

QCD: Theoretical Developments

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Lepton Photon 2003

QCD

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- QCD today is becoming precision physics

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LEP precision physics:

Electroweak processes

Tevatron/LHC precision physics:

QCD processes

Precision physics with QCD

- precise determination of
 - strong coupling constant
 - quark masses
 - electroweak parameters
 - parton distributions
 - LHC luminosity (!)

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 - strong coupling constant
 - quark masses
 - electroweak parameters
 - parton distributions
 - LHC luminosity (!)
- precise predictions for
 - new physics effects
 - and their backgrounds

Theoretical challenges in QCD

- QCD **describes** quarks and gluons; experiments **observe** hadrons
 - describe parton → hadron transition (**fragmentation**)
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- frequent **multiparticle** final states
- important **higher order** corrections
- observables involve different scales

$$m_Q, p_T, M_Z, m_T$$

→ large logarithms e.g. $\ln(p_T^2/M_Z^2)$

- reorder (**resum**) perturbative series

Outline

- Heavy Quarks
- Jets and Multiparticle Production
- Photons
- Gauge and Higgs Bosons

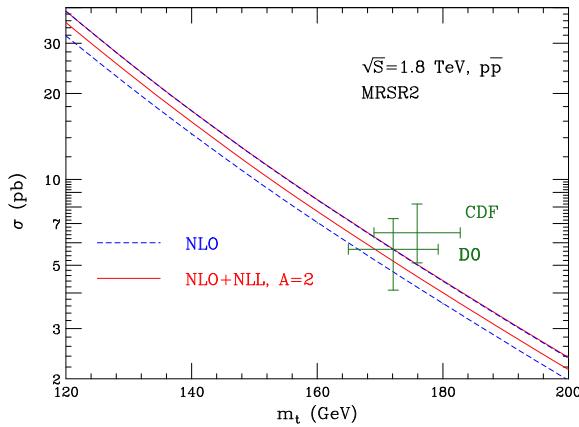
Heavy Quarks

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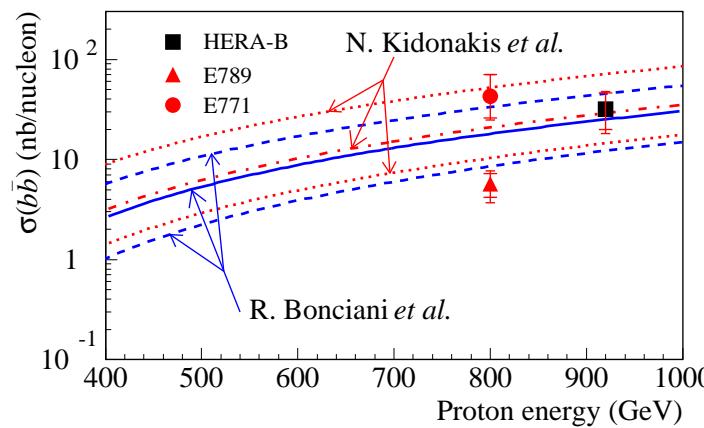
Total cross sections

status: NLO + NLL soft gluon resummation

W. Beenakker et al., R. Bonciani, S. Catani, M. Mangano, P. Nason
N. Kidonakis, E. Laenen, S. Moch, R. Vogt



Tevatron: $\frac{\sqrt{s}}{m_t} \simeq 10$



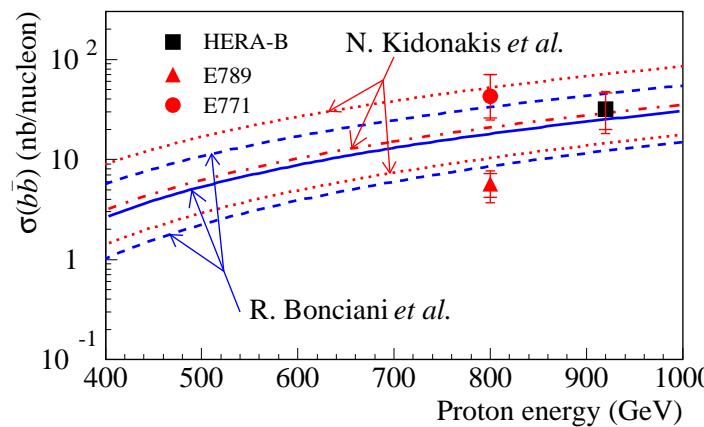
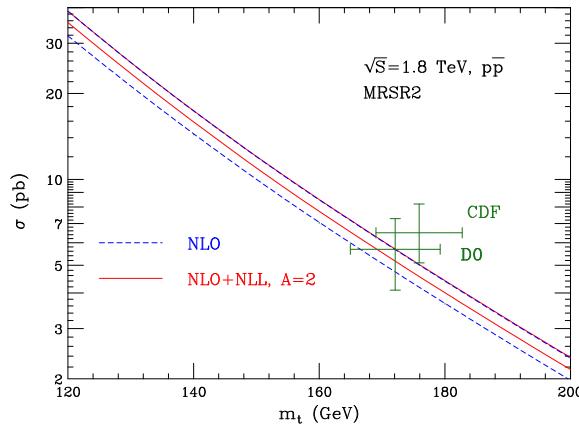
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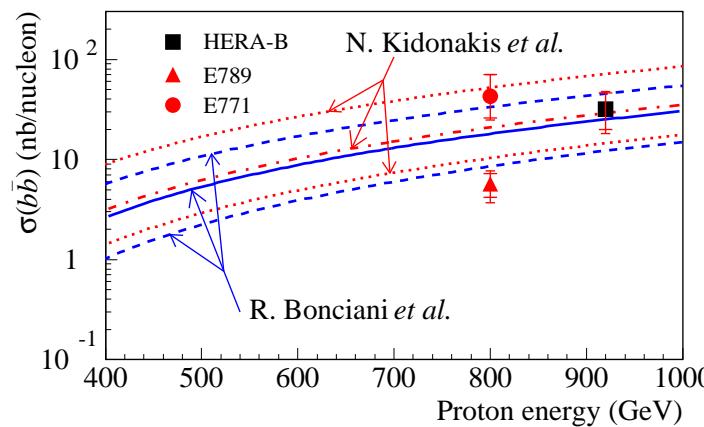
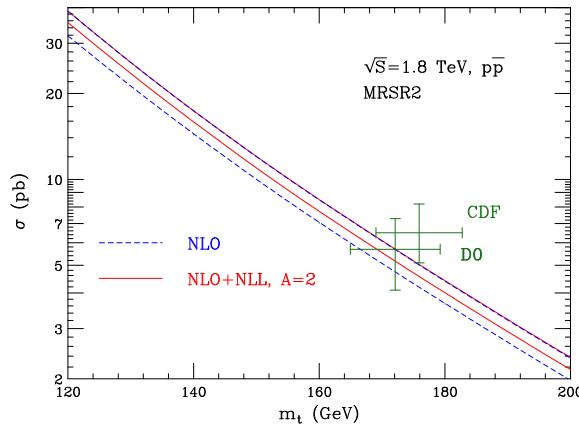
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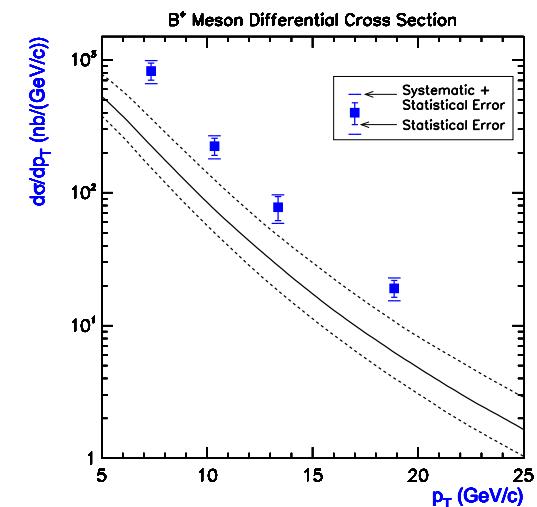
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- Soft gluon resummation enhances cross section and lowers theoretical uncertainty
- Theoretical uncertainty lower for $t\bar{t}$ than for $b\bar{b}$: $\alpha_s(m_b) \gg \alpha_s(m_t)$,
 $q\bar{q}$ dominance at Tevatron and gg dominance at HERA-B

