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NLO QCD corrections to $\mathrm{t\bar{t}}$Higgs production

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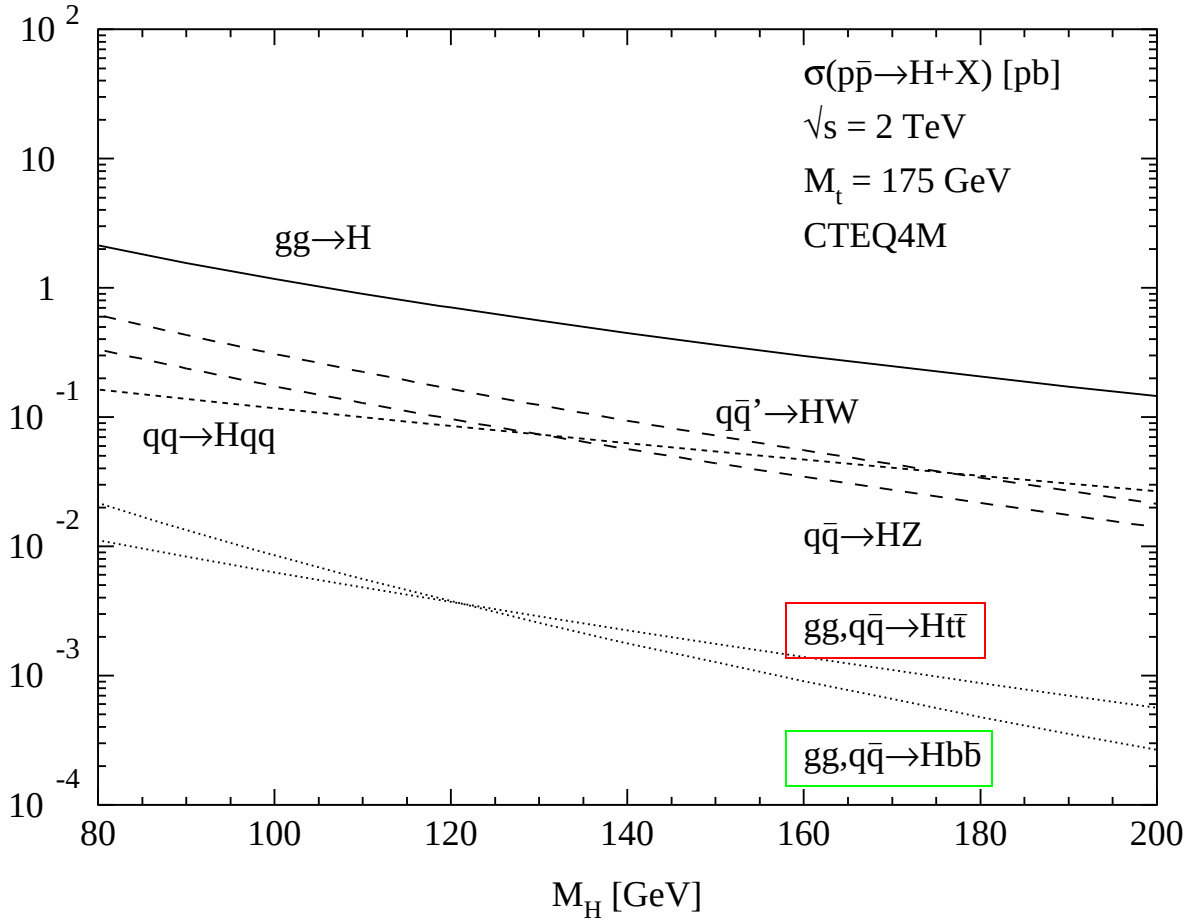
1 Motivation

Higgs-boson production at Tevatron

Spira '98



$t\bar{t}H$ production:

