Setting Limits in CDF New Particle Searches

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- Introduction
- Single-channel counting experiments
- Particle searches using spectrum fits
- Combining channels
- Remarks

Introduction

In Run I at CDF, we searched for many non-standard-model particles.

Despite our wishes, we see no clear evidences to claim a discovery of such a particle, so far.

This talk focuses on simply describing how we set limits in many of these studies.

(Not intended to focus on detail discussion of mathematical techniques & their validities)

note 1:

For Run II new particle search capability studies, see http://fnth37.fnal.gov/susy.html
http://runiicomputing.fnal.gov/strongdynamics/web/strongdynamics.html

note 2:

See John Conway's talk from Jan. CERN CL workshop for "What if there is an excess?"

note 3:

"e" "e" $\gamma\gamma$ $\not{E}_T$ event

→ no model, so simply estimate SM probability ... a posteriori
Phys. Rev. D 59, 092002 (1999)

Setting Limits in Single-Channel Counting Experiment

We search for a new particle & figure out:

n_0 : # of observed events in $\underline{L}$ pb^{-1}.

$\mu_b \pm \sigma_{\mu_b}$: # of expected background events and its uncertainty.

$\mu_s \pm \sigma_{\mu_s}$: # of expected signal events (From a model) and its uncertainty.

Using these numbers, how can we constrain the theoretical model quantitatively?

We want to use a method which

* is reasonable and well defined.
* takes into account uncertainties in background
* takes into account uncertainties in signal acceptance.

CDF uses the 'Standard' frequentist definition of a 95% C.L. limit with a (Bayesian) integration over background and Acceptance uncertainties

Poisson Distribution, $P(n_0|\mu)$

$$\boxed{P(n_0|\mu) = \frac{\mu^{n_0} e^{\mu}}{n_0!}}$$

Poisson distribution with mean value μ gives probability of observing n_0 events.

Confidence Limit (C.L.) & the Upper Limit (N_{limit})

N_{limit} : Number of expected events as that value of μ for which there is some probability ϵ to observe n_0 or fewer events.

$$\epsilon = \sum_{n=0}^{n_0} P(n|\mu)$$

then

$$CL = 1 - \epsilon$$

In practice, we calculate N_{limit} by varying μ until finding the value of ϵ corresponding to the desired CL

→ N_{limit} is the resultant value of μ.

example

with no (uncertainty / background)

n_0	95% C.L. N_{limit}
0	3.00
1	4.75
2	6.30
3	7.75

Example

→ Poisson distribution with mean value = 4.75 gives 5% probability that we observe 1 or 0 events.

We exlude a model which expects more than 4.75 events at 95% C.L.

$$\rightarrow \boxed{(\sigma \cdot B_r)_{limit} = \frac{N_{limit}}{\varepsilon_{tot} \cdot L}}$$ cross section Limit

note: N_{limit} is a smaller number (appears 'better') at 90% C.L. (< 95% C.L.) with the same experimental results.

ε_{tot}: Total signal efficiency, L: Inter luminosity

Setting Limits when we expect μ_b background events in n_0 observed events

(no uncertainties, yet)

$$\begin{cases} n_0 : \text{\# of observed events} \\ \mu_b : \text{expected \# of back ground events in } n_0 \\ n_b : \text{\# of back ground events in } n_0 \ (n_b \le n_0) \\ \mu_s : \text{mean \# of signal event expected} \end{cases}$$

$$\epsilon = \frac{\sum_{n=0}^{n_0} P(n \mid \mu_s + \mu_b)}{\sum_{n=0}^{n_0} P(n \mid \mu_b)}$$

$$= \frac{\sum_{n=0}^{n_0} \frac{(\mu_s+\mu_b)^n \cdot e^{-(\mu_s+\mu_b)}}{n!}}{\sum_{n=0}^{n_0} \frac{\mu_b^n \cdot e^{-\mu_b}}{n!}}$$

N_{limit} (for the signal) at C.L. $= \epsilon$ represents the value of μ_s for which there is some probability $1-\epsilon$ to observe more than n_0 events
(= ϵ prob. to observe n_0 or less evts)

and have $n_b \le n_0$.

note: For a given n_o (observed events), we get "better" limit for higher # of expected background μ_b.

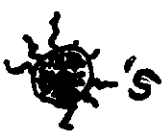
(downward fluctuation "Problem")

example:

Suppose we observe 1 event, when

1) expecting 1 background event
2) expecting 10 background events.

→ 2) → Gives 'better' limit!

→ Suggestion: Check  's

Incorporating Uncertainties

To incorporate

$$\begin{cases} \sigma_{\mu_b} & : \text{uncertainty in background} \\ \sigma_{\mu_s} & : \text{uncertainty in signal} \end{cases}$$

we integrate over all possible values of the true value $\mu_{s'}$ and $\mu_{b'}$.

This is a Bayesian technique, and assumes a Gaussian prior probability-density function for the true values $\mu_{s'}$ and $\mu_{b'}$.

N_{limit} at $1-\epsilon$ C.L. is equal to μ_s which satisfies:

$$\epsilon = \frac{\sum_{n=0}^{n_0} \frac{1}{\sqrt{2\pi\sigma_{\mu_s}^2}} \int_0^\infty \int_0^\infty P(n \mid \mu_{s'} + \mu_{b'}) e^{-\frac{(\mu_b - \mu_{b'})^2}{2\sigma_b^2}} e^{-\frac{(\mu_s - \mu_{s'})^2}{2\sigma_s^2}} d\mu_{b'} d\mu_{s'}}{\sum_{n=0}^{n_0} \int_0^\infty P(n \mid \mu_{b'}) e^{-\frac{(\mu_b - \mu_{b'})^2}{2\sigma_{\mu_b}^2}} d\mu_{b'}}$$

$\sigma_{\mu_s} = \mu_s \cdot \frac{\sigma_{\varepsilon_{tot}}}{\varepsilon_{tot}}$, ε_{tot} ← total efficiency for the

To obtain N_{limit} at $(1-\epsilon)$ c.l.

One could perform the formidable integral (in previous page).

Instead, we solve this by performing a large ensemble of random pseudo-experiments varying the expected number of signal and background about their nominal values according to a gaussian distribution.

→ standardized in program poilim.f

(J. Conway)
checked by various people.

Single-channel Counting Experiment

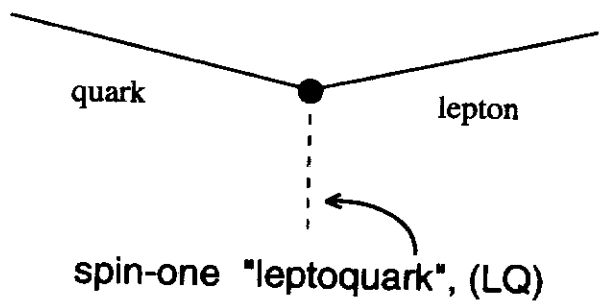
→ Used in many CDF new particle searches
SUSY, Charged Higgs, Leptoquarks
etc ....