Heavy Hadron Production

Differential distributions

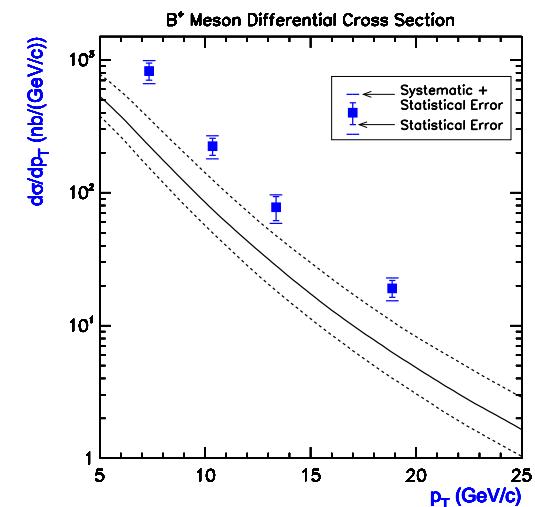
long-standing **excess** of b -quark transverse momentum
distributions in $p\bar{p}$, γp , $\gamma\gamma$
measured final state: B hadrons, e.g. CDF: B^\pm



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long-standing **excess** of b -quark transverse momentum distributions in $p\bar{p}$, γp , $\gamma\gamma$
measured final state: B hadrons, e.g. CDF: B^\pm



Theoretical prediction:

$$\frac{d\sigma^{B^\pm}}{dp_T} = f_{a/p} \otimes f_{b/\bar{p}} \otimes \frac{d\sigma^{ab \rightarrow b\bar{b}}}{dp_T} \otimes D_{b \rightarrow B^\pm}$$

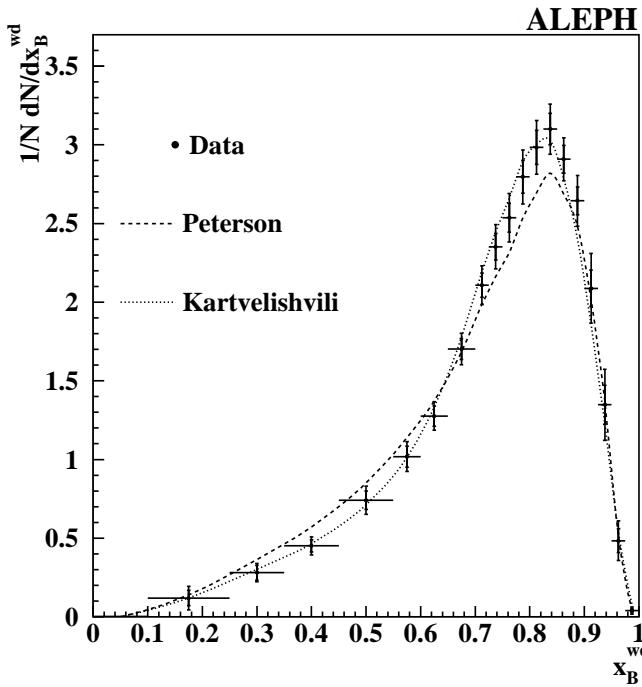
$f_{a/p}$: parton distribution function

$D_{b \rightarrow B^\pm}$: fragmentation function

Heavy Hadron Production

Fragmentation Functions

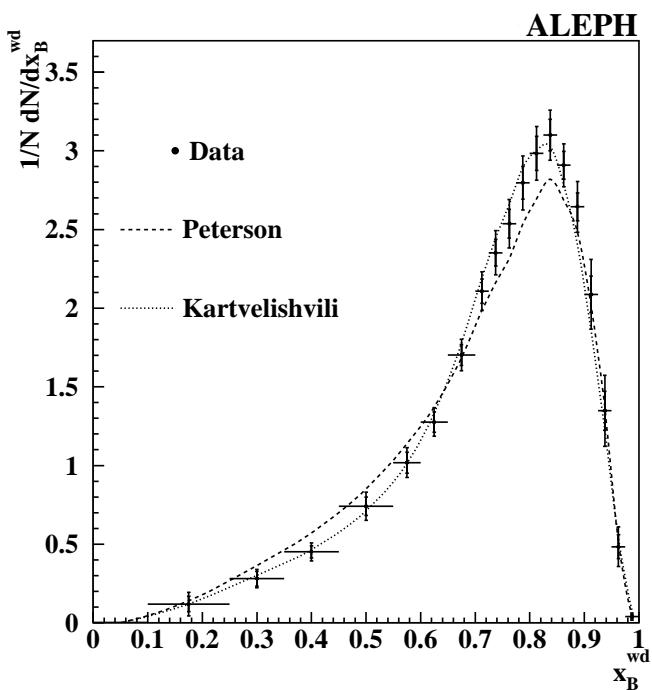
Extraction of $D_{b \rightarrow B^\pm}$ from LEP data on B meson spectra: many ambiguities



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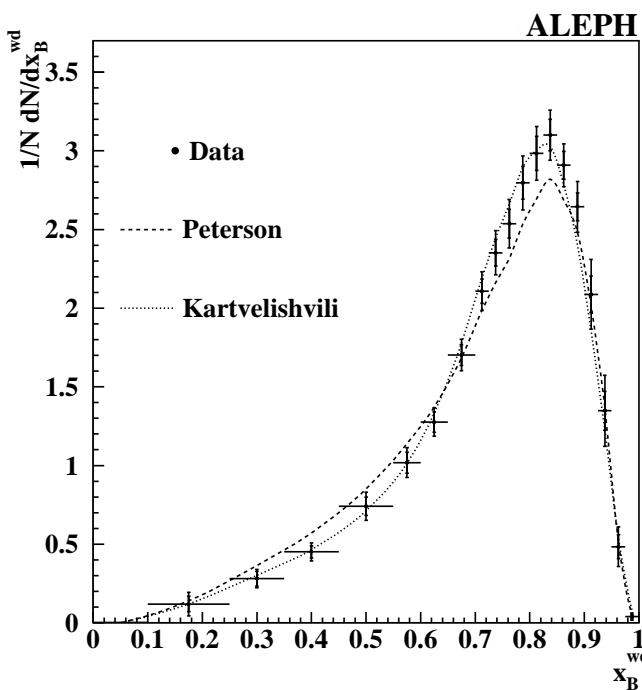
- order of perturbative calculation
- massless or massive hard matrix elements
- resummation of perturbative terms
- inclusion of power corrections $\mathcal{O}(\Lambda/m)$
- M. Cacciari, E. Gardi
- data corrected with parton showers ?
- parametric form of $D_{b \rightarrow B^\pm}$: e.g. Peterson:

$$D_{b \rightarrow B^\pm}(z, \mu_0) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2} \quad \text{fit } \epsilon, N$$

