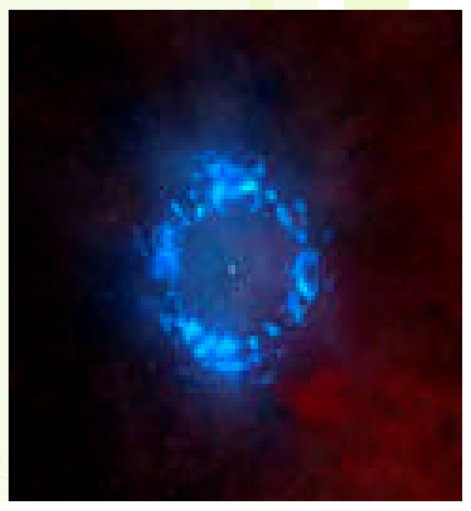
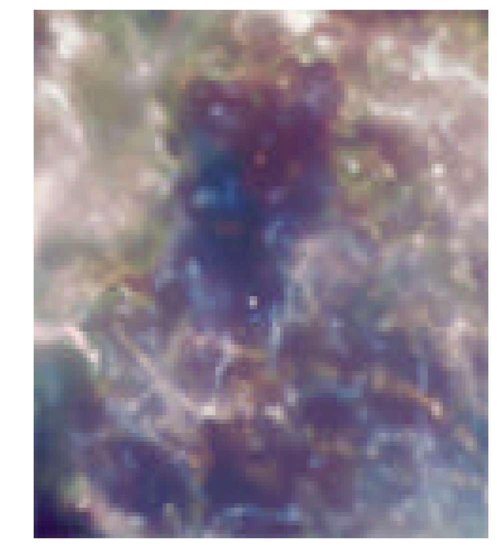
Quantum instability of magnetized stellar objects



A. Pérez Martínez, H. Pérez Rojas
Group of Theoretical Physics. ICIMAF. Calle E, esq. a 15. Vedado. La Habana. Cuba.



Abstract

The equations of state for degenerate electron and neutron gases are studied in the presence of magnetic fields. After including quantum effects, it is found that some magnetized stars can be unstable based on the criterium of stability of pressures. It is shown that strongly magnetized white dwarfs should collapse producing a supernova type Ia, whilst hypermagnetized neutron stars cannot stand their own super strong magnetic field and must implode, too. A comparison of our results with some of the available observational data for other stars is also presented.

Introduction

Many stellar objects are known to be endowed with large magnetic fields.

- white dwarfs with surface B from 10^5 to 10^9 G have been discovered J.C. Kempet al.(1970).
- Magnetic fields of strength of order 10^{20} G have been suggested to exist in the core of neutron stars or pulsars Chakrabarty et al PRL 78 (1997).

Electron (e) and neutron (n) gases in strong magnetic fields are considered with the aim to study the equation of state (EoS) of white dwarfs, neutron stars and supernovae. It was found some effect, which is described below, which can be understood as a manifestation of the Lorentz force in the extreme quantum regime. Such effect opens the possibility of an instability which may lead to a quantum magnetic collapse of the gas under consideration. This effect appears as a critical curve relating the density of the star and its magnetic field.

Why appear this effect?

The loss of rotational symmetry of the particle spectrum $\Longrightarrow$ an anisotropy in the thermodynamic properties of the system $\Longrightarrow$ an anisotropy in its pressure.
If the symmetry is AXIAL the anisotropy is AXIAL too.
That means $P_{\parallel} \neq P_{\perp}$
Two cases:

- Classical case (collapse may occur $\parallel$ to $\vec{B}$)

$$P_{\parallel} \leq P_{\perp}$$

- Quantum Case (collapse may occur $\perp$ to $\vec{B}$)

Energy-Momentum Tensor and Pressures

The Energy-momentum tensor of matter $\mathcal{T}_{\mu\nu}$, if $F_{\mu\nu}$ is the electromagnetic field tensor, is

$$\mathcal{T}_{\mu\nu} = (T\frac{\partial\Omega}{\partial T} + \sum \mu_i \frac{\partial\Omega}{\partial\mu_i})\delta_{4\mu}\delta_{\nu 4} + 4F_{\mu\rho}F_{\nu\rho}\frac{\partial\Omega}{\partial F^2} - \delta_{\mu\nu}\Omega$$

