Coherent Production of Parabosons of Order 2

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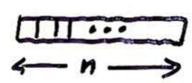
Physics Motivation:

Where are the fundamental particles corresponding to the other representations of the permutation group?

So Far:

Fermions go in totally anti-symmetric representations:

Bosons go in totally symmetric representations:



But

there are mixed representations

Some references on paraparticles and parastatistics:

[c.f. W. Pauli, Handbuch der Physik Vol 24 (1934)]

1. H.S. Green, Phys. Rev. 90, 270 (1953).
2. D.V. Volkov, Sov. Phys. - JETP 11, 375 (1960).
3. O.W. Greenberg and A.M.L. Messiah, Phys. Rev. 136, B248 (1964); 138, B1155 (1965).
4. Y. Ohnuki and S. Kamefuchi, "Quantum Field Theory and Parastatistics" (1982) (Univ. Press of Tokyo)
5. S. Jing and CAN, J. of Phys. A 32, 4131 (1999) "... order 2 conserved-charge parabose coherent states."

Paraquantization:

We consider the parafermi and parabose cases together:

The Basic Commutation Relations:

Now tri-linear not bi-linear

$$[a_k, [a_\ell^\dagger, a_m]_\mp] = 2\,\delta_{k\ell}\, a_m$$

$$[a_k, [a_\ell^\dagger, a_m^\dagger]_\mp] = 2\,\delta_{k\ell}\, a_m^\dagger \mp 2\,\delta_{km}\, a_\ell^\dagger$$

$$[a_k, [a_\ell, a_m]_\mp] = 0$$

[Upper signs ≡ parafermi case
Lower signs ≡ parabose case]

The Number Operator

$$N_k \equiv \tfrac{1}{2}[a_k^\dagger, a_k]_\mp \pm \tfrac{p}{2}$$

$p \equiv$ "order of the paraparticles"

$=$ Maximum Number of parafermions (parabosons) in a totally symmetric state (anti-symmetric state)

The vacuum state:

$$a_k |0\rangle = 0 \quad \text{for all } k$$

$$\langle 0|0\rangle = 1$$

$$a_k a_l^\dagger |0\rangle = p\, \delta_{kl} |0\rangle$$

Parabosons of order 2:

For only 2 kinds of parabosons

$$a \equiv a_1 \quad , \quad b \equiv a_2$$

$$[a, b^2] = [b, a^2] = [a^\dagger, b^2] = [b^\dagger, a^2] = 0$$

Order 2 is simpler because then

$$a_m a_\ell a_k^\dagger - a_k^\dagger a_\ell a_m = 2\delta_{k\ell} a_m$$

$$a_k a_\ell^\dagger a_m - a_m a_\ell^\dagger a_k = 2\delta_{k\ell} a_m - 2\delta_{\ell m} a_k$$

$$a_k a_\ell a_m - a_m a_\ell a_k = 0$$

Outline of Talk:

① Idea of parabosons of order 2.

② $p=1$ case:

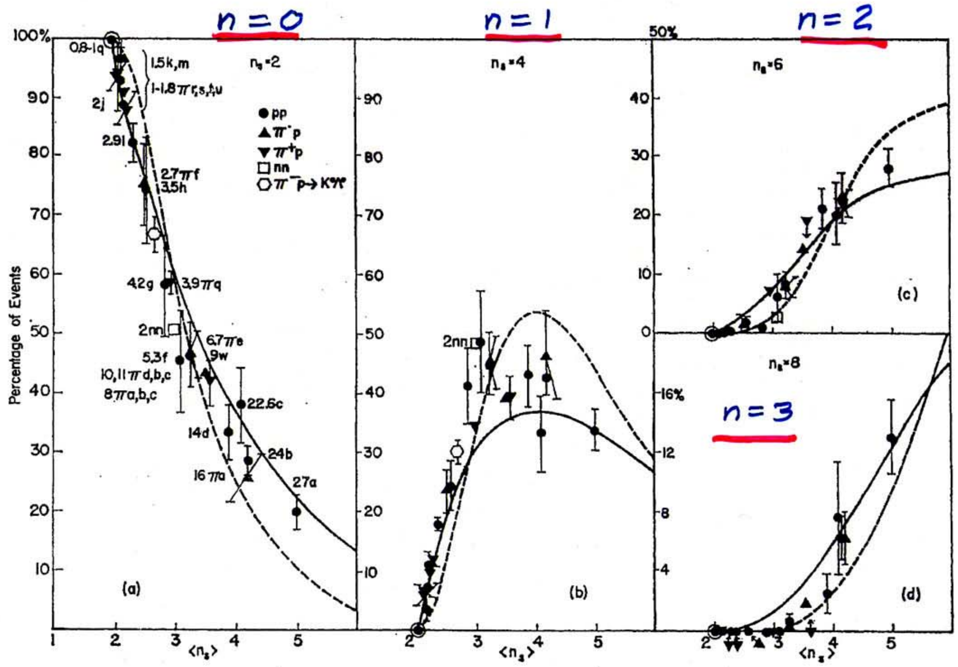
Empirical regularity: C.P. Wang (1969)
Inelastic $\pi^+\pi^-$ pair production from fixed targets; lab K.E. up to 27 GeV.

Parameter-free statistical model explanation:
Horn-Silver (1970).

③ $p=2$ case:

Construction of analogous model:

④ Comparison of multiplicity signatures for $p=2$ case (parabosons of order 2)
versus
$p=1$ case (usual bosons).



