gravity has been extensively studied ...



IT'S GETTING TOO CROWDED!!



Outline

- Unified framework for braneworld gravity
- Static Case: AdS_5/AdS_4 with localized curvature
- Phenomenological issues:
 - Ghosts and tachyons
 - Strong Coupling
 - vDVZ discontinuity
 - Cross-over behavior of gravity
- Time-dependent backgrounds: implications for cosmology
 - Modified Friedmann equation
 - Acceleration in AdS!
- Conclusions and future work



Can we do all in one?

- Can we study all these interesting features in a unified framework?

- We have to choose a general enough model:

AdS_5/AdS_4 with localized curvature terms

Karch, Randall '01

Dvali, Gabadadze, Porrati '00

- And then study all phenomenological implications:
 - **Spectrum**: masses and couplings
 - Instabilities due to **ghosts, tachyons**
 - **Strong coupling** effects
 - **Cosmological implications**



The Model

- The Action

$$\frac{S}{4M^3} = \int d^5x \sqrt{-G} \left[R + 12k^2 \right] + \int_{\partial\mathcal{M}} K$$

$\nearrow -\Lambda$

$$+ \frac{1}{2} \sum_i \int_{\Phi_i=0} d^4x^{(i)} \sqrt{-g^{(i)}} \left[\frac{v_i}{k} R^{(i)} - \frac{\mathcal{L}_i}{2M^3} \right]$$

- Equations of Motion

$$R_{MN} - \frac{G_{MN}}{2} (R + 12k^2) = 0$$

$$\frac{v_i}{k} \left(R_{\alpha\beta}^{(i)} - \frac{g_{\alpha\beta}^{(i)}}{2} R^{(i)} \right) + 2(K_{\alpha\beta}^{(i)} - g_{\alpha\beta}^{(i)} K^{(i)}) = \frac{T_{\alpha\beta}^{(i)}}{2M^3}$$



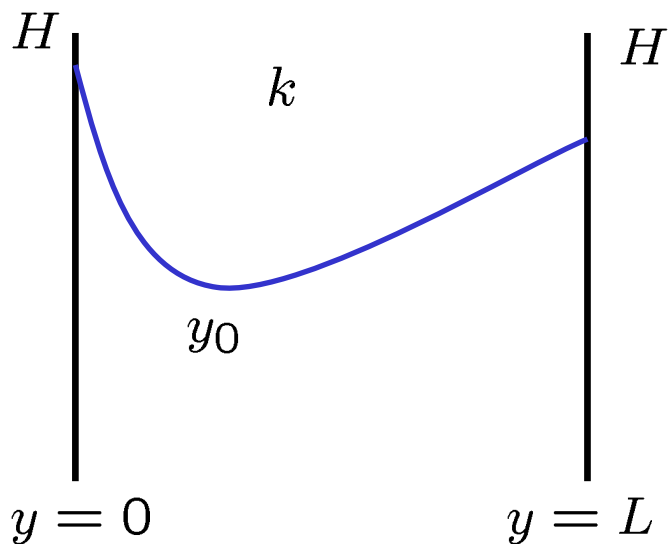
Static Case I: Background

- Brane EMT is pure tension $T_{\alpha\beta}^{(i)}/(2M^3) = \omega_i k g_{\alpha\beta}^{(i)}$

Karch, Randall '01

$$ds^2 = \frac{\cosh k(y - y_0)}{\cosh ky_0} \frac{\eta_{\mu\nu} dx^\mu dx^\nu}{1 - H^2 x^2/4} + dy^2$$

