General Properties of Total Cross Sections at High Energy and Gauge/String-Gravity Duality

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ABSTRACT

The COMPETE group has performed a series of exhaustive tests of the analytic parametrisations for the forward scattering amplitudes agaisnt the largest available data at $t = 0$, which includes all measured total cross sections as well as the ratios, ρ, of the real part to imaginary part of the elastic amplitude of $pp, \overline{p}p, \pi^{\pm}p, K^{\pm}p$, and total cross sections of $\gamma p, \gamma\gamma$ and $\Sigma^{-}p$. Applying a set of carefully designed criteria for measuring the quality of fits to differentiate the different parametrizations, beyond the usual χ^2/dof, they found $RRPL2_u$ and $RRP_{nf}L2_u$ to be the best analytic amplitude models. Common features of these models are: (1). total cross sections have a universal Heisenberg behavior in energy corresponding to the maximal energy behavior allowed by the Froissart bound, i.e., $A + B \quad \ln^2(s/s_0)$ with $B \sim 0.32 \quad mb$ and $s_0 \sim 34.41 \quad GeV^2$ for all reactions, and (2). the factorization relation among $\sigma_{pp,even}, \sigma_{\gamma p}$ and $\sigma_{\gamma\gamma}$ is well satisfied by experiments. I present theoretical arguments leading to such universal rise in cross sections including in particular the recent interesting application of the gauge/string-gravity duality of AdS/CFT correspondence with a deformed background metric so as to break the conformal symmetry and resulting black hole productions. In addition, I will show how the factorization relation can differentiate two different Monte Carlos used in unfolding the $\gamma\gamma$ cross section datas.

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OUTLINE

1. Global Description of High Energy Scattering - COMPETE results

1a. Universally Rising Behavior of $\sigma_{tot}(s)$

1b. Factorization Property of $\sigma(pp), \sigma(\gamma p)$, **and** $\sigma(\gamma\gamma)$

2. Theoretical Models for the Rising Cross Sections

2a. Heisenberg Model and Froissart Bound

2b. Analytic Amplitude Models

2c. Universal Rising Behavior of $\sigma_{tot}(s)$ **from Eikonal Models**

2d. Gauge/Gravity Duality and the Heisenberg Behavior

3. Factorization Test of $\sigma^{+}(pp)/\sigma(\gamma p) = \sigma(\gamma p)/\sigma(\gamma\gamma)$

2. Global Description of High Energy Scattering - COMPETE results

J. R. Cudell et al, Phys. Rev. Lett. **89**, 201801 (2002), hep-ph/0206172;

J. R. Cudell et al, Phys. Rev. **D 65**, 074024 (2002), hep-ph/0107219.

See also the COMPETE webpage http://nuclth02.phys.ulg.be/compete

Best Analytic Amplitude Models:

The best analytic amplitudes determined through an acceptance criteria based on a set of statistical quality indicators are:

$RRPL2_u$ and $RRP_{nf}L2_u$ out of 256 possible models:

$$\sigma_{\mp}^{ab} = \mathrm{R}^{+ab}(s) \pm \mathrm{R}^{-ab}(s) + \mathrm{P}^{ab} + \mathrm{H}^{ab}(s)$$

where

$$\mathrm{R}^{+ab}(s) = Y_1^{ab} \cdot (s/s_1)^{-\eta_1}, \quad \mathrm{R}^{-ab}(s) = Y_2^{ab} \cdot (s/s_1)^{-\eta_2}, \quad \mathrm{P}^{ab} = C^{ab}$$

and

$\mathrm{H}^{ab}(s)$ stands for:

$$\mathrm{E}^{ab} = X^{ab}(s/s_1)^{\Delta}, \mathrm{L}^{ab} = B^{ab}\ln(s/s_0), \mathrm{L2}^{ab} = B^{ab}\ln^2(s/s_0)$$

denoting the models as RRE, $RRPL$, $RRPL2$ etc.