ɜ. 1. Relative frequencies of events against mean multiplicity $\langle n_s \rangle$ of charged secondaries for pp, $\pi^{\pm}p$, and nn collisions at various
ies. The numbers next to the data points are the laboratory kinetic energies of the primaries in BeV, and the letters are the refer-
in Ref. 2. Solid and dashed curves are the parameterless distributions $W_{n_s(\mathrm{even})}{}^{\mathrm{I}}$ and $W_{n_s(\mathrm{even})}{}^{\mathrm{II}}$, respectively; see text. The neutral
ɔoints also fall on the curves within experimental errors. Because of charge neutrality, the nn data points are plotted against $\langle n_s \rangle+2$
(a)–(c) are $n_s=0$, 2, and 4.

C. P. Wang, Phys. Rev. 180, 1463 (1969).

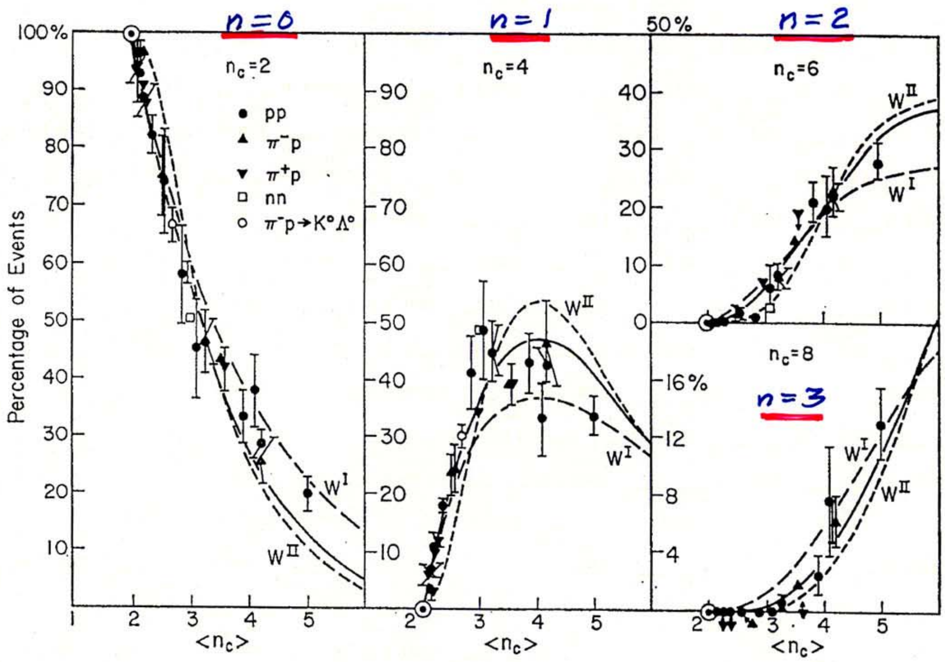
Wang plotted $P_m(q)$ for $q=0$ versus the equivalent mean number of charged prongs

$$\langle n_s \rangle = 2\langle n \rangle + 2$$
$$= \langle n_c \rangle \text{ (Horn-Silver)}$$

Horn - Silver, Phys. Rev. D2, 2082 (1970).

$P_m(q)$ for $q=0$ versus

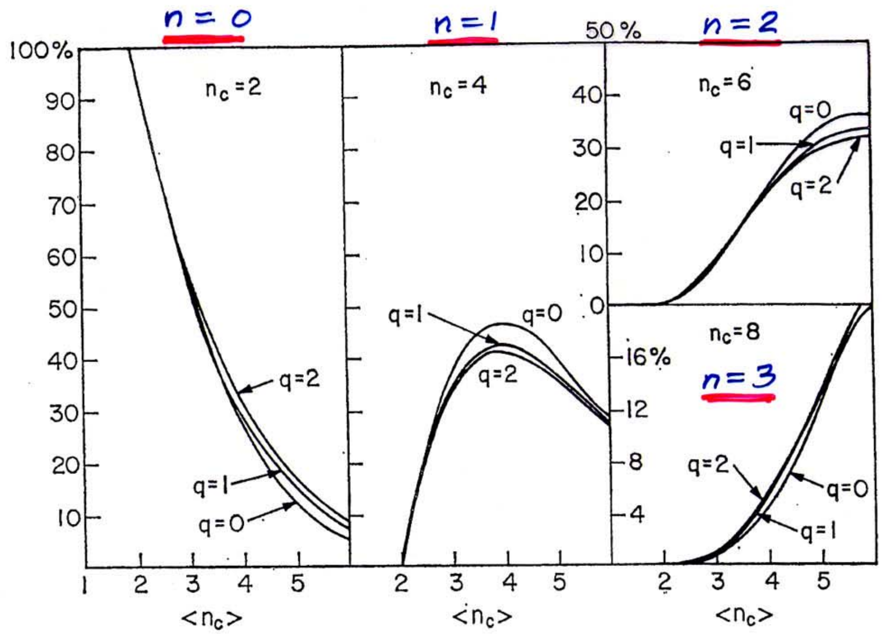
$\langle n_c \rangle = 2\langle n \rangle + 2 = \langle n_s \rangle$ (Wang)



$p=1$ (usual bosons)

$$\langle n \rangle = \frac{x\, I_1(2x)}{I_0(2x)}$$

$$P_m(0) = \frac{x^{2m}}{I_0(2x)\, m!\, m!}$$



$p = 1$, arbitrary q: (Horn - Silver)

$$P_m(q) = \frac{x^{2m+q}}{I_q(2x)\, m!\,(m+q)!}$$

$$\langle n \rangle = \frac{x\, I_{q+1}(2x)}{I_q(2x)}$$

$$\langle n^2 \rangle = \langle n \rangle + \frac{x^2\, I_{q+2}(2x)}{I_q(2x)}$$

$p = 2$ Conserved Charge Coherent States:

a annihilates one A quanta of charge $+1$

b " " B " " " -1

(A charged-parabose pair consists of one of each type but

$$ab \neq ba$$
)