example → **Pati-Salam Model and "leptoquark"** Search

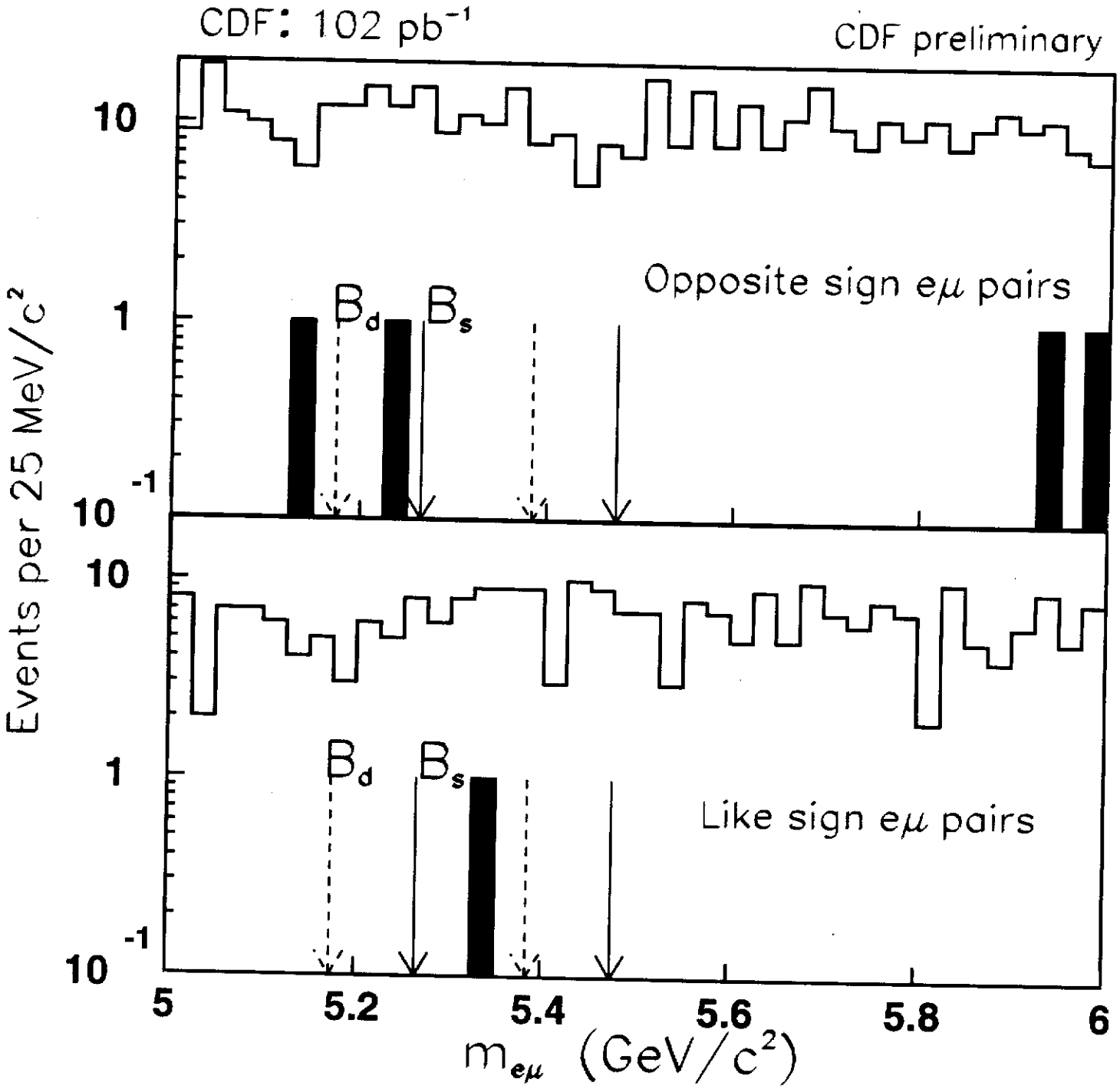
Pati-Salam Model: Simple gauge theory of quark-lepton unification
Lepton number as the Fourth "color"

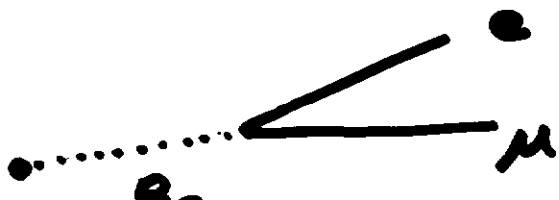
Symmetry breaking: $SU(4)_c \longrightarrow SU(3)_c$ at high E

the theory predicts: Heavy Pati-Salam Bosons (leptoquarks)



PRL Dec. 28, 1998
Vol 81, 26 pp. 5742-5747


CDF: 102 pb^{-1}
CDF preliminary
Opposite sign $e\mu$ pairs
B_d
B_s
Like sign $e\mu$ pairs
Events per 25 MeV/c^2
$m_{e\mu}$ (GeV/c^2)
10
1
10^{-1}
5
5.2
5.4
5.6
5.8
6


e
μ
e

N^{limit}

The limit is dominated by Poisson statistic. The Poisson limits are:

no uncertainties

- $N^{limit} = 2.3$ (3.0) events for observing zero events (B_s case).
- $N^{limit} = 3.9$ (4.7) events for observing one events (B_d case).

We combine the systematic uncertainties with the poisson limits using the convoluted-likelihood function method.

uncertainty on B-meson cross section	±23 %
uncertainty on acceptance and cut efficiencies	±10%
uncertainty on $\int \mathcal{L}dt$	±6.9%
Total	±26.0%

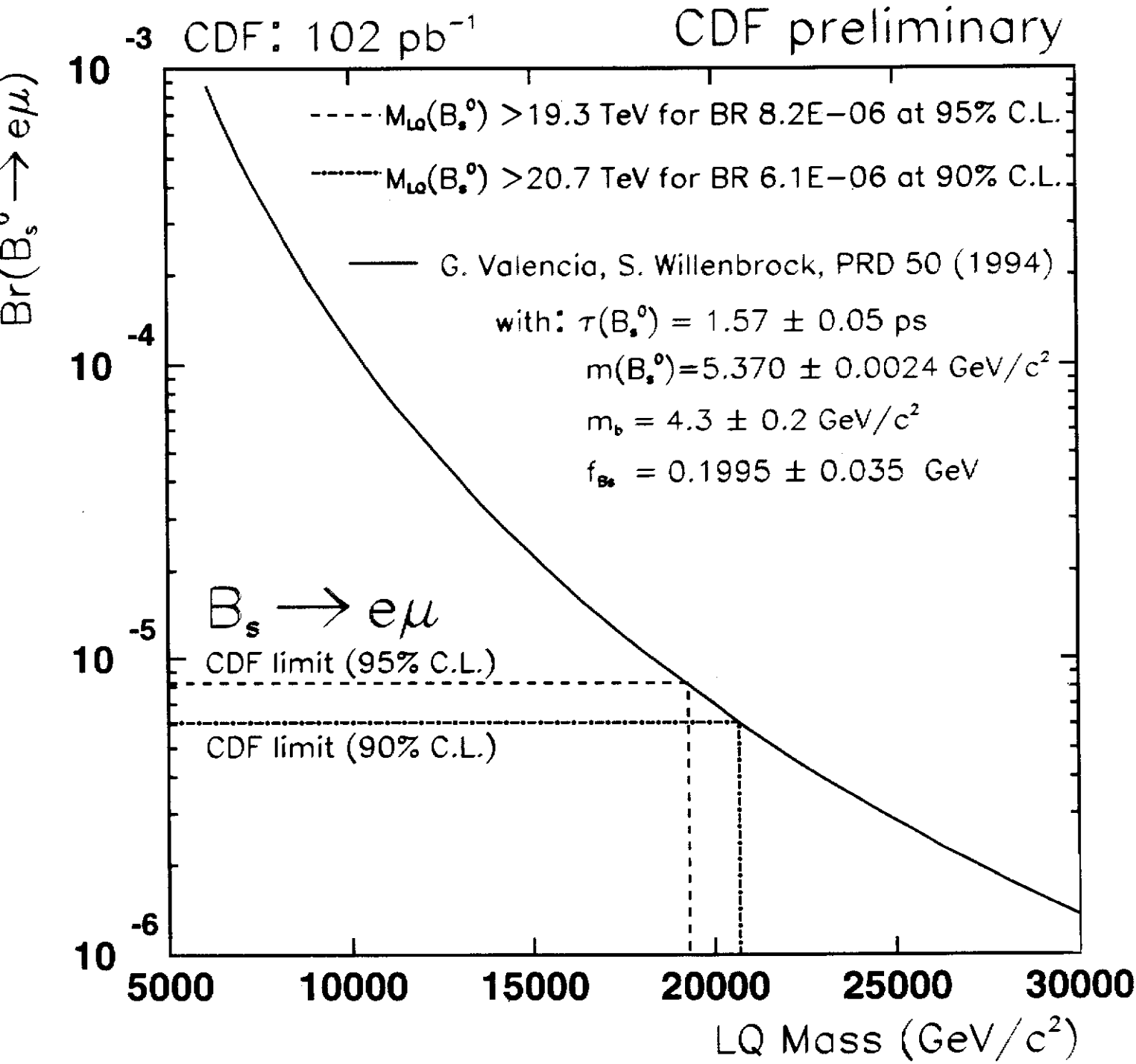
Table 1: Systematic errors in %

with uncertainties

- $N^{limit} = 2.52$ (3.38) events for observing zero events (B_s case).
- $N^{limit} = 4.34$ (5.52) events for observing one events (B_d case).