For $RRPL2$ models, the imaginary part is given by

$$\rho_{ab}\sigma_{ab} = \pi B \ln\left(\frac{s}{s_0}\right) - \frac{Y_1^{ab} s^{-\eta_1}}{\tan\left[\frac{1-\eta_1}{2}\pi\right]} - \frac{Y_2^{ab} s^{-\eta_2}}{\cot\left[\frac{1-\eta_2}{2}\pi\right]}$$

Common Features of $RRPL2$ models:

(1). the universal $B\ \log^2\ (s/s_0)$ for all total cross sections of $p^{\pm}p, \pi^{\pm}p, K^{\pm}p, \Sigma^{-}p$, with
$B = 0.315mb, s_0 = 34.41GeV^2$ for $RRPL2_u$; and
$B = 0.315mb, s_0 = 34.03GeV^2$ for $RRP_{nf}L2_u$;

(2). the result seems to be true irrespectively of degeneracy in Reggeon terms, though favoring non-degeneracy: for example,
$B = 0.328mb, s_0 = 49.06GeV^2$ for $(RR)_d PL2_u$.

(3). **factorization relation**, $(H_{\gamma p})^2 \rightarrow H_{\gamma\gamma} \times H_{pp}$,
is well satisfied by the $H = PL2$ terms. Numerically,
$\delta = (H_{\gamma p}/H_{pp}) = 0.0031$,
in good agreement with the generalized vector dominance.

2. Theoretical Models for the Rising Cross Sections

2a. Heisenberg Model and Froissart Bound

W. Heisenberg, Z. Phys. **133** (1952) 65; See also
H.G. Dosch, P. Gauron and B. Nicolescu, Phys. Rev **D 67** (2003) 077501

Heisenberg: The hadron-hadron scattering at high energies as a collision of two flat discs that produce and exchange a pair of mesons in the interaction region of the impact parameter space.

Assuming that
the portion of the energy density that is responsible for the non-renormalizable meson exchange interactions is high enough to create at least a pair of mesons
and
the portion is exponentially decreasing with the exchanged meson mass and impact distance in analogy to the shock wave process,
he has argued that the maximum impact distance for which the effective interaction takes place, corresponding to the minimum portion to create a pair of mesons, is $b_{max} = (1/2m) log(s/s_0)$, m being the exchanged meson pair mass.

From this the total cross section is given by

$$\sigma = \ (\pi/16m_\pi^2)\ log^2\ (s/s_0)$$

which corresponds to a saturating behavior of the Froissart bound.

The **Froissart** bound, $\sigma_{tot} \leq c \log^2 s$, in which $c \leq (\pi/m_\pi^2)$, a consequence of the unitarity and positivity of the imaginary part of the scattering amplitudes in the Lehmann ellipse.
M. Froissart ('61); L. Lukaszuk and A. Martin ('67).

Rigorous mathematical (axiomaic) proof of the Pomeranchuk theorem (Y. Ya. Pomeranchuk, JETP **7** (1958) 499) required the increasing behavior of the total cross sections in energy as a **necessary condition**.
R.J. Eden ('66); T. Kinoshita ('66); G. Grunberg and T. N. Truong ('73); etc

2b. Analytic Amplitude Models

(Kang-Nicolescu, '75): At high energies the s-u crossing even forward scattering amplitude is of the **PL2** form

$$A^+ \sim Z + B\ log^2(s/s_0)$$

as a solution to the derivative dispersion relation (DDR).
L. Lukaszuk and B. Nicolescu ('73); K. Kang, B. Nicolescu ('75); G. Bialkowski, K. Kang and B. Nicolescu ('75);

Numerous analytic amplitude models have been proposed and discussed over the years by
U. Amaldi and K.R. Schubert ('80); M.M. Block and R. Cahn ('85); M.M. Block, K. Kang and A.R. White ('92); and others.

2c. Universal Rising Behavior of $\sigma_{tot}(s)$ from Eikonal Models

K.K. and Bruce H.J. McKellar, hep-ph/0302085, KIAS-P02043

The Scattering Amplidude

$$f(s,t) = \imath \int_0^\infty b db J_0(b\sqrt{-t}) \left(1 - e^{-\chi(b,s)}\right)$$

The Total Cross-Section

$$\sigma_t = 4\pi \Im f(s,0) = 4\pi \int_0^\infty b db \left(1 - e^{-\chi(b,s)}\right)$$

The Eikonal is parametrized such that

$$\int \chi(b,s) d^2\mathbf{b} = \sigma_0(s)$$

Case 1. The Non-factorisable Gaussian Eikonal

$$\chi(b,s) = \frac{\pi\lambda}{B(s)} s^{\Delta} \exp\left\{-\frac{b^2}{B(s)}\right\}$$

where

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P(0)t.$$

$$\sigma_t(s) = 2\pi B(s)\left(\ln C(s) + \gamma + E_1(C(s))\right)$$

where

$$C(s) = (\pi\lambda/B(s))\, s^{\Delta} = \chi(b=0,s),$$

$$E_1(x) = \int_x^{\infty} e^{-t} dt/t$$

γ is Euler's constant, and $E_1(x) = \int_x^{\infty} e^{-t} dt/t$

The leading term gives

$$\sigma_t(s) = 4\pi\alpha'_P(0)\Delta \ln^2 s + O(\ln s)$$

Clearly B is **universal**.