Hermitian charge operator

$$Q \equiv N_a - N_b$$

$$[Q, ab] = [Q, ba] = [ab, ba] = 0$$

Conserved Charge Coherent State:

$$Q\,|q, z, z'\rangle = q\,|q, z, z'\rangle$$

$$ab\,|q, z, z'\rangle = z\,|q, z, z'\rangle$$

$$ba\,|q, z, z'\rangle = z'\,|q, z, z'\rangle$$

where

$$\left.\begin{array}{l} u \equiv |z| \\ v \equiv |z'| \end{array}\right\} \text{Two non-negative real parameters}$$

$p=2$ Conserved-Charge Coherent States:

$(q \geq 0)$

Where

$|n, m; i\rangle$ = State vector of n parabosons A and m parabosons B; and $1 \leq i \leq \{\min(n,m) + 1\}$

degeneracy index due to $ab \neq ba$

$$|q, z, z^*\rangle = N_q \sum_{m=0}^{\infty} \sum_{i=1}^{m+1} \frac{z^r (z^*)^s}{2^m \sqrt{\left[\frac{m+i}{2}\right]! \left[\frac{q+m+i}{2}\right]! \left[\frac{m+1-i}{2}\right]! \left[\frac{q+m+1-i}{2}\right]!}} |q+m, m; i\rangle$$

$$(N_q)^{-2} = \left(\frac{u}{2}\right)^{-\left[\frac{q}{2}\right]} I_{\left[\frac{q}{2}\right]}(u) \quad \left(\frac{v}{2}\right)^{-\left[\frac{q+1}{2}\right]} I_{\left[\frac{q+1}{2}\right]}(v)$$

$$r = \left[\frac{m - (-)^{q+m+i}_{i}}{2} + \frac{1 - (-)^q}{4}\right]$$

$$s = \left[\frac{m + (-)^{q+m+i}_{i}}{2} + \frac{1 + (-)^q}{4}\right]$$

and

$[x]$ = (largest integer $\leq x$)

Plots:

$p = 2$ Case (parabosons of order 2)

$P_m(q)$ = "the probability of
$\underline{m}$ paraboson charged-pairs
plus
$\underline{q}$ positive-charged parabosons"

versus

$\langle n \rangle$ = "mean number of
charged pairs"

$\langle n^2 \rangle$ = "mean of the square
of the number of
charged pairs"

Case: $q_{even} = 0, \pm 2, \pm 4, \cdots$

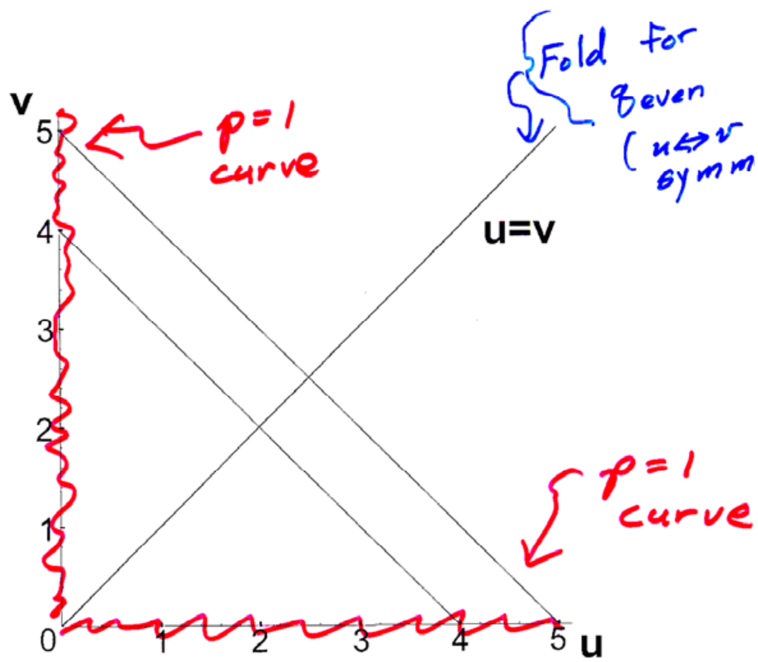


Fig. 8b

$\boxed{m=1,\ q=0 \quad \text{case}}$ $\quad u \leftrightarrow v$ symmetric

$\underline{p=1}$:

$$\langle n \rangle = \frac{x\, I_1(2x)}{I_0(2x)}$$

$$\langle n^2 \rangle = \langle n \rangle + \frac{x^2 I_2(2x)}{I_0(2x)}$$

$$P_1^{(1)}(0) = \frac{x^2}{I_0(2x)}$$

$\underline{p=2 \text{ ribbon}}$:

$$\langle n \rangle = \frac{1}{2}\left(\frac{u\, I_1(u)}{I_0(u)} + \frac{v\, I_1(v)}{I_0(v)} \right)$$

