Gauge Invariant Classes of Feynman Diagrams and Applications for Calculations

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Abstract. In theories like SM or MSSM with a complex gauge group structure the complete set of Feynman diagrams contributing to a particular physics process can be split into exact gauge invariant subsets. Arguments and examples given in this paper demonstrates that in many cases computations and analysis of the gauge invariant subsets are important.

The increase of collider energies requires computations of processes with more particles in the final state and with better precision (NLO, NNLO etc). At LEP1 the basic processes were 2 fermions (γ) production; LEP2 deals basically with 4 fermion (γ) processes; Tevatron, LHC and LC in many cases need in an analysis of the processes with 5,6,8 and so on fermions in the final state, for example, top pair production with decays - 6 fermions; single top production in W-gluon fusion mode - 5 fermions; strongly interacting Higgs sector in hadronic collisions- $pp \to q\bar{q}W^+W^-$ - 6 fermions; study of Yukawa coupling in $pp(e^+e^-) \to t\bar{t}H$ - 8 fermions etc. Typically for the processes with multi-particle final states number of contributing diagrams is large. For hadronic collisions not only number of diagrams but also number of partonic subprocesses is very large.

One of the problem in process computations is a gauge cancellation among many diagrams. Any method of calculation should preserve gauge invariance which is rather complicated in theories like SM with not a simple gauge group. The well known statement from the quantum theory of gauge fields is that the whole set of Feynman diagrams contributing to any physics process is exactly gauge invariant. However, in practical calculations it remains amazing how gauge cancellations take place. But in general the complete set of Feynman diagrams contributing to a particular physics process can be split in to exact gauge invariant subsets, and the gauge cancellations occur in each of the subset. It will be demonstrated that in many cases the idea to split the complete set of diagrams on exact gauge invariant subsets (1) could be useful in practice:

- any physics approximation should be based on gauge invariant classes of diagrams
- better precision of computations in many cases
- better understanding of physics parameters like running couplings or scales (QCD scale, ISR scale etc). Often different part of the same process need different values because different kinematical regions might be important
- any MC generator needs well behaved and compact matrix elements and the gauge invariance gives that

For simple cases it is very easy to find gauge invariant subclasses of diagrams. Take, for instance, the Bhabha-scattering. In that case the two s-channel and two t-channel SM diagrams are separately gauge invariant as immediately follows if one substitutes the final e^+e^- pair by the $\mu^+\mu^-$ pair. In fact, it gives the simplest example of so called "Flavor Flip" introduced in (1). There are two kinds of flips: Gauge and Flavor, which correspond to permutations the $2 \to 2$ subdiagrams with on- and/or off-shell legs (see the exact definitions of flips in (1)). For a concrete physical process one can get one Feynman diagram from another by a sequence of gauge and flavor flips. In correspondence to all the diagrams for the process one may put some graph called Forest. The Forest is a graph with each vertex representing a diagram and the edges given by the flips (gauge and/or flavor) of four-point sub-diagrams. The Gauge Forest is such a Forest or part of the Forest in which the points connected by the only gauge flips. The connected components of the Gauge Forest are called Groves. The general theorem has been proved in (1) by the mathematical induction method:

The Forest F(E) for an external state E consisting of gauge and matter fields is connected if the fields in E carry no conserved quantum numbers other than the gauge charges. The Groves are the minimal gauge invariant classes of Feynman diagrams.

A very simple forest as an example is shown in Fig. 1 for the process $u\bar{d} \to c\bar{s}\gamma$. All diagrams are connected

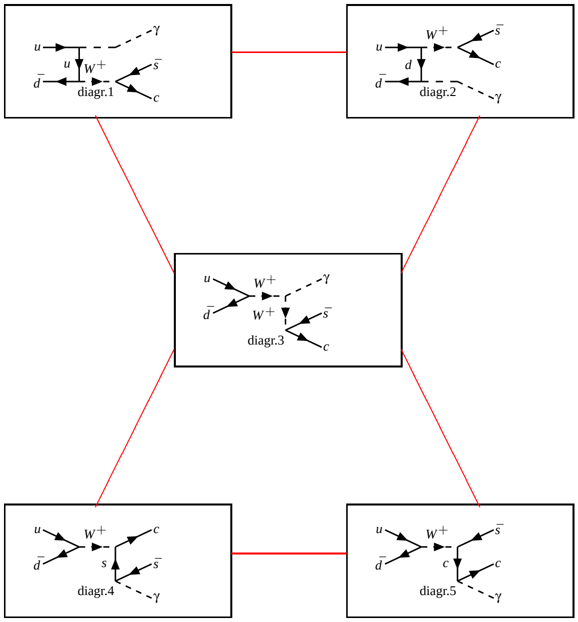

FIGURE 1. The Forest for the process $u\bar{d} \to c\bar{s}\gamma$ is the Grove.

by the gauge flips passing through the middle diagram 3. Therefore all of them form one Grove - one gauge invariant set of diagrams.