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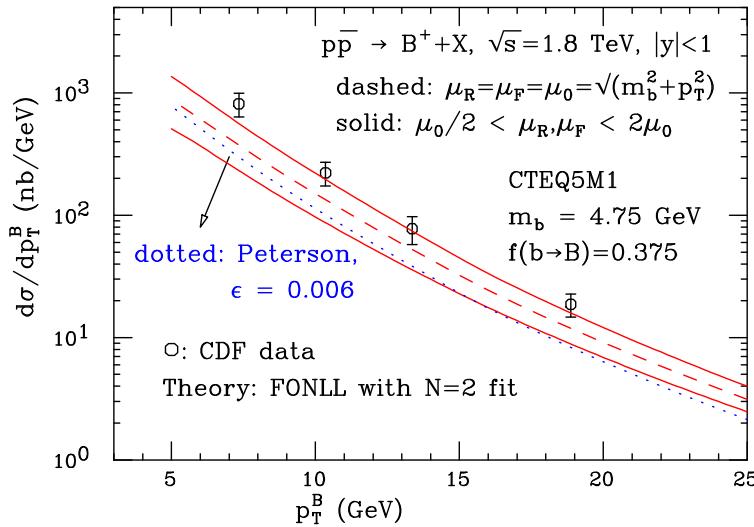
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Prediction for $p\bar{p}$ must use same assumptions as made in the extraction from e^+e^-

Heavy Hadron Production

Resummation of $\ln(m_b^2/p_T^2)$ and $\ln(m_b^2/s)$

M. Cacciari, P. Nason

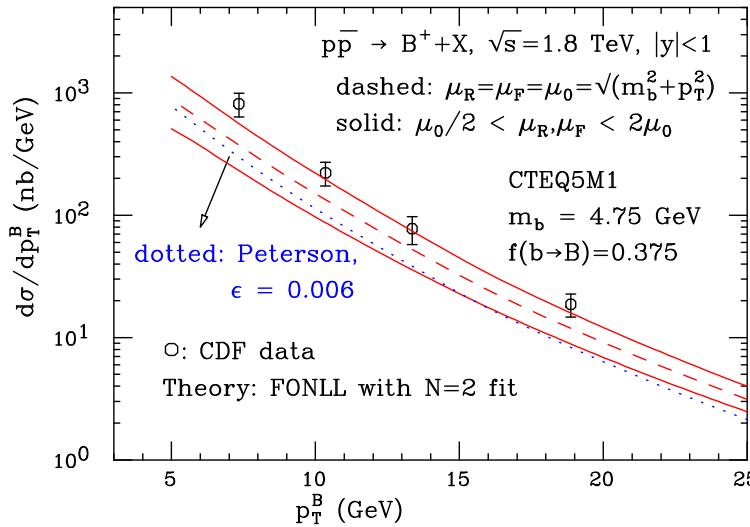


- must re-fit $D_{b \rightarrow B^\pm}$ from e^+e^- data
- done in moment space to expose information content relevant to $p\bar{p}$
- use of the resummed (FONLL) scheme enhances transverse momentum spectrum

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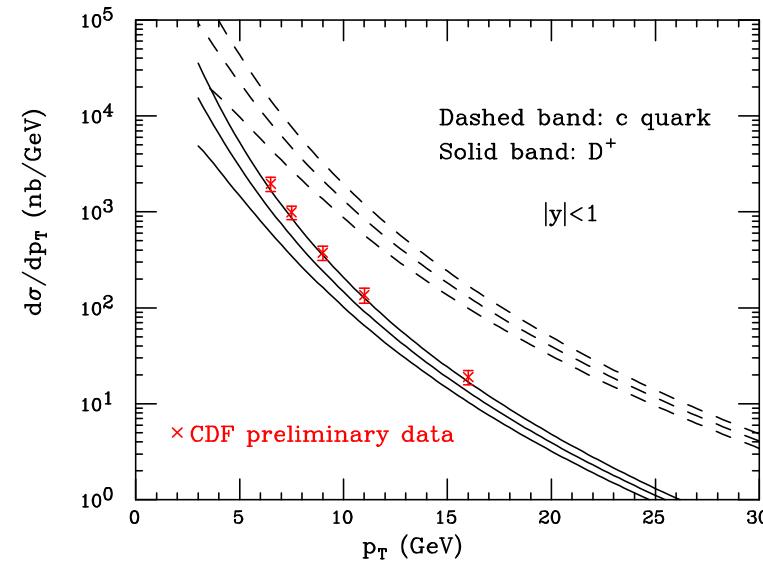
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recently extended to D hadrons:
considerable improvement between
data and theory



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- higher precision measurements (Tevatron-Run II, LHC) will require **NNLO** corrections

Jets and Multiparticle Production

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QCD jet final states very frequent at colliders

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- sensitive probe of standard model parameters
 - measure α_s from $e^+e^- \rightarrow 3j$, $ep \rightarrow (2+1)j$, $pp \rightarrow j + X$, $2j + X$
 - measure M_W , M_Z from $pp \rightarrow V + X$, $V + j + X$
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- want QCD prediction as precise as possible
- multijet final states frequent: $pp \rightarrow nj \sim \alpha_s^n$
 - important background for searches
- want robust QCD prediction as guidance
- also might want QCD to predict the full hadronic final state

Jets and Multiparticle Production

State-of-the-art: leading order

Jets and Multiparticle Production

State-of-the-art: leading order

- several efficient codes available for multiparton matrix element generation (from helicity amplitudes or fully numerically): $2 \rightarrow 8$ and beyond feasible on current computers
 - VECBOS W. Giele
 - COMPHEP E. Boos et al.
 - MADGRAPH T. Stelzer et al.
 - GRACE Minami–Tateya Group
 - HELAC C. Papadopoulos et al.
 - ALPHGEN M. Mangano et al.
 - AMEGIC++ F. Krauss et al.
- combined with automatic integration over multiparticle phase space
 - RAMBO R. Kleiss et al.
 - PHEGAS C. Papadopoulos
 - MADEVENT T. Stelzer et al.

Jets and Multiparticle Production

State-of-the-art: leading order

- generic procedure to interface partonic final state with **parton shower**:
modified matrix element plus vetoed parton shower
S. Catani, F. Krauss, R. Kuhn, B.R. Webber

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Leading order QCD is good tool to estimate relative magnitudes of processes and to design searches. Once precision is required (e.g. to identify a discovery with a particular model), it is not sufficient.