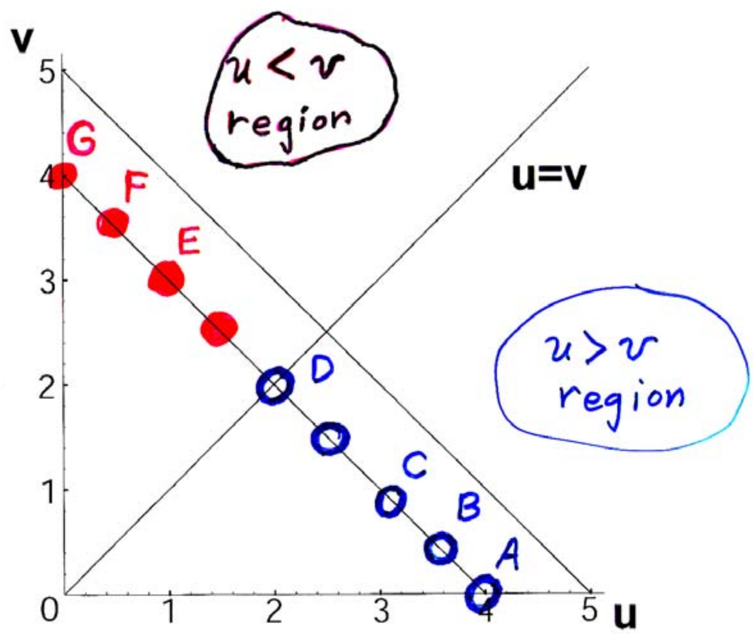
$$\langle n^2 \rangle = \langle n \rangle + \frac{1}{4}\left(\frac{u^2 I_2(u)}{I_0(u)} + \frac{2uv\, I_1(u)\, I_1(v)}{I_0(u)\, I_0(v)} + \frac{v^2\, I_2(v)}{I_0(v)} \right)$$