AdS_5/AdS_4



H, y_0 and L are function of k, v_i and w_i



We can reproduce most of known models



Static Case II: Linearized analysis

- We want to study the spectrum of linearized perturbations

- Expand around background metric

Carena, Lykken, Park '05

$$G_{MN} = G_{MN}^0 + h_{MN}$$

Bao, Carena, Lykken, Park, J.S. '05

- Perform a full gauge fixing

radion

Straight gauge, $h_{\mu 4} = 0, h_{44} = F(y) \overbrace{\psi(x)}$

- Compute the action at quadratic (+ leading cubic) level in h_{MN} and integrate out the extra dimension 4D effective action

$$h_{\mu\nu}(x, y) = a^2(y) \underbrace{h_{\mu\nu}^{(0)}(x)} + \sum_q \mathcal{Y}^{(q)}(y) \underbrace{h_{\mu\nu}^{(q)}(x)} + \dots$$

Massless zero mode (2 dof)

massive modes (5 dof)



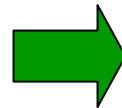
Static case II: Linearized analysis (cont'ed)

- Effective 4D Action:

$$\frac{S}{2M^3} = \int d^4x \sqrt{-g} \left\{ \overline{C_\psi} \psi [\nabla^2 + 4H^4] \psi \right. \\ \left. + \frac{C_g^{(0)}}{4} h^{(0)\mu\nu} [\nabla^2 + 2H^2] h_{\mu\nu}^{(0)} \right. \\ \left. + \sum_{q>0} \frac{C_g^{(q)}}{4} h^{(q)\mu\nu} [\nabla^2 + 2H^2 - \underbrace{m_q^2}_{\text{wavy}}] h_{\mu\nu}^{(q)} \right\}$$

- Kinetic coefficients and masses are functions of the parameters of the model:

$$k, v_i, w_i$$



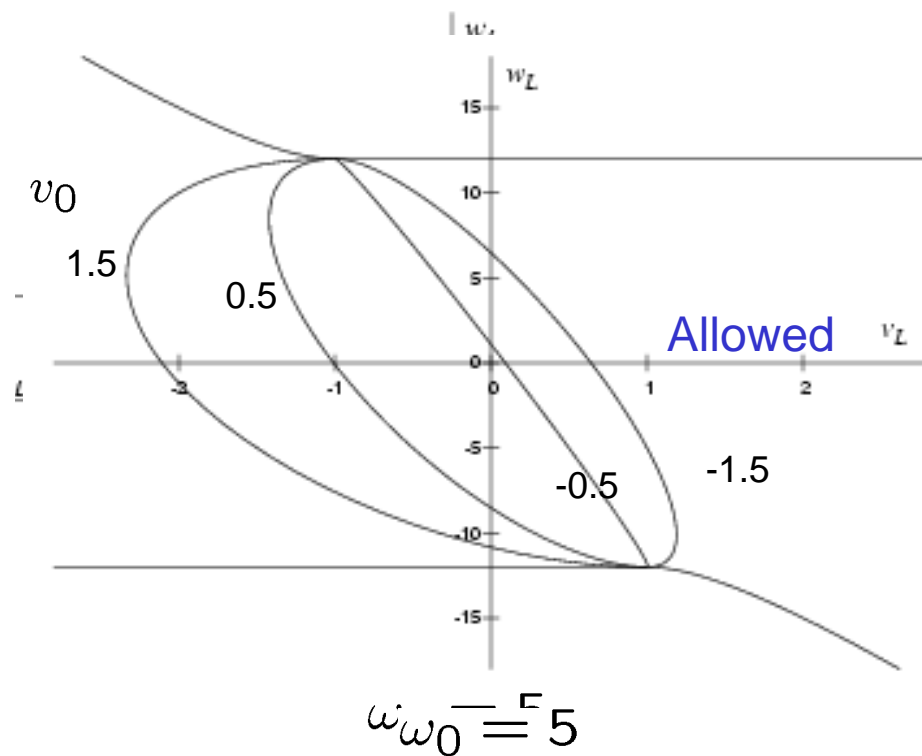
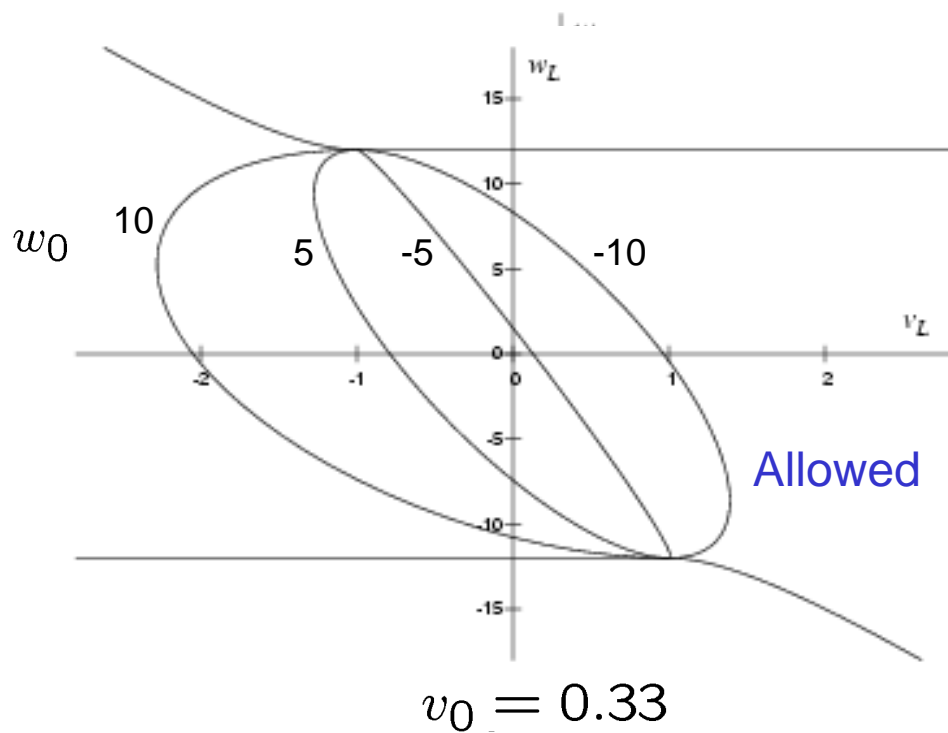
We can study the presence of ghosts, tachyons, strong coupling, or cross-over behavior of gravity as a function of them.



