# NNLL/NLO transverse momentum resummation for $gg \to \gamma\gamma$

Pavel Nadolsky

Argonne National Laboratory

April 18, 2007

**P. N.**, *C. Balazs, E. Berger, C.-P. Yuan, hep-ph/0702003; see also arXiv:0704.0001; Phys. Lett., B637, 235 (2006)*

# $\gamma\gamma$ pair production in QCD...

- ...constitutes a large irreducible background in searches for $H \to \gamma\gamma$
  - assumptions about the $\gamma\gamma$ transverse momenta $Q_T$ affect significance of Higgs discovery and measurements of Higgs production cross sections
- ...is measured with increasing precision in the Tevatron Run-2
- ... tests rich properties of QCD interactions!
  - large radiative contributions distributed in a complex kinematical pattern

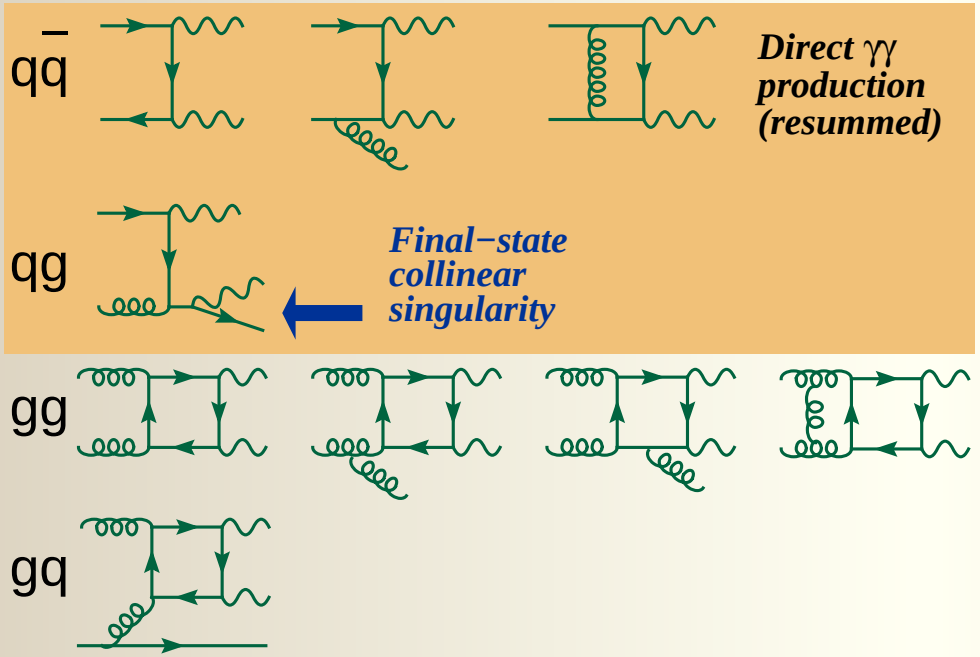
# Theoretical studies of $\gamma\gamma$ production

- Perturbative contributions to direct $\gamma\gamma$ production
  *Aurenche et al.; Berger, Braaten, Field; Bailey, Owens, Ohnemus; Balazs, **PN**, Schmidt, Yuan; de Florian, Kunzst; Bern, De Freitas, Dixon; Bern, Dixon, Schmidt; Del Duca, Maltoni, Nagy, Trocsanyi;...*
  - Resummation of initial-state $Q_T$ logarithms
    *Balazs, Berger, Mrenna, Yuan; Balazs, **PN**, Schmidt, Yuan; Catani, de Florian, Grazzini; **PN**, Schmidt;...*
  - $\gamma\gamma$ production via final-state fragmentation at NLO
    *Binoth, Guillet, Pilon, Werlen;...*
- Our new study
  - computes NNLL/NLO resummed $Q_T$ distributions for direct production
  - includes essential information about photon fragmentation
  - Numerical implementation: improved MC integrator program ResBos

# Direct diphotons

The dominant production mode; evaluated up to NLO in $\alpha_s$

Balazs, Berger, Nadolsky, Yuan, 2006



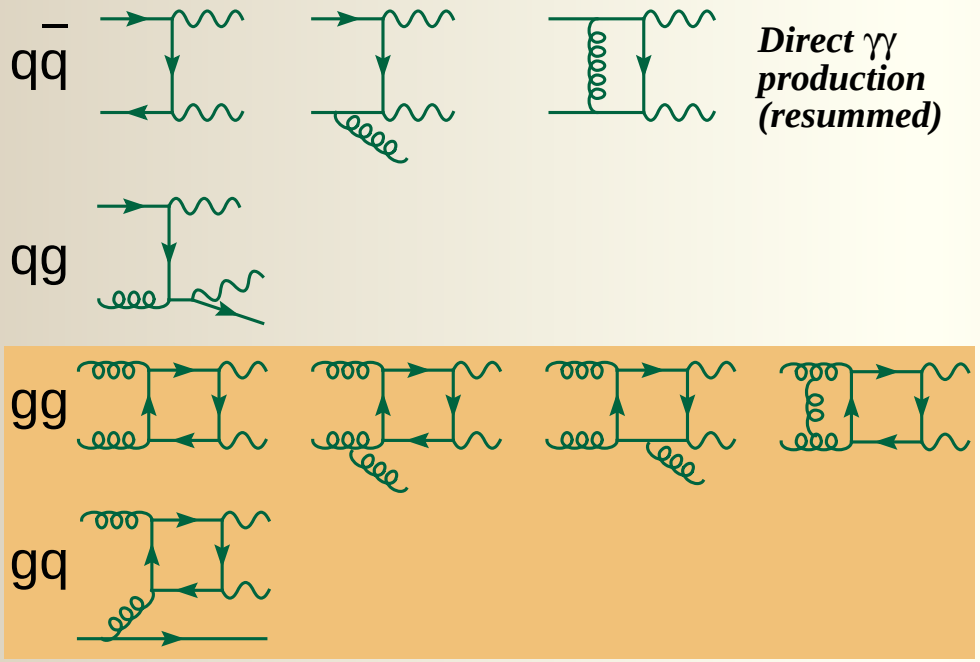

## $q\bar{q} + qg$ channel

- NLO matrix elements:
  *Aurenche et al.; Bailey, Owens, Ohnemus*
- $q\bar{q}$ scattering dominates at the Tevatron
- $qg$ scattering is strongly enhanced at the LHC by photon radiation off final-state quarks

# Direct diphotons

The dominant production mode; evaluated up to NLO in $\alpha_s$

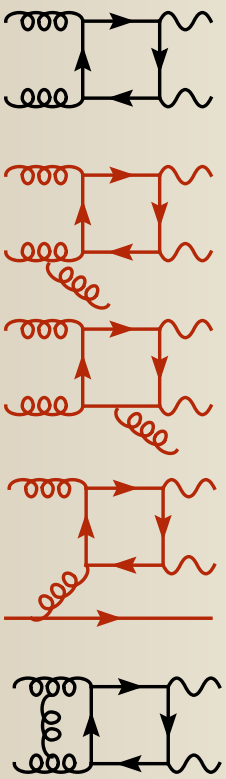
Balazs, Berger, Nadolsky, Yuan, 2006




**$gg + gq$ channel (via the quark box)**

- contributes $\sim 20\%$ at the LHC
- NLO $gg$ matrix elements: *Balazs, PN, Schmidt, Yuan; de Florian, Kunzst; Bern, De Freitas, Dixon; Bern, Dixon, Schmidt*
- $gq$ matrix element: derived by our group from the $q\bar{q}ggg$ matrix element

# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$



## 1-loop $2 \to 3$ diagrams

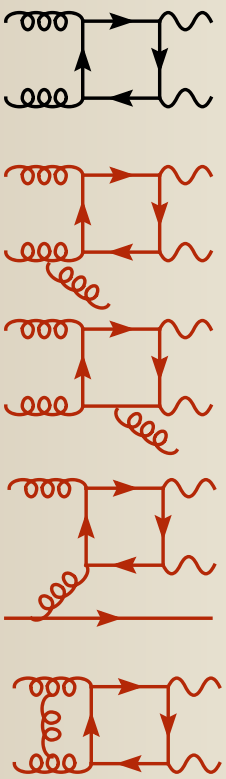
- obtained in the helicity amplitude formalism from $gggg$ and $q\bar{q}ggg$ diagrams *(Bern, Dixon, Kosower)*

$$\mathcal{M}_5(q_{1i}, \bar{q}_{2j}, g_3^{a_3}, g_4^{a_4}, g_5^{a_5}) \sim g^5 \sum_{\sigma \in S_3^{(345)}} (T^{\sigma_3} T^{\sigma_4} T^{\sigma_5})_{ij} \, A_{5;1}^{L,[1/2]}(1_q, 2_{\bar{q}}; \sigma_3, \sigma_4, \sigma_5) + \ldots$$

  - to obtain $\mathcal{M}_5(q_{1i}, \bar{q}_{2j}, \gamma_3, \gamma_4, g_5^{a_5})$, replace $gT_{cd}^{a_{3,4}} \to ee_i\sqrt{2}\,\mathcal{I}_{cd}$

- $gg\gamma\gamma g$ checked against the "sector decomposition" calculation *(Binoth, Guillet, Mahmoudi, 2003)*

# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$



## Soft and collinear limits

- Derived at the matrix-element level with the splitting amplitude method *(Bern, Chalmers, Dixon, Dunbar, Kosower;...)*

$$A_{5;1}^{L,[1/2]}(\ldots,5,1,\ldots) \xrightarrow{5\|1} \sum_{\lambda_I=\pm 1} \mathrm{Split}_{-\lambda_I}^{tree}(5,1) A_{4;1}(\ldots,I,\ldots)$$

$$+ \text{ subleading terms}$$

- **Soft singularities** are cancelled against 2-loop box diagrams (*Bern, De Freitas, Dixon;* ***P.N.****, Schmidt)*
- **Collinear singularities** are factorized into gluon PDF's

# $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude in the $5||1$ collinear limit
**(see also Bern, Dixon, Schmidt)**

- includes an unusual log term proportional to interference between $2 \to 2$ matrix elements with opposite spins of gluon 1 and a universal spin-flip splitting function $P'_{g/g}(x)$
- also arises in the Catani-Seymour dipole formalism
- affects dependence on the azimuthal ($\varphi_*$) and polar ($\theta_*$) angles of photons in the Collins-Soper $\gamma\gamma$ rest frame
- constitutes up to $\sim 25\%$ of the conventional $gg + gq$ collinear term
- may need all-order resummation in the future

# $g_1 g_2 \longrightarrow \gamma_3 \gamma_4 g_5$ amplitude in the $5||1$ collinear limit

**(see also Bern, Dixon, Schmidt)**

$$|\mathcal{M}_5(1,2,3,4,5)|^2 \xrightarrow{5||1} \frac{\sigma_g^{(1)}}{2\widehat{x}_1 p_1 \cdot p_5} \left\{ P_{g/g}(\widehat{x}_1) L_g(\theta_\star) + P'_{g/g}(\widehat{x}_1)\, L'_g(\theta_\star)\, \cos 2\varphi_\star \right\}$$

$P_{g/g} L_g$ is the usual collinear term, with

$$, \qquad P_{g/g} = 2C_A \left[ \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) \right] + \beta_0 \delta(1-x),$$

$$L_g(\theta_\star) \equiv \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} |\mathcal{M}_4(\lambda_1,\lambda_2,\lambda_3,\lambda_4)|^2$$

$P'_{g/g} L'_g \cos 2\varphi_\star$ is the interference (spin-flip) term, with

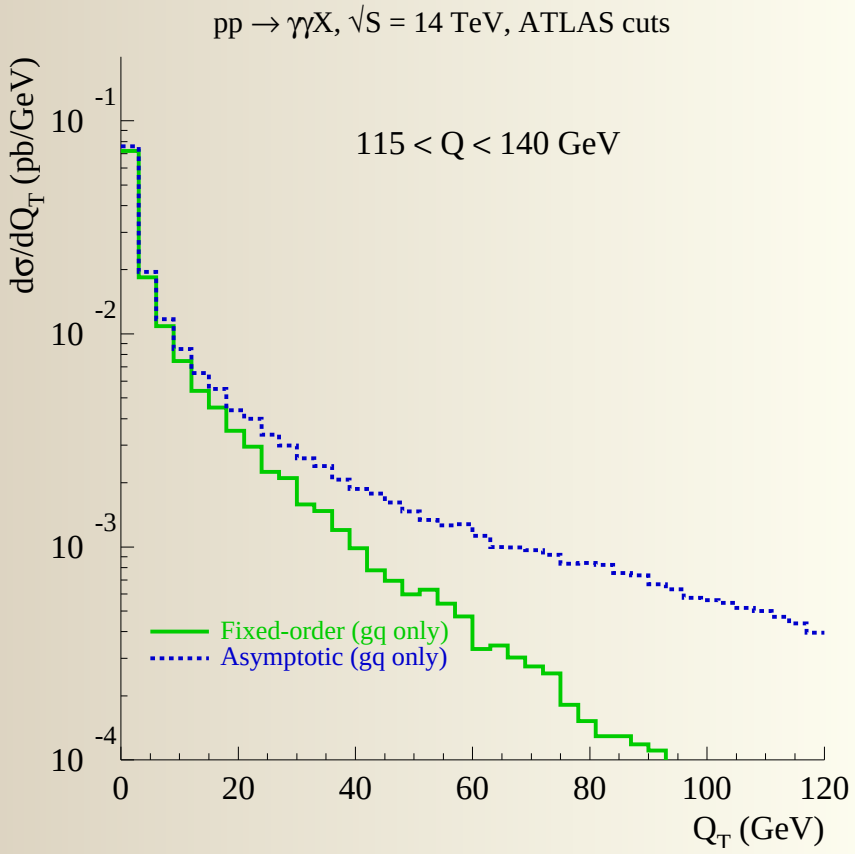
$$P'_{g/g}(x) = 2C_A(1-x)/x$$

$$L'_g(\theta_\star) \cos 2\varphi_\star \quad = \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4=\pm 1} \mathcal{M}_4^*(\lambda_1,\lambda_2,\lambda_3,\lambda_4) \mathcal{M}_4(-\lambda_1,\lambda_2,\lambda_3,\lambda_4)$$