$$P_1^{(2)}(0) = \frac{u^2 + v^2}{4\, I_0(u) I_0(v)}$$

NOTE: $p=1$ curve corresponds to

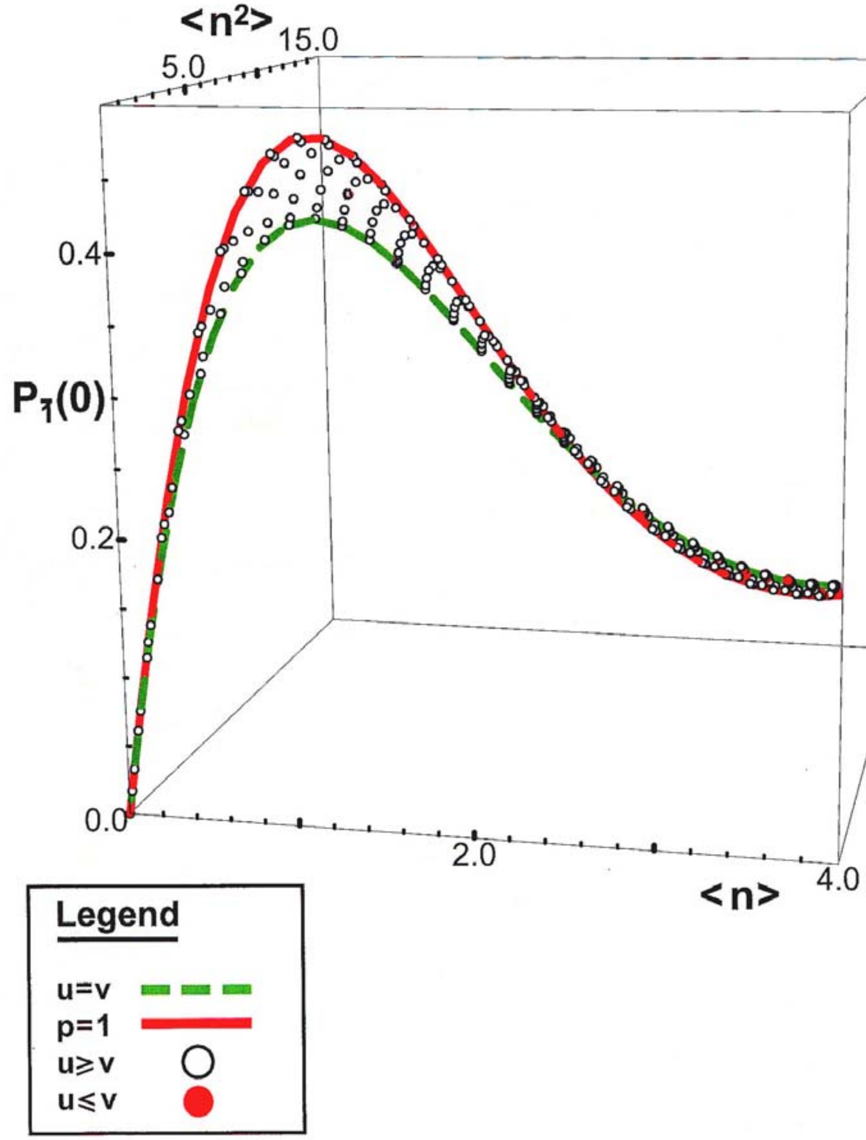
$$\{u, v\} = \{0, 2x\}$$

and $\quad \{u, v\} = \{2x, 0\}$

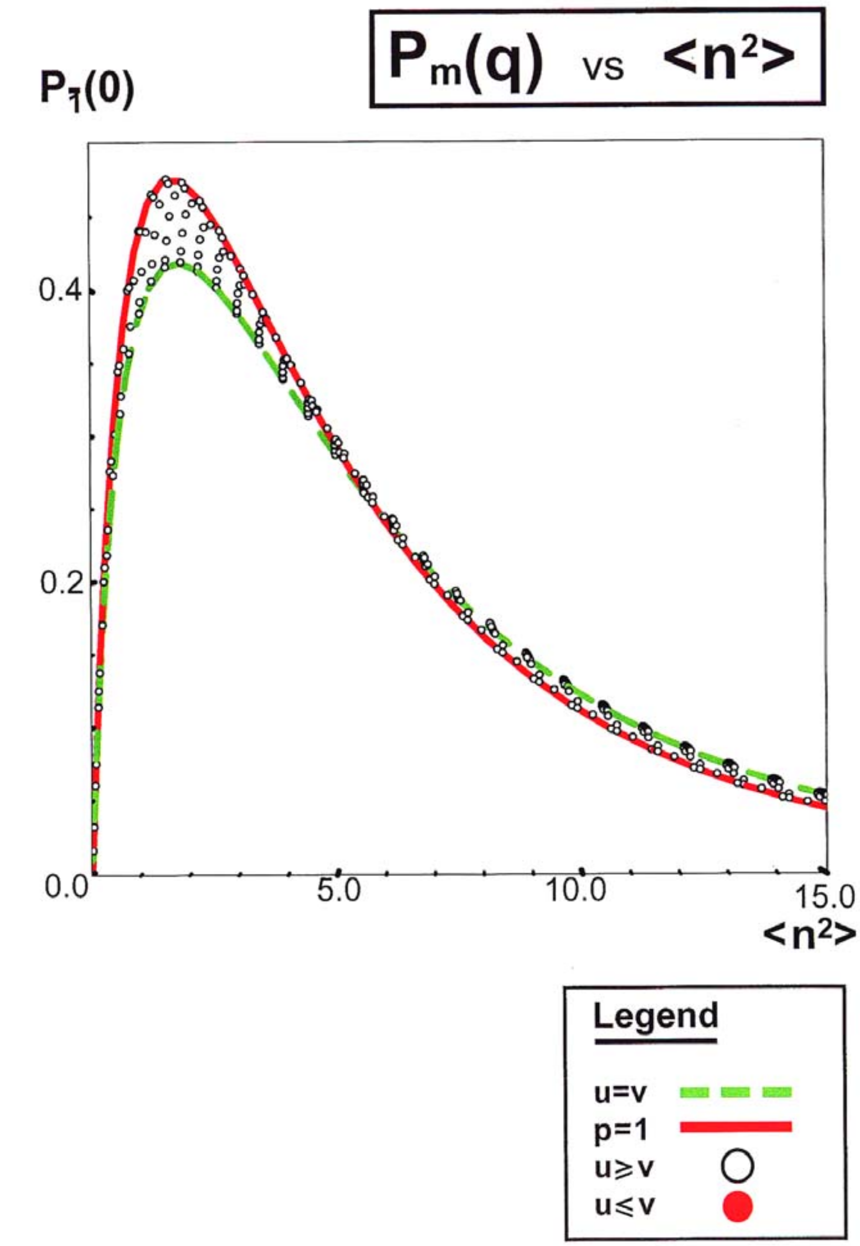


$q_{even} \Rightarrow \left\{ \begin{array}{l} \text{Ribbon is } u \leftrightarrow v \\ \text{symmetric} \end{array} \right.$

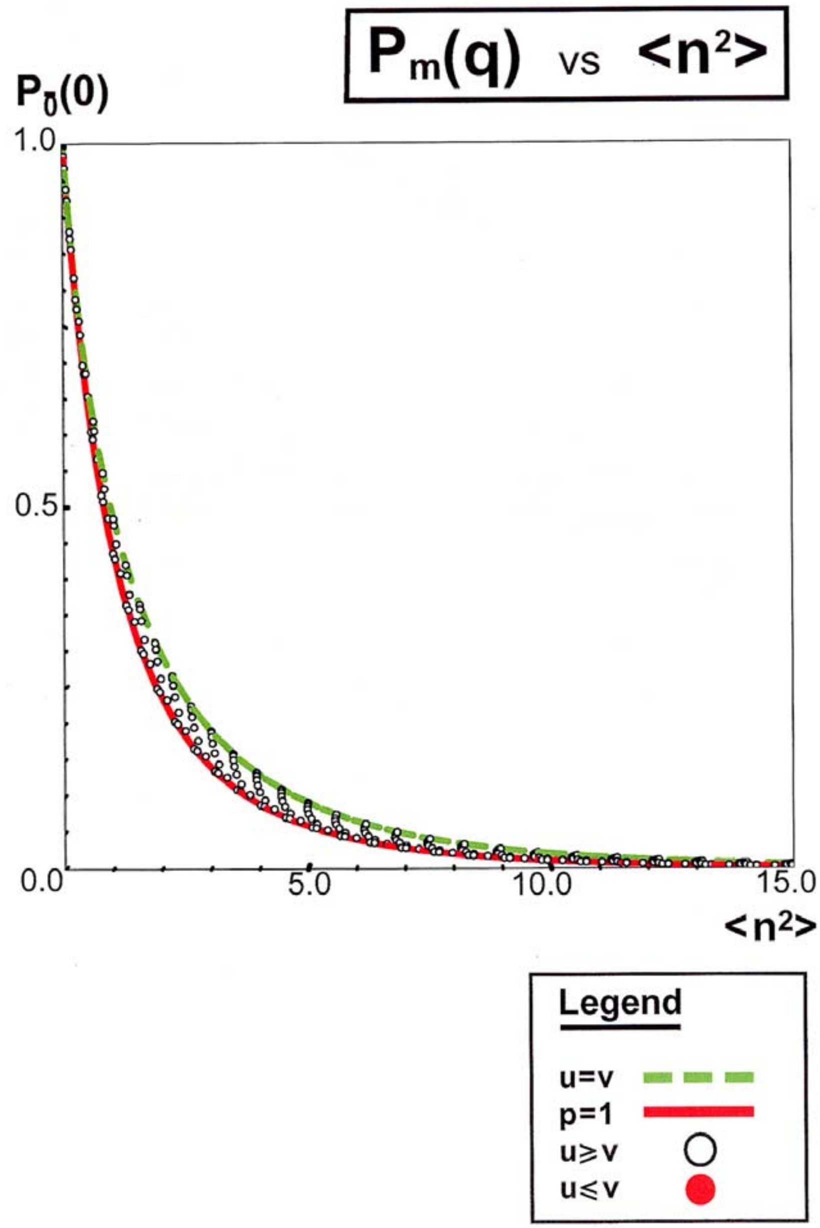
$q_{odd} \Rightarrow \left\{ \begin{array}{l} \text{Ribbon is } u \leftrightarrow v \\ \text{asymmetric} \end{array} \right.$



