On $t\bar{t}$-threshold and top quark mass definition

Oleg Yakovlev (Univ. of Michigan)

LCWS-2000 Oct. 25

§1. Introduction

§2. $R_{t\bar{t}}$ at LO, NLO, NNLO

§3. mass definitions, $\overline{\text{PS}}$

§4. Non-factorizable QCD

§5. Outlook

Cross-section at NNLO

1) O. Yakovlev PLB 457 (1999) 170

2) A. Hoang, M. Beneke, O. Yakovlev + ..
Eur. Phys. J. (2000) 1

mass definition : $m_{\overline{PS}}$

3) O. Yakovlev, S. Groote hep-ph/0008156
4) — , — hep-ph/0009014

Non-factorizable correction

5) R. Akhoury, O. Yakovlev ..
(in progress)

§1 Introduction

- The top quark physics is one of the main subjects of future e^+e^- - colliders → 1) Higgs 2) Top 3) Supersymmetry 4) Extra-dim 5) Giga-Z ... → LC
- The goals are to measure and determine the properties of the top quark

  m_t (mass), Γ_t (width), $g_{t\bar{t}H}$ (Yukawa), $|V_{tb}|$ (CKM m.e.), ... $\alpha_s(m_t)$..
- t-quark has been discovered (1995) at Tevatron (Fermilab) by CDF & DØ with a mass

$$\boxed{m_t = 174.3 \pm 5 \text{ GeV}}$$

CDF & DØ : hep-ex/0005030

- t-quark will be studied at LHC (CERN) and at Tevatron RUN-II exp.

– expected accuracy $\boxed{2 \text{ GeV}}$

↳ most accurate measurement → NLC (100M)

- t-quark mass will be measured at NLC with 0.1% accuracy (0.2 GeV)
it is 10 times better than at LHC.

- Why do we need high accuracy in m_t ?

Lessons from LEP:

The precision study of Z and W bosons gave predictions on $m_{top} = 175 \pm 10$ GeV (confirmed by Tevat. CDF & DØ)

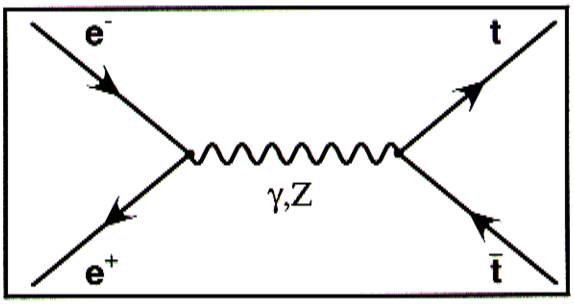
Similar at NLC

studing m_t, m_W, m_Z, (m_H) we may reconstract a signal of NEW PHYSICS.

M : new particle
interaction
dimension
⋮

§ 2. The leading order (LO) cross section

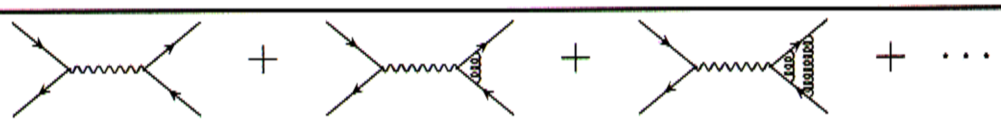
- to understand the main problem, let us consider LO cross section of $e^+e^- \to t\bar{t}$ process.
  The basic diagram is:



- we focus on the non-relativistic region, where the velocity is small

$$v = \sqrt{1 - 4m_t^2/s} \ll 1, \qquad v \simeq \alpha_s = 0.1 - 0.3$$

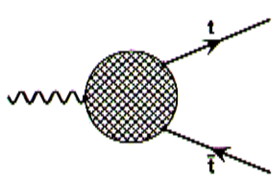
- One has to resum all Coulomb singularities ($\frac{\alpha_s}{v} \simeq 1$)