Case 2. The Factorizable Exponential Eikonal

$$\chi(b,s) = \frac{\lambda}{2\pi b_0^2} s^\Delta \exp\left\{-\frac{b}{b_0}\right\},$$

$$\sigma_t(s) = 2\pi b_0^2 \Big(\ln^2 C(s) + \Big[\frac{\pi^2}{6} + \gamma^2 + 2\ln C(s)\gamma\Big]$$

$$- \left[\Gamma_1(0, C(s)) - 2\ln C(s)\Gamma(0, C(s))\right]\Big)$$

where
$C(s) = \frac{\lambda}{2\pi b_0^2} s^\Delta$ and $\Gamma(\alpha, x) = \int_x^\infty t^{\alpha-1} e^{-t} dt$

is the upper incomplete gamma function, and $\Gamma_1(\alpha, s)$ is its partial derivative with respect to α.
For large s,

$$\sigma_t(s) = 2\pi b_0^2 \Delta^2 \ln^2 s + O(\ln s)$$

with the coefficient $B = 2\pi b_0^2$.
In this case b_0^{-1} is the mass of the exchanged meson, which can be **universally taken to be the lightest glueball**.

Case 3. The QCD Inspired Eikonal

Block *et al* introduced the eikonal

$$\chi = \sigma_{gg}(s)W(b;\mu_{gg}) + \sigma_{qg}W(b;\mu_{qg}) + \sigma_{qq}W(b;\mu_{qq})$$

where σ_{gg}, σ_{qg} and σ_{qq} represent gluon-gluon, quark-gluon and quark-quark interaction cross sections, and $W(b;\mu)$ are overlap functions normalised so that

$$2\pi \int_0^\infty bdbW(b;\mu) = 1$$

and is given by the Fourier transform of a dipole form factor squared,

$$W(b;\mu) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b),$$

where $K_3(x)$ is the modified Bessel function of the second kind.

For small x,

$$x^3 K_3(x) = 8 + O(x^2)$$

and for large x

$$x^3 K_3(x) = \sqrt{\frac{\pi}{2}} x^{5/2} e^{-x}(1 + O(x^{-1}).$$

We approximate the eikonal with an exponential form,

$$\chi_a(b,s) = C(s) e^{-b/b_0(s)},$$

reducing the calculation to the previous case.
6.$C(s)$ is chosen to be the opacity of the eikonal we are approximating, and $b_0(s)$ is determined so that

$$\int_0^\infty bdb\chi(b,s) = \int_0^\infty bdb\chi_a(b,s),$$

giving

$$C(s) = \sigma_{gg}(s)\frac{\mu_{gg}^2}{12\pi} + \sigma_{qg}(s)\frac{\mu_{qg}^2}{12\pi} + \sigma_{qq}(s)\frac{\mu_{qq}^2}{12\pi},$$

and

$$2\pi b_0^2(s) = \left(\sigma_{gg}(s) + \sigma_{qg}(s) + \sigma_{qq}(s)\right)/C(s).$$

If a single elementary collision process dominates over other terms, b_0 becomes $\sqrt{6}/\mu$, independent of s, where μ is the scale parameter of the dominant process.
The leading large s behaviour of the total cross section is

$$\sigma_t(s) = 2\pi b_0^2(s) \ln^2 C(s).$$

In the large s region, the eikonal is dominated by the gluon-gluon scattering term, which has the structure

$$\sigma_{gg}(s) \approx \lambda \left(\frac{s}{m_0^2}\right)^{\epsilon} \ln \left(\frac{s}{m_0^2}\right),$$

where ϵ is obtained from the gluon structure function

$$f_g(x) = N_g(1-x)^5 x^{-(1+\epsilon)}$$

.

The cross section again has the Heisenberg $\ln^2 s$ dependence at high energies, with the universal coefficient
$B = 12\pi\mu_{gg}^{-2}\epsilon^2$.
Block *et al* use $\epsilon = 0.05$, $\mu_{gg} = 0.73$GeV, which give $B = 0.069$mb. However, recent results on ϵ give $\epsilon \in (0.08, 0.4)$ which give $B \in (0.18, 4.42)$mb, covering the empirical value.