95% C.L.

90% C.L.


CDF: 102 pb^{-1}
CDF preliminary
$M_{LQ}(B_s^0)$ >19.3 TeV for BR 8.2E−06 at 95% C.L.
$M_{LQ}(B_s^0)$ >20.7 TeV for BR 6.1E−06 at 90% C.L.
G. Valencia, S. Willenbrock, PRD 50 (1994)
with: $\tau(B_s^0)$ = 1.57 ± 0.05 ps
$m(B_s^0)$=5.370 ± 0.0024 GeV/c^2
m_b = 4.3 ± 0.2 GeV/c^2
f_{Bs} = 0.1995 ± 0.035 GeV
$B_s \rightarrow e\mu$
CDF limit (95% C.L.)
CDF limit (90% C.L.)
$Br(B_s^0 \rightarrow e\mu)$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
5000
10000
15000
20000
25000
30000
LQ Mass (GeV/c^2)

Advantages

- Simple extension of standard frequentist method → easy to understand
- No issue of choice of Prior Prob. density function.
- Can be easily extended to take into account correlated uncertainties.
- Can always quote an upper limit even if there is an excess (no flip-flopping)

Disadvantages

- Suffers from downward fluctuation "problem".
- How do we combine channels?
- Only works for single-channel counting experiment, not fit to spectra.

Limits from Spectra and Combining Channels

In searching for new particles using a fit to spectra, CDF typically uses binned likelihood fit method, allowing the signal cross section multiplier, f, to float.

$$\boxed{\mu_i = \mu_{b_i} + f \cdot \mu_{s_i}}$$

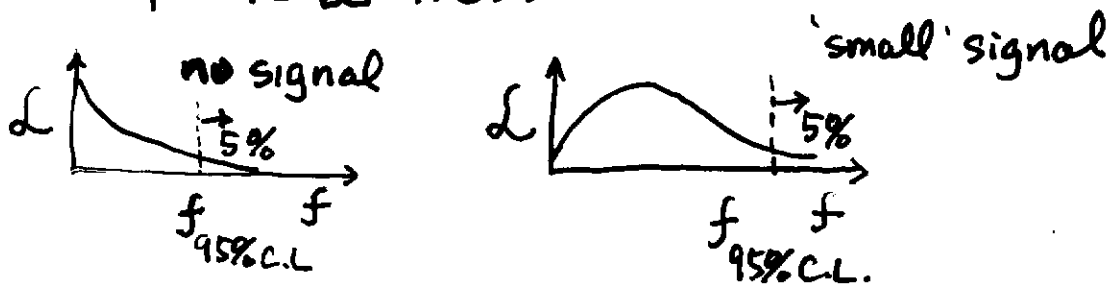
Likelihood function

$$\boxed{\mathcal{L} = (\text{Norm fac}) * \prod_{i=\text{bin}}^{N_{\text{bins}}} \frac{\mu_i^{n_i} \cdot e^{-\mu_i}}{n_i}}$$

Norm fac $\leftarrow$ $\boxed{1.0 = \int_0^\infty \mathcal{L}(f)\, df}$

To Obtain 95% C.L. Limit

Plot the likelihood function $\mathcal{L}(f)$, then find value of f beyond which 5% of the total integral of the $\mathcal{L}$ lies.



If $f_{95\%\,C.L} < 1$ then the theoretical prediction is ruled out at (at least) 95% C.L.

- Uncertainties and signal & background are taking into account by integrating over them in the same way as described for single channel counting experiment

- multiple channel results can be combined by multiplying the likelihoods for the different channels.

In a case,
Single Channel, single bin, no uncertainty
$\Longrightarrow$ equivalent to the frequentist method

<u>Analyses used this techniques</u>

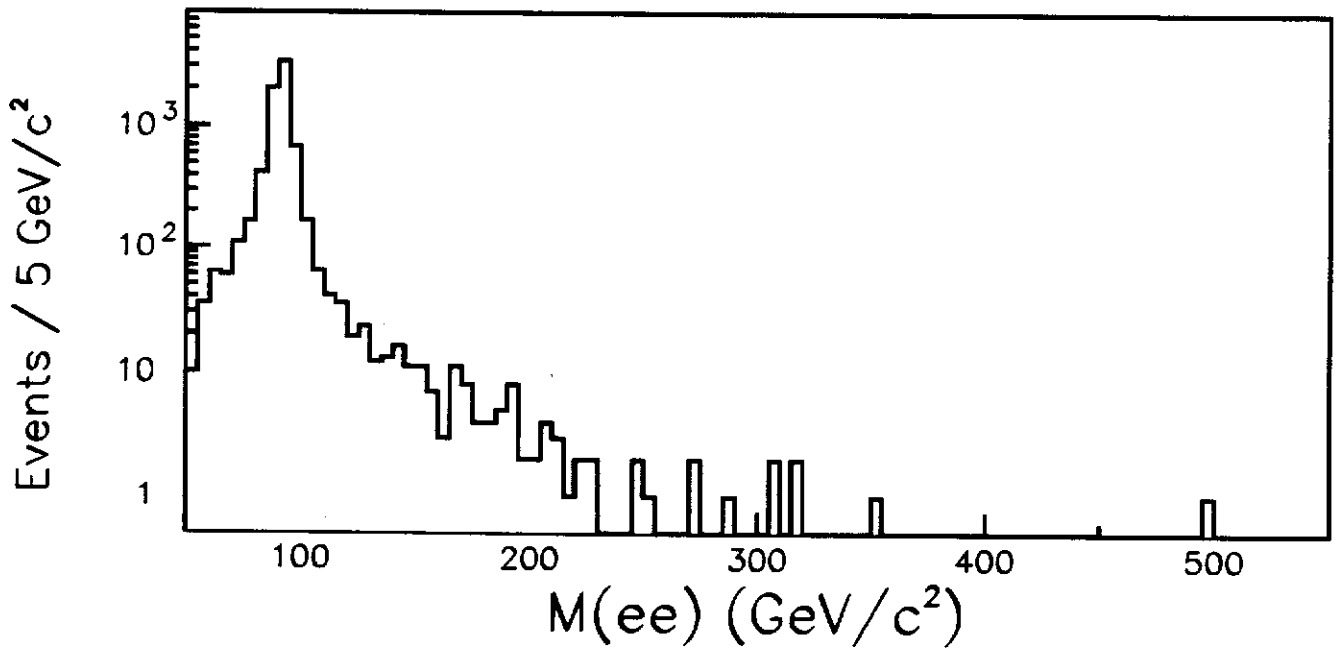
many .....