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- nature of calculations: **parton level Monte Carlo integrator**

Jets and Multiparticle Production

State-of-the-art: NLO recent results

- $pp \rightarrow V + 2j$

J.M. Campbell, R.K. Ellis

- $ep \rightarrow (3 + 1)j$

Z. Nagy, Z. Trocsanyi

- $pp \rightarrow 3j$

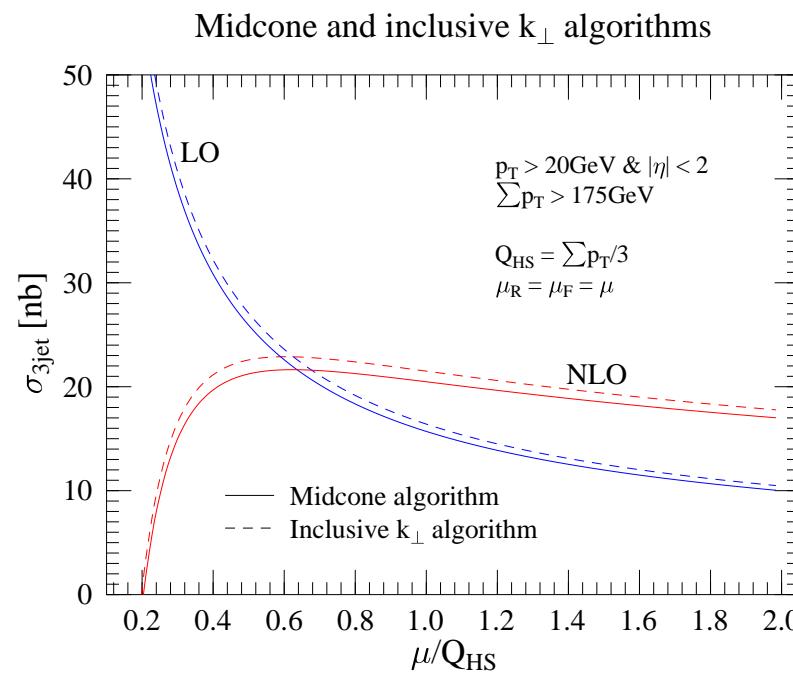
Z. Nagy

- $pp \rightarrow t\bar{t}H$

W. Beenakker, S. Dittmaier,
M. Krämer, B. Plumper,
M. Spira, P. Zerwas
S. Dawson, L. Orr, L. Reina,
D. Wackerlo

- $pp \rightarrow H + 2j$

C. Oleari, D. Zeppenfeld



Automatizing NLO

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- similar programs would be desirable for NLO calculations

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- ingredients to NLO calculation for n parton final state
 - virtual one-loop matrix element n partons
 - real matrix element $n + 1$ partons
 - procedure to extract infrared singularities from both and to combine them
- last two points solved for arbitrary n some time ago

W. Giele, E.W.N. Glover

S. Catani, M. Seymour

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- no generic procedure for automatic computation of one-loop integrals
- but many ideas (CERN workshop MC for LHC)
 - analytic reduction of hexagon integrals T. Binoth, J.P. Guillet, G. Heinrich
 - subtraction formalism for virtual corrections Z. Nagy, D. Soper
 - numerical evaluation of hexagon amplitudes T. Binoth, N. Kauer

Jets and Multiparticle Production

Interfacing NLO calculations with parton showers MC@NLO approach

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- introduces modified NLO subtraction method
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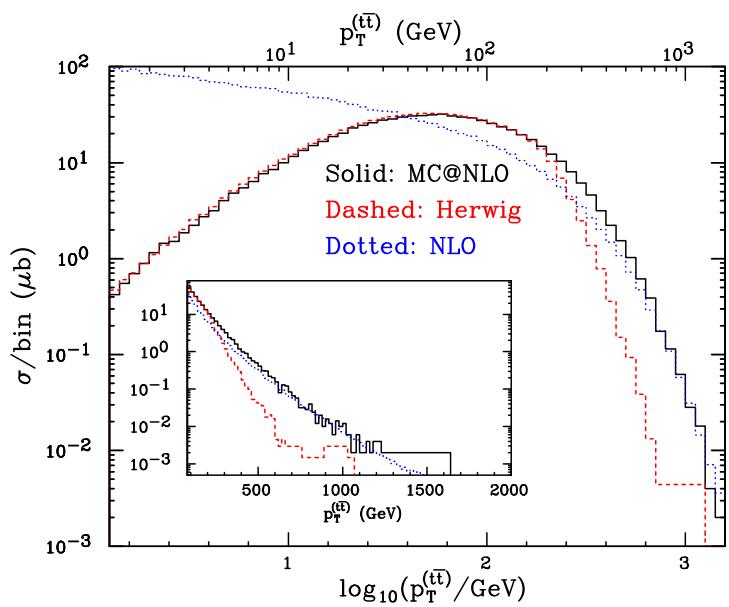
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- so far applied to VV , $b\bar{b}$, $t\bar{t}$

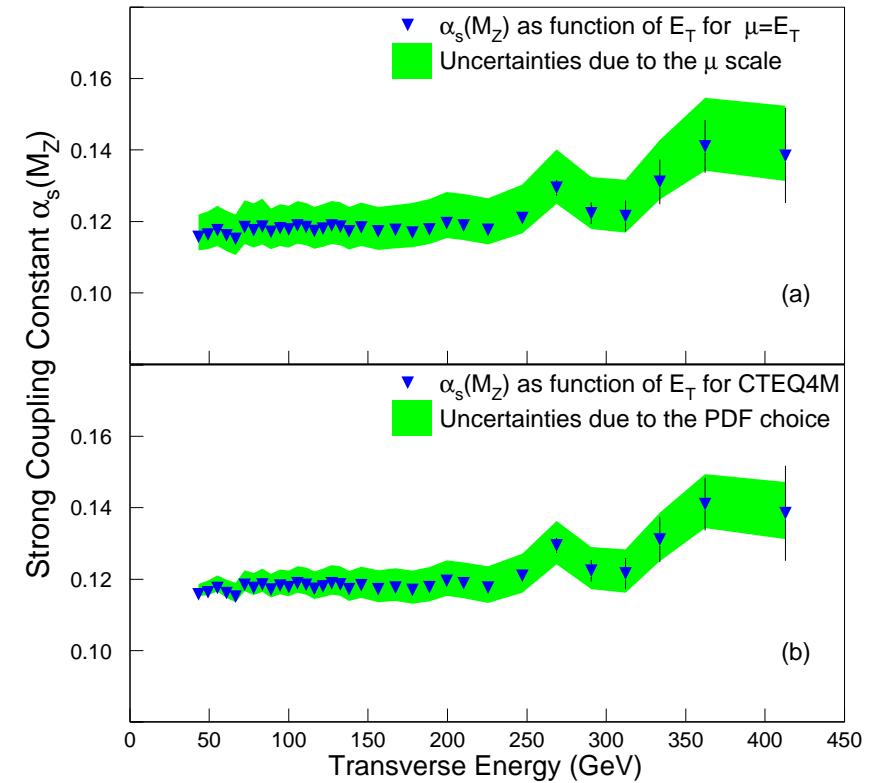
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NLO jet physics precise enough ?