- only very few events, but low background

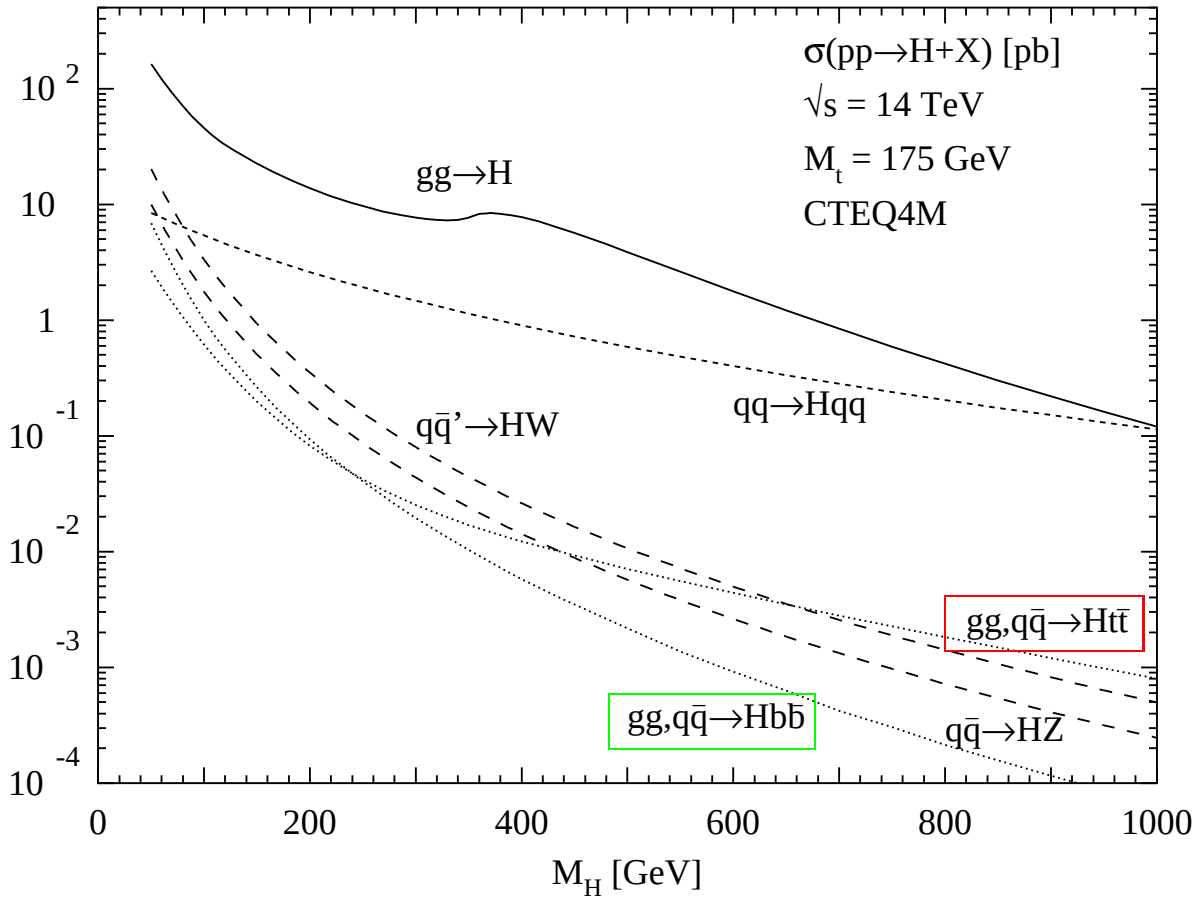
Goldstein, Hill, Incandela
Parke, Rainwater, Stuart '00

$b\bar{b}H$ production:

- enhanced in SUSY models for large $\tan\beta$

Higgs-boson production at the LHC

Spira '98



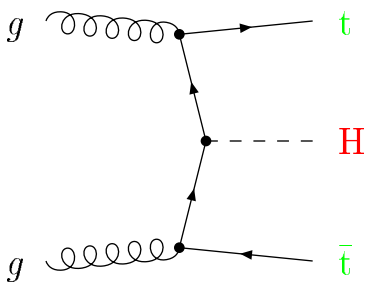
$t\bar{t}H$, $b\bar{b}H$ production:

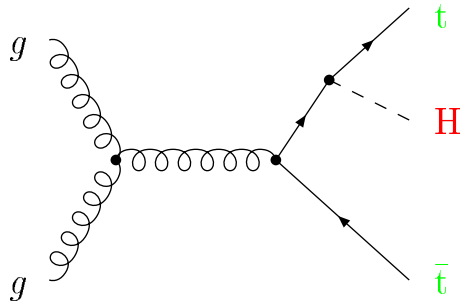
- cross sections by factor $\sim 10^2$–10^3 larger than at Tevatron
- $t\bar{t}H$ channel represents substantial contribution to 5σ discovery for $M_H \lesssim 120\,\mathrm{GeV}$
- determination of t and b Yukawa couplings

2 $\mathrm{t\bar{t}H}$ production in hadron collisions

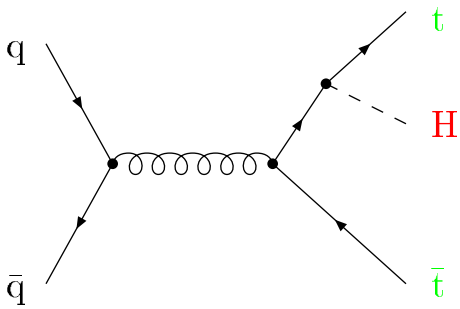
Typical LO diagrams:

...via gg fusion:





...via $\mathrm{q\bar{q}}$ annihilation:



Present status of predictions:

- LO predictions — Kunszt '84, Dicus, Willenbrock '89; Gunion '91, Marciano, Paige '91

  $\hookrightarrow$ large scale dependence

  $\Rightarrow$ NLO prediction very important ! — Beenakker, S.D., Krämer, Plümper, Spira, Zerwas; Dawson, Orr, Reina, Wackeroth