# $g_1 g_2 \to \gamma_3 \gamma_4 g_5$ amplitude in the $5||1$ collinear limit
**(see also Bern, Dixon, Schmidt)**

- includes an unusual log term proportional to interference between $2 \to 2$ matrix elements with opposite spins of gluon 1 and a universal spin-flip splitting function $P'_{g/g}(x)$
- also arises in the Catani-Seymour dipole formalism
- affects dependence on the azimuthal ($\varphi_*$) and polar ($\theta_*$) angles of photons in the Collins-Soper $\gamma\gamma$ rest frame
- constitutes up to $\sim 25\%$ of the conventional $gg + gq$ collinear term
- may need all-order resummation in the future

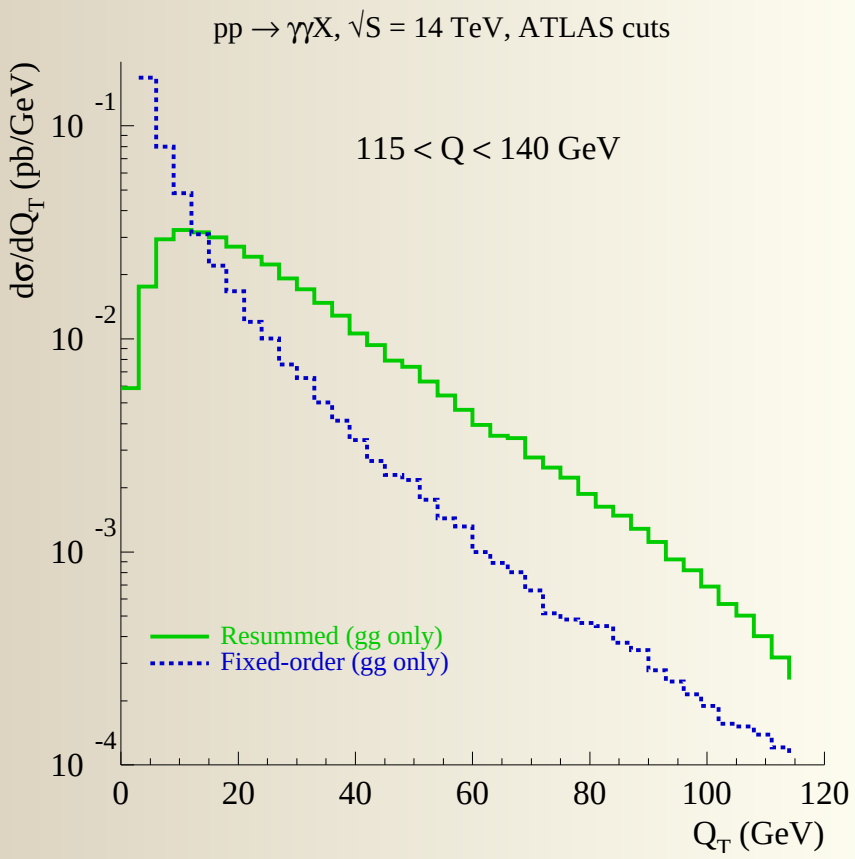
# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$




## $gq$ 5-leg matrix element

- $\mathcal{M}_5(gq \to \gamma\gamma q)$ correctly reproduces its known behavior in the collinear asymptotic limit ($Q_T \to 0$)
- It does not confirm an earlier calculation *(Yasui)* , which does not agree with this asymptotic behavior

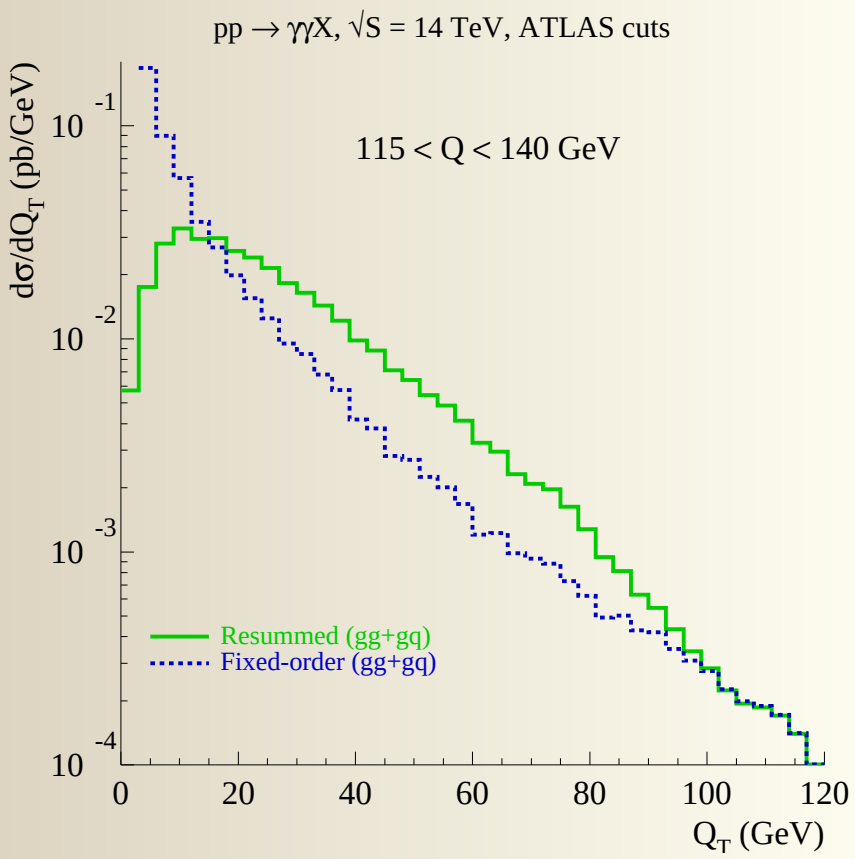
# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$




## $gq$ 5-leg matrix element

- The $gq$ contribution is subleading in the inclusive rate, but has more pronounced effect at high $Q_T$ (improves matching)

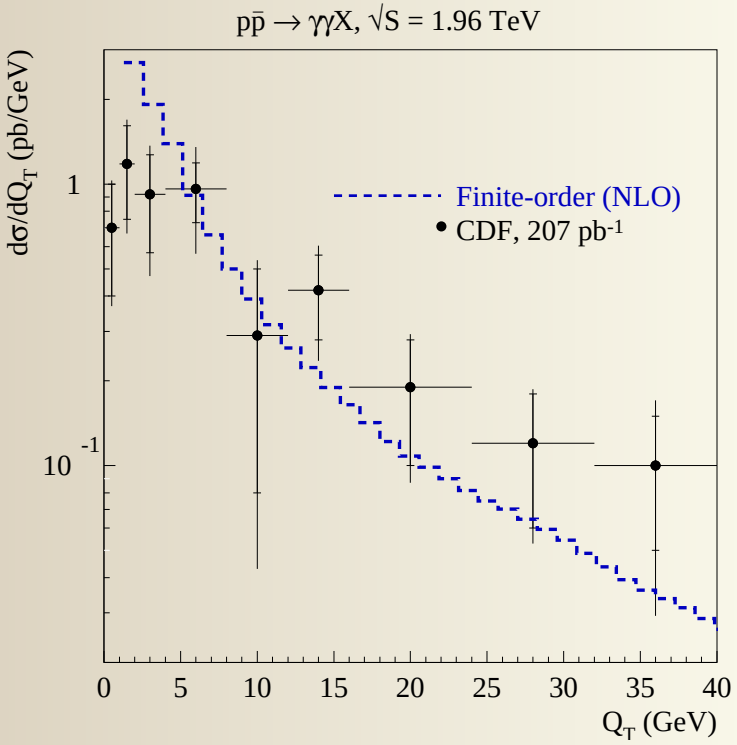
# Resummation for $gg + gq \to \gamma\gamma X$ at $\mathcal{O}(\alpha_s^3)$



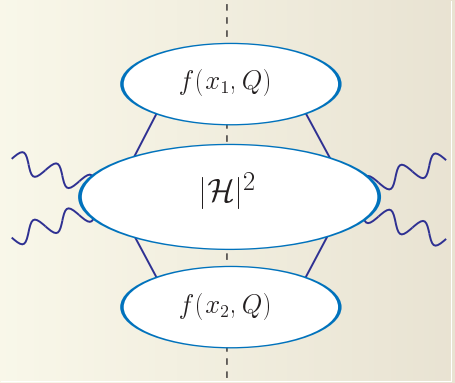

## $gq$ 5-leg matrix element

- The $gq$ contribution is subleading in the inclusive rate, but has more pronounced effect at high $Q_T$ (improves matching)

# $Q_T$ resummation in impact parameter space




$$Q_T^2 \sim Q^2$$

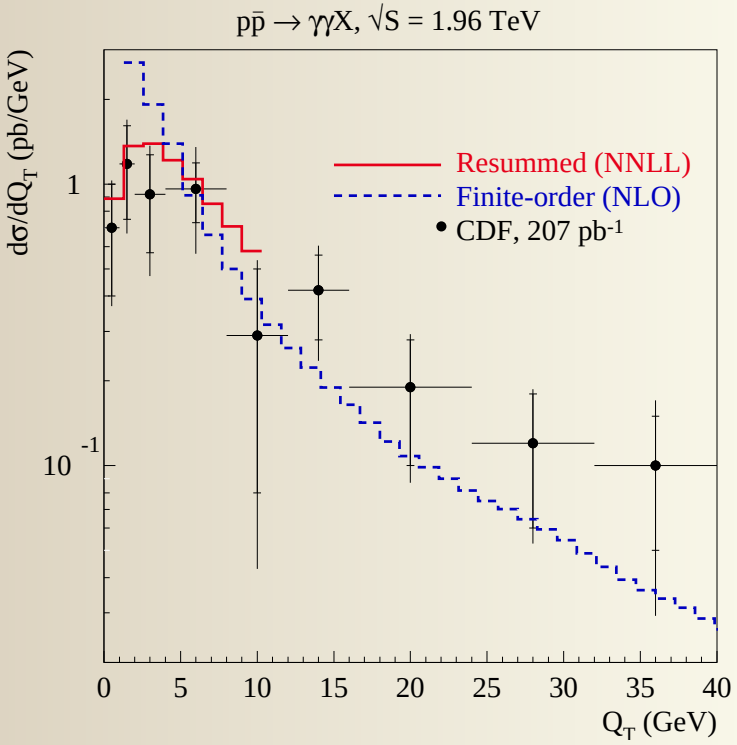



Collinear factorization at NLO; unstable at $Q_T^2 \ll Q^2$

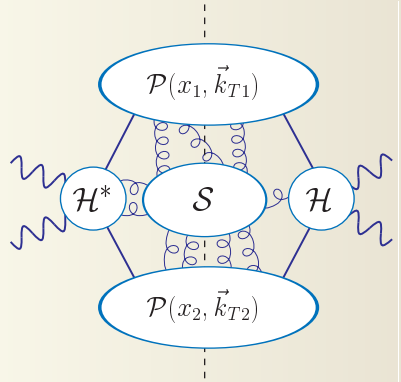
$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int dx_1 dx_2 f_a(x_1, Q) f_b(x_2, Q) \left|\mathcal{H}(ab \to V)\right|^2$$

$$\left(\frac{d\sigma}{dQ_T^2}\right)_{Q_T^2 \ll Q^2} \approx \sum_{k=0}^{\infty} \alpha_s^k \left[ c_k \delta(\vec{Q}_T) + Q_T^{-2} \cdot \sum_{n=0}^{2k-1} d_{nk} \ln^n (Q^2/Q_T^2) \right]$$