More complex examples of the Forests and Groves are shown in (1). They have been obtained by means of the program *bocages* (2) in which the algorithm of tree diagrams generation based on the flips has been realised. Table shows how the number of diagrams for various processes with 6 external fermions splits into minimal gauge invariant subsets (Groves).

E	$\sum$	classes
$u\bar{u}u\bar{u}u\bar{u}$	144	$18 \cdot 8$
$u\bar{u}u\bar{u}u\bar{u}\gamma$	1008	$18 \cdot 24 + 36 \cdot 16$
$u\bar{u}u\bar{u}d\bar{d}$	92	$4 \cdot 11 + 6 \cdot 8$
$u\bar{u}u\bar{u}d\bar{d}\gamma$	716	$4 \cdot 95 + 6 \cdot 24 + 12 \cdot 16$
$\ell^+\ell^- u\bar{u}d\bar{d}$	35	$1 \cdot 11 + 3 \cdot 8$
$\ell^+\ell^- u\bar{u}d\bar{d}\gamma$	262	$1 \cdot 94 + 3 \cdot 24 + 6 \cdot 16$
$\ell^-\nu d\bar{u}d\bar{d}$	20	$2 \cdot 10$
$\ell^-\nu d\bar{u}d\bar{d}\gamma$	152	$2 \cdot 76$
$\ell^+\ell^-\ell^-\nu d\bar{u}$	20	$2 \cdot 10$
$\ell^+\ell^-\ell^-\nu d\bar{u}\gamma$	150	$2 \cdot 75$
$\ell^-\nu\ell^+\bar{\nu}d\bar{d}$	19	$1 \cdot 9 + 2 \cdot 4 + 1 \cdot 2$
$\ell^-\nu\ell^+\bar{\nu}d\bar{d}\gamma$	107	$1 \cdot 59 + 2 \cdot 12 + 2 \cdot 8 + 2 \cdot 4$
$\ell^-\bar{\nu}\ell^+\nu\ell^+\ell^-$	56	$4 \cdot 9 + 4 \cdot 4 + 2 \cdot 2$
$\ell^-\bar{\nu}\ell^+\nu\ell^+\ell^-\gamma$	328	$4 \cdot 58 + 4 \cdot 12 + 4 \cdot 8 + 4 \cdot 4$
$\ell^+\nu\ell^-\bar{\nu}\nu\bar{\nu}$	36	$4 \cdot 6 + 6 \cdot 2$
$\ell^+\nu\ell^-\bar{\nu}\nu\bar{\nu}\gamma$	132	$4 \cdot 26 + 2 \cdot 6 + 4 \cdot 4$
$\nu\bar{\nu}\nu\bar{\nu}\nu\bar{\nu}$	36	$18 \cdot 2$

The familiar LEP2 gauge invariant classes (CC09, CC10, CC11, etc.) appear here automatically. However now we know that these classes are not only gauge invariant but they are minimal classes.

Let us take as an example the CC20 process (so called "single W") $e^+e^- \to e^-\nu d\bar{u}$ which splits to t- and s-channel CC10 gauge invariant subclasses. By means of the CompHEP (3) one can compute the contributions of the classes and their interference as shown in the Table below (4) (quark phase space cuts: $E_q \geq 3$ GeV, $M_{ud} \geq$ 5 GeV and the lepton phase space cut: $\cos\theta_e \geq 0.997$).

$\sqrt{s}$	$\sigma(CC10-t)$	$\sigma(CC10-s)$	$\sigma(t-sinterf.)$
quark phase space cuts, no ISR			
190	**147(0)**	**680(1)**	**5(0)**
350	**635(1)**	**420(1)**	**21(0)**
500	**1127(2)**	**270(0)**	**19(0)**
800	**1981(4)**	**143(0)**	**16(0)**
lepton and quark phase space cuts, no ISR			
190	**116(0)**	**2(0)**	**0.0(0)**
350	**513(1)**	**7(0)**	**0.2(0)**
500	**928(2)**	**10(0)**	**0.3(0)**
800	**1671(4)**	**15(0)**	**0.4(0)**