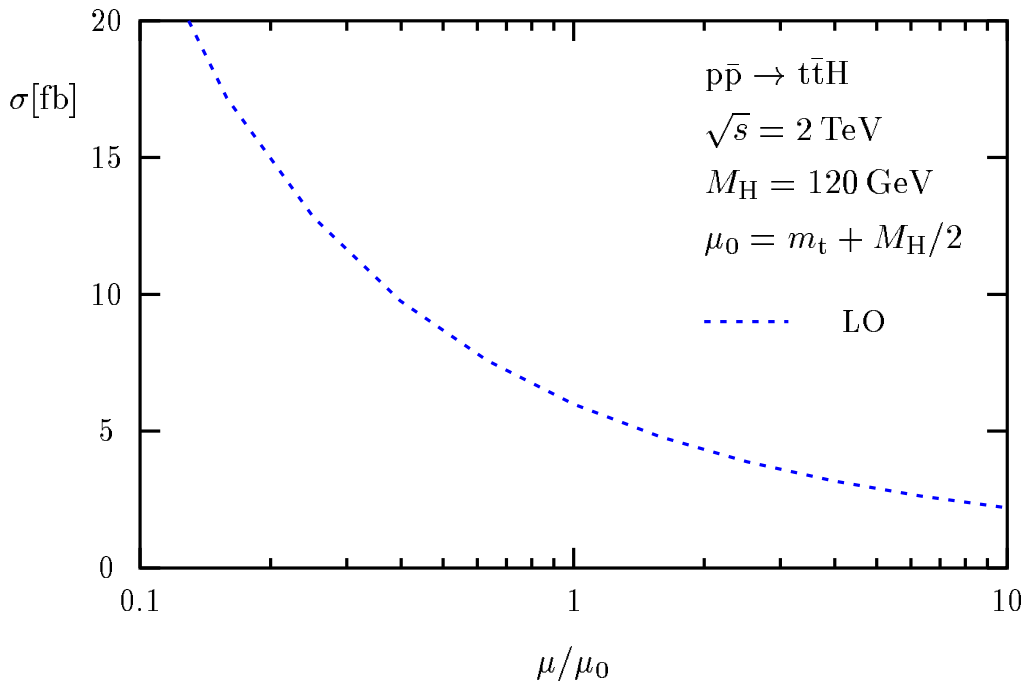
  Work in progress by two groups

- Estimate of NLO corrections for $M_{\mathrm{H}} \ll m_{\mathrm{t}}$ — Dawson, Reina '98

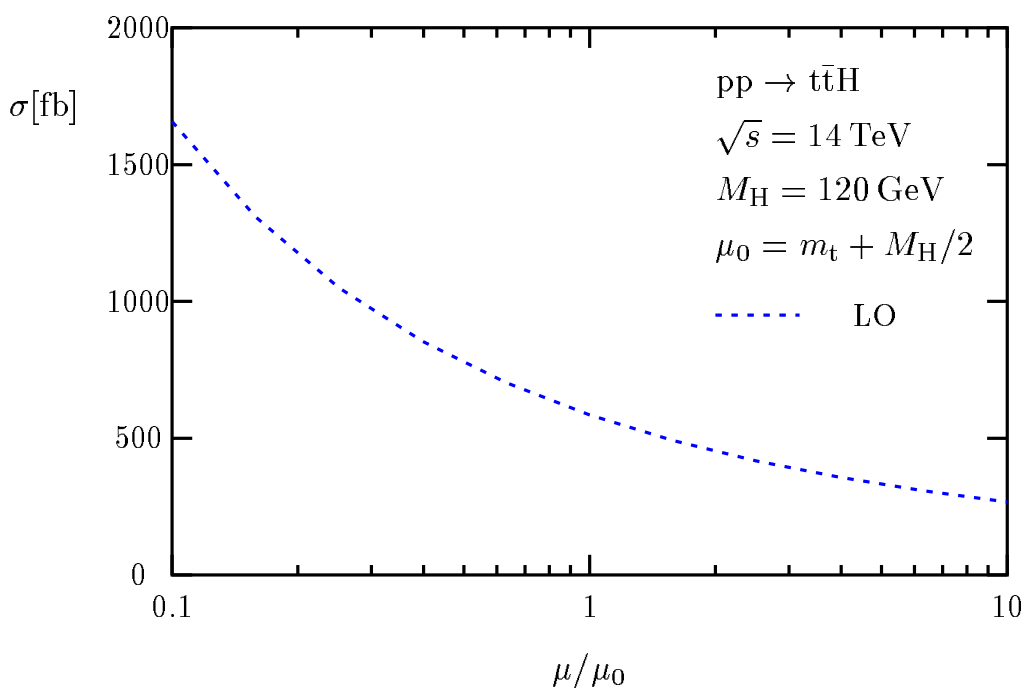
  in "equivalent Higgs approximation"

Scale dependence of LO cross section

... at Tevatron: $\mu = \mu_{\rm ren} = \mu_{\rm fact}$



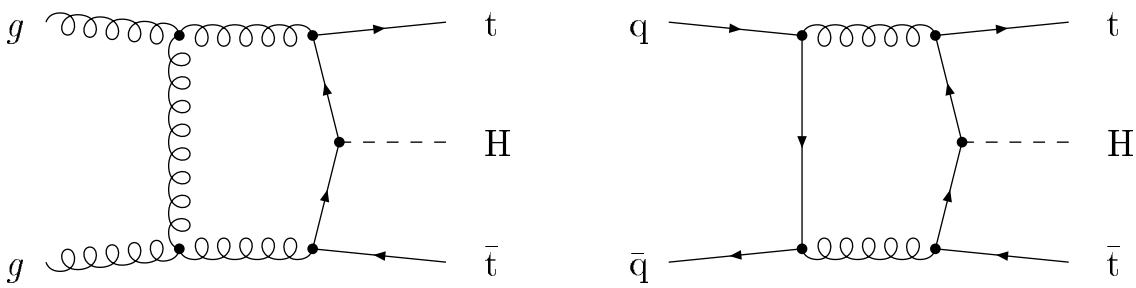
... at the LHC:



Large scale dependence !