# $Q_T$ resummation in impact parameter space




$$Q_T^2 \ll Q^2$$

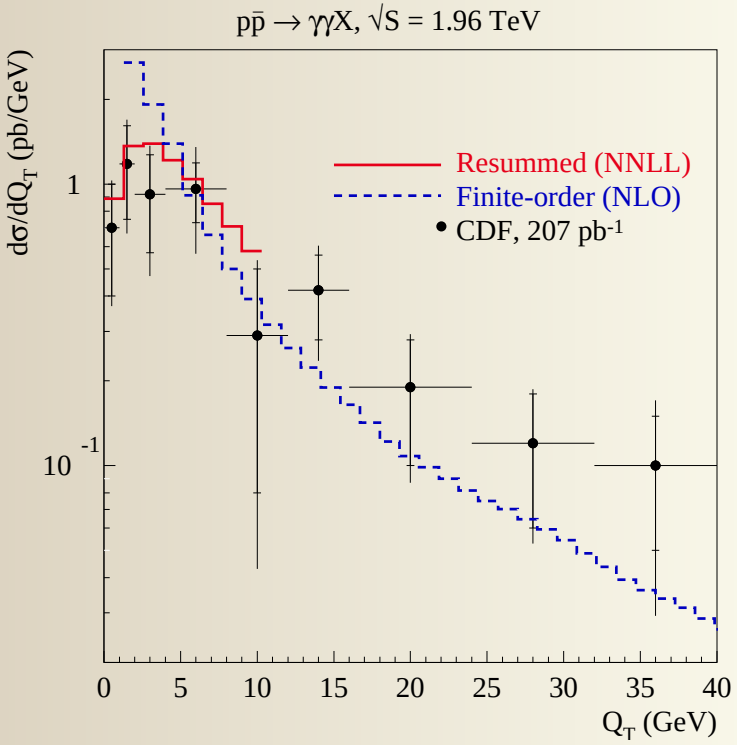



Factorization in terms of a soft factor $S$, $\vec{k}_T$-dependent PDF's $\mathcal{P}_a$, and LO hard vertex $\mathcal{H}_{ab}^{LO}$

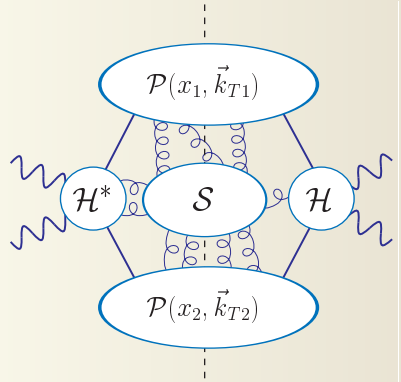
$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{Q}_T\cdot\vec{b}} \widetilde{W}_{ab}(b, Q, x_1, x_2)$$

$$\widetilde{W}_{ab}(b, Q, x_1, x_2) = \left|\mathcal{H}_{ab}^{LO}\right|^2 e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_1, b) \overline{\mathcal{P}}_b(x_2, b)$$

# $Q_T$ resummation in impact parameter space




$$Q_T^2 \ll Q^2$$

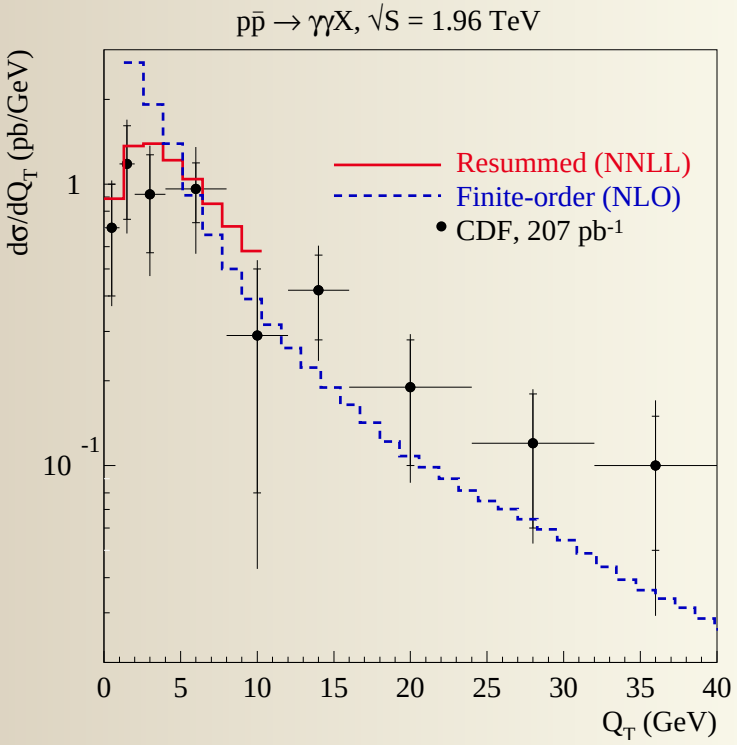



The region $b \lesssim 1$ GeV$^{-1}$ , where $\mathcal{S}$ and $\overline{\mathcal{P}} \approx [\mathcal{C} \otimes f]$ are computed in PQCD, dominates at $Q \gtrsim 10$ GeV

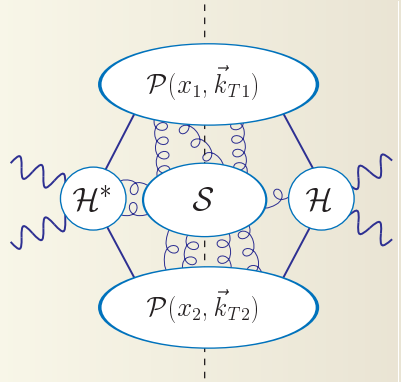
$$\frac{d\sigma}{dQ^2 dy dQ_T^2} = \sum_{partons} \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{Q}_T \cdot \vec{b}} \widetilde{W}_{ab}(b, Q, x_1, x_2)$$

$$\widetilde{W}_{ab}(b, Q, x_1, x_2) = \left|\mathcal{H}_{ab}^{LO}\right|^2 e^{-\mathcal{S}(b,Q)} \overline{\mathcal{P}}_a(x_1, b) \overline{\mathcal{P}}_b(x_2, b)$$

# $Q_T$ resummation in impact parameter space



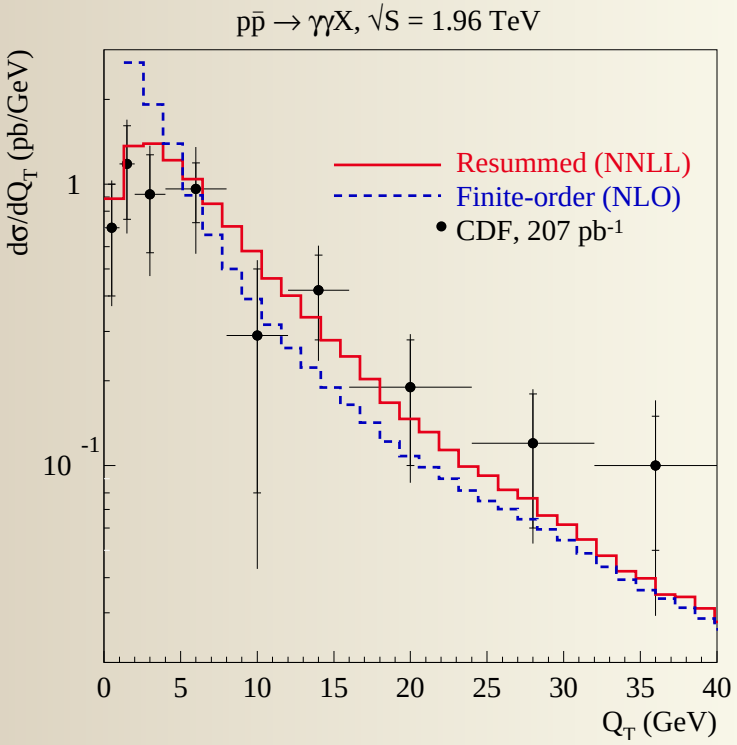

$$Q_T^2 \ll Q^2$$




The region $b \lesssim 1$ GeV$^{-1}$ , where $\mathcal{S}$ and $\overline{\mathcal{P}} \approx [\mathcal{C} \otimes f]$ are computed in PQCD, dominates at $Q \gtrsim 10$ GeV

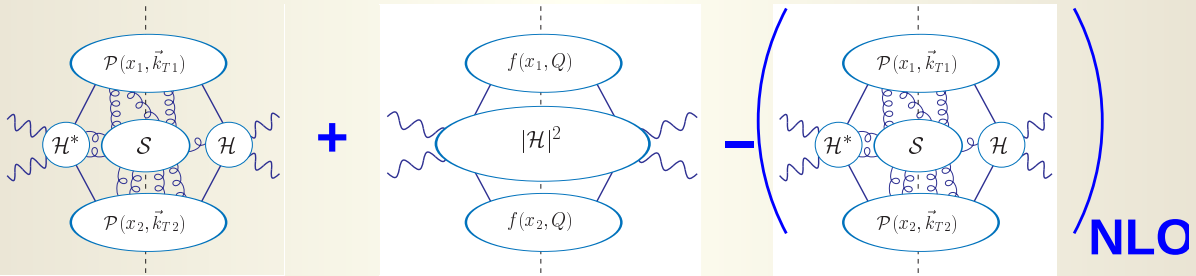
At NNLL accuracy, we include perturbative coefficients up to orders $\mathcal{A}^{(3)}$ *(from Moch, Vermaseren, Vogt, 2004)*; $\mathcal{B}^{(2)}$ ; and $\mathcal{C}^{(1)}$