Measurement of strong coupling constant α_s from single jet inclusive cross section

CDF collaboration

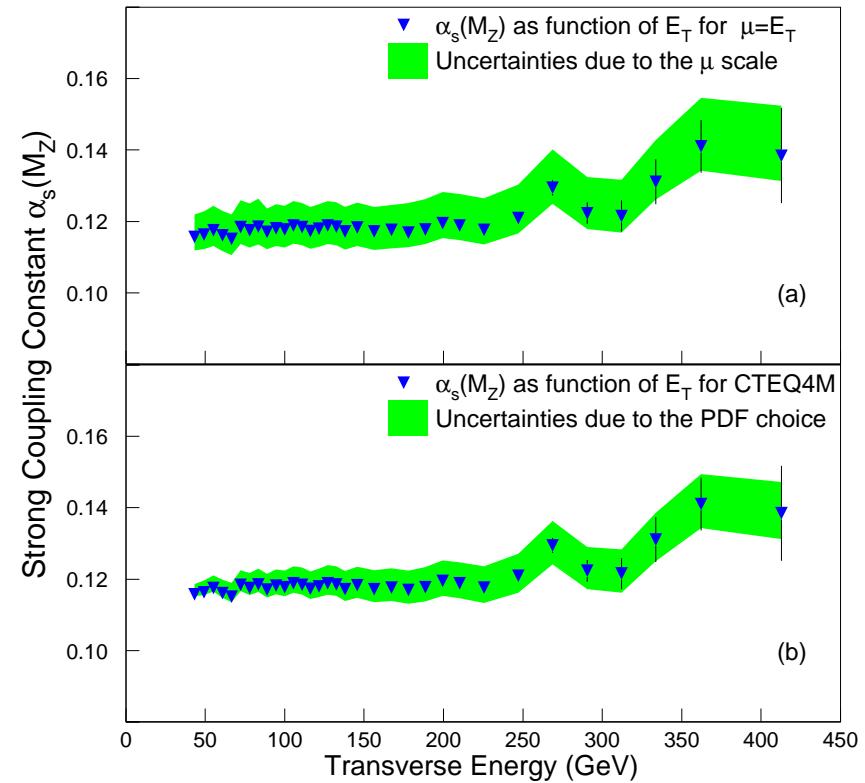


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$$\begin{aligned}\alpha_s(M_Z) &= 0.1178 \pm 0.0001(\text{stat})^{+0.0081}_{-0.0095}(\text{sys}) \\ &\quad {}^{+0.0071}_{-0.0047}(\text{scale}) \pm 0.0059(\text{pdf})\end{aligned}$$



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- 3-jet type observables at LEP (preliminary)

$$\begin{aligned}\alpha_s(M_Z) &= 0.1202 \pm 0.0003(\text{stat}) \pm 0.0009(\text{sys}) \\ &\quad \pm 0.0009(\text{had}) \pm 0.0047(\text{scale})\end{aligned}$$

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- (2+1) jet observables in DIS at HERA

$$\begin{aligned}\alpha_s^{\text{ZEUS}}(M_Z) &= 0.1190 \pm 0.0017(\text{stat})^{+0.0049}_{-0.0023}(\text{sys}) \pm 0.0026(\text{th}) \\ \alpha_s^{\text{H1}}(M_Z) &= 0.1186 \pm 0.0030(\text{exp})^{+0.0039}_{-0.0045}(\text{scale}) \pm 0.0023(\text{pdf})\end{aligned}$$

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NLO scale uncertainty dominates the error on α_s from jets at colliders

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- better description of **transverse momentum** of final states at hadron colliders due to double radiation in the initial state
- modified **power corrections** as higher perturbative powers $1/\ln(Q^2/\Lambda^2)$ can mimic genuine power corrections Q/Λ

NLO jet physics precise enough ?

More reasons to go beyond NLO

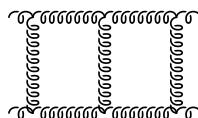
- better matching of **parton level** jet algorithm with experimental **hadron level** jet algorithm
- better description of **transverse momentum** of final states at hadron colliders due to double radiation in the initial state
- modified **power corrections** as higher perturbative powers $1/\ln(Q^2/\Lambda^2)$ can mimic genuine power corrections Q/Λ
- allow full NNLO global fits to parton distributions → lower error on **benchmark processes** at LHC and Tevatron

Towards NNLO jet physics

Ingredients to NNLO n -jet calculations:

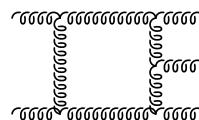
- Two-loop **matrix elements**

$$|\mathcal{M}|_{2\text{-loop}, n \text{ partons}}^2$$



- One-loop **matrix elements**

$$|\mathcal{M}|_{1\text{-loop}, n+1 \text{ partons}}^2$$



- One-loop one-particle **subtraction terms**

$$\int |\mathcal{M}^{R,1}|_{1\text{-loop}, n+1 \text{ partons}}^2 d\Phi_1$$

D. Kosower, P. Uwer

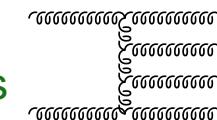
S. Catani, M. Grazzini

Z. Bern et al.

S. Weinzierl

- Tree level **matrix elements**

$$|\mathcal{M}|_{\text{tree}, n+2 \text{ partons}}^2$$



- Tree-level one-particle **subtraction terms**

$$\int |\mathcal{M}^{R,1}|_{\text{tree}, n+2 \text{ partons}}^2 d\Phi_1$$

W. Giele, N. Glover

S. Catani, M. Seymour

- Tree-level two-particle **subtraction terms**

$$\int |\mathcal{M}^{R,2}|_{\text{tree}, n+2 \text{ partons}}^2 d\Phi_2$$

remain to be calculated

Towards NNLO jet physics

Virtual two-loop corrections to jet observables have seen enormous progress in the past years

Technical breakthroughs:

- algorithms to reduce the ~ 10000 's of integrals to a few (10 – 30) master integrals
 - Integration-by-parts (IBP)
K. Chetyrkin, F. Tkachov
 - Lorentz Invariance (LI)
E. Remiddi, TG
 - and their implementation in computer algebra
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- new methods to compute master integrals
 - Mellin-Barnes Transformation V. Smirnov, O. Veretin; B. Tausk
 - Differential Equations E. Remiddi, TG
 - Sector Decomposition (numerically) T. Binoth, G. Heinrich
 - Nested Sums S. Moch, P. Uwer, S. Weinzierl

Towards NNLO jet physics

Virtual two-loop matrix elements have recently been computed for:

- Bhabha-Scattering: $e^+e^- \rightarrow e^+e^-$
Z. Bern, L. Dixon, A. Ghinculov
- Hadron-Hadron 2-Jet production: $qq' \rightarrow qq'$, $q\bar{q} \rightarrow q\bar{q}$, $q\bar{q} \rightarrow gg$, $gg \rightarrow gg$
C. Anastasiou, N. Glover, C. Oleari, M. Yeomans-Tejeda
Z. Bern, A. De Freitas, L. Dixon [SUSY-YM]
- Photon pair production at LHC: $gg \rightarrow \gamma\gamma$, $q\bar{q} \rightarrow \gamma\gamma$
Z. Bern, A. De Freitas, L. Dixon
C. Anastasiou, N. Glover, M. Yeomans-Tejeda
- Three-jet production: $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}g$
L. Garland, N. Glover, A. Koukoutsakis, E. Remiddi, TG
S. Moch, P. Uwer, S. Weinzierl
- DIS (2+1) jet production: $\gamma^* g \rightarrow q\bar{q}$, Hadronic (V+1) jet production: $qg \rightarrow Vq$
E. Remiddi, TG