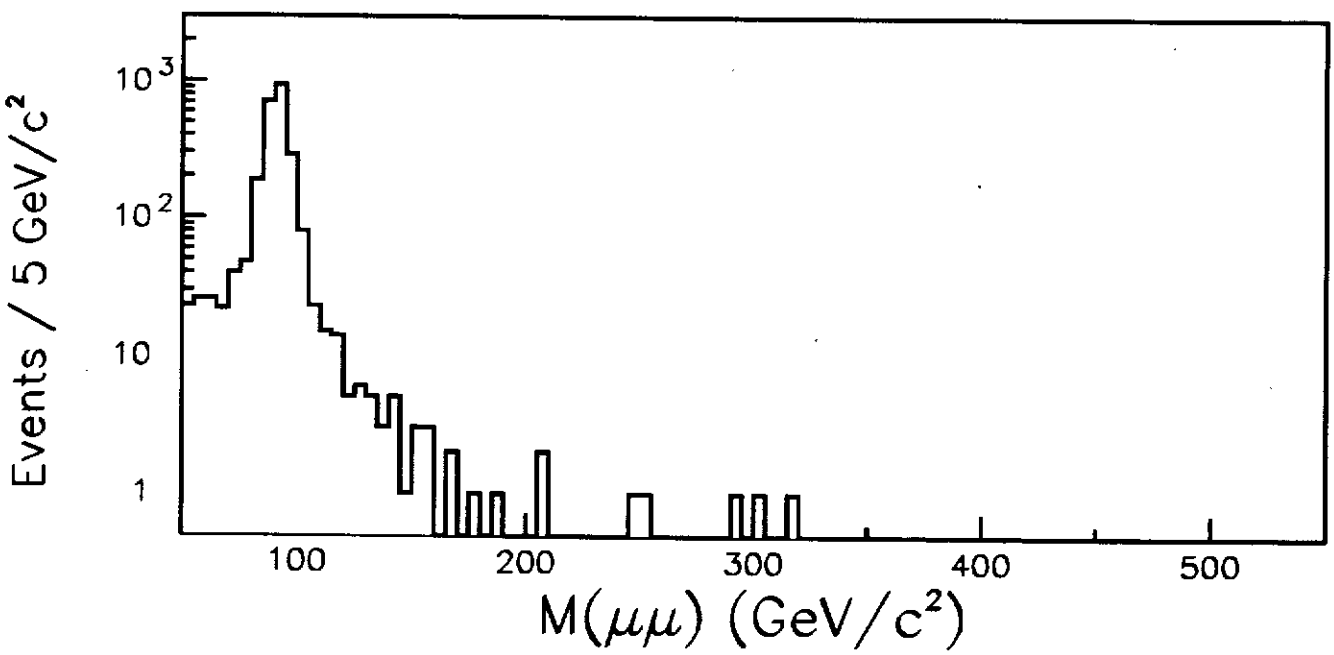
for example,

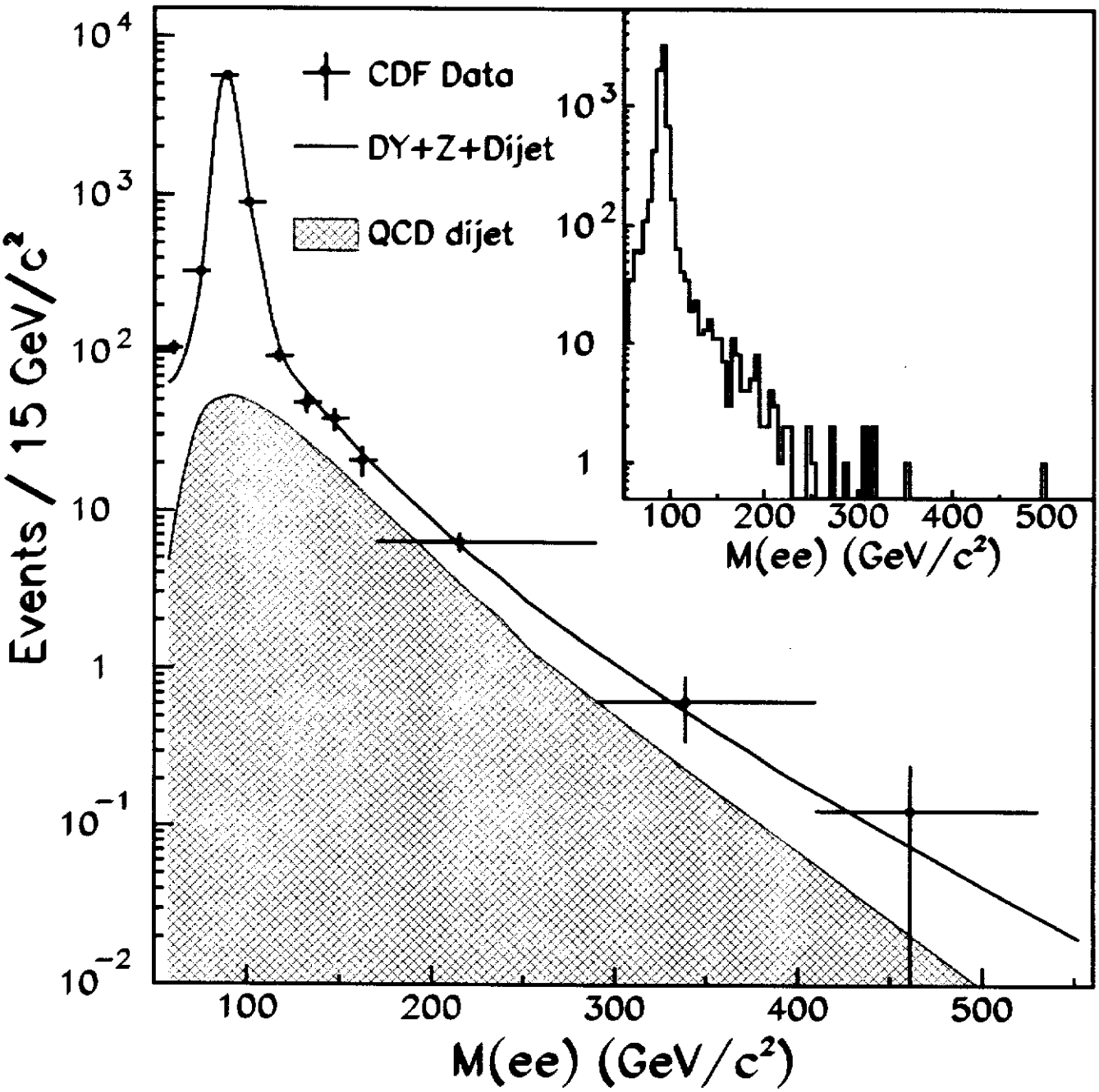
* Z' search (ee) & $(\mu\mu)$ channels
* TC (π_T, ρ_T) search, (leptonic)+(Hadronic)
* SM. H_0 search, (leptonic)+(Hadronic)
* 4th-gen. $b' \to bZ$ search