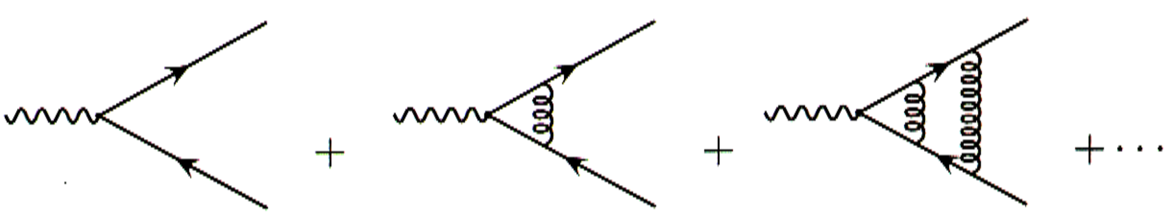


$$1 + \frac{\alpha_s}{v} + \left(\frac{\alpha_s}{v}\right)^2 + \cdots + \left(\frac{\alpha_s}{v}\right)^n$$

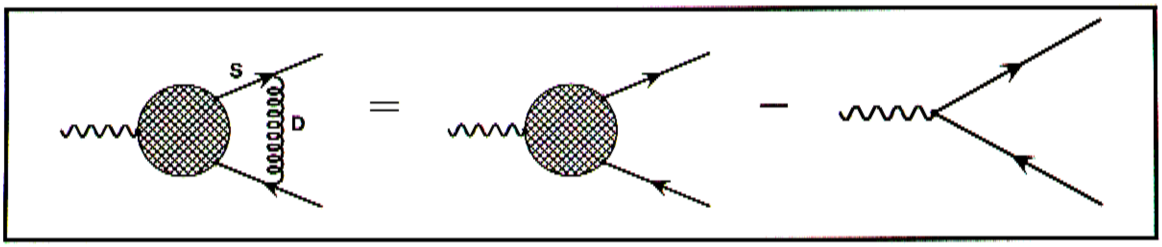
How ?

- First, we define the vertex $\Gamma(p)$ by

$\Gamma(p) \equiv$  $=$



- Second, we construct the integral equation: (which has simple interpretation)



- by using the non-relativistic form of the t- propagator and Coulomb gauge for the gluon fields

$$S(p) = \frac{1+\gamma^0}{2}\frac{i}{\varepsilon - \vec{p}^2/(2m) + i0}, \quad D(k) = \frac{i}{|\vec{k}|^2}\delta^{\mu,0}\delta^{\nu,0}$$

we obtain a Schrödinger equation. The final result reads

$$\vec{\Gamma}(p) = \vec{\gamma}\cdot G(\vec{p}, r=0|E)(\frac{\vec{p}^2}{m} - E - i0)$$

V. Fadin
V. Khoze
M. Peskin
'90

- where $G(r, r'|E)$ is a non-relativistic Green function, which is a solution of the Sch.Eq-n

$$(\frac{\vec{p}^2}{m} - \frac{C_F\alpha_s(\mu)}{r} - E - i0)G(r, r'|E) = \delta(\vec{r} - \vec{r}')$$

