Valence Quarks Distributions at Small-x and Saturation

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GOALS:

$\Longrightarrow$ Include saturation effects in the valence quark distribution functions of hadrons and nuclei:

$\Rightarrow$ Include multiple rescatterings in the valence quark distributions in the quasi-classical approximation (McLerran-Venugopalan model).

$\Rightarrow$ Include the effects of linear and non-linear small-x evolution at the double logarithmic level ($\alpha_S \ln^2 1/x$).

$\Longrightarrow$ Make a connection between the perturbative reggeon and baryon stopping in heavy ion collisions at RHIC.

Light-Cone Wave Function

To construct lowest order light cone wave function consider diagrams like

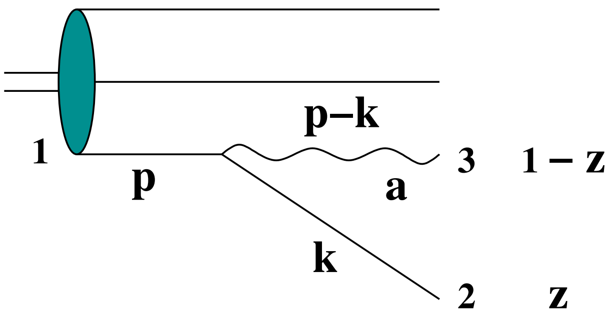

Figure 1: Soft valence quark wave function of a nucleon.

Defining $z = k_+/p_+$ the graph above gives valence quark distribution

$$\left.\frac{dn_{val}\,(\underline{x}-\underline{y})}{dy}\right|_{LO} = \frac{z\,N_c}{2}\,A\,\frac{\alpha_S\,C_F}{\pi}\,\ln\frac{1}{(\underline{x}-\underline{y})^2\Lambda^2}$$

or, in momentum space

$$f_{val}(x,\underline{k}^2)\Big|_{LO} = \frac{z\,N_c}{2}\,A\,\frac{\alpha_S\,C_F}{\pi}\,\frac{1}{\underline{k}^2}.$$

For a nucleus one has a more complicated picture since other nucleons can modify the valence quark wave function by gluon exchanges. To construct light cone wave function follow the standard prescription (Yu.K. '96, Jalilian-Marian, Kovner, McLerran, Weigert '97):

In covariant $(\partial \cdot A = 0)$ gauge $\psi_N(\underline{x}, x_-) \sim \delta(x_-)$

$$\psi_A^{cov}(\underline{x}, x_-) \, = \, \sum_{i=1}^{A} \psi_{N_i}^{cov}(\underline{x}, x_-).$$

Then we gauge-transform to light cone gauge

$$\psi_A^{LC}(\underline{x}, x_-) \, = \, S(\underline{x}, x_-) \, \sum_{i=1}^{A} \psi_{N_i}^{cov}(\underline{x}, x_-)$$

with the matrix of gauge transformation

$$S(\underline{x}, x_-) \, = \, P \exp \left\{ -ig \int_{-\infty}^{x_-} dx'_- \, A_+(\underline{x}, x'_-) \right\}.$$

The valence quark distribution would then be given by

$$\frac{dn_{val}\,(\underline{x} - \underline{y})}{dy} \, = \, \frac{1}{4\pi} \frac{z}{2} \int \frac{dx_- \, dy_-}{(2\pi)^2} \, e^{ik_+(x_- - y_-)} \times \left\langle \bar{\psi}_A^{LC}(\underline{y}, y_-) \, \frac{1}{2} \gamma_+ \, \psi_A^{LC}(\underline{x}, x_-) \right\rangle .$$

The resulting valence quark light cone wave function of the nucleus corresponds to a sum of diagrams like