<n²>
15.0
5.0
0.4
P₁(0)
0.2
0.0
2.0
<n>
4.0
Legend
u=v
p=1
u≥v
u≤v



$P_m(q)$ vs $<n^2>$
$P_1(0)$
0.4
0.2
0.0
5.0
10.0
15.0
$<n^2>$
Legend
u=v
p=1
u≥v
u≤v



$P_m(q)$ vs $\langle n^2 \rangle$
$P_0(0)$
1.0
0.5
0.0
5.0
10.0
15.0
$\langle n^2 \rangle$
Legend
u=v
p=1
u≥v
u≤v

$\boxed{m = 2, \quad q = 1 \quad \text{case}}$ $\quad u \leftrightarrow v$ asymmetric

$p=1$:

$$\langle n \rangle = \frac{x\, I_2(2x)}{I_1(2x)}$$

$$\langle n^2 \rangle = \langle n \rangle + \frac{x^2 I_3(2x)}{I_1(2x)}$$

$$P_2^{(1)}(1) = \frac{x^5}{12\, I_1(2x)}$$

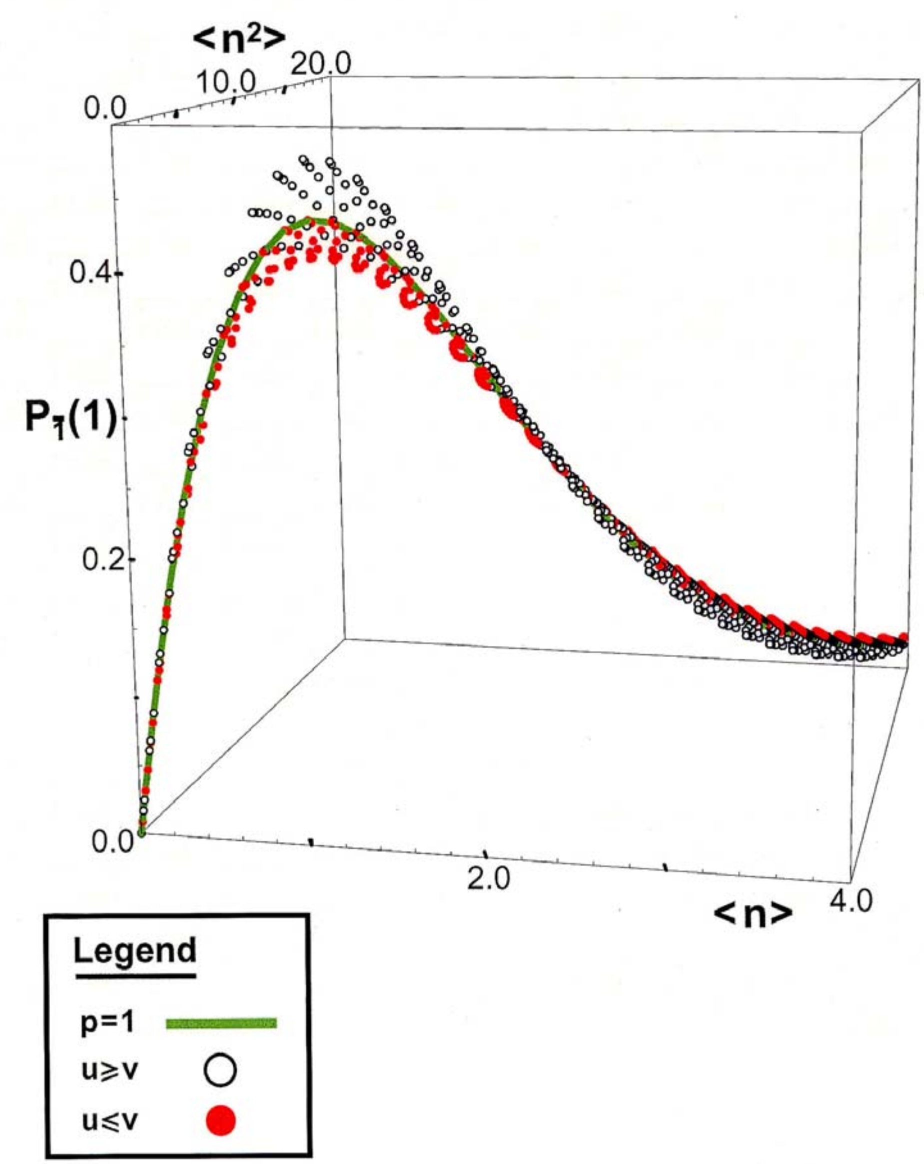
$p=2$ ribbon:

$$\langle n \rangle = \frac{1}{2}\left(\frac{u\, I_1(u)}{I_0(u)} + \frac{v\, I_2(v)}{I_1(v)} \right)$$