! Basic equation

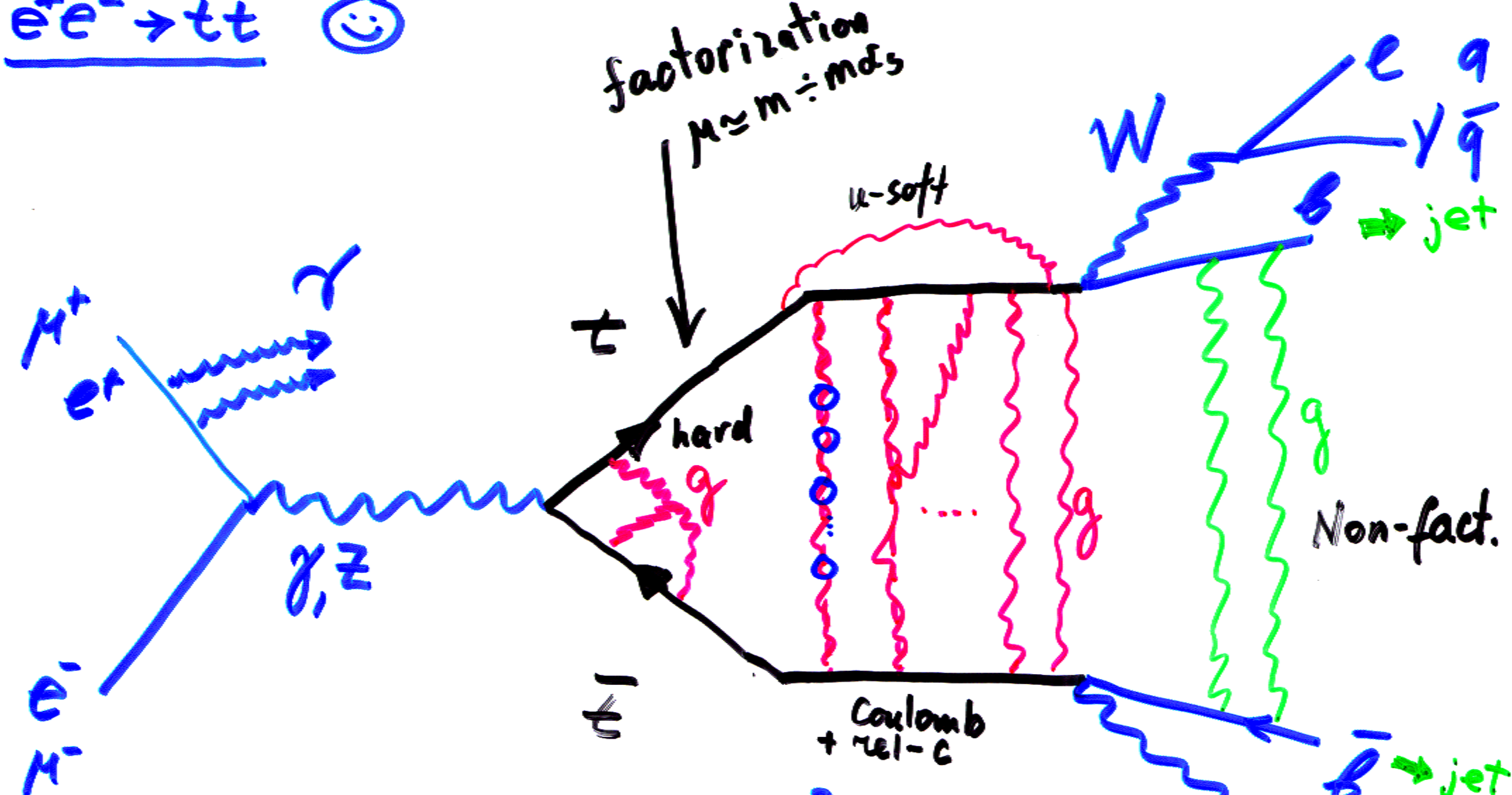
We have to include the width, through the SELF ENERGY of the top quark.

$$S(p) = \frac{1+\gamma^0}{2}\frac{i}{\varepsilon - \vec{p}^2/(2m) + i\frac{\Gamma}{2}}$$