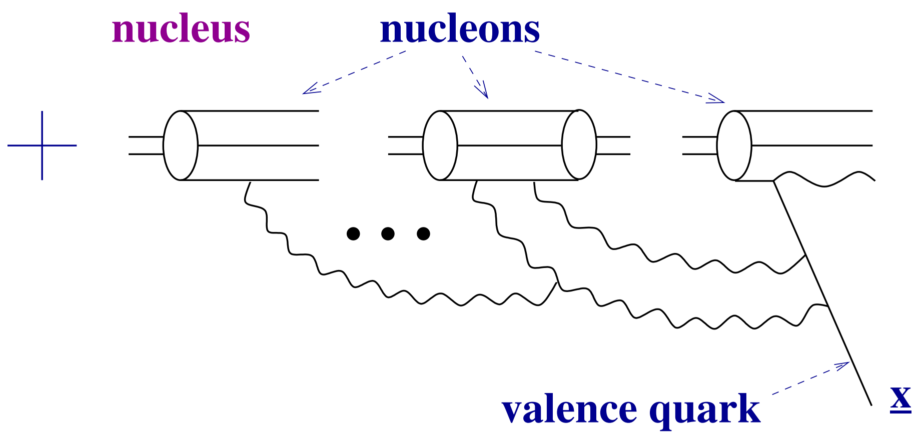

Figure 2: Soft valence quark wave function of a nucleus.

and is given by

$$\frac{dn_{val}\,(\underline{x}-\underline{y})}{dy} = \frac{z\,N_c^2\,S_\perp}{\alpha_S\,\pi^2\,(\underline{x}-\underline{y})^2}\left(1-e^{-(\underline{x}-\underline{y})^2 Q_s^{quark\,2}/4}\right)$$

with the saturation scale

$$Q_s^{quark\,2} = \frac{2\pi\alpha_S^2\,C_F}{N_c}\,\frac{A}{S_\perp}\,\ln\frac{1}{(\underline{x}-\underline{y})^2\,\Lambda^2}.$$

This distribution is more IR safe: in momentum space

$$f_{val}(x,\underline{k}^2) = \frac{z\,N_c^2\,S_\perp}{4\,\alpha_S\,\pi^2}\,\ln\frac{Q_s^{quark\,2}}{\underline{k}^2}, \qquad k_\perp \ll Q_s^{quark}.$$

It is only logarithmically divergent!
(cf. gluon distribution)

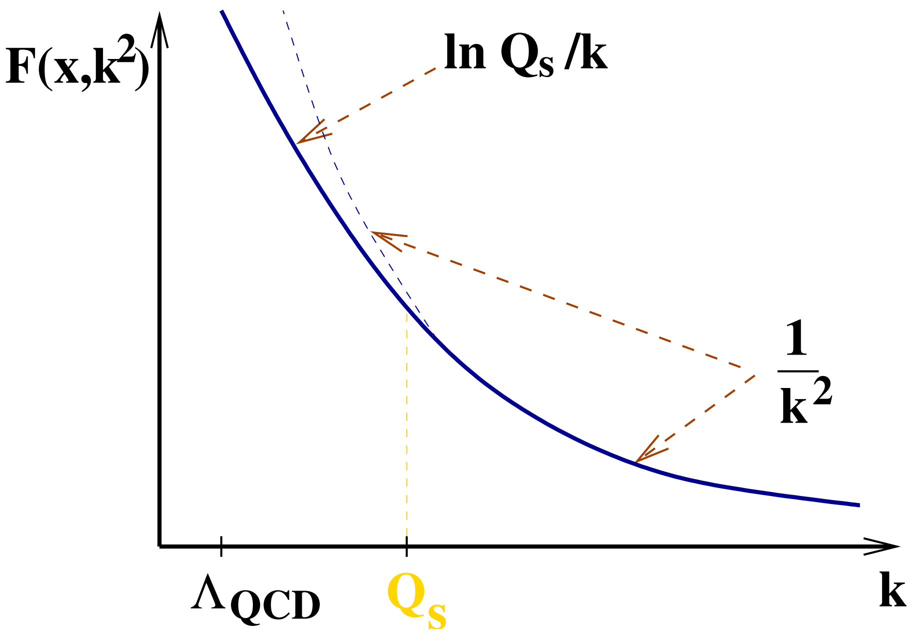

Figure 3: Quasi-classical valence quark distribution of a nucleus plotted as a function of transverse momentum k_T (sketch).