2d. Gauge/Gravity Duality and the Heisenberg Behavior

Attempts to find the scattering amplitude at high energy in the gauge sector from supersymmetric CFT in AdS, following the (Maldacena) gauge/gravity duality of the AdS_{d+1}/CFT_d correspondence, i.e.,
Superstrings in AdS_{d+1} dual to Conformal (SUSY) YM Gauge Theory in d, i.e., weak coupling gravity in AdS space of d+1 dimension is dual to strong coupling supersymmetric gauge theory in d dimension.

But there are several **difficulties** with the superstring theory formulation of strong interaction:
— Conformal and diffeomorphism invariance leads to stringent constraint on the space, i.e., d=26 and 10 repectively for bosonic and super strings in flat space,
— Supersymmetry,
— Zero mass gauge and gravitational fields in the string spectra of asymptotic states, thus no mass gap due to the presense of zero mass states (gauge/graviton), and
— No hard scattering recovered in a string theory framework i.e.,

$$A_{string}(s,t) \sim \exp[\frac{-\alpha'}{2}(s\ln s + t\ln t + u\ln u)]$$

to be contrasted to the partonic or hard Regge behavior in gauge theory,

$$A_{qcd}(s,t) \sim s^{2-\frac{n}{2}}$$

Can one find a consistent picture of gauge field properties and Regge amplitude in the strong gauge coupling regime from a suitable string theory via AdS/CFT corrspondence ?

J. Polchinski and M. J. Strassler, Phys. Rev. Lett. **88** (2002) 03160; R. Peschanski, hep-ph/0302257;
S.B. Giddings, Phys. Rev. **D 67** (2003) 126001, hep-th/0203004.

Anti-deSitter metrics provides a way to overcome some of these difficulties:
deform the AdS/gravity background metric to get the QCD universality class,

$$ds^2 = (r/R)^2 \eta_{\mu\nu} dx^\mu dx^\nu + (R/r)^2 (1 - (b/r)^d)^{-1} dr^2 + R^2 ds_Y^2$$

whereby breaking the conformal (and SUSY) symmetry with an IR cut-off at $r_{min} = b$. Here R is the anti-deSitter radius and ds_Y^2 is the metric for 5 (or 6) compact dimensions of 10-d string (or 11-d M-) theory.

Once the conformal symmetry is broken and a mass gap produced, the theory has an ordinary 4-d S-matrix.

Polchinski and Strassler have argued that with the extra dimensional branes of either warped or non-warped large space-time geometry, the amplitude can be treated essentially as a 10-d scattering that takes place at a point in transverse dimensions integrated coherently over this transverse position, so that the soft behavior of the strings would conspire the shape of the bulk wave functions and produce the correct power behavior of the confining gauge theory.

Basically in 10-d string theory, due to the gravitational **Red Shift** in the warped co-ordinate r, $\Delta s = (R/r)\Delta x$ and $p = (r/R)p_s$, so that a state with a characteristic 10-d energy scale, $p_s \sim (1/R)$, corresponds to a 4-d energy scale $p \sim (r/R^2)$,
The mass gap determined by the lightest glue ball (KK mode), $p \sim \Lambda_{KK}$ will cut off r at $r_{min} \sim \Lambda_{KK} R^2$, and a plane wave state $\phi(r, Y)exp(ix_\mu p^{\mu)}$ scatters with a local proper momentum $\bar{p}_s(r) = (R/r)p$, i.e., UV shifted in the IR.

Thus in the bulk region $r \in [r_{min}, r_{scatt}]$, where $\sqrt{\alpha'}p_s = l_s(R/r_{scatt})p > 1$, the wide angle scattering is exponentially suppressed. **However** there is a small remaining amplitude at large r (but not so far away from r_{min}) that gives the correct conformal scaling behavior of the parton model,

$$\phi(r) \sim \left(\frac{r_{scatt}}{r_{min}}\right)^{-\Delta_4} \sim \left(\sqrt{\alpha'_{qcd}}p\right)^{-\Delta_4}$$

e.g., a scalar glue ball $\phi \sim r^{-4}$ corresponding to $n = 4$ for the YM operator, $Tr[F^2]$.