Complications in the virtual corrections

Pentagon diagrams:



etc.

$\rightarrow$ One-loop integrals of the form

$$E_{\mu\nu\ldots} = \frac{(2\pi\mu)^{4-D}}{\mathrm{i}\pi^2} \int \mathrm{d}^D q \, \frac{q_\mu q_\nu \cdots}{[(q+p_1)^2 - m_1^2] \cdots [(q+p_5)^2 - m_5^2]}$$

Problems:

- complicated singularity stucture (IR and collinear)

  trick: $\left[E - E^{\mathrm{sing}}\right]\big|_{\mathrm{dim\,reg.}}$ = finite = $\left[E - E^{\mathrm{sing}}\right]\big|_{\mathrm{mass\,reg.}}$

  – E^{sing} relatively easy calculable in both regularizations
  $\hookrightarrow$ combinations of 3-point functions

  – $E|_{\mathrm{mass\,reg.}}$ calculable in 4 dimensions
  $\hookrightarrow$ combinations of 4-point functions

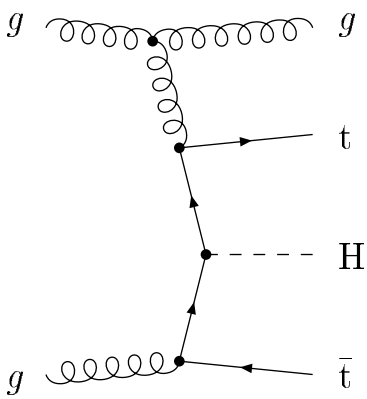
  $\rightarrow$ solve for $E|_{\mathrm{dim\,reg.}}$

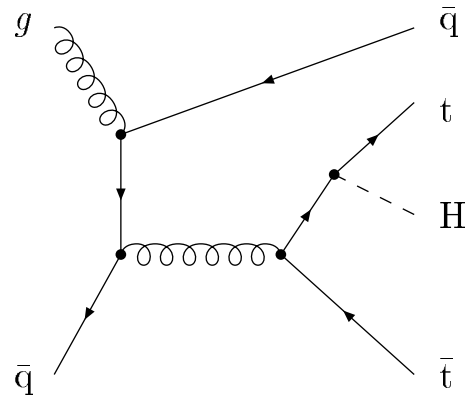
- numerical instabilities in Passarino–Veltman decomposition
  origin:
  linear dependence of momenta on phase-space boundary

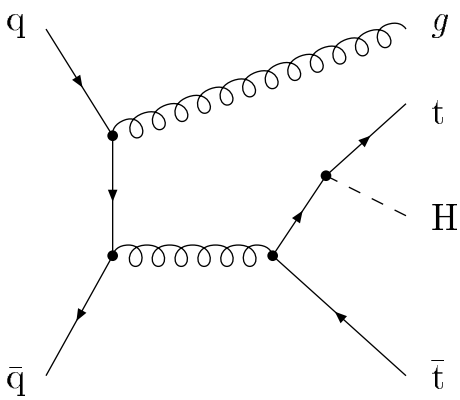
  solution:
  numerical extrapolation from safe to problematic regions

Complications in the real corrections

Typical diagrams:







etc.

Problems:

- complexity of matrix elements
- extraction of collinear/IR divergences and factorization

  $\rightarrow$ use generalization of dipole subtraction formalism

Dipole subtraction formalism

→ process-independent treatment of singularities in real NLO corrections

worked out for

- QCD with massless partons
  Catani, Seymour '96
- γ radiation off massive fermions
  S.D. '99

} QCD with massive partons
Phaf, Weinzierl '01
Catani, S.D., Trócsányi '01
(in preparation)

basic idea: NLO correction to process with m partons

$$\sigma^{\rm NLO} = \underbrace{\int_{m+1}\left[{\rm d}\sigma^{\rm real} - {\rm d}\sigma^{\rm sub}\right]}_{\text{finite}} + \underbrace{\int_{m}\left[{\rm d}\sigma^{\rm virtual} + {\rm d}\bar{\sigma}_1^{\rm sub}\right]}_{\text{finite}}$$

$$+ \int_0^1 {\rm d}x \underbrace{\int_{m}\left[{\rm d}\sigma^{\rm fact}(x) + \left({\rm d}\bar{\sigma}^{\rm sub}(x)\right)_+\right]}_{\text{finite}}$$

conditions on ${\rm d}\sigma^{\rm sub}$:

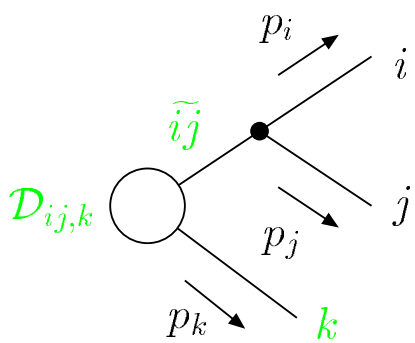
- sum rule:

$$-\int_{m+1} {\rm d}\sigma^{\rm sub} + \int_m {\rm d}\bar{\sigma}_1^{\rm sub} + \int_0^1 {\rm d}x \int_m \left({\rm d}\bar{\sigma}^{\rm sub}(x)\right)_+ = 0$$

- asymptotics:

$$\sigma^{\rm sub} \sim \sigma^{\rm real} \quad \text{in all collinear/IR regions}$$

On the construction of $\mathrm{d}\sigma^{\mathrm{sub}}$:



"dipole" $(\widetilde{ij}, k)$

$$\int \mathrm{d}\sigma^{\mathrm{sub}} = \sum_{\text{dipoles}} \int \mathrm{d}\sigma^{\mathrm{LO}} \otimes \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

Color and spin correlations in $\mathrm{d}\sigma^{\mathrm{LO}}$

singular limits:

- IR limit: $p_i \to 0$

$$\mathcal{D}_{ij,k} + \mathcal{D}_{ik,j} \sim \text{const.} \times \frac{p_j \cdot p_k}{(p_i \cdot p_j)(p_i \cdot p_k)}$$

"eikonal factor"

- collinear limit: $p_i \cdot p_j \to 0,\ m_{i,j,ij} \to 0$

$$\mathcal{D}_{ij,k} \sim \frac{\text{const.}}{(p_i + p_j)^2 - m_{ij}^2} \times \hat{P}_{\widetilde{ij},i}(p_i, p_j)$$

Altarelli–Parisi function

main difficulties:

- phase-space embedding: $\int \mathrm{d}\Phi_{m+1} = \int_0^1 \mathrm{d}x \int \mathrm{d}\Phi_m(x) \int [\mathrm{d}p_i(x)]$
- analytical integration of $\int_0^1 \mathrm{d}x \int [\mathrm{d}p_i(x)]$:

$$\int \mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x) = \int \mathrm{d}\sigma^{\mathrm{LO}} \otimes \int [\mathrm{d}p_i(x)]\, \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

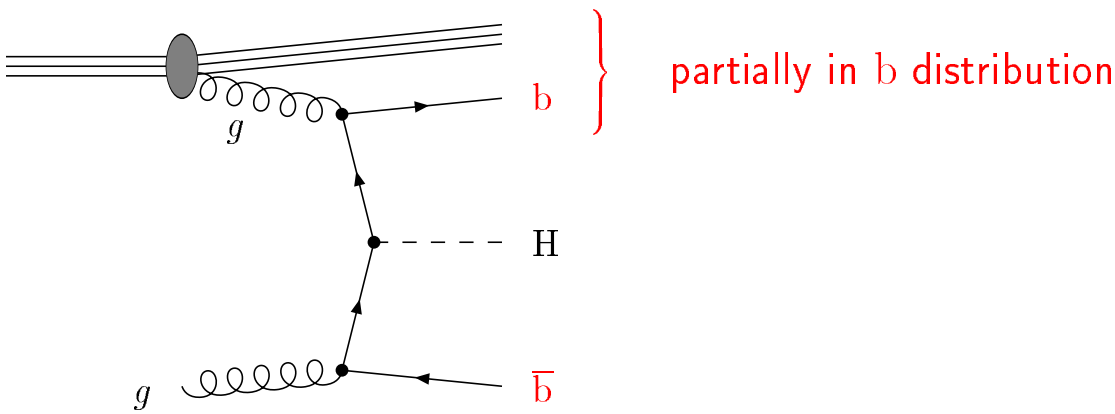
$$\mathrm{d}\bar{\sigma}_1^{\mathrm{sub}} = \int_0^1 \mathrm{d}x\, \mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x)$$

Remarks on $\mathrm{b\bar{b}H}$ production

Pure total cross section

LO process: $\mathrm{b\bar{b}} \to \mathrm{H}$

- b's from $g \to \mathrm{b\bar{b}}$ splitting inside the proton
  $\hookrightarrow$ b-quark distribution with DGLAP evolution
- NLO prediction known — Dicus, Stelzer, Sullivan, Willenbrock '98
- but: b's have very small p_T
- $\ln(m_\mathrm{b}/M_\mathrm{H})$ part of the $\mathrm{q\bar{q}}, gg \to \mathrm{b\bar{b}H}$ cross section already contained in $\mathrm{b\bar{b}} \to \mathrm{H}$
  $\Rightarrow$ avoid double counting !