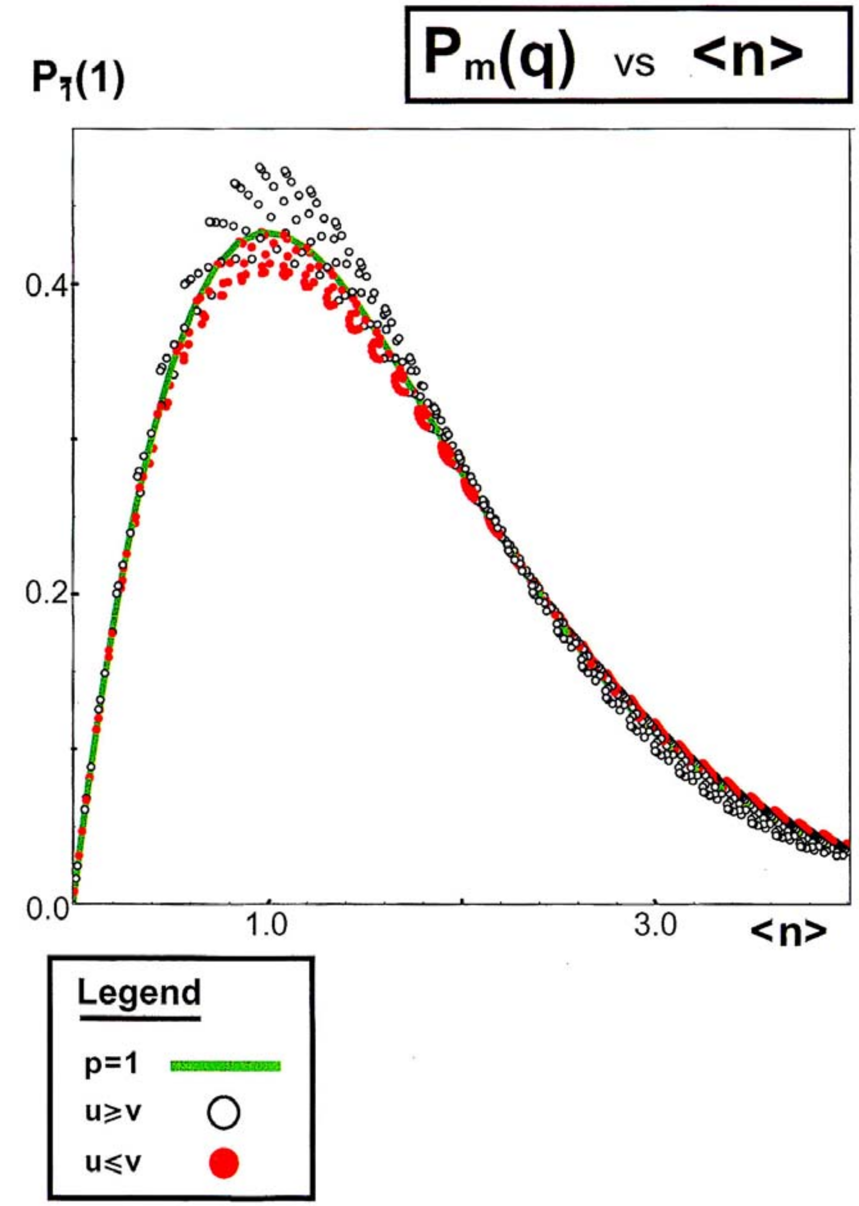
$$\langle n^2 \rangle = \langle n \rangle + \frac{1}{4}\left(\frac{u^2 I_2(u)}{I_0(u)} + \frac{2uv\, I_1(u)\, I_2(v)}{I_0(u)\, I_1(v)} + \frac{v^2 I_3(v)}{I_1(v)} \right)$$

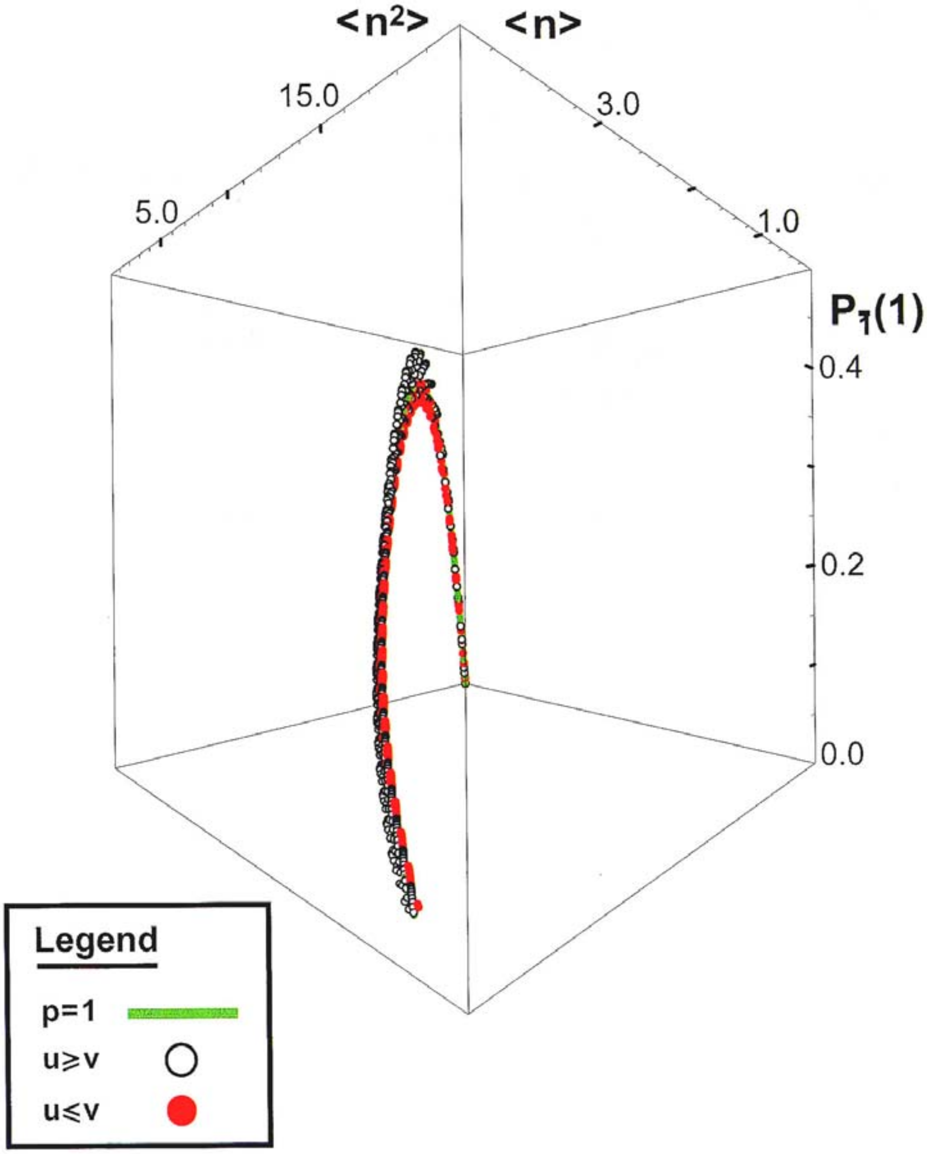
$$P_2^{(2)}(1) = \frac{3u^4 v + 6u^2 v^3 + v^5}{384\, I_0(u)\, I_1(v)}$$