Subsequently **Giddings** studied other bulk perspectives and in particular argued that the effect of strong-gravity processes, such as black hole formation, to the high energy behavior of the total cross sections is important in the dual dynamics. Here the key point is that in TeV scale gravity scenario, black holes should be produced once the energy passes the fundamental Planck scale near a TeV.
He argues that this strong gravity effect is a dominant feature of the high energy scattering in gauge theory and the relevant cross sections can well be approximated by the geometric cross section of the black hole formation, $\sigma = \pi r_h^2(M)$ where r_h is the Schwarzschild radius of the black hole of mass M.

He estimated the size of r_h from the black hole solution projected onto the infrared boundary in a linearized analysis of gravity for a mass near an IR brane with the Neumann boundary conditions from a perturbed deformed AdS/black hole background metric and showed at high energies,

$$\sigma = \ (1/M^2)\ log^2\ (s/s_0)$$

where M is the lightest Kaluza-Klein (KK) excitation mass.
This is in agreement with the Heisenberg behavior of the total cross sections .

But see the criticsms on the semiclassical description of black hole production cross section by:
M.B. Voloshin, Phys. Lett. **B 518** (2001) 137; ibid **B 524** (2002) 376; A. Jevicki and J. Thaler, Phys. Rev. **D 66** (2002) 024041; G. Landsberg, hep-ph/0211043.

3. Factorization Test of $\sigma^+(pp)/\sigma(\gamma p) = \sigma(\gamma p)/\sigma(\gamma\gamma)$

Global test of factorization relation with $RRPL2_u$ by the COMPETE collaboration showed that the **factorization relation**
$(H_{\gamma p})^2 \to H_{\gamma\gamma} \times H_{pp}$
is well satisfied by the $H = PL2$ terms. Numerically,
$\delta = (H_{\gamma p}/H_{pp}) = 0.0030641$
in good agreement with the generalized vector dominance.

But the OPAL and L3 total cross sections for $\gamma\gamma$ scattering are different depending on the Monte Carlo programs used, i.e., between PHOJET and PYTHIA, for unfolding the datas.

"In most of distributions, both Monte carlo models descrivethe data equally well and there is no reason for preferring one model over another for unfolding of the data. We therefore average the results of of the unfolding. The difference between this cross section and the results obtained by using PYTHIA or PHOJET alone are taken as the systematic error due to the Monte Carlo model dependence of the unfolding."

—– OPAL Coll. G. Abbiendi et al, Eur. Phys. J. **C 14**, 199 (2000).

The Particle Data Group (K. Hagiwara et al, Phys. Rev. **D 66**, 01001 (2002)) used the average of two independent analyses performed by both OPAL and L3 groups, using the two different Monte Carlo programs, PHOJET and PYTHIA. The dominant error quoted is half the difference between the two different values.

Because of the PDG $\gamma\gamma$ data (i.e., the quoted values of cross sections and errors), the COMPETE fit used the $\gamma\gamma$ data only up to $\sqrt{s} \leq 100$ GeV where the two different Monte Carlo programs agree.

M.M. Block and A. Kaidalov (Phys. Rev. **D 64** (2001) 076002) showed that the factorization relation for the total cross sections,

$$\sigma^+(pp)/\sigma(\gamma p) = \sigma(\gamma p)/\sigma(\gamma\gamma)$$

from the eikonal model using the additive quark model and vector meson dominance and assuming that the opacity defined by the ratio of elastic and total cross sections is process independent and the same.

Block-Kaidalov factorization relation was tested in the energy range $8 \leq \sqrt{s} \leq 2000$ GeV by
M. M. Block and K. Kang (hep-ph/0302146 and N.U.H.E.P. Report No. 1101)
using $(RR)_d PL2$ amplitude with $\alpha_{rho}(0) = \alpha_f(0) = 0.5, \delta = 0.003985, B = 0.304, s_0 = 34.3 GeV^2, Y_1 = 55.5 mb, Y_2 = 35.1 mb$.
This is to be compared to the global COMPETE fit of $(RR)_d PL2$, that gives
$\alpha_-(0) = \alpha_+(0) = 0.47, \delta = 0.003810, B = 0.328, s_0 = 49.056 GeV^2$, $Y_1 = 44.318 mb, Y_2 = 30.819 mb$.

When unfolding the data in each Monte Carlo, the PHOJET results of both OPAL and L3 are in reasonable agreement with ether factorization relation in both shape and normalization, whereas the PYTHIA is in distinct disagreement.