Realistic situation:

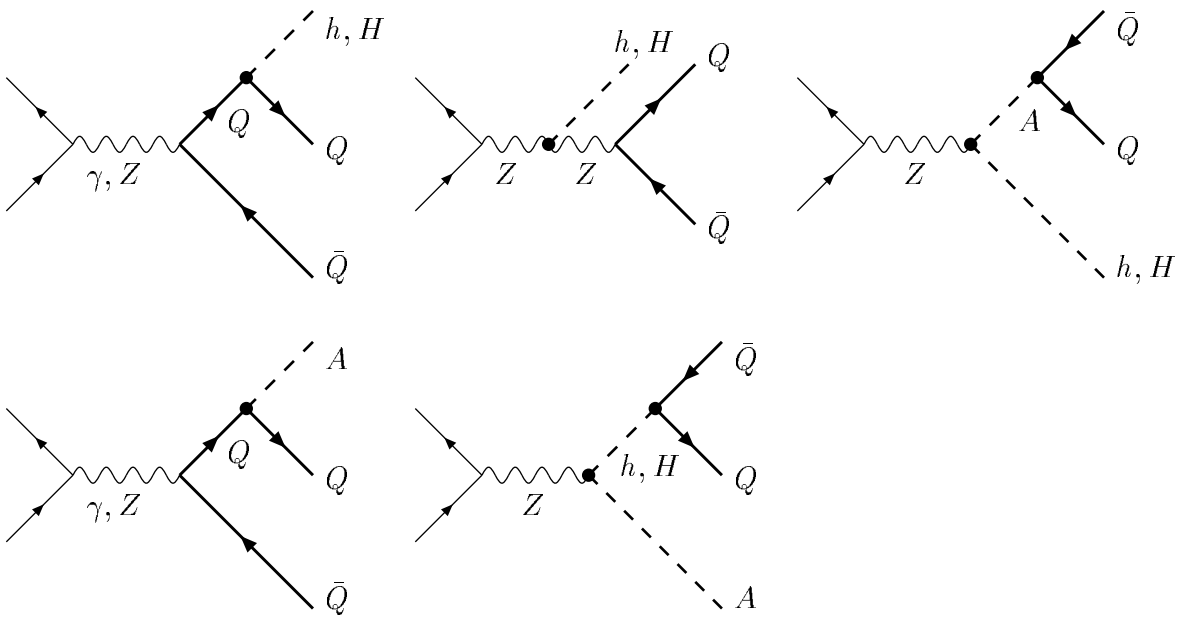
at least one b quark with finite p_T required to define signal

$\Rightarrow$ $\mathrm{q\bar{q}}, gg \to \mathrm{b\bar{b}H}$ is LO process for $\mathrm{b\bar{b}H}$ for b's with finite p_T

- NLO prediction not known
- p_T distribution for b's required in NLO
  $\to$ resummation of corrections $\alpha_\mathrm{s} \ln(p_\mathrm{T}^2/M_\mathrm{H}^2)$ desirable

3 $\mathrm{t\bar{t}H}, \mathrm{b\bar{b}H}$ production in $\mathrm{e^+e^-}$ annihilation

Typical LO diagrams in SM and MSSM:



Important features:

- $\mathrm{t\bar{t}H}$: Yukawa coupling measureable with 2–3% at an $800\,\mathrm{GeV}$ linear collider Juste, Merino '99; S.Moretti '99 Baer, Dawson, Reina '00 TESLA Collaboration '01

- $\mathrm{b\bar{b}H}$: contributions from resonances $\mathrm{h, H, A} \to \mathrm{b\bar{b}}$ in MSSM if kinematically possible

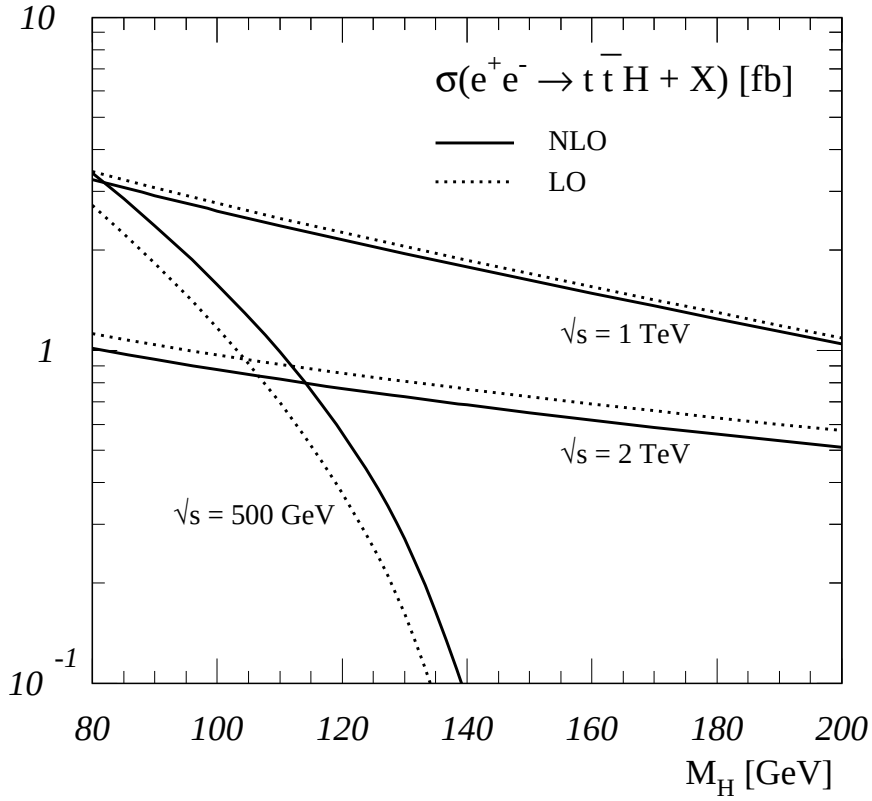
  $\Rightarrow$ use continuum contribution to determine Yukawa couplings