$i = n, p, e$. The spatial components are

$$\mathcal{T}_{33} = P_3, P_3 = -\Omega, P_{\parallel} = P_3$$

Quantum Case

$$\mathcal{M} > 0 \Rightarrow P_{\perp} < P_{\parallel}$$

$\nwarrow\swarrow\searrow$

with leads to a Ferromagnetic behavior in the sense that $\mathcal{M} = f(B)$ is non linear (but no spin-spin coupling is assumed).
$\star$ charged fermions (electrons) (used for describing White Dwarfs, Chaichian et al PRL.84, 5261 (2000))

$\star$ neutral fermions (neutrons) (used for describing Neutron Stars A. Perez Martinez et al Eur. Phys. J. C 29, 111 (2003))

Our model of stars assume;

- Newtonian gravitational potential (no general relativity) to compensate the pressure exerted by quantum gas.
- Relativistic Quantum Statistical Physics: Dense gas of fermions with magnetic moment $\neq 0$.

Calculations Procedure: to obtain

- the particle spectrum
- thermodynamical potential of the system
- magnetization of the system

these quantities allow to infer the equation of state (EoS) of the system

Degenerate Magnetized e-gas

Thermodynamical Potential

$$\star\Omega_e = -\sum_0^{n_\mu} \frac{e a_n B}{2\pi^2}\left(\mu_e\sqrt{\mu_e^2 - m_e^2 - eBn} - (m_e^2 + 2eBn)\right.$$

Chemical Potential

$$\left.\times \ln\frac{\mu_e + \sqrt{\mu_e^2 - m_e^2 - eBn}}{\sqrt{m_e^2 + 2eBn}}\right) ,$$

Density of particles

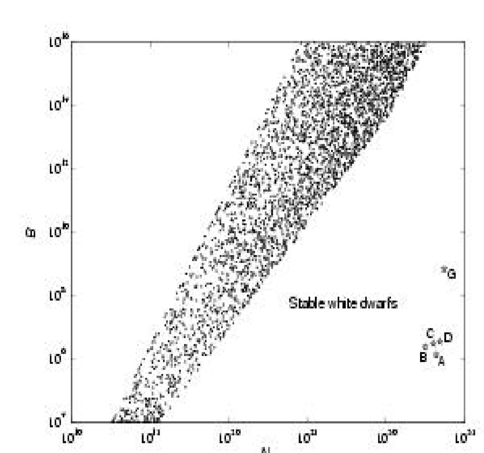
$(2 - \delta_{0n})$

$$\star N_e = \frac{eB}{2\pi^2}\sum_0^{n_\mu} a_n \sqrt{\mu_e^2 - m_e^2 - 2eBn} .$$

Magnetization

$$\star\mathcal{M}_e = \sum_0^{n_\mu}\frac{e a_n}{2\pi^2}\left(\mu_e\sqrt{\mu_e^2 - m_e^2 - eBn} - (m_e^2 + 4eBn)\right.$$

$$p_{\perp} = \frac{2e^2B^2}{\pi^2}\sum_{n=0}^{n_\mu} n \ln\frac{\mu_e + \sqrt{\mu_e^2 - m_e^2 - eBn}}{\sqrt{m_e^2 + 2eBn}} .$$



The instability region (dotted area) in the (N_e, B)-plane for a magnetized election gas. The labelled points represent some stable white dwarfs A = PG 0136 + 251, B = PG 2329 + 267, C = 1RXS J0823.6-2525, D = PG 1658 + 441, E = LB 11146B, F = Grw + 70° 8247, G = RE J0317-858, H = PG 1031 + 234, I = GD 229 Mathews.

Magnetized neutron gas

Most of the observed neutron stars are pulsars, i.e., fast rotating neutron stars with strong magnetic fields. They consist mainly of neutron matter with a high central density. As in the case of WDs, a similar study can be performed for neutron stars if we assume they are well described by a degenerate neutron gas.