It's better to multiply by phase space:

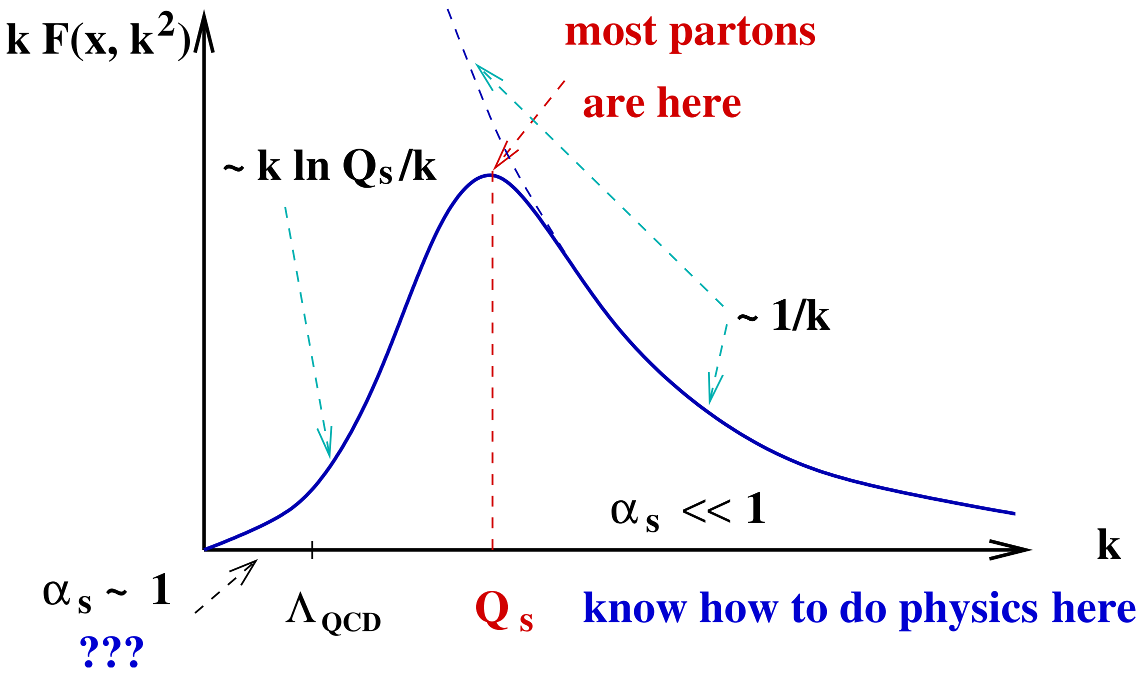

Figure 4: Distribution of valence quarks in the nuclear wave function.

Valence Quarks in DIS: Multiple Rescatterings

To set up the stage for quantum evolution one has to construct initial conditions by resumming multiple rescatterings in the valence quark structure function

$$F_2^{val}(x_{Bj}, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{EM}} \int \frac{d^2x\, dz}{4\pi} \, \Phi^{\gamma^* \to q\bar{q}}(\underline{x}, z) \; 2 \int d^2b \; R(\underline{x}, \underline{b}, z_1).$$