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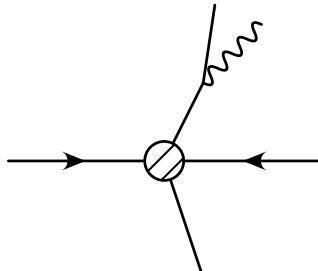
Ongoing:

- Matrix elements with internal masses: $\gamma^* \rightarrow Q\bar{Q}$
R. Bonciani, P. Mastrolia, E. Remiddi; U. Aglietti, R. Bonciani

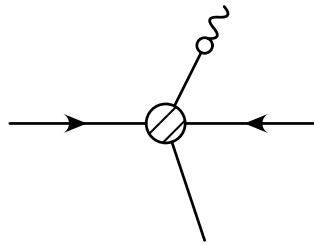
Photons

Photons

Two production processes for photons



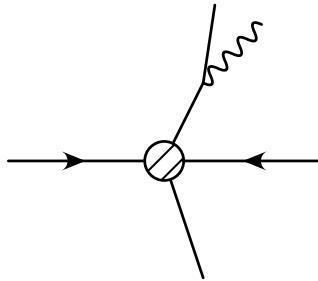
direct



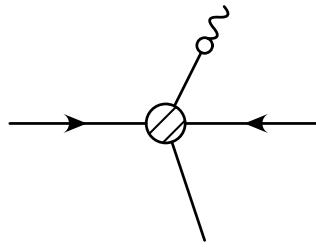
fragmentation

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Two production processes for photons



direct



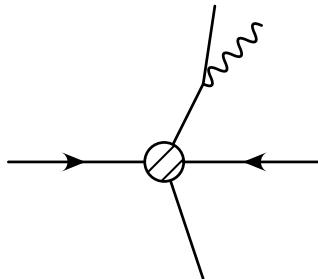
fragmentation

Photons never fully isolated from hadrons

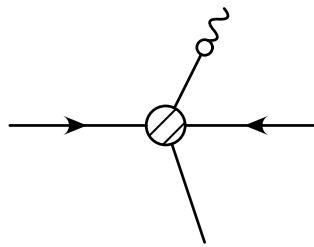
- isolation cone: $E^{\text{had}} < E^{\text{isol}}$ for $R < R^{\text{isol}}$
- cluster photon into jet: $E^\gamma > z_{\text{cut}} E^{\text{jet}}$ N. Glover, A. Morgan

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Both isolation criteria

- are infrared safe
- induce contribution from non-perturbative quark-to-photon fragmentation function

Cone-based isolation fails for small cones:

S. Catani, M. Fontannaz, J.P. Guillet, E. Pilon

$$\sigma^{\text{isol}} > \sigma^{\text{incl}} \quad \text{for } R \leq 0.1 \quad (\alpha_s \ln R^{-2} \sim 1)$$

Photon Pairs

$pp \rightarrow \gamma\gamma X$: background to Higgs search
in $H \rightarrow \gamma\gamma$ decay mode

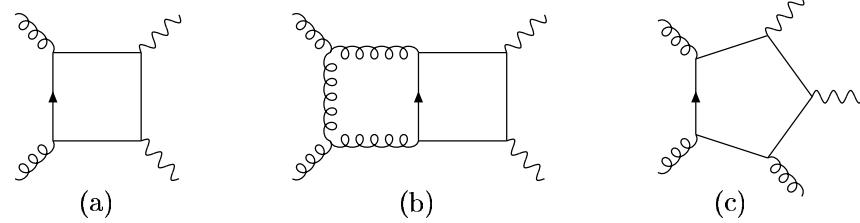
- Subprocesses: $q\bar{q} : \mathcal{O}(\alpha_s^0)$, $qg : \mathcal{O}(\alpha_s^1)$, $gg : \mathcal{O}(\alpha_s^2)$
comparable magnitude due to large gluon luminosity

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- NLO corrections now complete
DIPHOX: T. Binoth, J.P. Guillet, E. Pilon, M. Werlen

Z. Bern, L. Dixon, C. Schmidt

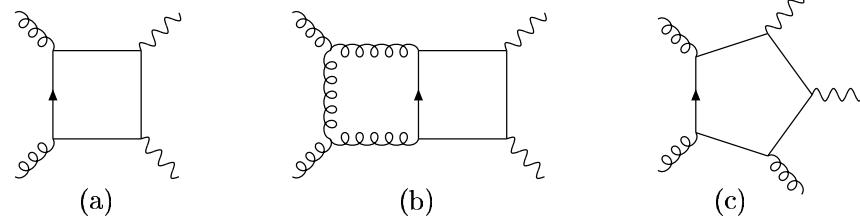


Photon Pairs

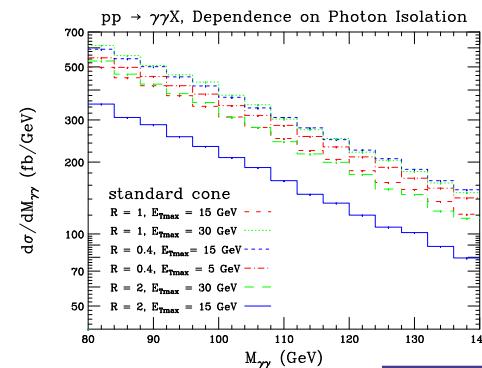
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- cross section depends on photon isolation

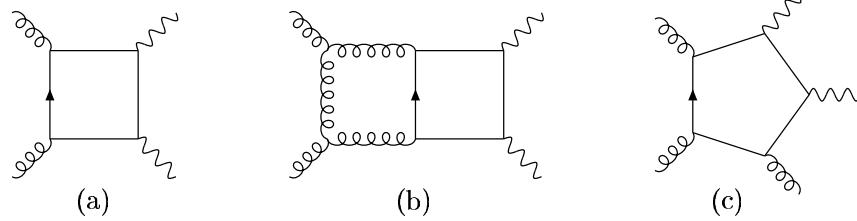


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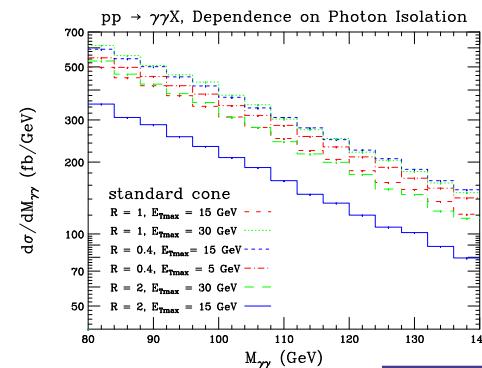
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- cross section depends on **photon isolation**
- sensitive on **photon fragmentation function**
 $D_{q \rightarrow \gamma}$ at large momentum transfer
(only experimental constraint: LEP)



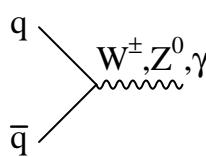
Gauge and Higgs Boson Production

Gauge and Higgs Boson Production

Inclusive Production: NNLO corrections

Drell–Yan process

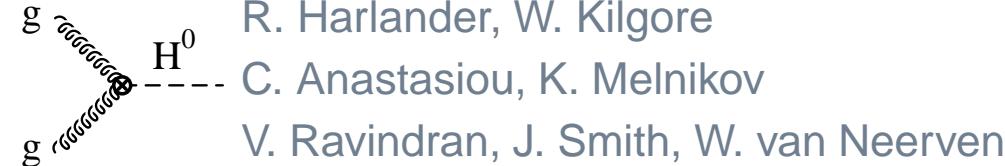
$$q\bar{q} \rightarrow W^\pm, Z^0, \gamma^*$$