Neutron Gas in a background of protons and electrons

For free neutrons (with the anomalous magnetic moment) in a B field the Dirac equation reads (Ternov et al 1990)

the spin tensor

$$(\gamma_\mu\partial_\mu + m + iq\sigma_{\mu\lambda}F_{\lambda\mu})\psi = 0$$

$q = 1.91M_n$. M_n: nuclear magneton. The eigenvalues are
are the $\sigma_3 \pm 2$-orientations of the anomalous magnetic moment $\uparrow$ or $\downarrow$ to B

$$E_n(p, B, \eta) = \sqrt{p_3^2 + (\sqrt{p_\perp^2 + m_n^2} + \eta qB)^2}$$

Degenerate neutron gas

Thermodynamical Potential

$$\star\Omega_n = -\Omega_0\sum_{\eta=1,-1}\left[\frac{xf_\eta^3}{12} + \frac{(1+\eta y)(5\eta y - 3)xf_\eta}{24}\right.$$

$$\left. + \frac{(1+\eta y)^3(3-\eta y)}{24}L_\eta - \frac{\eta y x^3}{6}s_\eta\right]$$

Density of Particles

$$\star N_n = N_0 \sum \left[\frac{f_\eta^3}{3} + \frac{\eta y(1+\eta y)f_\eta}{2} - \frac{\eta y x^2}{2}s_\eta\right]$$

Magnetization

$$\star\mathcal{M}_n = -\mathcal{M}_0\sum_{\eta=1,-1}\eta\left[\frac{(1-2\eta y)xf_\eta}{6}\right.$$

$$\left. - \frac{(1+\eta y)^2(1-\eta y/2)}{3}L_\eta + \frac{x^3}{6}s_\eta\right] ,$$

where $N_0 = m_n^3/4\pi^2 \simeq 2.73\times10^{39}$, $\Omega_0 = N_0 m_n \simeq 4.11\times10^{36}$ and $\mathcal{M}_0 = N_0 q_n \simeq 2.63\times10^{16}$. $x = \frac{\mu_n}{m_n}$, $y = \frac{B}{B_n}$

We have introduced the notations:

$$f_\eta = \sqrt{x^2 - (1+\eta y)^2}, \quad s_\eta = \frac{\pi}{2} - \arcsin\left(\frac{1+\eta y}{x}\right)$$

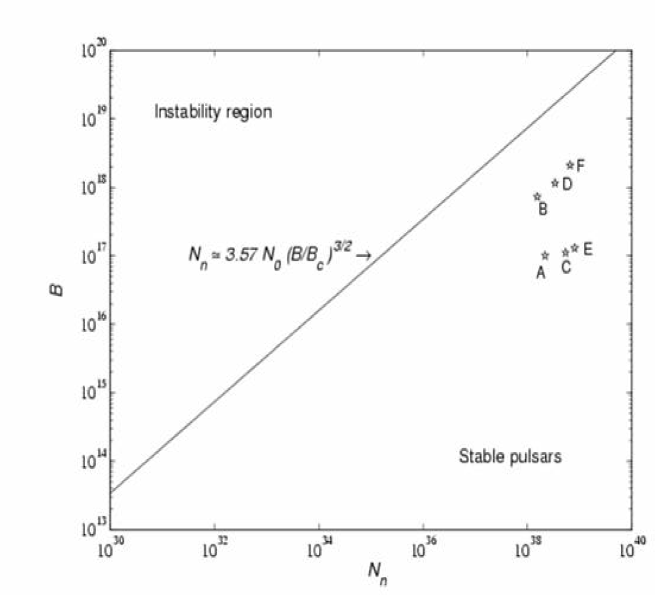
$$L_\eta = \ln\left(\frac{x + f_\eta}{1+\eta y}\right) .$$

For $x < 1 + y \Rightarrow \Omega_n, M$, etc become complex. The condition $x = 1 + y$ defines a curve in the (N_n, B)-plane which delimits the region where the pressure becomes complex

EOS neutron gas

The solution of the equation $p_{\perp} = 0$ is given by

$$N_n = \simeq 3.57 N_0\, y^{3/2} .$$



The instability region in the (N_n, B)-plane for a magnetized neutron gas. A star whose configuration lies above the solid curve should collapse due to the vanishing of the transverse pressure $p_{\perp} = -\Omega_n - B\mathcal{M}_n$. The labelled points represented by stars correspond to the neutron star configurations and computed using different EoS.

Concluding remarks

- We have presented a consistent theory which could be applied discuss the stability of main sequence and compact remnant stars whose structure is dictated by a combination of quantum and magnetic effects.
- We have shown that in general results of the theory are consistent with the current observational data. A major outcome is the possibility of some special configurations of highly magnetized WDs could collapse and trigger explosions similar to a SNIa.
- Another consequence is that some neutron stars endowed with superstrong magnetic fields would be naturally unstable, and therefore should collapse for their quoted surface magnetic fields.