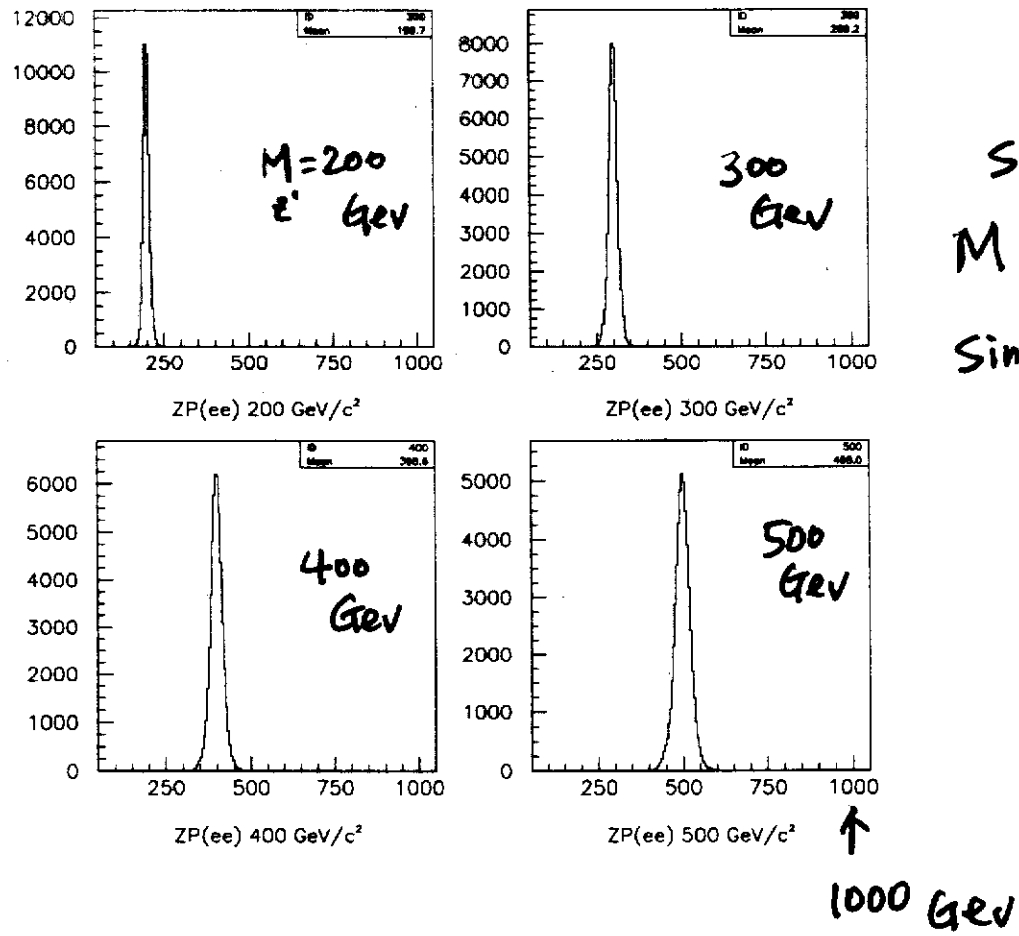
--- etc.

Z' Search


Events / 5 GeV/c²
10³
10²
10
1
100
200
300
400
500
M(ee) (GeV/c²)


Events / 5 GeV/c²
10³
10²
10
1
100
200
300
400
500
M(μμ) (GeV/c²)


CDF Data
DY+Z+Dijet
QCD dijet
Events / 15 GeV/c²
M(ee) (GeV/c²)
M(ee) (GeV/c²)



signal
M(Z'→ee)
simulation

Fig. 3. The figure shows distributions of Z' events in dielectron invariant mass for events after the detector smearing effects are accounted for. The distributions are shown for a Standard model Z' of mass 200, 300, 400 and 500 GeV respectively.

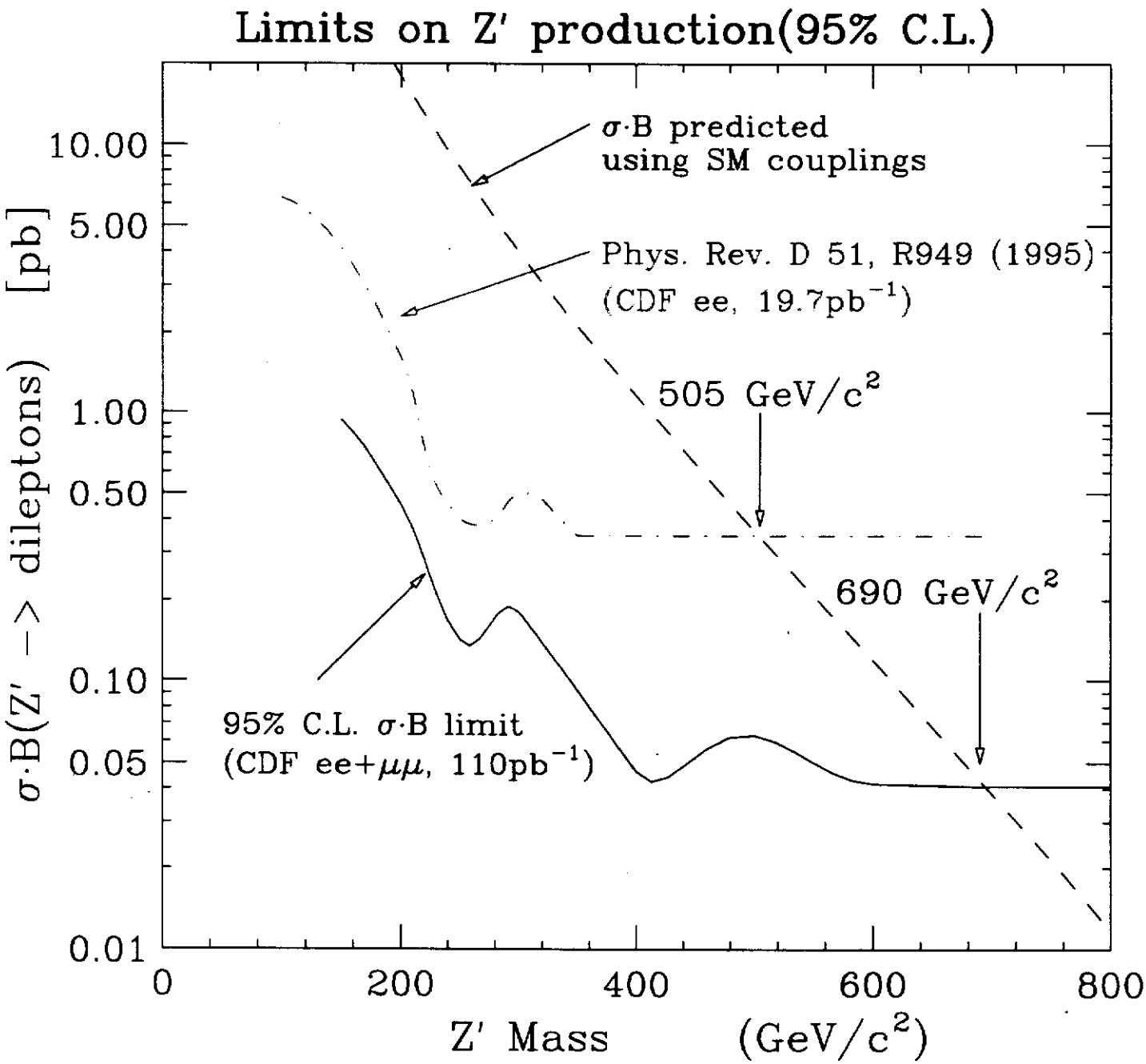
$$Lik(\mu_i) = (Normfac) \times \prod_{i=bins} \frac{\mu_i^N}{N!} e^{-\mu_i} \tag{3}$$

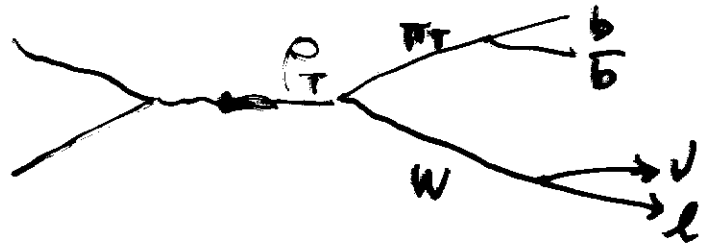
where the product over the index i runs over different mass bins and $Normfac$ is defined below. This likelihood function is just a sum over products of likelihood in each bin.