NOTE: $p=1$ curve corresponds to $\{u, v\} = \{0, 2x\}$



<n2>
20.0
10.0
0.0
0.0
0.4
P1(1)
0.2
0.0
2.0
<n>
4.0
Legend
p=1
u⩾v
u⩽v


P_m(q) vs <n>
P_1(1)
0.4
0.2
0.0
1.0
3.0
<n>
Legend
p=1
u≥v
u≤v



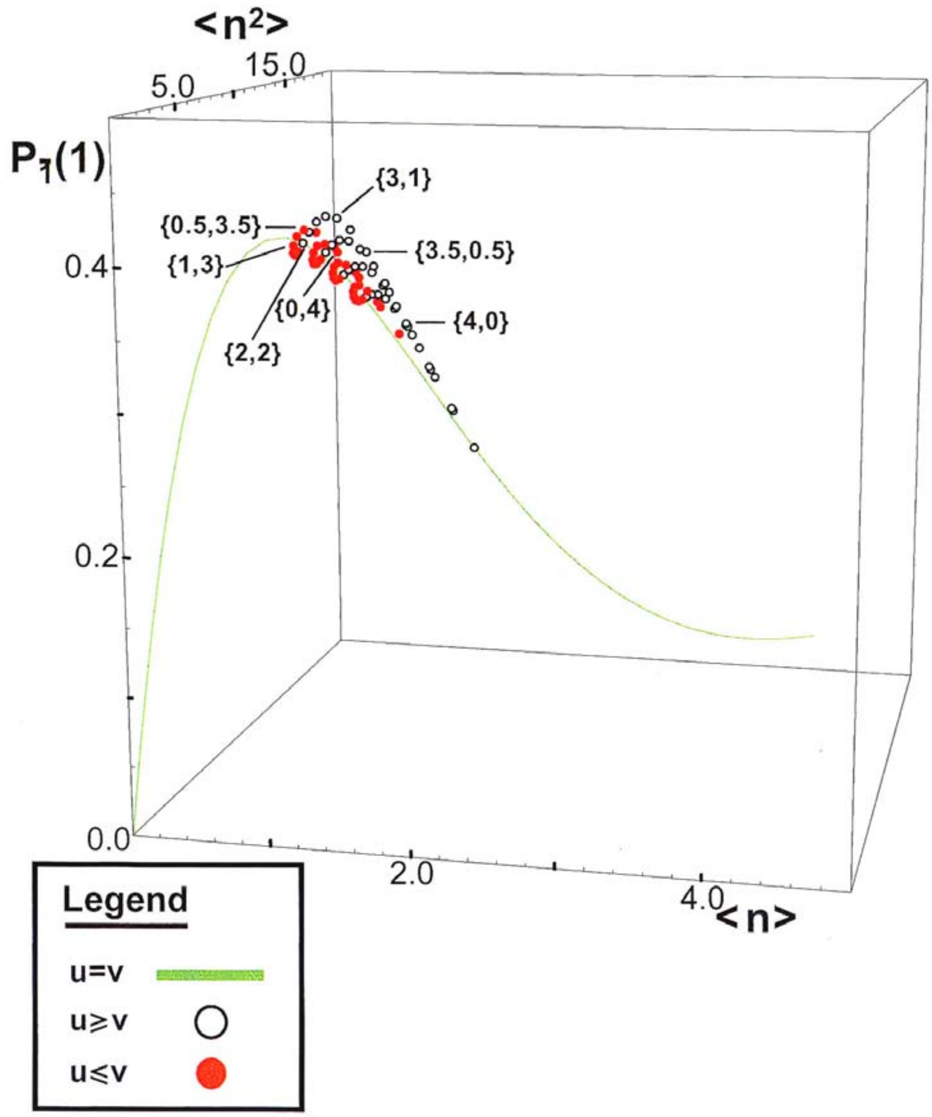
<n2>
<n>
15.0
3.0
5.0
1.0
P₁(1)
0.4
0.2
0.0
Legend
p=1
u≥v
u≤v

Discussion:

① From this parameter-free statistical model, a signature for $p=2$ parabosons is "bands" instead of curves in plots of $P_m(q)$'s versus $\langle n \rangle$, due to the projection of the varying-width folded ribbons.

② Physical observables are not always $u \leftrightarrow v$ symmetric when q odd.
(Could be used to identify A quanta with $q>0$).

③ If $U(1)$ charge is hidden and conserved, can still use results with $\sum_{m=0}^{\infty} P_m(0) = 1$.
(that is only $q=0$ final states will occur)



<n2>
5.0
15.0
P1(1)
0.4
0.2
0.0
2.0
4.0
<n>
{3,1}
{0.5,3.5}
{1,3}
{3.5,0.5}
{0,4}
{2,2}
{4,0}
Legend
u=v
u≥v
u≤v