One has to resum the following diagrams

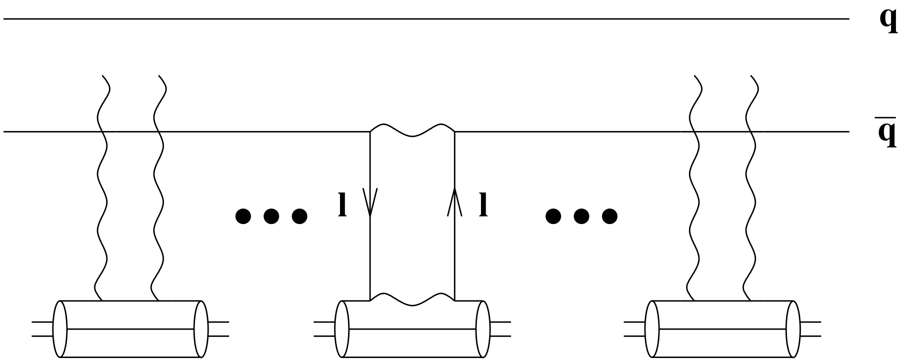

Figure 5: Forward amplitude of a $q\bar{q}$ pair interaction with the nucleus with one flavor-exchange interaction and all orders in gluon exchange rescatterings.

A simple calculation yields

$$R_0(\underline{x}, \underline{b}, z_1) \; = \; \frac{\alpha_S^2 \, C_F^2 \, \pi}{z_1 \, s} \, \frac{A}{S_\perp} \, \ln \frac{z_1 \, s}{\Lambda^2} \, e^{-\underline{x}^2 Q_s^{quark\,2}/4}.$$

where z_1 is the fraction of the virtual photon's light cone momentum carried by the anti-quark.

$\Rightarrow$ R_0 gives the initial condition for the small-x evolution. It's also IR safe!

Valence Quarks in DIS: Small-x Evolution

Linear small-x evolution equation for non-singlet distributions was written down and solved in the DLA by Kirschner and Lipatov in '83 and then solved in LLA by Kirschner in '86. Evolution is given by the ladder graphs like

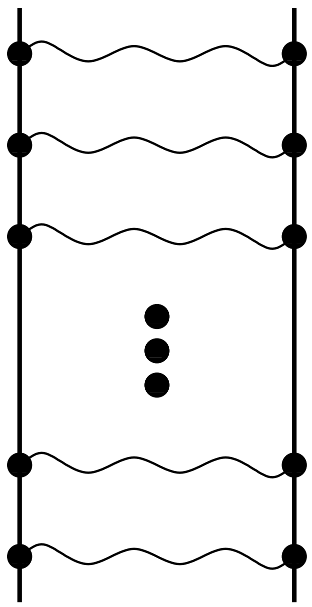

Figure 6: Linear evolution for non-singlet structure functions with reggeized quarks and effective quark-gluon vertices.

⇒ Parametrically this graph is of the order

$$f(\alpha_S^2 A^{1/3})\, s^{-1+C\,\sqrt{\alpha_S}+C'\,\alpha_S}$$

where the two terms in the power of s correspond to DLA $(\alpha_s \ln^2 s)$ and LLA $(\alpha_s \ln s)$. $f(\alpha_S^2 A^{1/3})$ resums multiple rescatterings on the nucleus.

⇒ We want to construct a dipole model equivalent of this evolution to include the effects of merging of ladders (saturation).