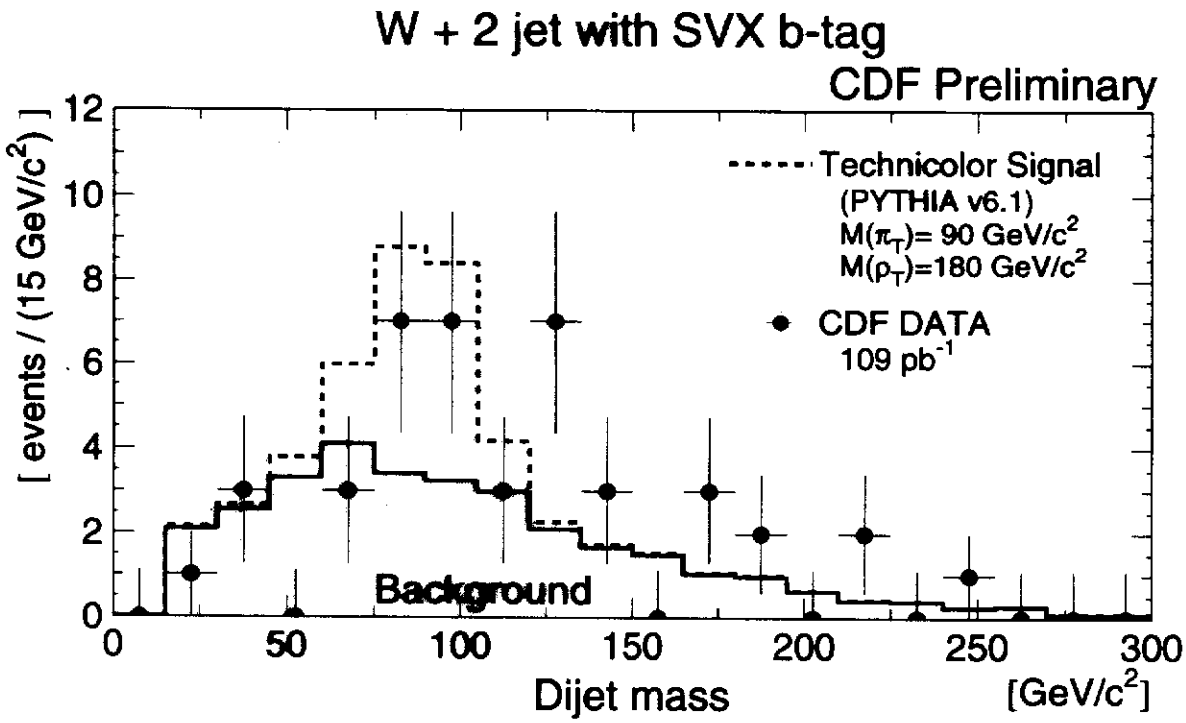
The quantity μ_i is given by :

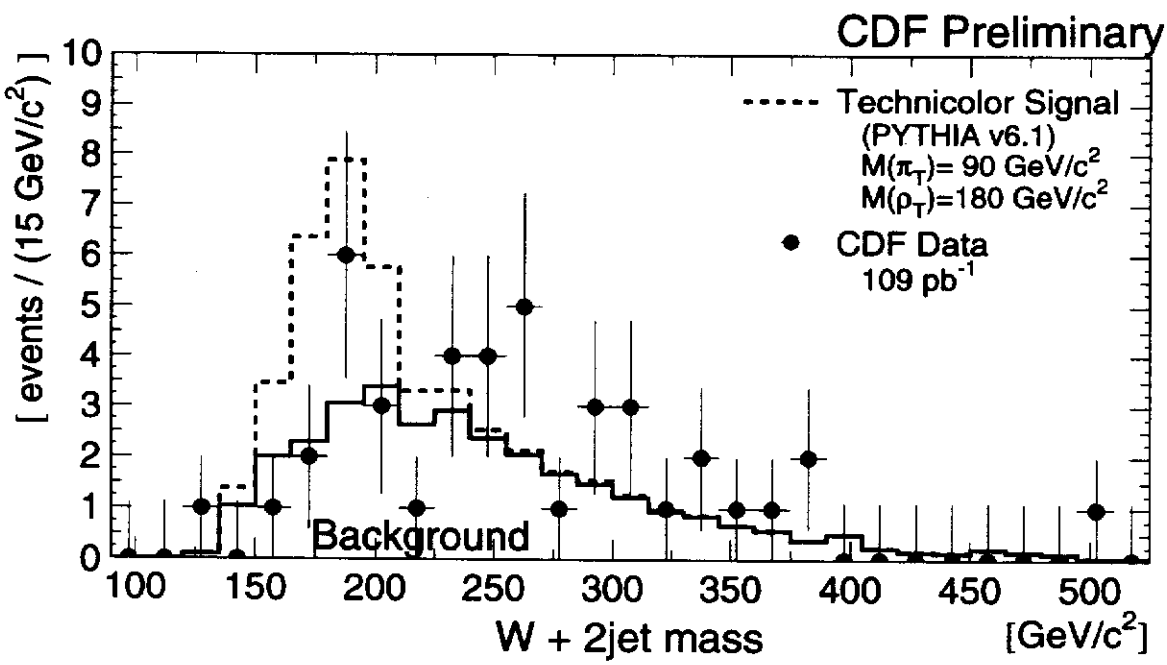
$$\mu_i = ZDY_i + \alpha \times Z'_i \tag{4}$$

Since the μ_i are constrained by the equation 4 the quantities μ_i are no longer independent, and the likelihood is a function of a single parameter α. The quantity α is the level of the Z' content in the data with respect to the prediction from the model. The


Limits on Z′ production(95% C.L.)
σ·B predicted
using SM couplings
Phys. Rev. D 51, R949 (1995)
(CDF ee, 19.7pb^{-1})
505 GeV/c^2
690 GeV/c^2
95% C.L. σ·B limit
(CDF ee+μμ, 110pb^{-1})
σ·B(Z′ -> dileptons) [pb]
10.00
5.00
1.00
0.50
0.10
0.05
0.01
0
200
400
600
800
Z′ Mass (GeV/c^2)