- NLO corrections known in SM and MSSM

  S.D., Krämer, Liao, Spira, Zerwas '98, '00
  Dawson, Reina '98,'99

Total SM cross section for $e^+e^- \to t\bar{t}H$ in NLO:

S.D., Krämer, Liao, Spira, Zerwas '98



Asymptotic behaviour of $K = \sigma_{\rm NLO}/\sigma_{\rm LO}$:

- near threshold: Coulomb rescattering correction

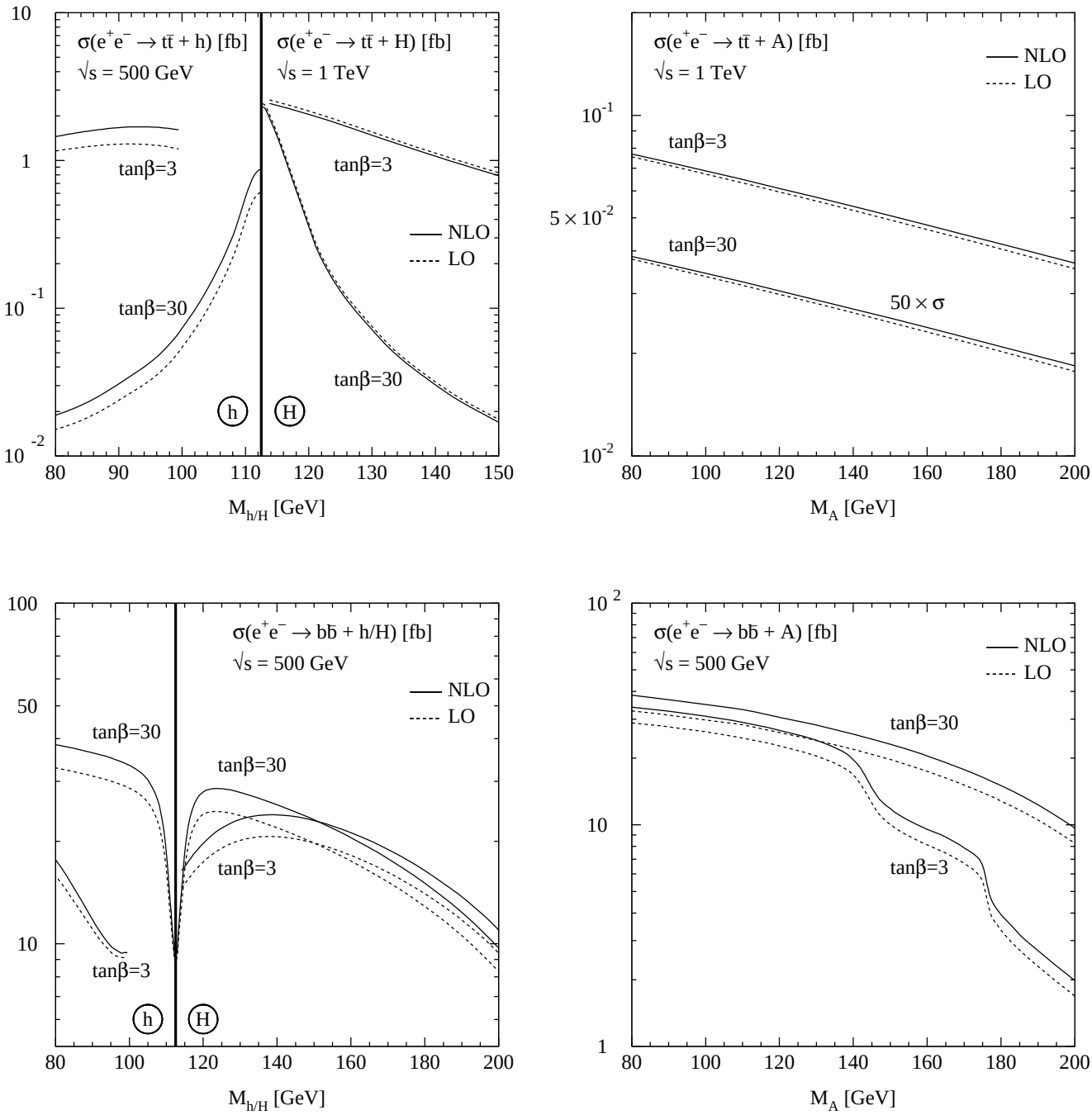
$$\Rightarrow \quad K \sim 1 + \frac{32\alpha_s}{9\beta_t} \qquad \text{with } \beta_t = \sqrt{(\sqrt{s} - M_{\rm H})^2 - 4m_{\rm t}^2}/2m_{\rm t}$$