Combining together, we have the cross section

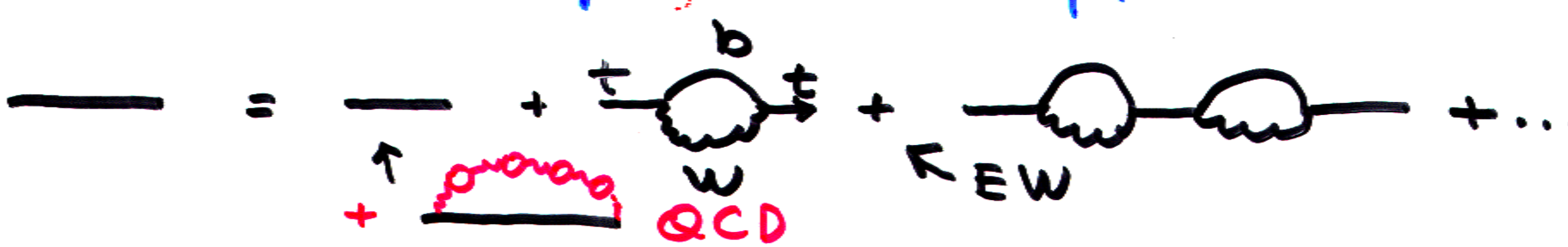
$$\sigma(e^+e^- \to t\bar{t}) = e_t^2 N\frac{24\pi}{s} Im[G(0,0|E + i\Gamma)].$$

$e^+e^- \to t\bar{t}$ ☺

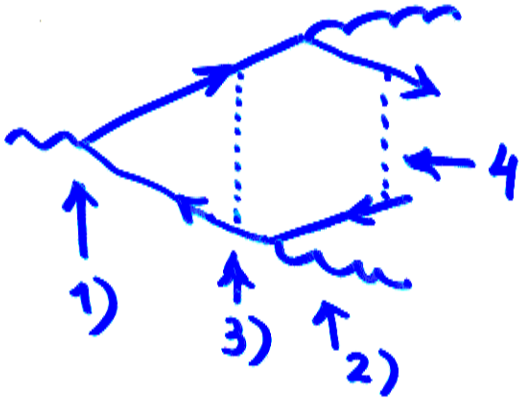


$m \gg m\alpha_s \gg m\alpha_s^2, \Gamma$

hard soft ultrasoft



NLO rad-ve corrections

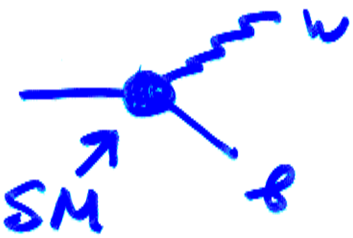


There are 4 sources at 1 loop:

1) production: $t\bar{t}\gamma^*$ vertex (Schwinger 1973)

 $\Rightarrow \left(1 - 2\frac{\alpha_s C_F}{\pi}\right)\vec{\gamma}\, e_Q$ $\quad v \to \phi$

2) $t \to Wb$ decay



QCD+EW

Y.P. Yao et al.
J. H. Kühn et al.
A. Denner et al.
('89 – '90)

3) Coulomb potential

Fishler '70
Billoire '80

4) Non-factorizable corrections

K. Melnikov + O. Yakovlev '94–96
Sumino; V Fadin + V. Khoze '94

§ · NNLO QCD correction

The cross section of $e^+e^- \to t\bar{t}$ process in near threshold region is given by optical theorem | $\langle 0|J_\mu(x) J^+_\nu|0\rangle$

- $$R = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)} = e_Q^2 N_c \frac{24\pi}{S} \, \mathrm{Im}\left[\left(1 - \frac{\vec{p}^{\,2}}{3m^2}\right) G(r, r'|E)\right]_{r\to 0} \cdot C(r)$$

charge color $i\frac{\partial}{\partial r}$ Green function

➡ Schrödinger Eq.

- $$\left(\hat{H} - (E + i\Gamma)\right) G(\vec{r}, \vec{r}\,'|E) = \delta(\vec{r} - \vec{r}\,')$$

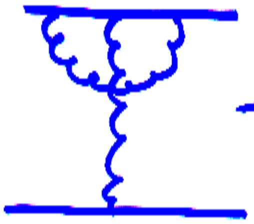
- Hamiltonian of $t\bar{t}$ system

$$H = H_0 + W(r), \qquad H_0 = \frac{\hat{\vec{p}}^{\,2}}{m} + V(r)$$

Coulomb-like system Breit-Fermi Hamiltonian Coulomb Potential

$$V(r) = -\frac{C_F \alpha_s}{r} \cdot \Big[1 + \left(\frac{\alpha_s C_F}{4\pi}\right) \cdot \left(2\beta_0 \ln(\mu_1 r) + a_1\right)$$
$$+ \left(\frac{\alpha_s C_F}{4\pi}\right)^2 \cdot \Big\{ \beta_0^2 \left(4\ln^2(\mu_1 r) + \frac{\pi^2}{3}\right) + 2(\beta_0 a_1 2 + \beta_1) \ln \mu_1 r + \underbrace{+\, a_2}_{\text{non-trivial part}} \Big\} \Big]$$