# $Q_T$ resummation in impact parameter space



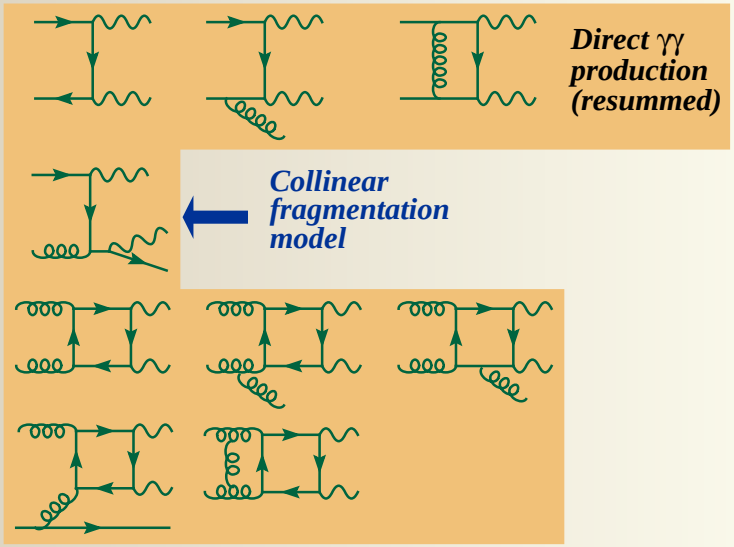

$$Q_T^2 \lesssim Q^2$$

Add small-$Q_T$ and large-$Q_T$ X-sections, subtract the overlap

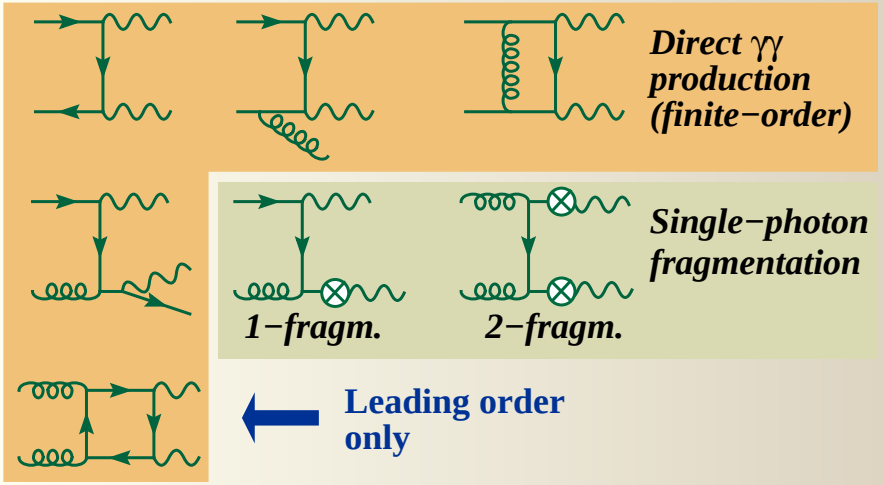


# Our calculation vs. DIPHOX (Binoth et al. )

Balazs, Berger, Nadolsky, Yuan, 2006



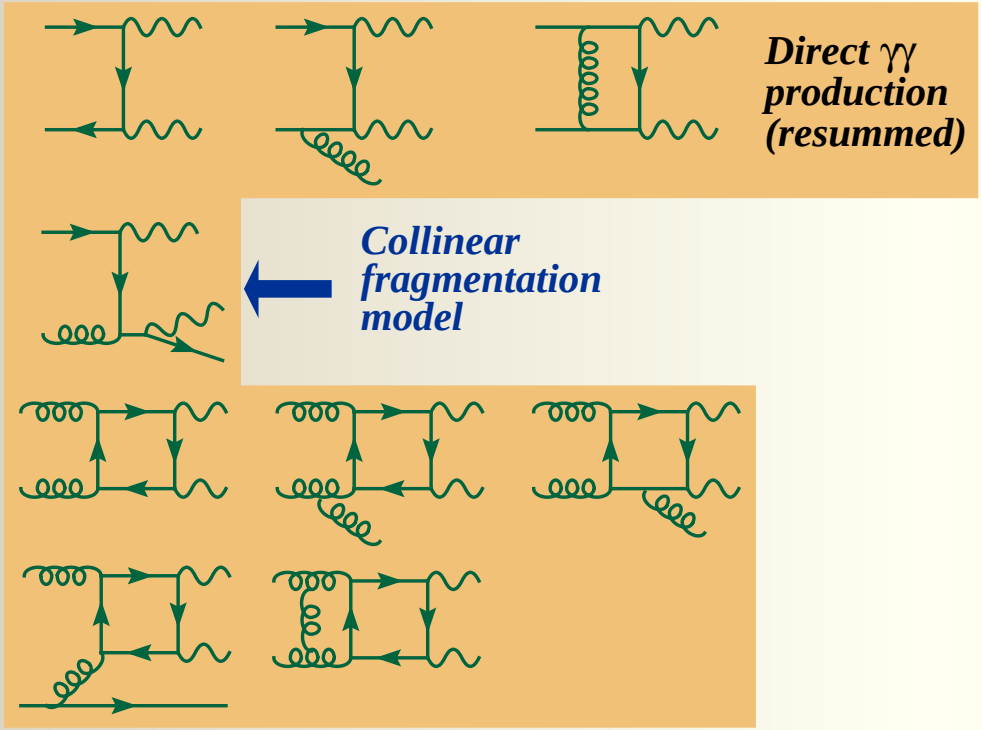

Binoth, Guillet, Pilon, Werlen, 2001



# Fragmentation model

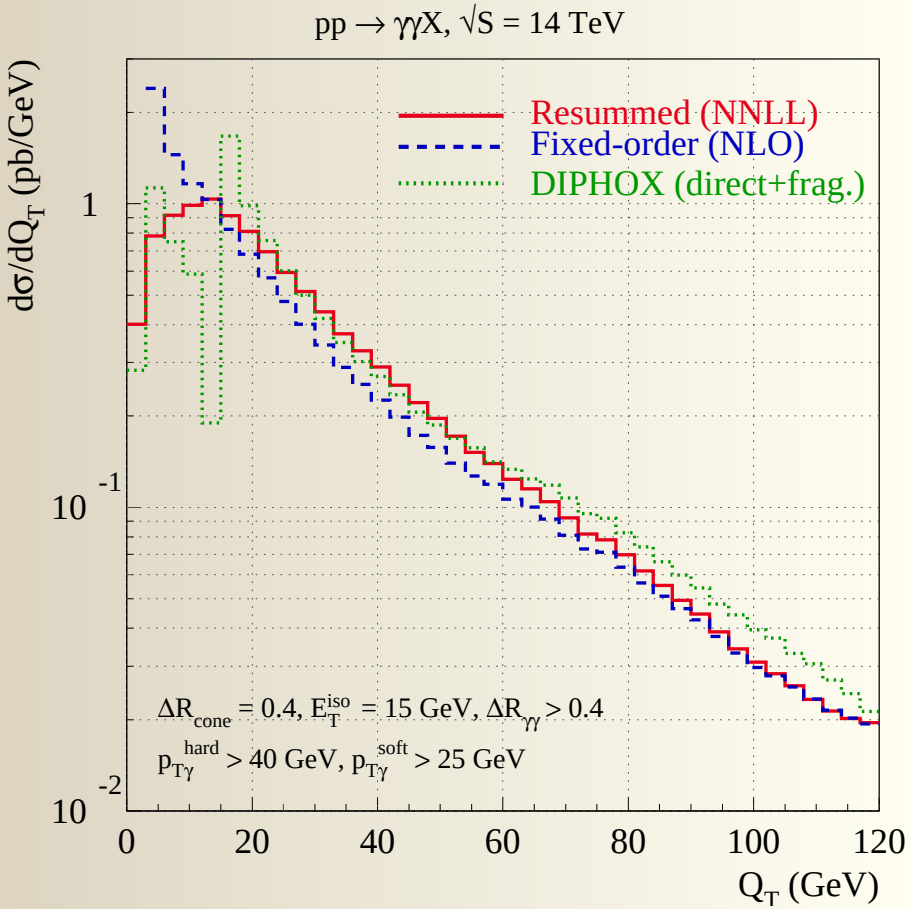
- Typical isolation: hadronic $E_T < E_T^{iso}$ in a $\Delta R$ cone around each $\gamma$
- The final-state singularity is removed by quasi-experimental isolation at $Q_T > E_T^{iso}$; by subtraction or smooth-cone isolation at $Q_T \leq E_T^{iso}$

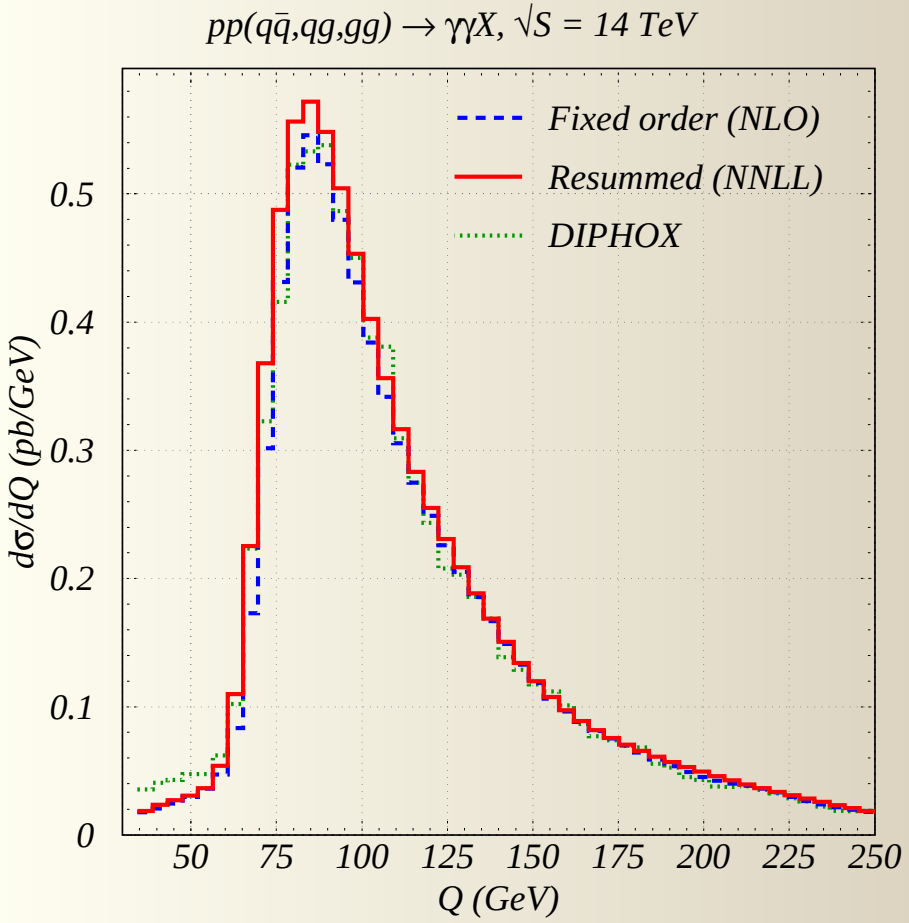
Balazs, Berger, Nadolsky, Yuan, 2006




- sufficient for description of fragmentation contributions at the Tevatron and LHC
- approximately reproduces the inclusive NLO fragmentation rate *(Binoth et al.; DIPHOX program)* at both colliders
- good agreement with the Tevatron data

# $\gamma\gamma$ production at the LHC (ATLAS cuts, $E_T^{iso} = 15$ GeV, $\Delta R = 0.4$)



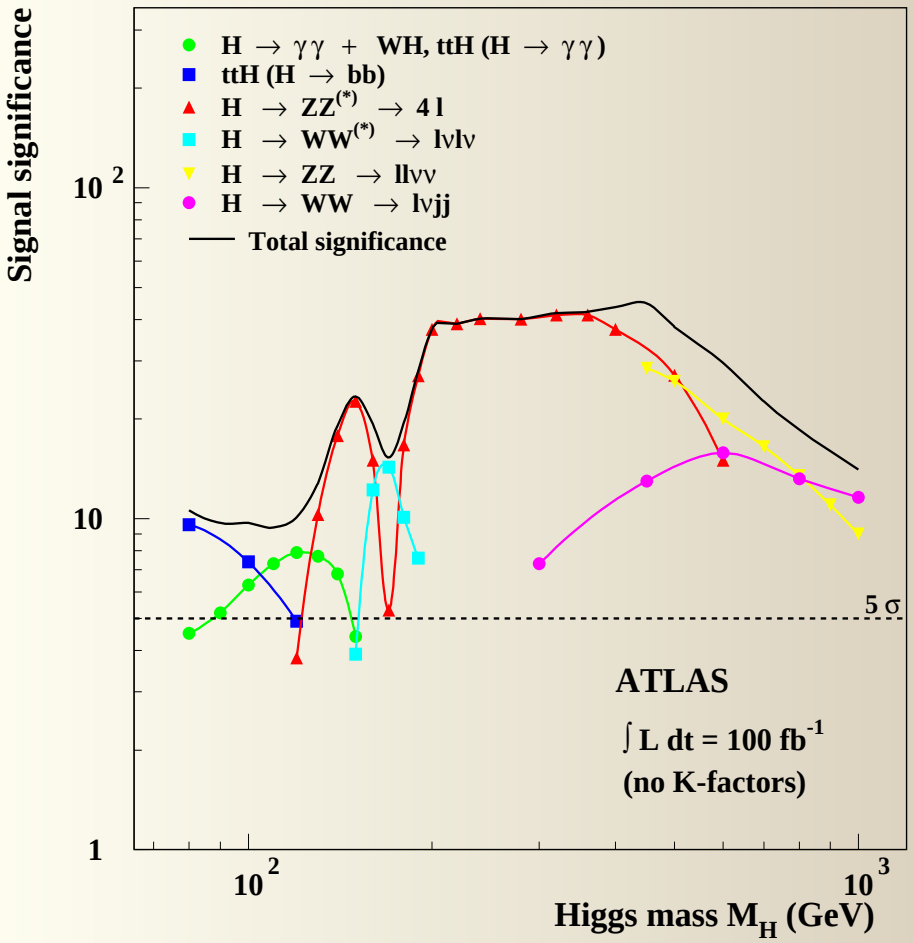

- DIPHOX agrees with the resummation at large $Q_T$; exhibits integrable logarithmic singularities at $Q_T < E_T^{iso}$
- RESBOS shows a mild discontinuity at $Q_T = E_T^{iso}$

# Light Higgs boson search at the LHC

- $gg \to h \to \gamma\gamma$ (via $t$-quark loop) is the leading search mode for $115 \lesssim M_H \lesssim 140$ GeV

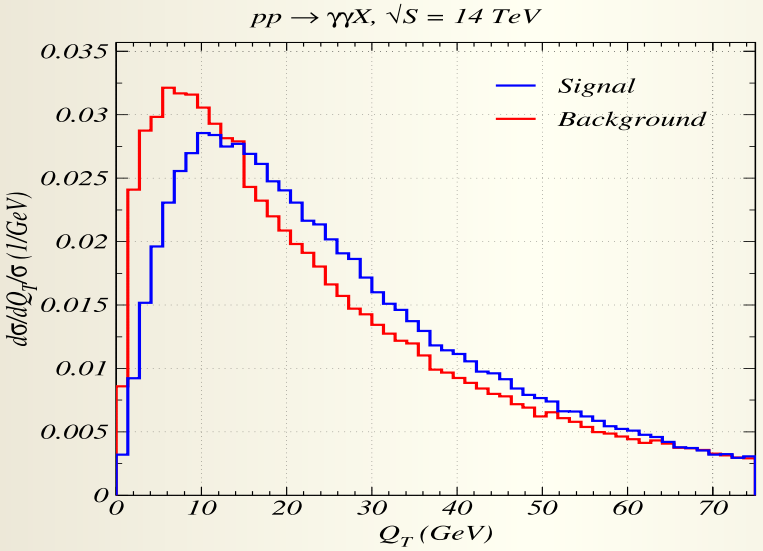
- A $5\sigma$ discovery of SM Higgs boson may be possible with $\mathcal{L} = 10 - 30$ fb$^{-1}$

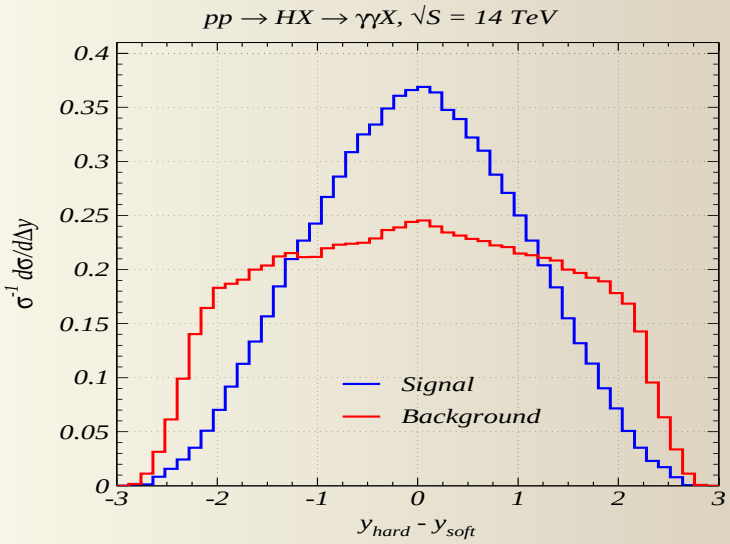
- Statistical significance depends on the (new) physics model, QCD contributions, etc.