We want to resum diagrams like

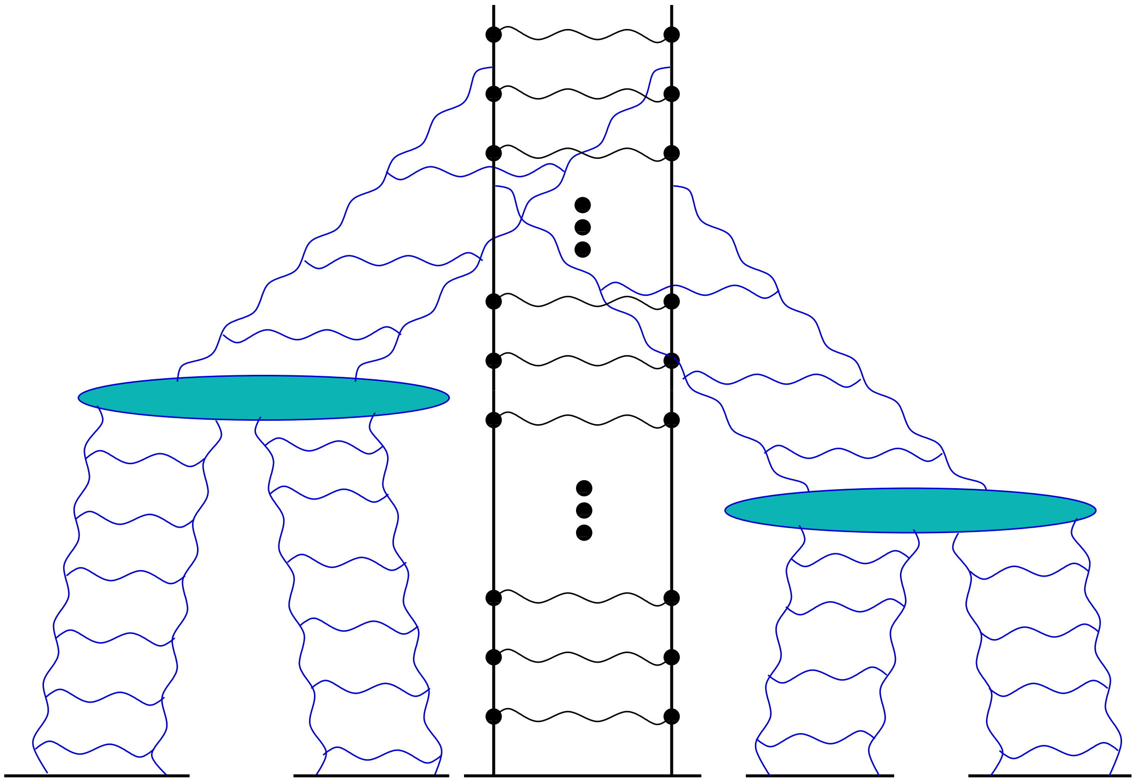

Figure 7: Fan diagrams for a single quark ladder "dressed" in many gluon ladders.

In the double logarithmic approximation (DLA), the quark ladder is parametrically of the order of $f(\alpha_S^2 A^{1/3})\, s^{-1+C\,\sqrt{\alpha_s}}$, while gluon ladders in the LLA approximation are of the order $h(\alpha_S^2 A^{1/3})\, s^{\tilde{C}\,\alpha_s}$. We thus want to resum one power of $f(\alpha_S^2 A^{1/3})\, s^{-1+C\,\sqrt{\alpha_s}}$ and all powers of $h(\alpha_S^2 A^{1/3})\, s^{\tilde{C}\,\alpha_s}$.

Non-linear Evolution Equation

Similarly to Mueller's dipole model we can write down an evolution equation in the large-N_c limit. Double line indicates a gluon.