$$\begin{cases} a_1 = \frac{31}{9} C_A - \frac{20}{9} T_R N_f \\ a_2 = C_A^2 \left(\frac{4343}{162} + \underset{\boxed{4\pi^2}}{6\pi^2} - \frac{\pi^4}{4} + \frac{22}{3}\zeta(3)\right) + \ldots \end{cases}$$


+ ...

a_2 { • M. Peter '96
• Y. Schröder '98 }

a_1 { W. Fishler '77
A. Billoire '80 }

The Breit-Fermi Hamiltonian

$$W(\vec{r}) = -\frac{\vec{p}^{\,4}}{4m^3} + \frac{11\pi C_F \alpha_s}{3m^2}\delta(\vec{r}) - $$
$$- \frac{C_F \alpha_s}{2m^2}\left\{\frac{1}{r}, \vec{p}^{\,2}\right\} - \frac{C_A C_F \alpha_s^2}{2mr^2} .$$

$$W(\vec{r}) = -\frac{H_0^2}{4m} + \frac{3}{4m}\left\{V_0(r), H_0\right\} +$$
$$\frac{11\pi C_F \alpha_s}{3m^2}\delta(\vec{r}) - \left(\frac{5}{2} + \frac{C_A}{C_F}\right)\frac{V_0^2}{2m}$$

$$[H_0, i p_r] = \frac{4\pi\delta(\vec{r})}{m} - \frac{C_F \alpha_s}{r^2}$$

★ S-D part & matching.

We begin with introducing some notations. The cross section of the reaction $e^+e^- \to \gamma^* \to \bar{Q}Q$ is written as

$$\sigma_{e^+e^-\to QQ} = \sigma^{(0)}\left[1 + C_F\left(\frac{\alpha_s}{\pi}\right)\Delta^{(1)} + C_F\left(\frac{\alpha_s}{\pi}\right)^2\Delta^{(2)}\right], \qquad (1)$$

where $\alpha_s \ll v \ll 1 \Rightarrow$ no resumation

$$\sigma^{(0)}(s) = \frac{4}{3}\frac{\alpha^2}{s} N_c e_Q^2 \frac{\beta(3-\beta^2)}{2}, \qquad \beta = \sqrt{1 - \frac{4m^2}{s}}, \qquad (2)$$

$$\Delta^{(1)} = \frac{\pi^2}{2\beta} - 4 + \mathcal{O}(\beta). \qquad (3)$$

(∗)

$$\Delta_A^{(2)} = \frac{\pi^4}{12\beta^2} - 2\frac{\pi^2}{\beta} + \frac{\pi^4}{6} + \pi^2\left(-\frac{35}{18} - \frac{2}{3}\log\beta + \frac{4}{3}\log 2\right) + \frac{39}{4} - \zeta_3, \qquad (6)$$

← Hoang '97 + ..

$$\Delta_{NA}^{(2)} = \frac{\pi^2}{\beta}\left(\frac{31}{72} - \frac{11}{12}\log 2\beta\right) + \pi^2\left(\frac{179}{72} - \log\beta - \frac{8}{3}\log 2\right) - \frac{151}{36} - \frac{13}{2}\zeta_3, \qquad (7)$$