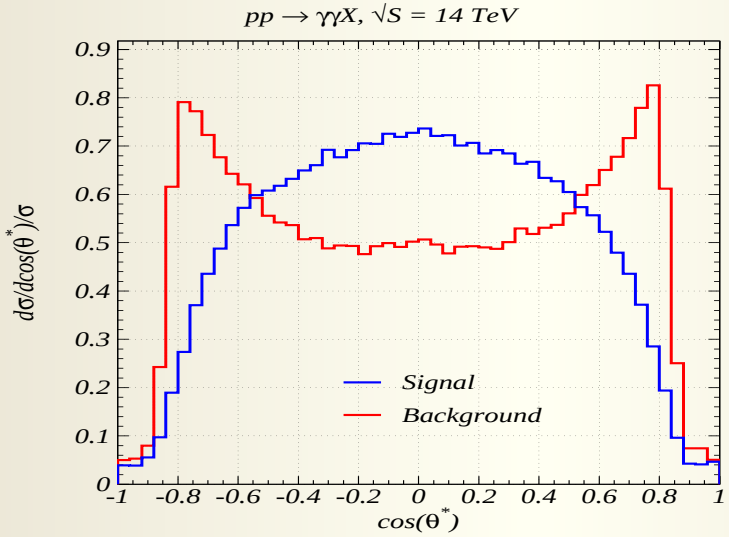
# NNLL/NLO distributions for Higgs $\to \gamma\gamma$ signal and background (ResBos, normalized; $M_H = 130$ GeV, $128 < Q < 132$ GeV)

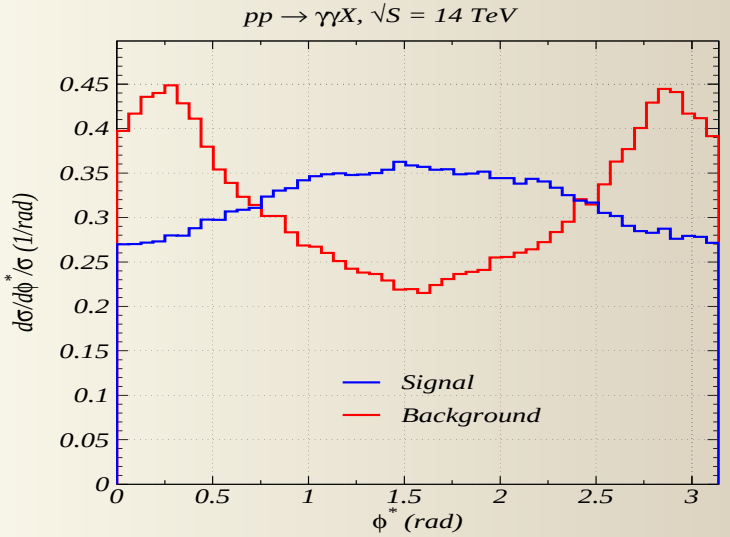
$Q_T$ and $y_{\gamma_1} - y_{\gamma_2}$ in the lab frame




Decay angles $\theta_*$, $\varphi_*$ in the $\gamma\gamma$ rest frame




no singularities, in contrast to the fixed-order rate

# Summary of new features

- NNLL resummation for $gg + gq \to \gamma\gamma$ at two loops
- combined with the resummed **direct** $q\bar{q} + qg$ cross section
- improved treatment of the **fragmentation** region
- improved model for nonperturbative resummed contributions *(A. Konychev, **P. N.**, PLB 633, 710 (2006))*
- resummed form factor in two resummation schemes
- predictions for differential distributions in the $H \to \gamma\gamma$ search

# Backup slides

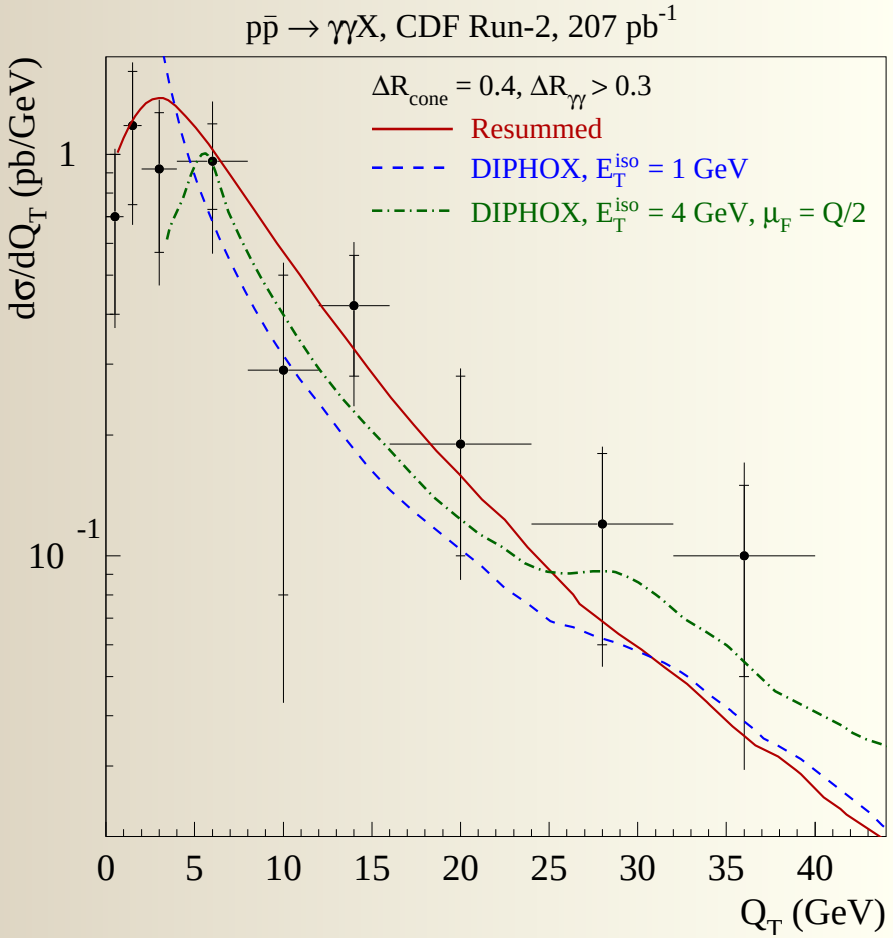
# Impact on the Higgs boson search

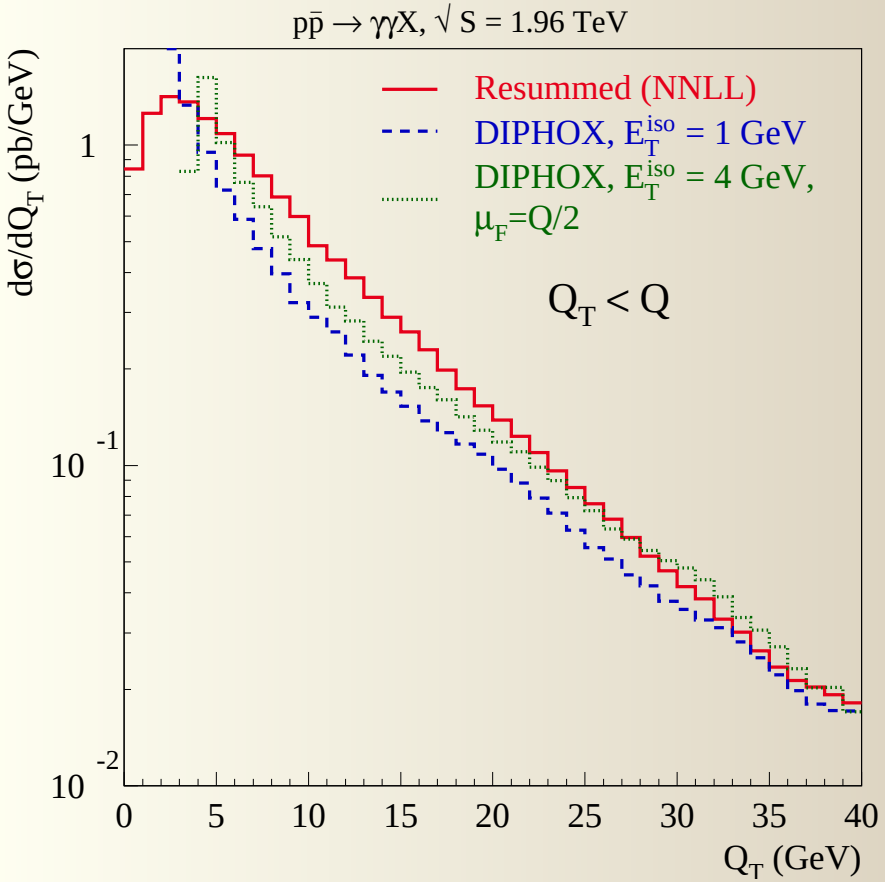
- Information about the shape of diphoton $Q_T$, $\theta_*$, and $\varphi_*$ distributions increases discrimination power of the likelihood analysis

- Radiative corrections have strong kinematic dependence. Selection of $\gamma\gamma$ events at $E_T^{iso} \lesssim Q_T \lesssim Q$, central $\theta_*$ and $\varphi_*$ increases the signal significance, while reducing theory uncertainties

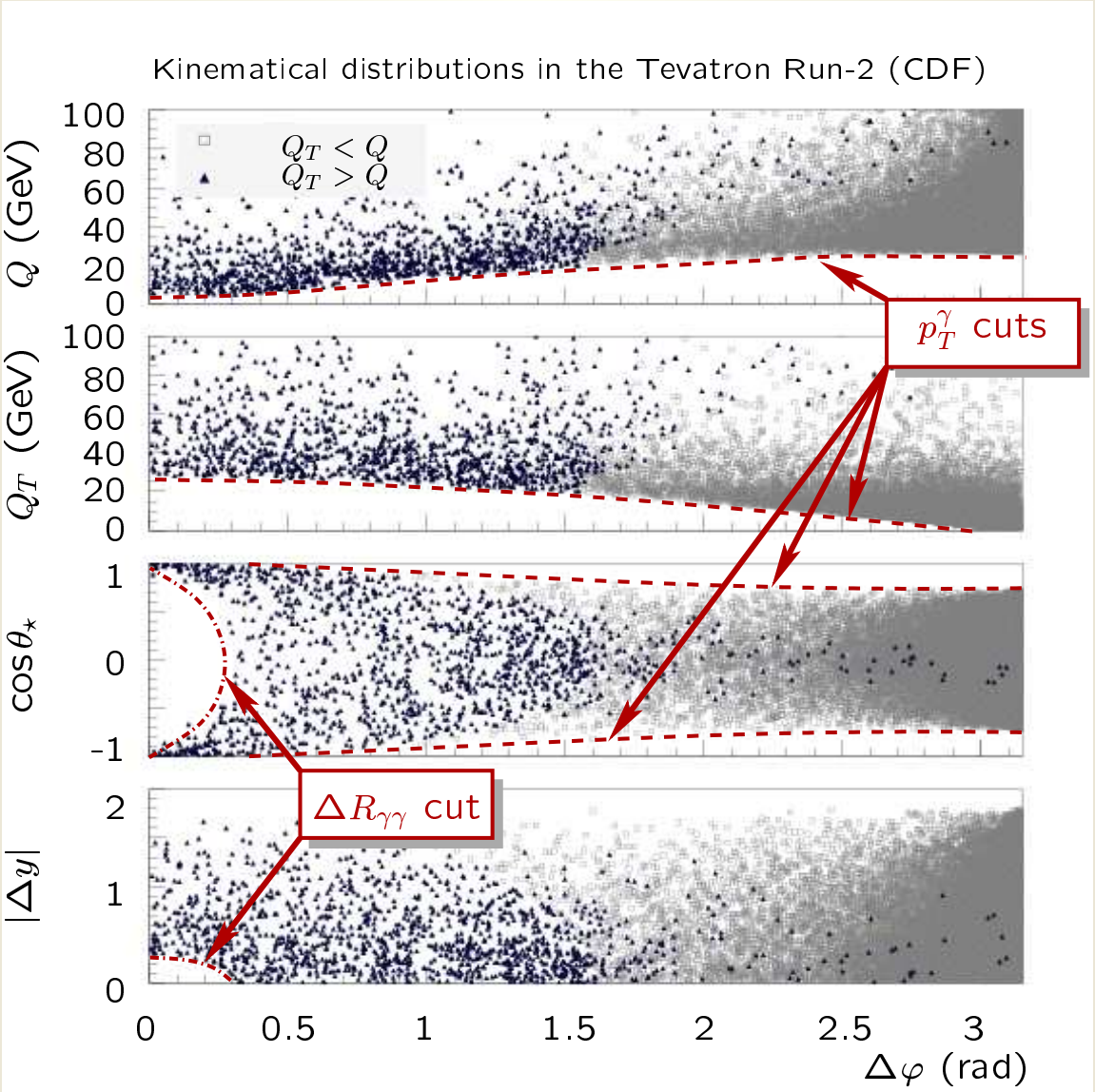
- The resummation calculation improves our understanding of the Higgs signal and background. It will be nevertheless important to experimentally examine $\gamma\gamma$ distributions in a wide range of $Q$ to facilitate further improvements in theory