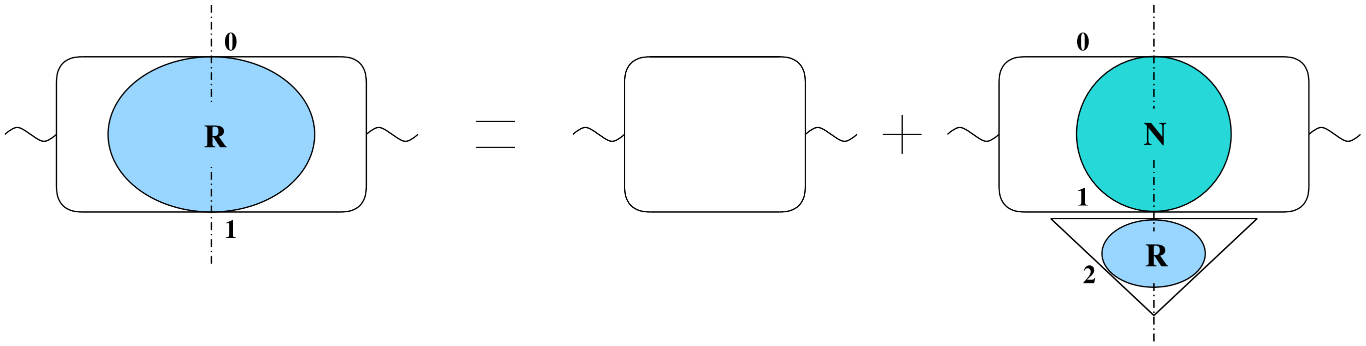

Figure 8: Evolution equation for the reggeon amplitude $R(\underline{x}_{01}, \underline{b}, z_1)$ in the double logarithmic approximation. $R(\underline{x}_{01}, \underline{b}, z_1)$ is the forward amplitude of a $q\bar{q}$ dipole interacting with the nucleus by a single $\bar{q}$ exchange and many gluon exchanges, while $N(\underline{x}_{01}, \underline{b}, z_1)$ is the forward amplitude of a $q\bar{q}$ dipole interacting with the nucleus by gluon exchanges only.

Defining

$$\tilde{R}(\underline{x}_{01}, \underline{b}, z_1) \equiv z_1 s \, R(\underline{x}_{01}, \underline{b}, z_1)$$

we write the nonlinear equation

$$\tilde{R}(\underline{x}_{01}, \underline{b}, z_1) = \tilde{R}_0(\underline{x}_{01}, \underline{b}, z_1) +$$

$$+ \frac{\alpha_S \, C_F}{2\,\pi^2} \int_{z_i}^{z_1} \int^{\min\{1, x_{01}^2/x_{21}^2\}} \frac{dz_2}{z_2} \frac{d^2 x_2}{x_{21}^2} \, \tilde{R}(\underline{x}_{12}, \underline{b} + \frac{1}{2}\underline{x}_{20}, z_2)$$

$$\times \left[1 - N(\underline{x}_{01}, \underline{b}, z_1)\right].$$

BK Evolution Equation

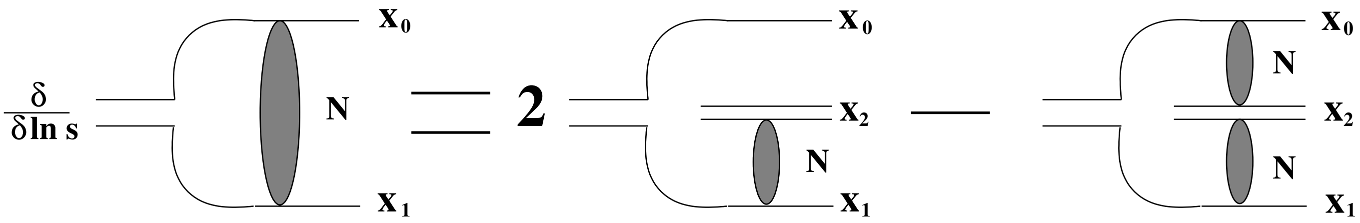

Figure 9: An equation resumming dipole cascade in the leading logarithmic approximation $\alpha_S \ln s$.

Defining rapidity $Y = \ln s$ we write an equation for the dipole amplitude on the target

$$\frac{\partial N(\mathbf{x}_{01}, Y)}{\partial Y} = \frac{2\alpha_S C_F}{\pi^2} \int_\rho d^2 x_2 \left[\frac{x_{01}^2}{x_{02}^2 x_{12}^2} - 2\pi \delta^2(\mathbf{x}_{01} - \mathbf{x}_{02}) \ln\left(\frac{x_{01}}{\rho}\right) \right]$$
$$\times\, N(\mathbf{x}_{02}, Y) - \frac{\alpha_S C_F}{\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2 x_{12}^2} N(\mathbf{x}_{02}, Y)\, N(\mathbf{x}_{12}, Y)$$

Yu. Kovchegov, '99 (large N_c QCD)
I. Balitsky, '96 (HE effective lagrangian)

The initial conditions are given at low energy by Glauber-Mueller formula to account for multiple rescatterings of the dipoles in the target

$$N(x_\perp, Y = 0) = 1 - e^{-x_\perp^2 Q_{s0}^2 \ln(1/x_\perp \Lambda)/4}.$$

$\Rightarrow$ The linear part of the BK equation corresponds to BFKL evolution equation, contributing to the growth of the amplitude N with rapidity Y.