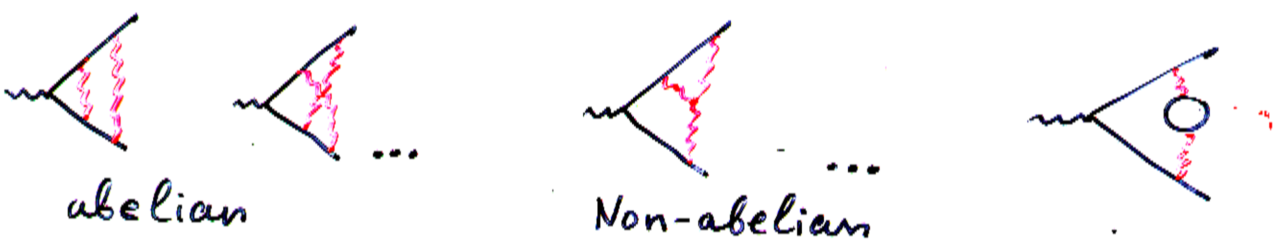
← Melnikov + ... Czarnecky

$$\Delta_L^{(2)} = \frac{\pi^2}{\beta}\left(\frac{1}{3}\log 2\beta - \frac{5}{18}\right) + \frac{11}{9},$$

← Kühn et al '96 + ...

$$\Delta_H^{(2)} = \frac{44}{9} - \frac{4\pi^2}{9}. \qquad (9)$$

← Abalmasov '95 + ...



$R = C(\mu_f) \cdot \mathrm{Im}\, G(0,0;E) \to C(r_0) \cdot \mathrm{Im}\, G(r_0, r_0; E)$

- we compare expansion of this formula with (∗) and fix up unknown coefficient $C(\mu_f)$
- terms $\frac{1}{\beta^2}$, $\frac{1}{\beta}$, $\log(\beta)$ are from $G(0,0)$
- terms without β belongs to $C(\mu_f)$

$$C(r) = 1 - \frac{C_F\alpha_s(\mu_h)}{\pi} + C_2(r)\Big(\frac{C_F\alpha_s(\mu_h)}{4\pi}\Big)^2, \tag{21}$$

$$C_2(r) = A_1\log(r/a) + A_2\log(m/\mu_h) + A_3 \tag{22}$$

with

$$A_1 = \pi^2(C_A + \frac{2C_F}{3}), \quad a = \frac{e^{2-\gamma_E}}{2m}, \quad A_2 = 2\beta_0, \tag{23}$$

$$A_3 = C_F C_2^A + C_A C_2^{NA} + T_R N_L C_2^L + T_R N_H C_2^H, \tag{24}$$

with

(Hoang Teubner)
(Melnikov Yelkovsky)

$$\begin{aligned} C_2^A &= \frac{39}{4} - \zeta_3 + \pi^2\left(\frac{4}{3}\ln 2 - \frac{35}{18}\right), \\ C_2^{NA} &= -\frac{151}{36} - \frac{13}{2}\zeta_3 + \pi^2\left(\frac{179}{72} - \frac{8}{3}\ln 2\right), \\ C_2^H &= \frac{44}{9} - \frac{4}{9}\pi^2, \\ C_2^L &= \frac{11}{9}. \end{aligned} \tag{25}$$

(O.Ya)