# Comparison with the CDF Run-2 data ($E_T^{iso} = 1$ GeV, $\Delta R = 0.4$)

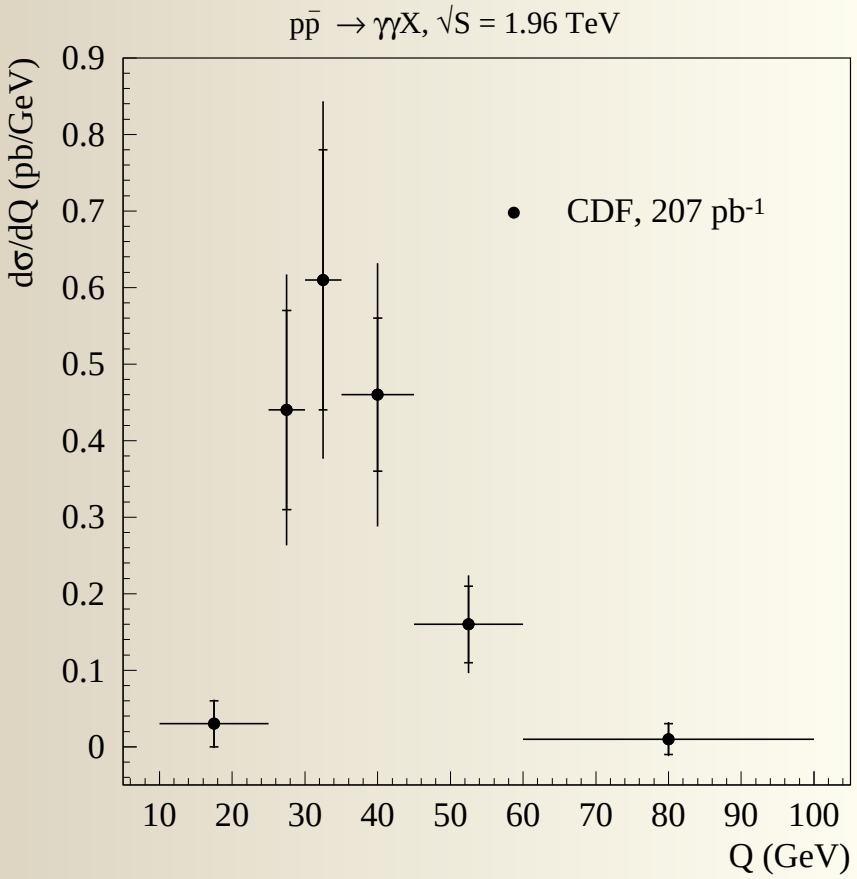


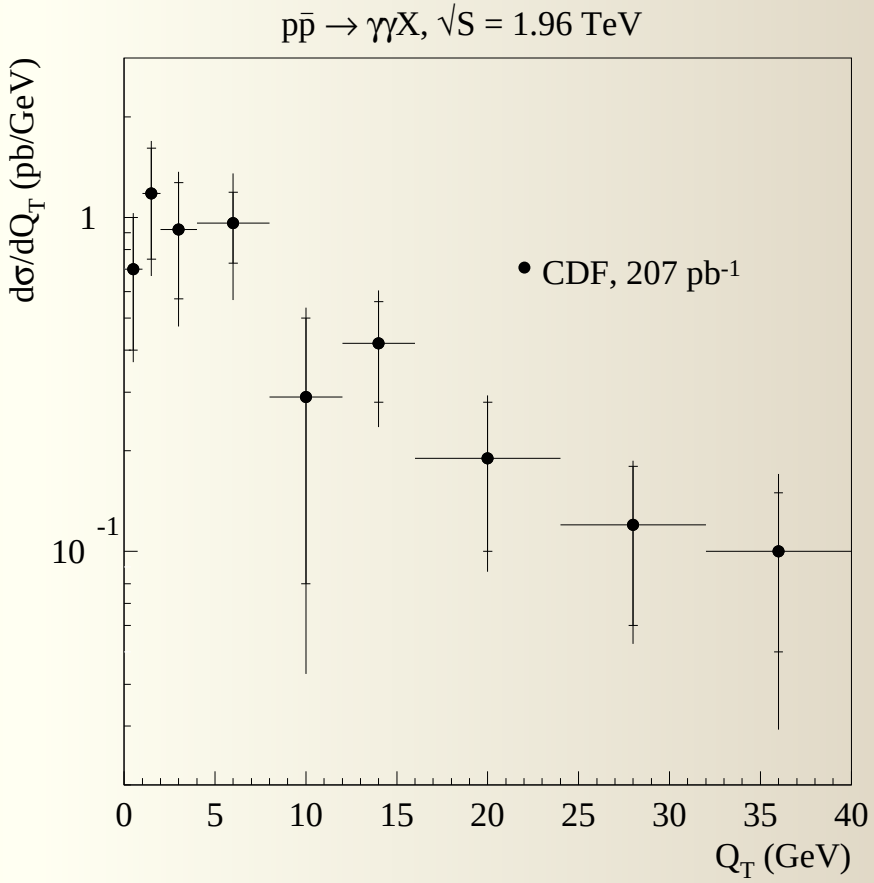



# Kinematical distributions at the CDF

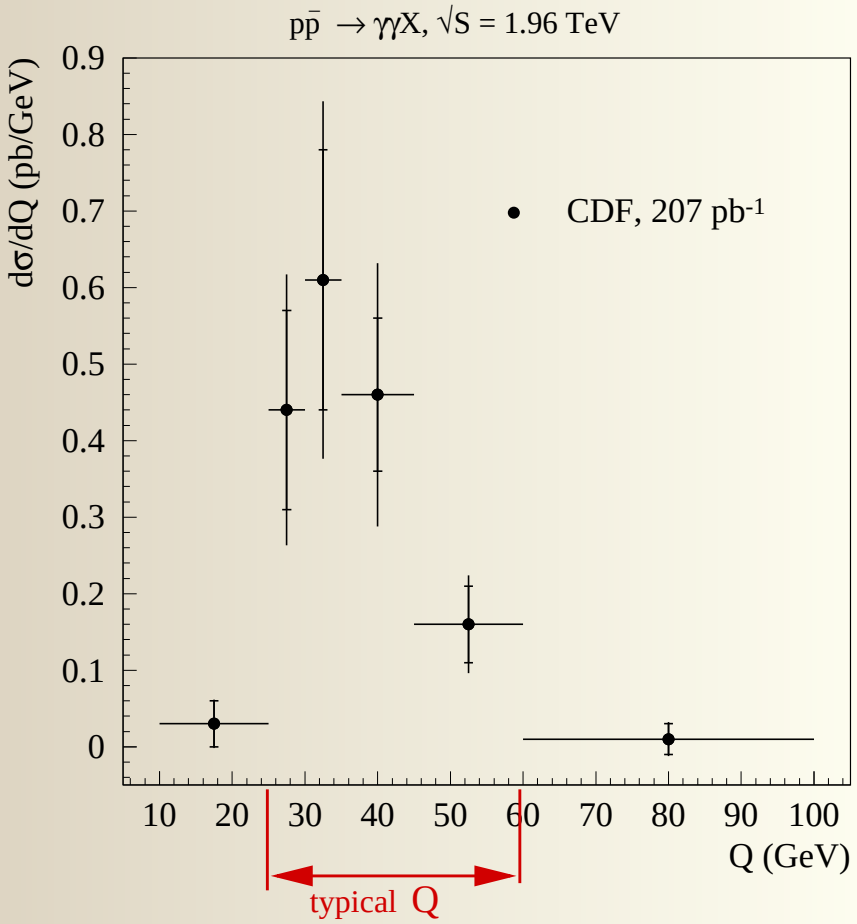


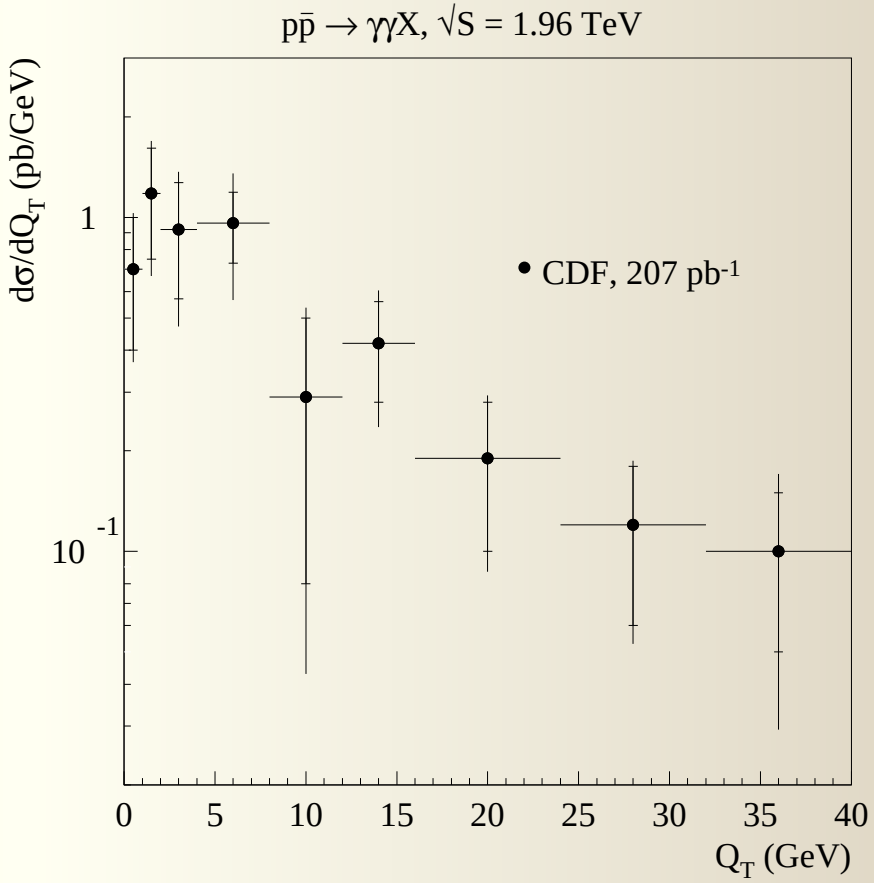
# $\gamma\gamma$ invariant mass ($Q$) and transverse momentum ($Q_T$) distributions at CDF