Ghostbusting

- $C_g^{(q)}, C_\psi < 0 \implies$ physical spectrum contains ghosts

Bao, Carena, Lykken, Park, J.S. '05

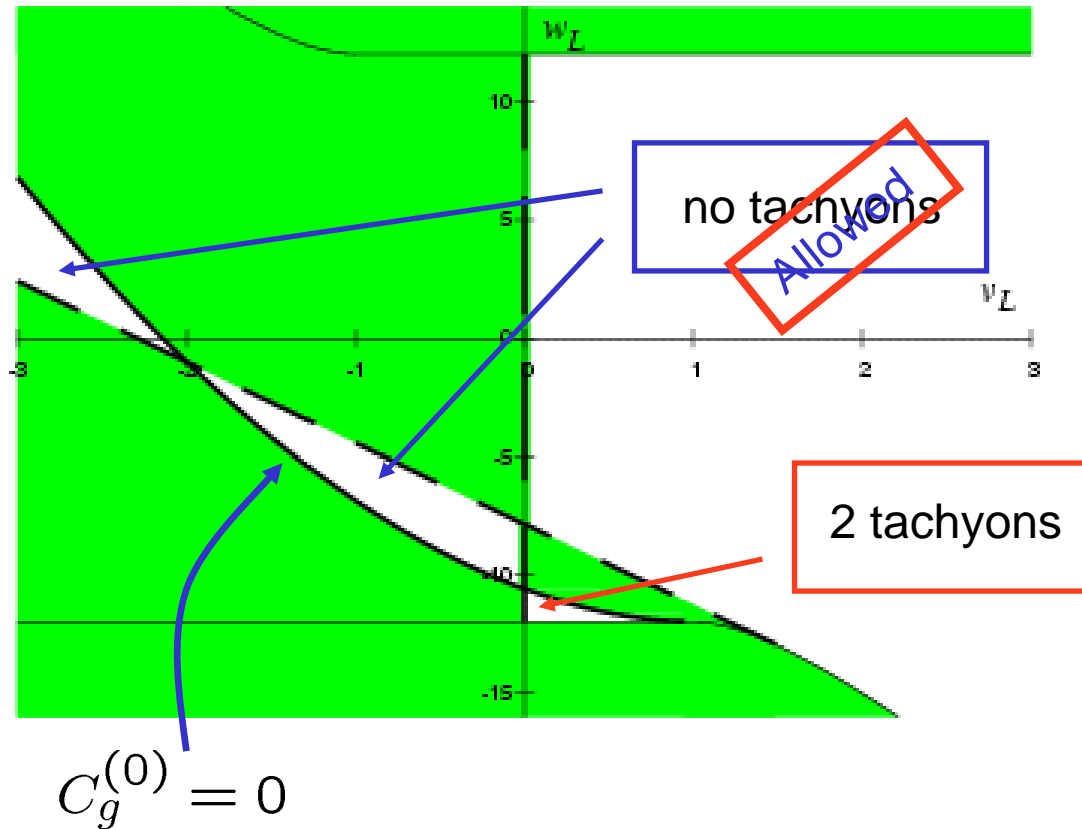




Tachyons

- It is also possible that $m_q^2 < 0$

Bao, Carena, Lykken, Park, J.S. '05



$$v_0 = 2$$

$$\omega_0 = 1$$



So what?

This AdS5/AdS4 model is very interesting but what is it *really* useful for?

We don't live in Anti de Sitter space, do we?

Maybe *yes*!!

Carena, Lykken, Park, J.S. *in progress*



Time-dependent case

- Let's consider a time dependent background (FRW-like)

$$ds^2 = -n^2(t, y)dt^2 + a^2(t, y)\gamma_{ij}dx^i dx^j + b^2(t, y)dy^2$$

- Brane energy-momentum tensor $T_\nu^{i\mu} = \text{diag}(\rho, -p, -p, -p)$
- Equations of motion

Binetruy, Deffayet, Ellwanger, Langlois '00

Bulk

$$\frac{n'\dot{a}}{n a} + \frac{a'\dot{b}}{a b} - \frac{\dot{a}'}{a} = 0$$

$$\frac{\dot{a}^2}{a^2 n^2} + \frac{K}{a^2} + k^2 - \frac{C}{a^4} = \frac{a'^2}{a^2 b^2}$$

Brane

$$3\frac{v_i}{k} \left(\frac{\dot{a}_i^2}{a_i^2 n_i^2} + \frac{K}{a_i^2} \right) = \boxed{6\frac{a'_i}{a_i b_i}} = \frac{\rho_i}{2M^3}$$