- $s \gg 4m_{\rm t}^2 \gg M_{\rm H}^2$: $e^+e^- \to t\bar{t}$ with soft-Higgs radiation

$$\Rightarrow \quad K \sim 1 - 3\frac{\alpha_s}{\pi} + \text{small corrections}$$

Total MSSM cross sections in NLO:

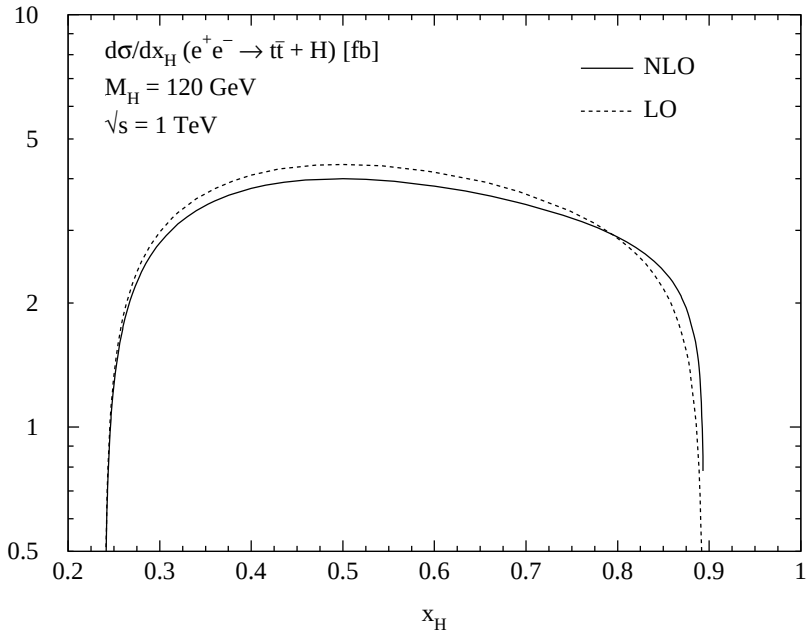
S.D., Krämer, Liao, Spira, Zerwas '00



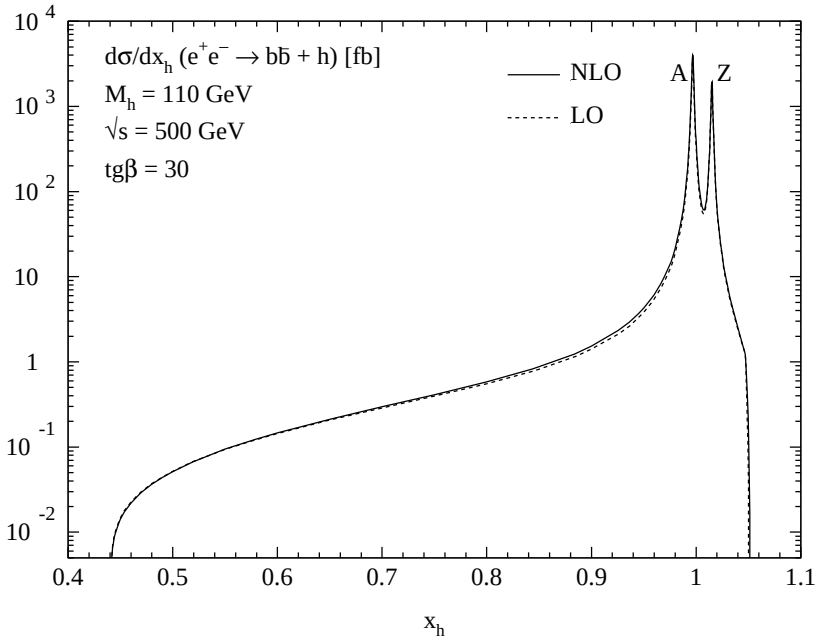
Higgs energy distributions in NLO

S.D., Krämer, Spira, Zerwas '01

... for $t\bar{t}H$ production in the SM:



... for $b\bar{b}h$ production in the MSSM:



4 Summary

$\mathrm{t\bar{t}H}$ production:

- important channel for Higgs searches at Tevatron and the LHC
- LO predictions have large scale uncertainties
- NLO predictions in progress

  $\hookrightarrow$ significant reduction of scale uncertainties

$\mathrm{b\bar{b}H}$ production:

- particular importance in SUSY models for large $\tan\beta$
- total cross section partially contained in $\mathrm{b\bar{b}} \to \mathrm{H}$

  but: $\mathrm{q\bar{q}}, gg \to \mathrm{b\bar{b}H}$ relevant for b's with finite p_{T}
- NLO predictions still missing

Situation for $\mathrm{e^+e^-}$ annihilation:

- precision measurement of Yukawa couplings possible at future colliders
- NLO predictions known in SM and MSSM