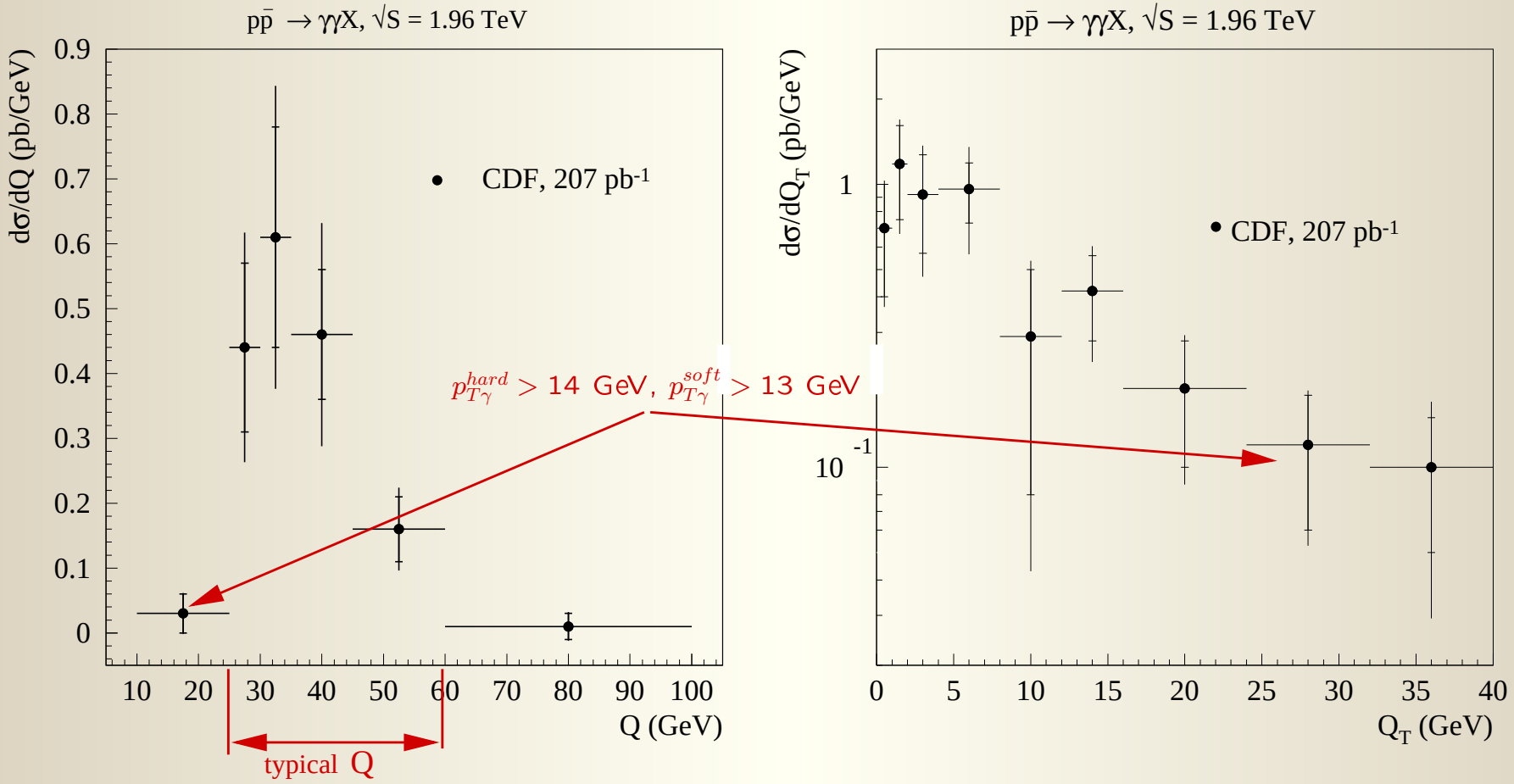



# $\gamma\gamma$ invariant mass ($Q$) and transverse momentum ($Q_T$) distributions at CDF

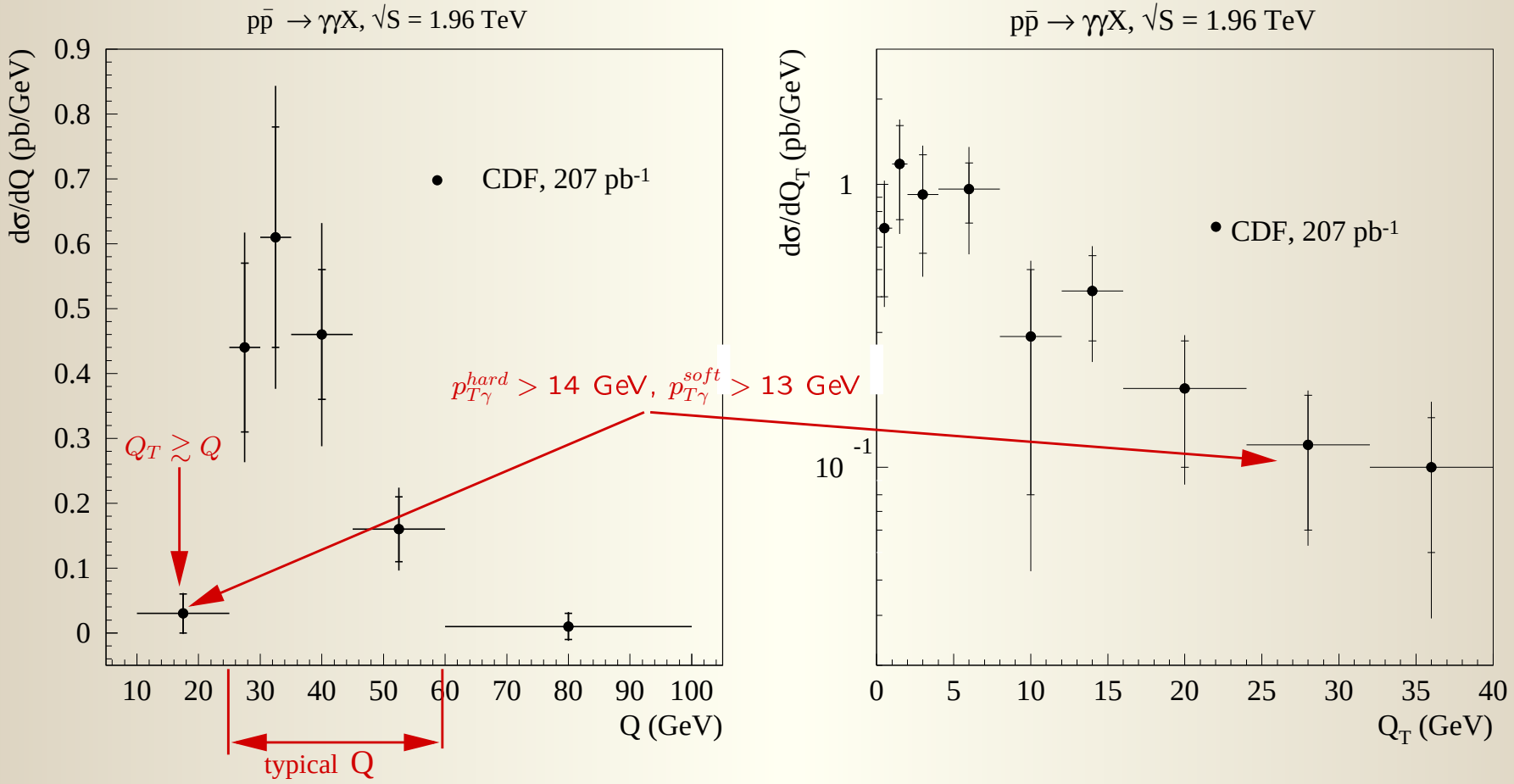


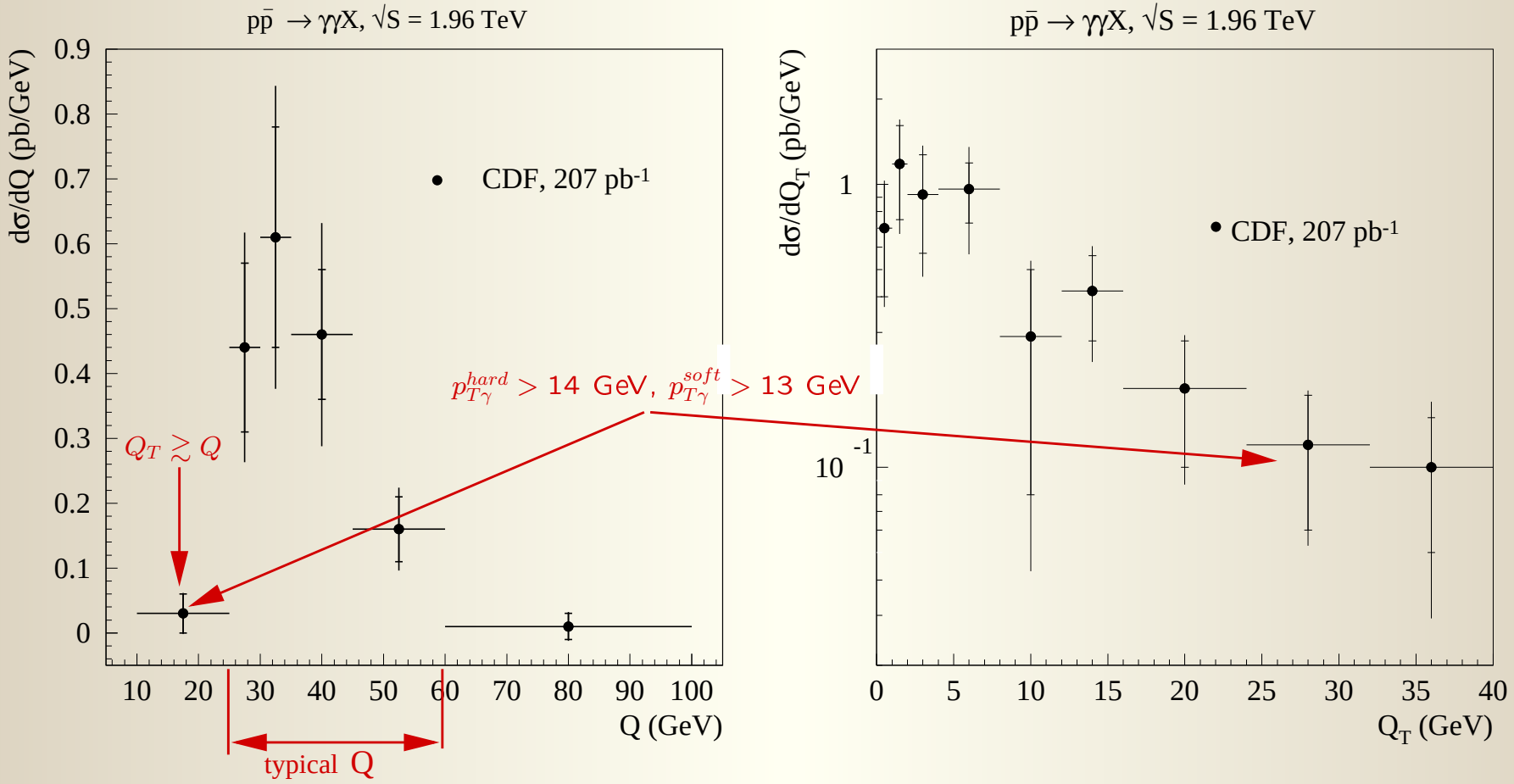
# $\gamma\gamma$ invariant mass ($Q$) and transverse momentum ($Q_T$) distributions at CDF



# $\gamma\gamma$ invariant mass ($Q$) and transverse momentum ($Q_T$) distributions at CDF

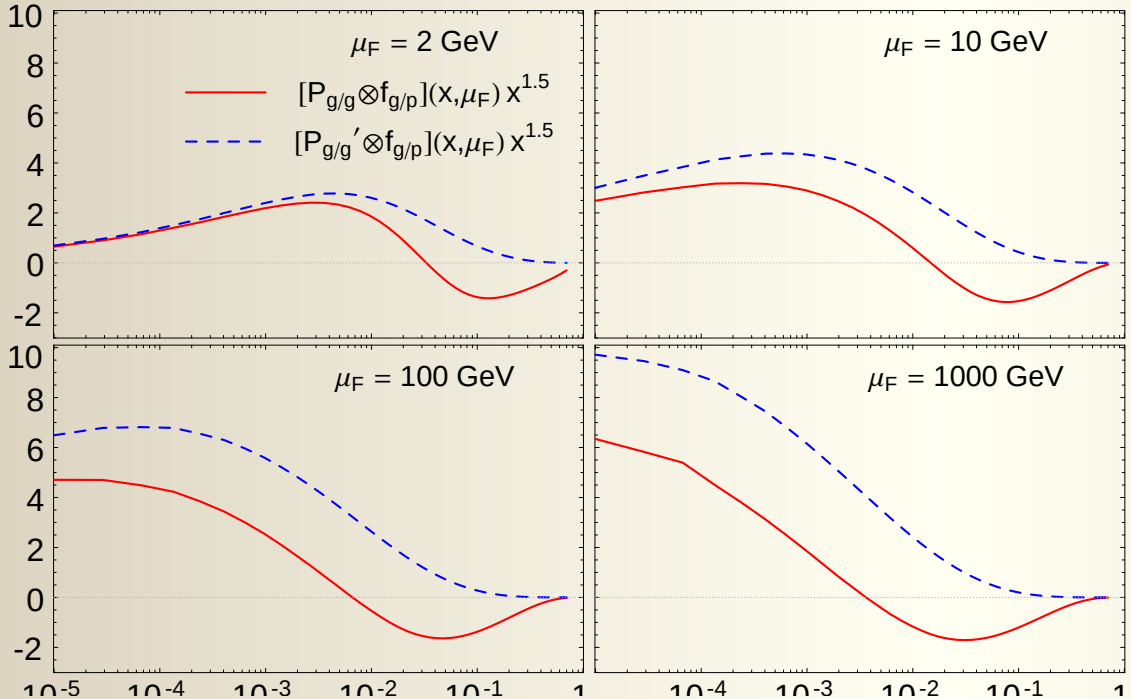


# $\gamma\gamma$ invariant mass ($Q$) and transverse momentum ($Q_T$) distributions at CDF




Predictions for $d\sigma/dQ_T$ affect the behavior of $d\sigma/dQ$ and other distributions