The CC10 t-channel part contains the single W boson production. It grows with the collision energy and it starts to dominate the CC10 s-channel part (W boson pair production) at about 320 GeV. One should stress a few points here

- a good precision of computations is obtained only if one splits the complete CC20 set of diagrams to CC10 subsets because in that case one can use different kinematical variables of integration for different subsets with different mapping of singularities (the interference contribution is small)

- the "overall" scheme of the W-boson width treatment could be used only for separate classes, otherwise there will be an artificial suppression of the CC10 t-channel part by the factor related to the second W pole

- for the CC10 s-channel part obviously a scale of the order of energy should be used for the electromagnetic α and ISR while the CC10 t-channel part has a very small characteristic virtuality of the soft virtual photon, and therefore a typical scale for the corresponding α and ISR should be taken much smaller, of the order electron momentum transfer (4, 5)

The same general statements are also true for the process of so called "single Z" production $e^+e^- \to e^+e^-\nu\bar{\nu}$. In this case there are 56 Feynman diagrams which split to 10 gauge invariant classes or 10 Groves $56D = 4*9D + 4*4D + 2*2D$ (6). Two classes of

4 diagrams each contribute to the single Z as given in the second column of the Table below. In the Table the contributions of the gauge invariant subsets in fb are given at the energy $\sqrt{s}$ =200 GeV. First row - with angular cuts, second row - no angular cuts for e^-, e^+. The following angular and lepton energy cuts are used: $\cos\theta_{e^-} \geq 0.997$ and $\cos\theta_{e^+} \leq 0.997$ and $E_l \geq 15$ GeV.

	$18W$	$8Z$	$9W^+W^-$	$4ZZ$
θ_e,E_l	36.1	16.4	0.91	0.02
only E_l	106.6	153.6	240.5	44.9

If the energy and angular cuts are applied still there are significant contributions from both single W (first column) and Z (second column) productions. One should be careful in interpretation of experimental measurements in this channel. (Contributions of other gauge invariant subsets are very small and we do not show them here).

One more example of applications of the gauge classes is related to the method of simplification of flavour combinatorics in the evaluation of hadronic processes (7). Here a serious computational problem is the large number of partonic subprocesses due to a presence of many quark partons with different flavors in the colliding hadrons and contributions of many additional diagrams for each subprocess because of the CKM quark mixing. However in the approximation when CKM matrix is reduced to the CK matrix without a mixing with the 3d quark generation

$$V_{CKM} \Longrightarrow \begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}, \qquad V = \begin{pmatrix} \cos\vartheta_c & \sin\vartheta_c \\ -\sin\vartheta_c & \cos\vartheta_c \end{pmatrix}$$

where ϑ_c is the Cabbibo angle and neglecting masses of the quarks from the first two generations $M_u = M_d = M_s = M_c = 0$ the problem can be simplified drastically. In this case diagrams contributing to the process can be split into the gauge invariant classes with different topologies of the incoming and outgoing quark lines. Then one can make a rotation of down quarks in all vertices of Feynman diagrams thus, transporting the mixing matrix elements from the diagrams to the parton distribution functions . As a result a number of rules for a convolution with quark distribution functions appear depending on the topology of the gauge invariant class (see details in (7); the method has been realized in the CompHEP version for hadron collisions V41.10).

In this talk we have discussed the method which allows splitting the complete set of Feynman diagrams contributing to a physics process into gauge invariant subclasses. Well known gauge invariant classes of diagrams like CC10, CC11, CC09 etc naturally appear in such an approach. It was demonstrated the above classes are the minimal invariant classes ("Groves"). For a concrete physical process one creates the graph - "Forest" in which vertices represent diagrams and edges show the connection between diagrams by possible flips, flavor and gauge. The vertices of the graph (diagrams) connected by the only gauge flips form connected subgraphs, "Groves", and the corresponding diagrams form minimal gauge invariant classes. The flavor flips connect diagrams from different gauge invariant classes.

Separation into gauge invariant classes in some cases allows to better understand properties of processes, to get better precision of calculations, to make in tree level computations a natural choice of characteristic scales for ISR, structure functions, running couplings etc. it also makes possible reasonable approximations, and leads to a simpfication of flavor combinatorics etc.

In some cases for processes with multi-particle final states the number of gauge invariant classes is much smaller than the number of physical reactions (8). So one can compute, in principle, amplitudes for gauge invariant subclasses of diagrams and then compute processes by taking different combinations of that amplitudes for classes.

The analysis was done for the tree level Feynman diagrams. A consideration at loop level is in progress (9).

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