T. Matsuura, R. Hamberg,
W. van Neerven
R. Harlander, W. Kilgore

Higgs production

$$gg \rightarrow H^0$$



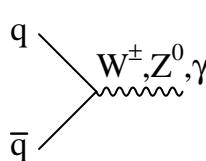
R. Harlander, W. Kilgore
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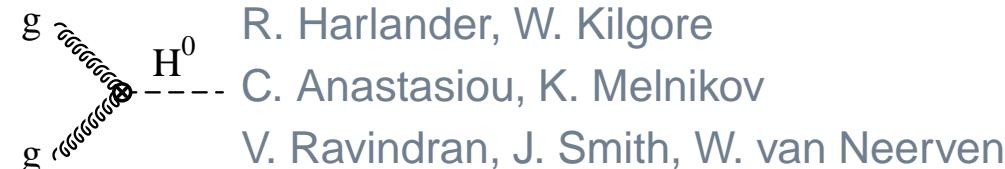
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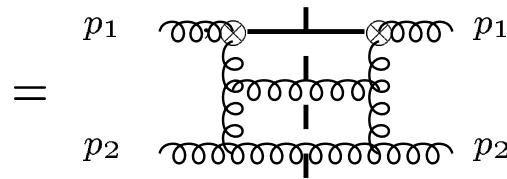
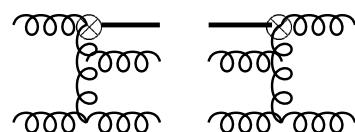
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R. Harlander, W. Kilgore
C. Anastasiou, K. Melnikov
V. Ravindran, J. Smith, W. van Neerven

New calculational techniques:

- expansion around the **soft limit**
R. Harlander, W. Kilgore
- extension of **multi-loop techniques** (IBP and LI, differential equations) to phase space integrals



C. Anastasiou, K. Melnikov

Inclusive Higgs Boson Production

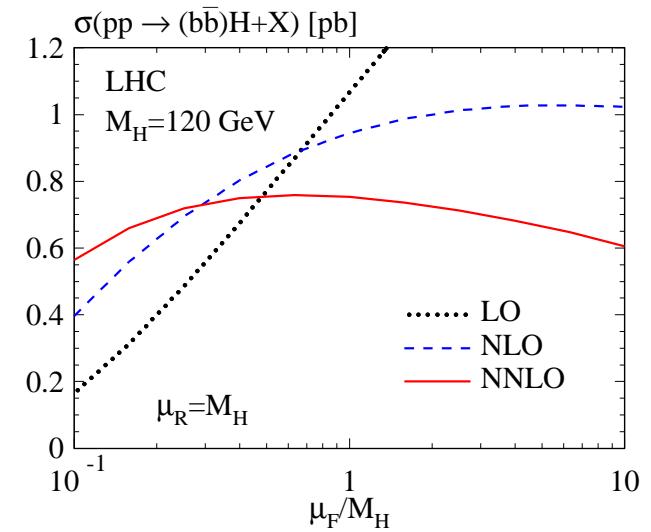
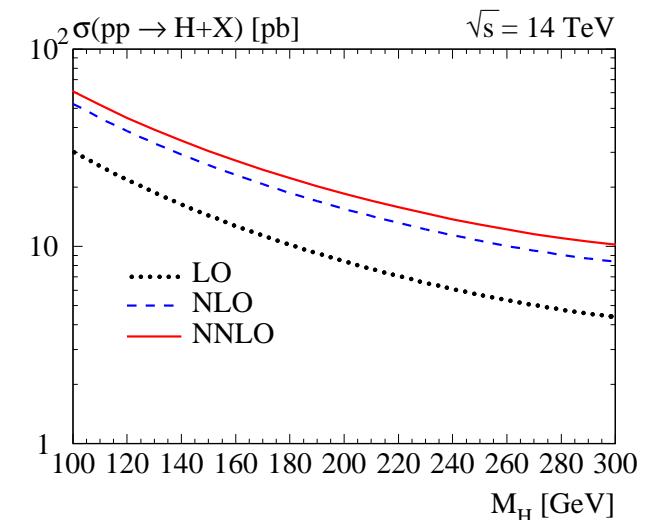
Recent NNLO results

- Standard model Higgs from gluon fusion
 $gg \rightarrow H + X$
R. Harlander, W. Kilgore
C. Anastasiou, K. Melnikov
- including NNLL soft gluon resummation
S. Catani, D. de Florian, M. Grazzini, P. Nason
- Pseudoscalar Higgs from gluon fusion
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R. Harlander, W. Kilgore
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- Higgs from bottom quark fusion $b\bar{b} \rightarrow H + X$
R. Harlander, W. Kilgore
supports b -density approach
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O. Brein, A. Djouadi, R. Harlander

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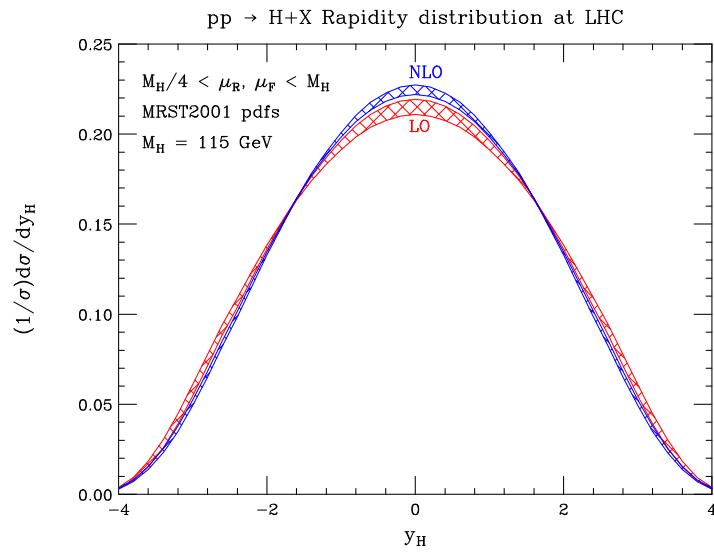
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Higgs Boson Spectra

Rapidity distribution

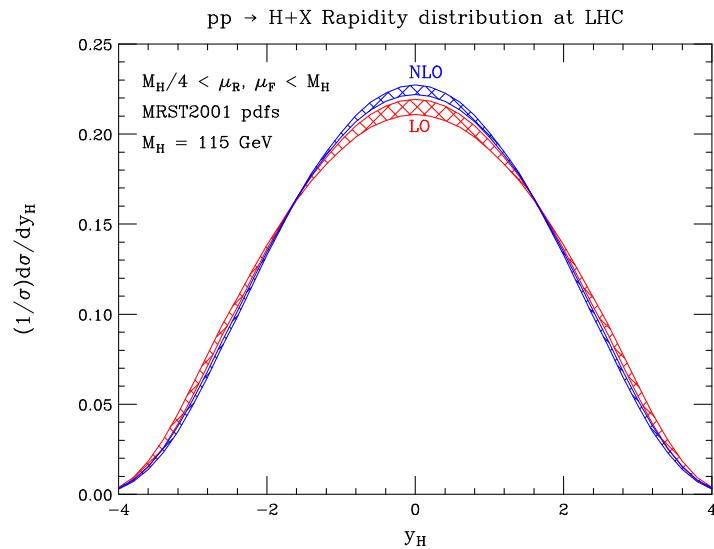
- experiments cover only limited range in rapidity
- recently computed to NLO $\mathcal{O}(\alpha_s)$
C. Anastasiou, L. Dixon, K. Melnikov