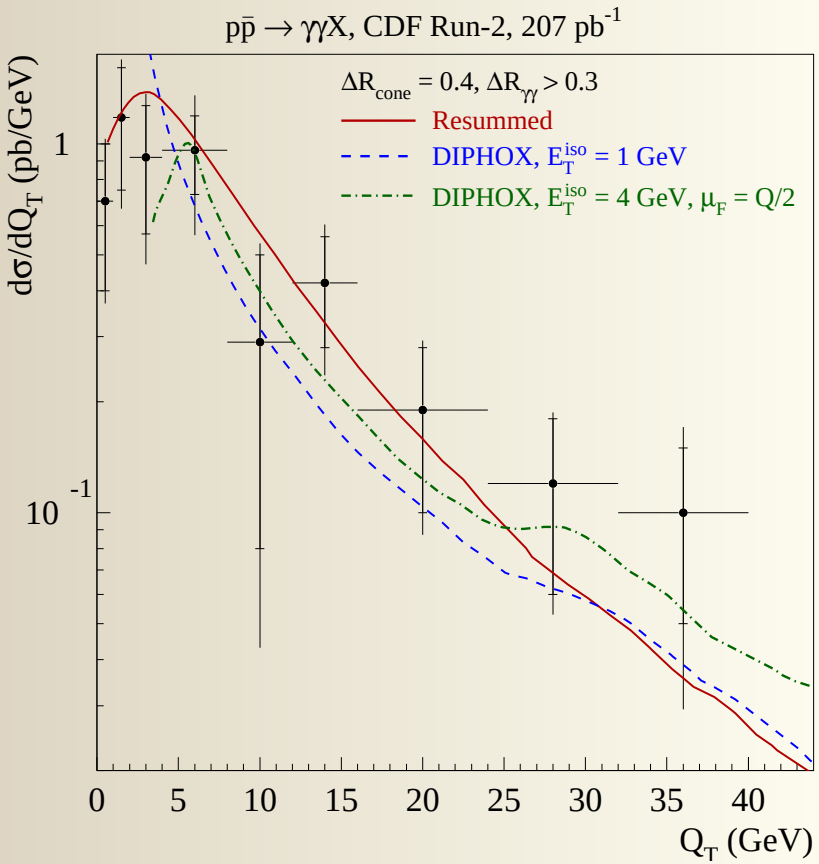
# DGLAP evolution of $\overline{\mathcal{P}}_a(x,b)$ and $\overline{\mathcal{P}}'_a(x,b)$




Scaling violation in $\overline{\mathcal{P}}'_a(x,b)$ due to mixing with gluon PDF $g(x,\mu_F)$ is determined by $(\mathcal{P}'_{gg} \otimes g)\,(x,\mu_F)$, with $P'_{gg}(x) = 2C_A(1-x)/x$

- $(\mathcal{P}'_{gg} \otimes g)\,(x,\mu_F) > (\mathcal{P}_{gg} \otimes g)\,(x,\mu_F)$

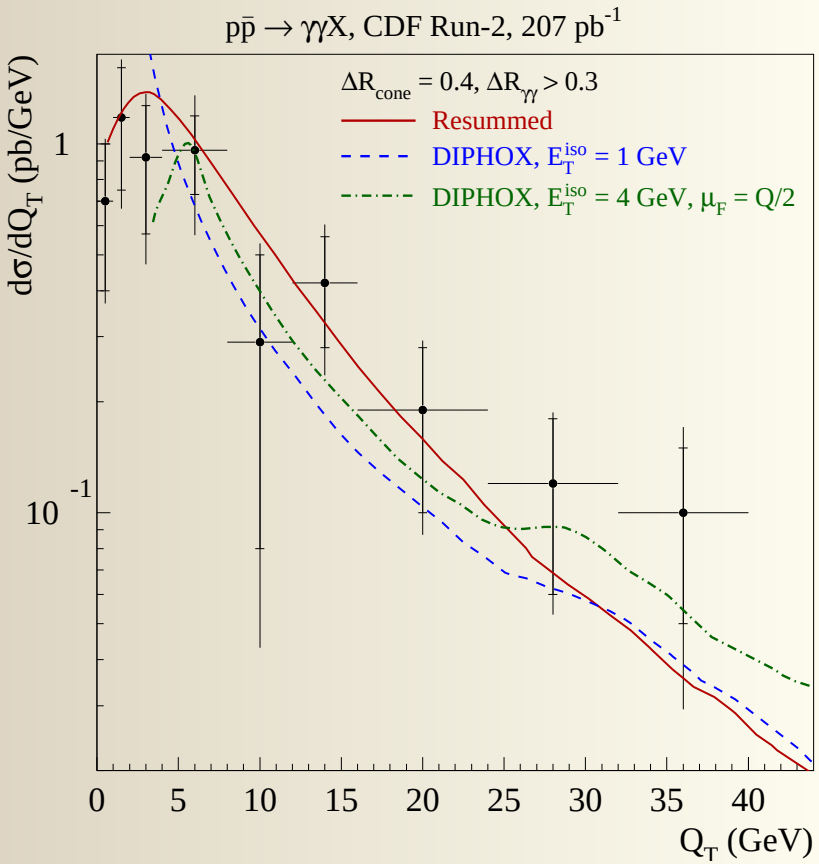
# Comparison with the CDF Run-2 data ($E_T^{iso} = 1$ GeV, $\Delta R = 0.4$)



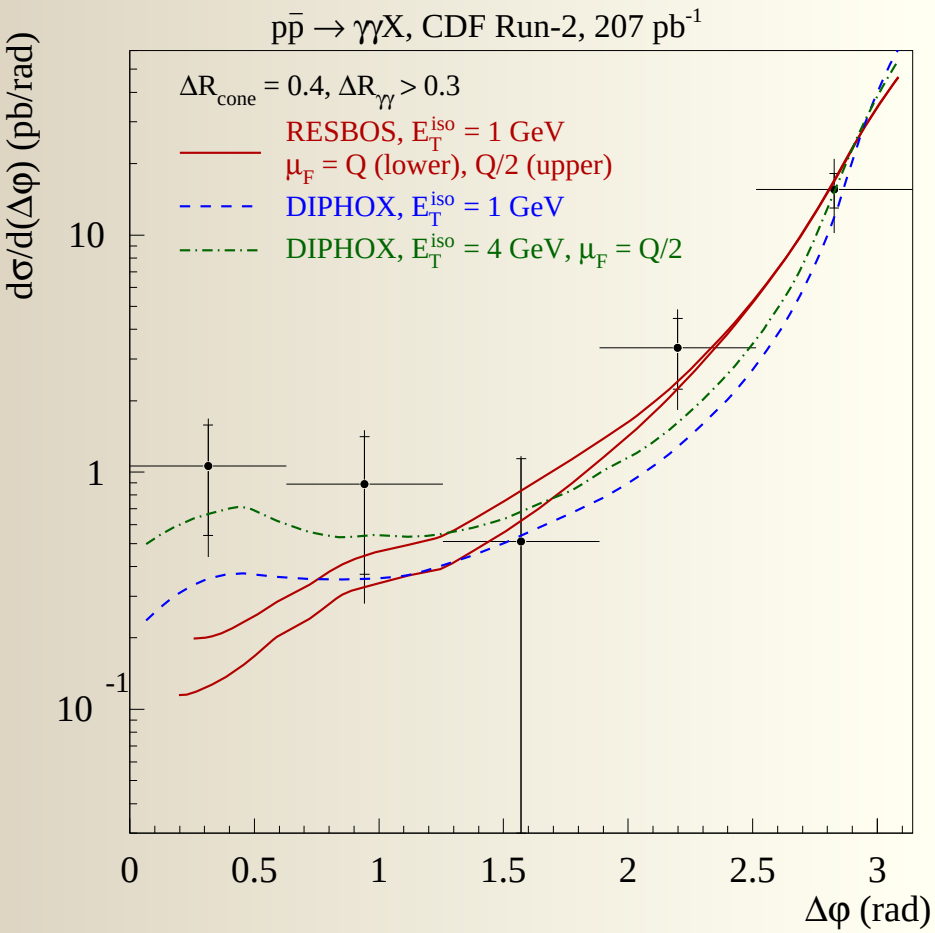
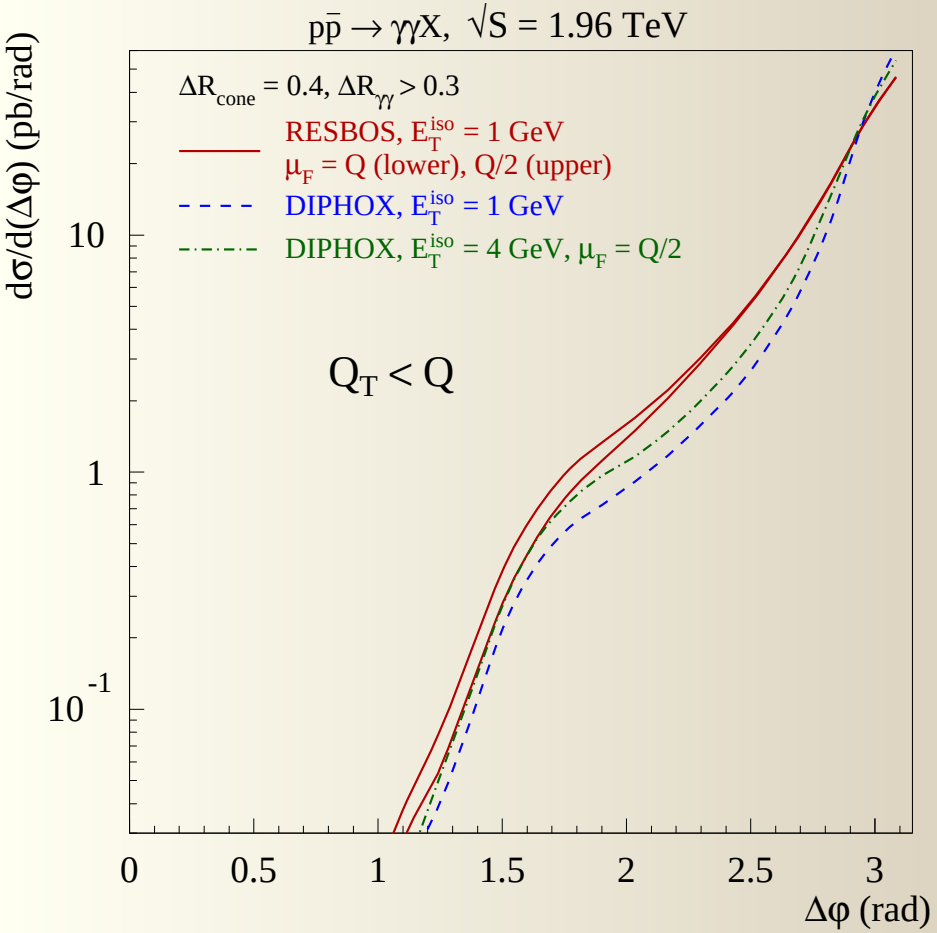

- The DIPHOX cross section...
  - ...agrees with NLO RESBOS for the nominal $E_T^{iso} = 1$ GeV, same $\mu_F$
  - ...can reproduce the "shoulder" by choosing $E_T^{iso} = 4$ GeV, smaller $\mu_F$ to increase the 1-fragmentation rate by 400%
- The "shoulder" occurs at $Q \lesssim 25$ GeV, $Q \lesssim Q_T$, $\Delta\varphi < \pi/2$, where large unknown corrections are present

The "shoulder" is a low-$Q$ effect, not relevant for the LHC Higgs searches; can be removed by a $Q_T \leq Q$ cut

# Comparison with the CDF Run-2 data ($E_T^{iso} = 1$ GeV, $\Delta R = 0.4$)




- The DIPHOX cross section...
  - ...agrees with NLO RESBOS for the nominal $E_T^{iso} = 1$ GeV, same $\mu_F$
  - ...can reproduce the "shoulder" by choosing $E_T^{iso} = 4$ GeV, smaller $\mu_F$ to increase the 1-fragmentation rate by 400%
- The "shoulder" occurs at $Q \lesssim 25$ GeV, $Q \lesssim Q_T$, $\Delta\varphi < \pi/2$, where large unknown corrections are present

The "shoulder" is a low-$Q$ effect, not relevant for the LHC Higgs searches; can be removed by a $Q_T \leq Q$ cut

# Azimuthal angle separation $\Delta\varphi$