$$\dot{\rho}_i + 3\frac{\dot{a}_i}{a_i}(\rho_i + p_i) = 0$$



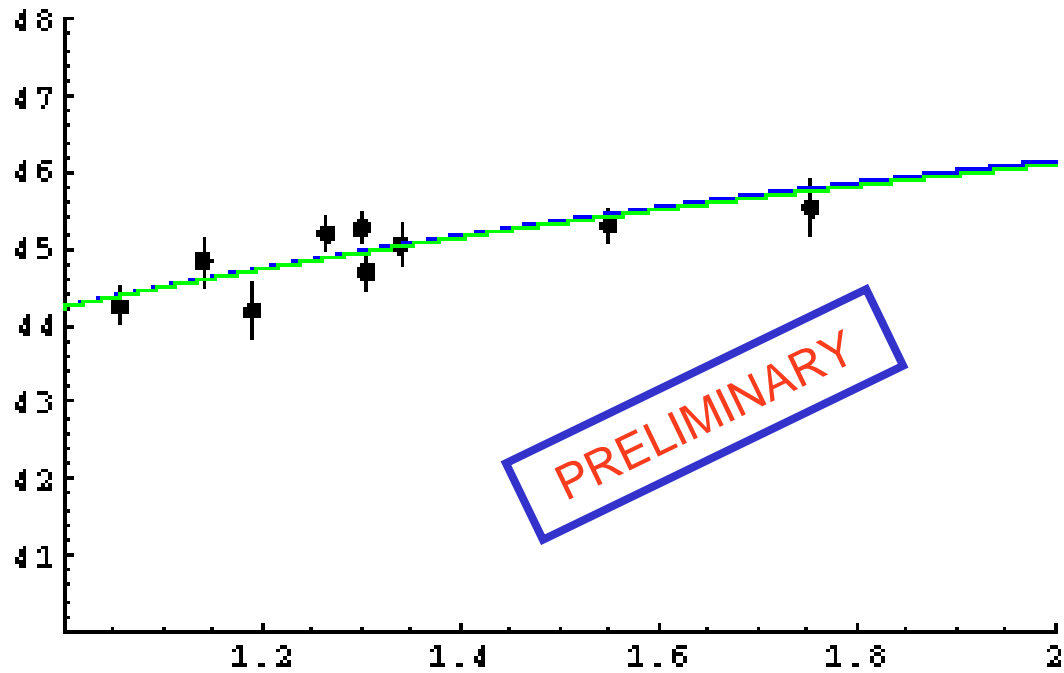
Time-dependent case (Cont'ed)

- The modified Friedmann equation has two branches

Deffayet '01; Deffayet, Dvali, Gabadadze '02; ...

$$H_i^2 = \underbrace{\frac{1}{6} \frac{k}{v_i M^3} \rho_i}_{\text{Standard branch (-)}} + \underbrace{2 \frac{k^2}{v_i^2} \pm 2 \frac{k}{v_i} \sqrt{\frac{k^2}{v_i^2} + \frac{1}{6} \frac{k}{v_i M^3} \rho_i}}_{\text{Self-accelerating branch (+)}} + \dots$$

- Standard branch (-): Universe tends to **AdS** as in usual gravity
- **Self-accelerating branch (+)**: Tends to a **dS** phase as matter dilutes
 - Very good fit to SNe data





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$$H_i^2 = \frac{1}{6} \frac{k}{v_i M^3} \rho_i + 2 \frac{k^2}{v_i^2} \pm 2 \frac{k}{v_i} \sqrt{\frac{k^2}{v_i^2} + \frac{1}{6} \frac{k}{v_i M^3} \rho_i} + \dots$$

- Standard branch (-): Universe tends to **AdS** as in usual gravity
- Self-accelerating branch** (+): Tends to a **dS** phase as matter dilutes
 - Very good fit to SNe data
 - Possible issues with **BAO** Fairbairn, Goobar '05
 - Ghosts** in self-accelerating branch?

} (work in progress)

Luty, Porrati, Rattazzi '03; Nicolis, Rattazzi '04; Koyama '05



Acceleration in AdS!

- AdS_5/AdS_4 with localized curvature terms can thus explain the current acceleration of the Universe
- All necessary ingredients are present in string theory (that naturally prefers AdS vacua)
- Generic to models of infrared modifications of gravity to explain acceleration of the Universe
- Still many things to work out (but AdS curvature may help)
 - spectrum in time-dependent backgrounds (ghosts, tachyons, ...)
 - Strong coupling effects
 - Cosmological perturbations and non-geometrical tests



Conclusions and Outlook

- AdS₅/AdS₄ with localized curvature is a unified description that parametrizes a wide class of interesting braneworld gravity models
- It is possible to study once and for all phenomenological issues such as ghosts, tachyons, strong coupling, cross-over behavior, etc.
- Negative tensions allowed as long as positive and large localized curvature terms are included
- Very exciting cosmological implications: acceleration in AdS!!!
- **Work in progress:**
 - Study of spectrum in time-dependent backgrounds
 - Search of viable models that interpolate between DGP and RS
 - Check consistency with non-geometrical cosmological tests