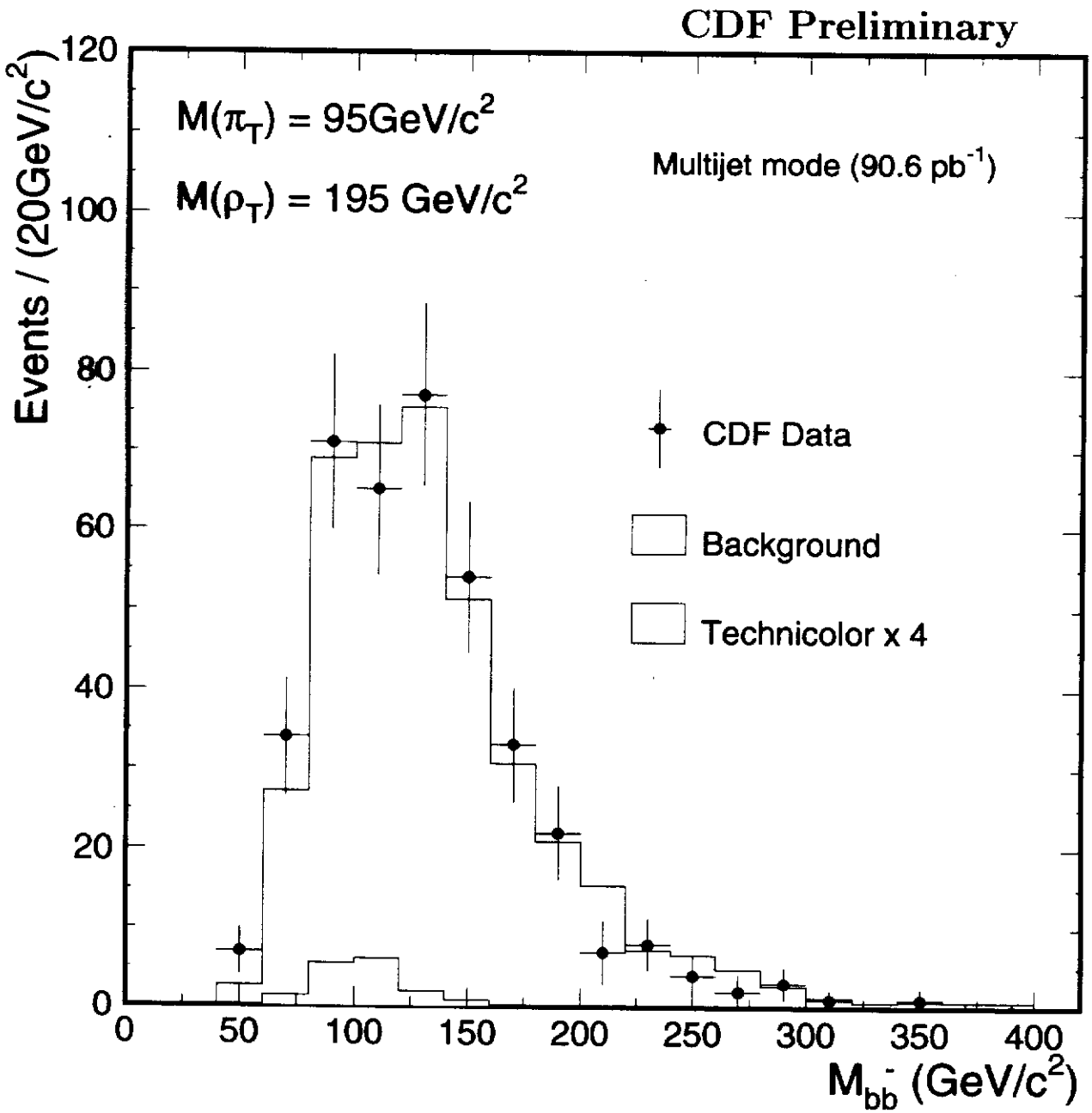

ρ_T
π_T
b
$\bar{b}$
W
ν
ℓ

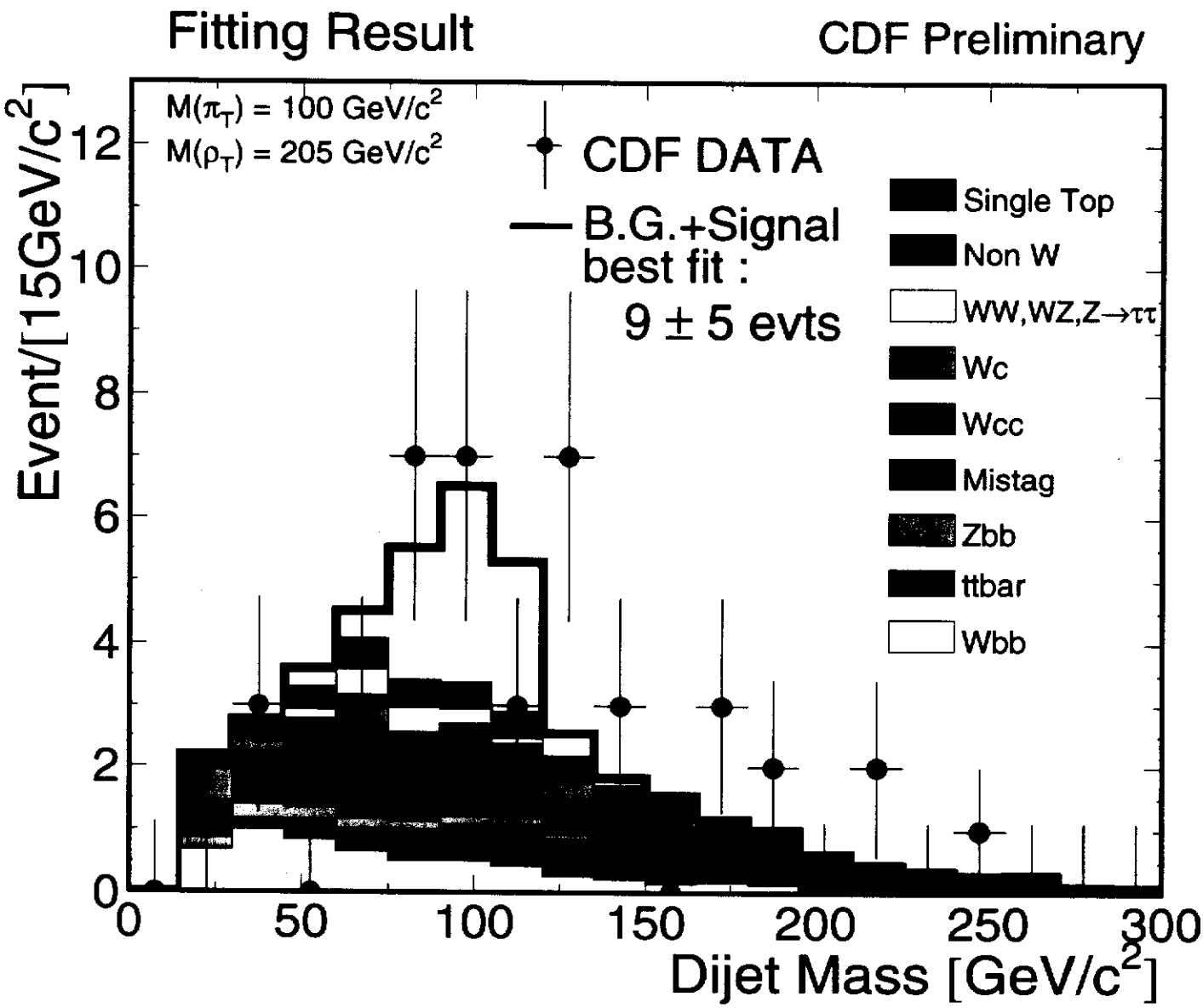

W + 2 jet with SVX b-tag
CDF Preliminary
Technicolor Signal
(PYTHIA v6.1)
M(π_T)= 90 GeV/c^2
M(ρ_T)=180 GeV/c^2
CDF DATA
109 pb^{-1}
Background
[events / (15 GeV/c^2)]
Dijet mass
[GeV/c^2]