Higgs Boson Spectra

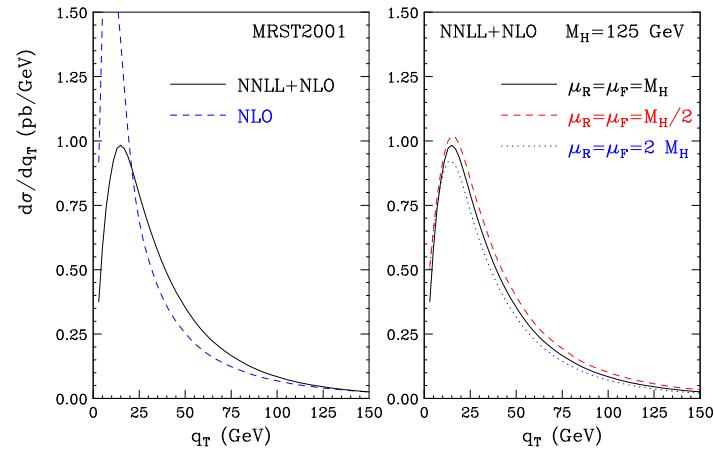
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C. Anastasiou, L. Dixon, K. Melnikov



Transverse momentum distribution

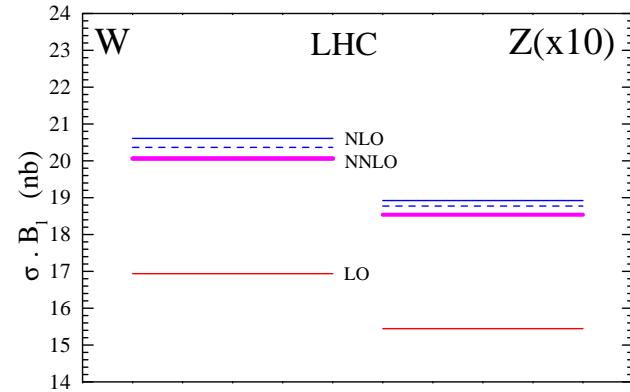
- fixed order NLO $\mathcal{O}(\alpha_s^2)$ reliable only for $q_T \geq M_H$
D. de Florian, M. Grazzini, Z. Kunszt
V. Ravindran, J. Smith, W. van Neerven
- small $q_T \ll M_H$ require soft gluon resummation (NNLL) of $\ln(q_T/M_H)$
G. Bozzi, S. Catani, D. de Florian,
M. Grazzini



Vector Boson Production

Inclusive cross section

- can be measured precisely
- are theoretically well understood
 - NNLO corrections known
 - relevant partons well constrained
- benchmark reaction for LHC
(luminosity monitor)

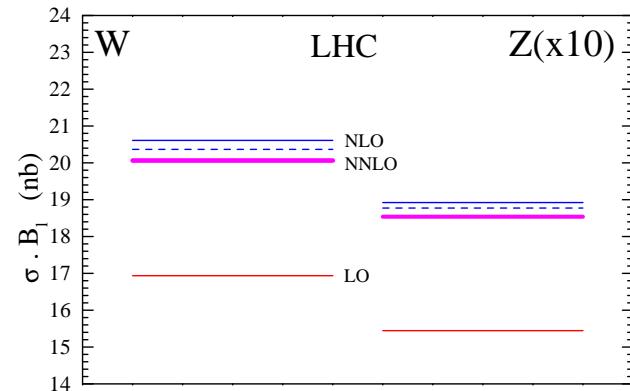


A. Martin, J. Stirling, R. Roberts, R. Thorne

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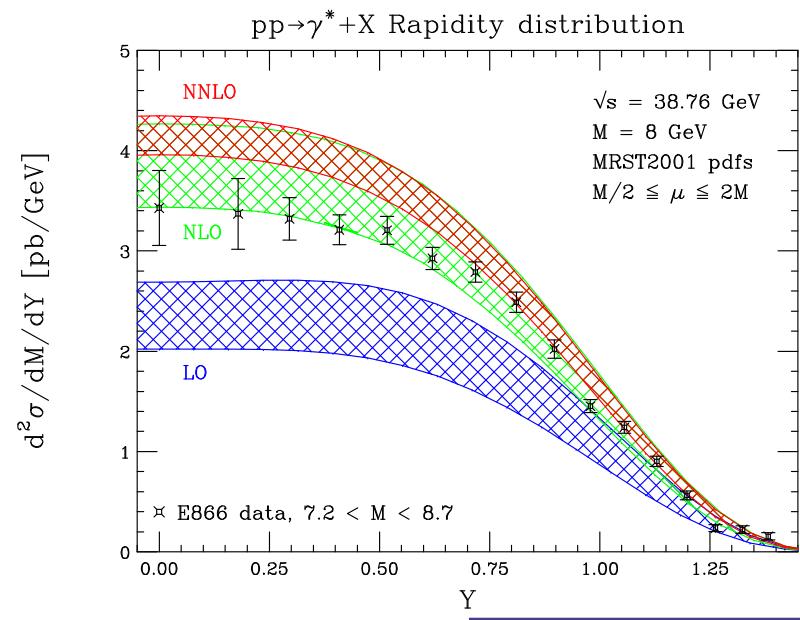


A. Martin, J. Stirling, R. Roberts, R. Thorne

Rapidity distribution

- probes antiquark distribution
- recently computed to NNLO
(further extension of multi-loop tools)

C. Anastasiou, L. Dixon, K. Melnikov,
F. Petriello



Parton Distributions at NNLO

Parton distributions from global fit (\rightarrow R. Thorne)

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- need coefficient functions to NNLO
 - deep inelastic scattering: neutral and charged current
 - Drell-Yan process
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Parton distributions from global fit (\rightarrow R. Thorne)

- need coefficient functions to NNLO
 - deep inelastic scattering: neutral and charged current
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 - jet production
 - (direct photon production)
- need splitting kernels (Altarelli-Parisi splitting functions) to NNLO
 - technique: evaluate $\gamma^* q \rightarrow \gamma^* q$ and $\phi g \rightarrow \phi g$ at three loops (in moment space)
 - some fixed moments known
S. Larin, P. Nogueira, T. van Ritbergen, J. Vermaseren; A. Retey
 - allow approximate reconstruction
W. van Neerven, A. Vogt
 - non-singlet n_f piece known
S. Moch, J. Vermaseren, A. Vogt
 - remaining pieces under way

Summary and Conclusions

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- QCD is **ubiquitous** at high energy colliders

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- QCD is becoming **precision physics**
 - LO **is an estimate to design search strategies**
 - many generic tools available
 - interface to parton shower and hadronization
 - NLO **important to refine searches and to identify signals**
 - current frontier $2 \rightarrow 3; 2 \rightarrow 4$ requires new tools
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- resummation often important
- many predictions require **input** from LEP data
(e.g. fragmentation functions)