$\Rightarrow$ The quadratic part slows down the growth eventually making N constant, leading to unitarization of the amplitude.

Solution of the linear DLA equation

The linear part of our non-linear equation can be solved to give (cf. Kirschner and Lipatov, '83)

$$R \sim s^{\alpha_R - 1} \sim s^{\sqrt{2 \frac{\alpha_S C_F}{\pi}} - 1}.$$

Solution of the non-linear equation

The full non-linear equation can be solved for a simple model for N:

$$N(\underline{x}_{01}, \underline{b}, \tau) = \theta(\underline{x}_{01}^2 Q_s^2(\tau) - 1).$$

where

$$Q_s^2 = \Lambda^2 e^{\kappa \tau} \, ,$$

and

$$\tau = \ln \frac{z \, s}{\Lambda^2} \, .$$

The resulting amplitude scales with energy as

$$R \sim e^{(\alpha_R - 1)\,\tau} \sim e^{\left(\sqrt{2 \frac{\alpha_S C_F}{\pi} (1-\kappa)} - 1\right)\tau}.$$

$\Rightarrow$ **Saturation reduces the intercept!**

Relation to RHIC data on Baryon Stopping

At RHIC people measure the # of baryons — of anti-baryons at mid-rapidity. This gives the total baryon number at mid-rapidity - "baryon stopping":

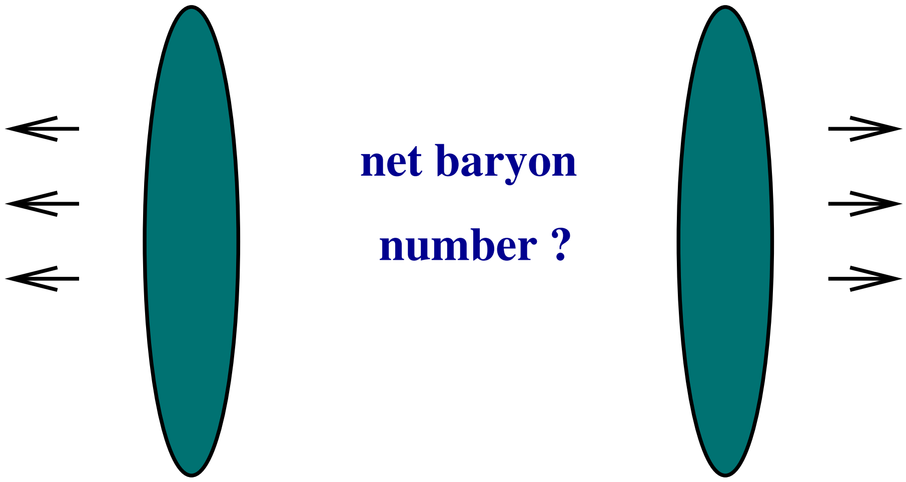

Figure 10: Heavy ion collision.

Perturbative baryon transport is proportional to distribution functions

$$\frac{dN_B^{net}}{dk^2 dy} \sim x_R f_{val}(x_R, Q_s^2) + x_L f_{val}(x_L, Q_s^2)$$

so that

$$\frac{dN_B^{net}}{dk^2 dy} \sim e^{-\Delta_R (Y_B - y)} + e^{-\Delta_R (Y_B + y)} \, .$$

with

$$\Delta_R \equiv 1 - 2 \sqrt{\frac{\alpha_s C_F}{2\pi}} \, .$$

Taking $\alpha_S = 0.33$ gives $\Delta_R = 0.47$, which gives a good fit of the existing data:

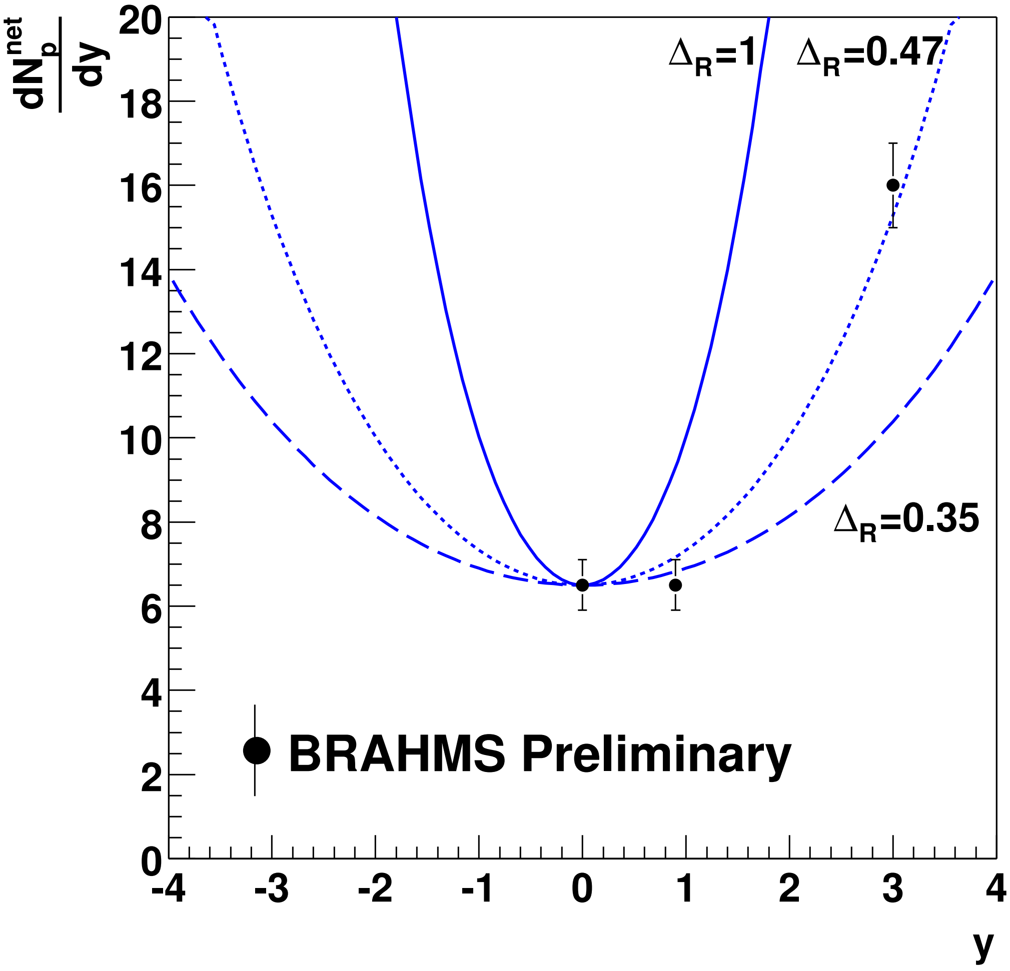

Figure 11: A comparison to preliminary BRAHMS data on net-proton rapidity distributions.

$\Rightarrow$ DLA approximation (or Regge theory ?) appear to be in good agreement with the existing RHIC data on baryon stopping.

SUMMARY

⇒ We have constructed an equivalent of the McLerran-Venugopalan model for the valence quark distributions. Multiple rescatterings tend to regulate the IR divergence, similar to gluon case.

⇒ We have constructed a dipole model equivalent of the linear DLA evolution equation for non-singlet structure functions and reproduced the intercept found by Kirschner and Lipatov.

⇒ We have included the effects of saturation in the small-X evolution by writing down a non-linear evolution equation. We showed that saturation reduces the Reggeon's intercept.

⇒ We have shown that there might be a connection between the small-x evolution and RHIC data on baryon stopping.