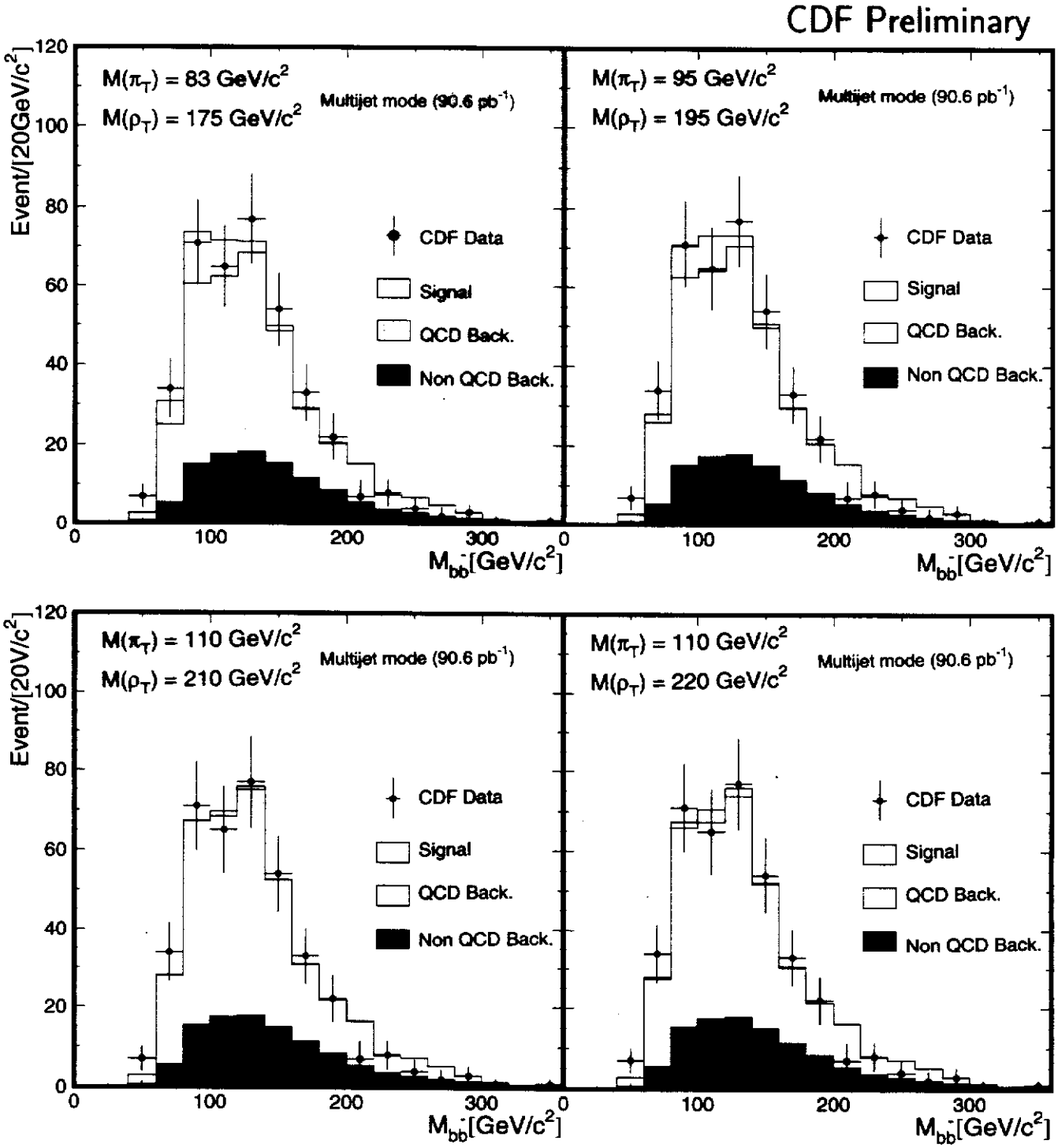

CDF Preliminary
Technicolor Signal
(PYTHIA v6.1)
M(π_T)= 90 GeV/c^2
M(ρ_T)=180 GeV/c^2
CDF Data
109 pb^{-1}
Background
[events / (15 GeV/c^2)]
W + 2jet mass
[GeV/c^2]

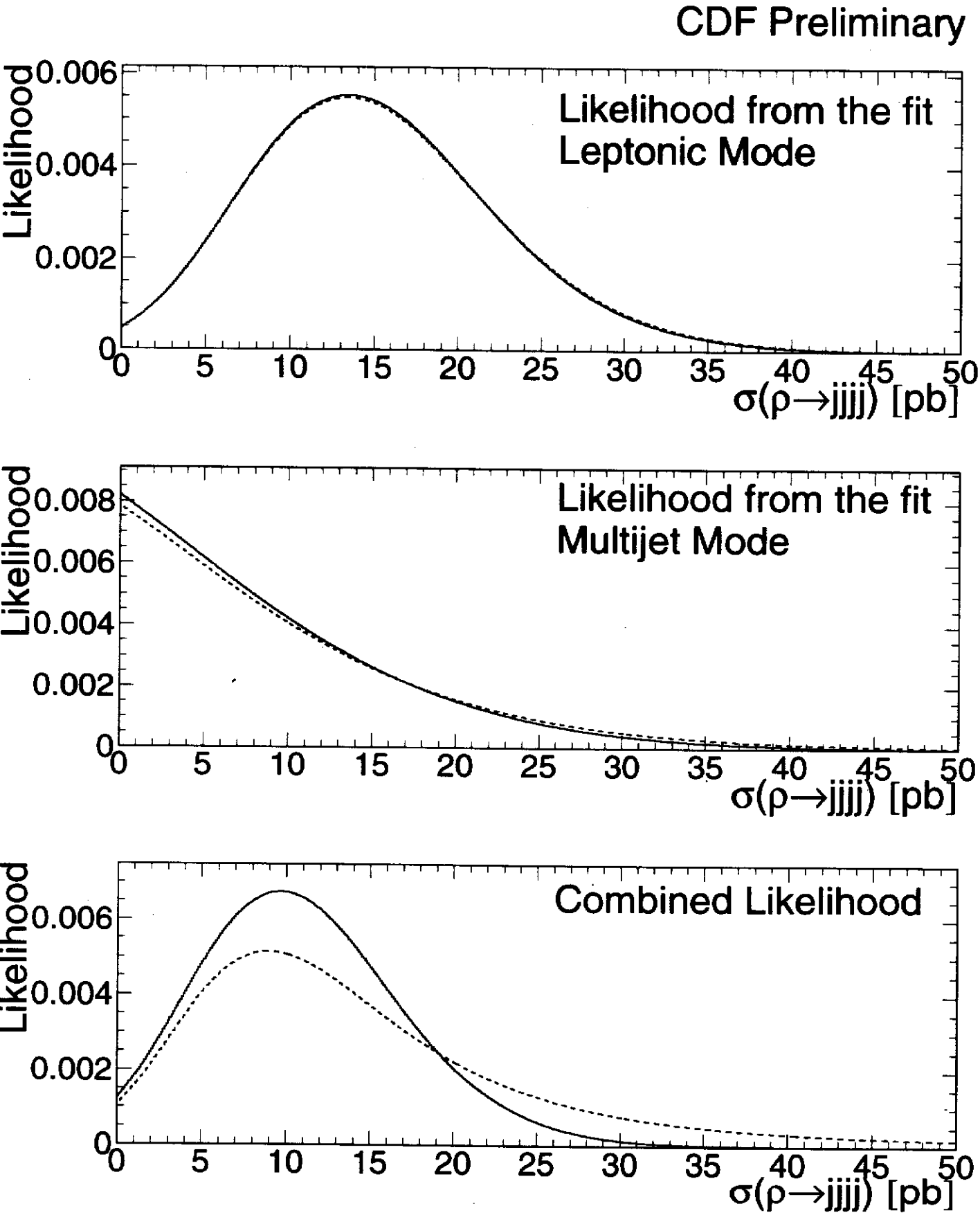
Hadronic Channel

4 jets + 2 btags




Fitting Result
CDF Preliminary
M(πT) = 100 GeV/c2
M(ρT) = 205 GeV/c2
CDF DATA
B.G.+Signal
best fit :
9 ± 5 evts
Single Top
Non W
WW,WZ,Z→ττ
Wc
Wcc
Mistag
Zbb
ttbar
Wbb
Event/[15GeV/c2]
Dijet Mass [GeV/c2]
0
2
4
6
8
10
12
0
50
100
150
200
250
300


CDF Preliminary
M(π_T) = 83 GeV/c²
M(ρ_T) = 175 GeV/c²
Multijet mode (90.6 pb⁻¹)
M(π_T) = 95 GeV/c²
M(ρ_T) = 195 GeV/c²
Multijet mode (90.6 pb⁻¹)
M(π_T) = 110 GeV/c²
M(ρ_T) = 210 GeV/c²
Multijet mode (90.6 pb⁻¹)
M(π_T) = 110 GeV/c²
M(ρ_T) = 220 GeV/c²
Multijet mode (90.6 pb⁻¹)
CDF Data
Signal
QCD Back.
Non QCD Back.
Event/[20GeV/c²]
Event/[20V/c²]
M_bb̄[GeV/c²]
0
20
40
60
80
100
120
100
200
300


CDF Preliminary
Likelihood from the fit
Leptonic Mode
Likelihood
0.006
0.004
0.002
0
0 5 10 15 20 25 30 35 40 45 50
σ(ρ→jjjj) [pb]
Likelihood from the fit
Multijet Mode
Likelihood
0.008
0.006
0.004
0.002
0
0 5 10 15 20 25 30 35 40 45 50
σ(ρ→jjjj) [pb]
Combined Likelihood
Likelihood
0.006
0.004
0.002
0
0 5 10 15 20 25 30 35 40 45 50
σ(ρ→jjjj) [pb]

Remarks

Expected Limit & Limit we actualy set

Using pseudo-experiments with already known quantities fed in (n_0, "$\sigma_{\mu b}$", "μ_b", etc), we can estimate (prob. of) a range of limit we expect to obtain.

$\Rightarrow$ example, TC (hadronic) search.

In this case, we found that the final limits we obtained were consistent with what we expect.

But that is not always the case in some analyses.

Often the problems which cause significant differences in results are not due to different statistical techniques we use.

more likely, wrong ε_{tot} estimate, wrong background estimate, etc, etc ....