$$\frac{r}{a} \to \frac{m}{\mu_f}$$

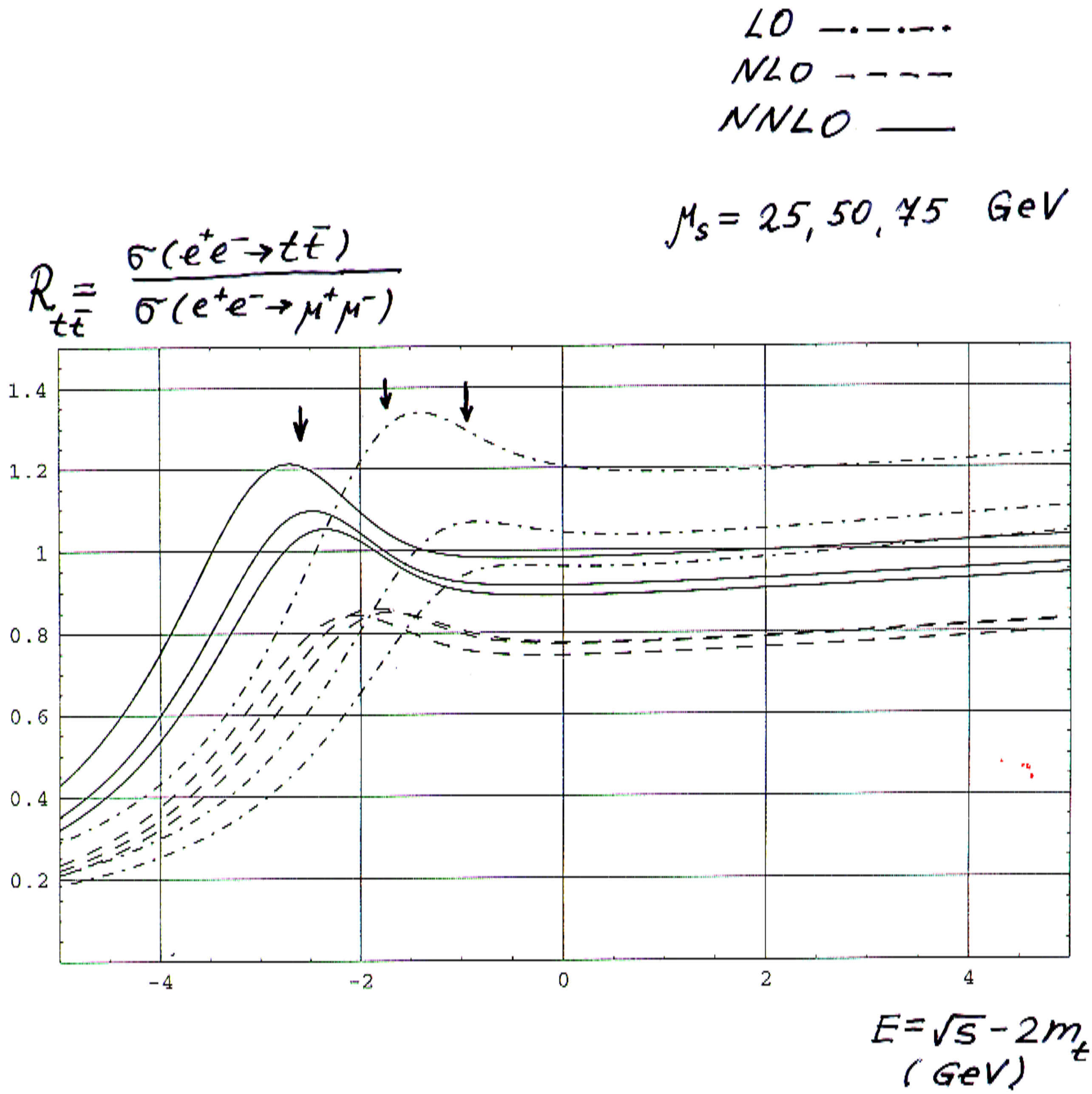
Final result ('98)

$$R(E) = \Big(1 - \frac{C_F\alpha_s}{\pi} + C_2(\mu_f)\Big(\frac{\alpha_s C_F}{4\pi}\Big)^2\Big)\frac{8\pi}{m^2}\,\mathrm{Im}\Big(1 - \frac{4E}{3m}\Big)G^{NNLO}(\mu_f/E)$$

in agreement with

A. Hoang, T. Teubner '98

K. Melnikov, A. Yelkhovsky '98



LO
NLO
NNLO
$\mu_s = 25, 50, 75$ GeV
$R_{t\bar{t}} = \frac{\sigma(e^+e^- \to t\bar{t})}{\sigma(e^+e^- \to \mu^+\mu^-)}$
1.4
1.2
1
0.8
0.6
0.4
0.2
-4
-2
0
2
4
$E = \sqrt{s} - 2m_t$
(GeV)

LO —·—·—·

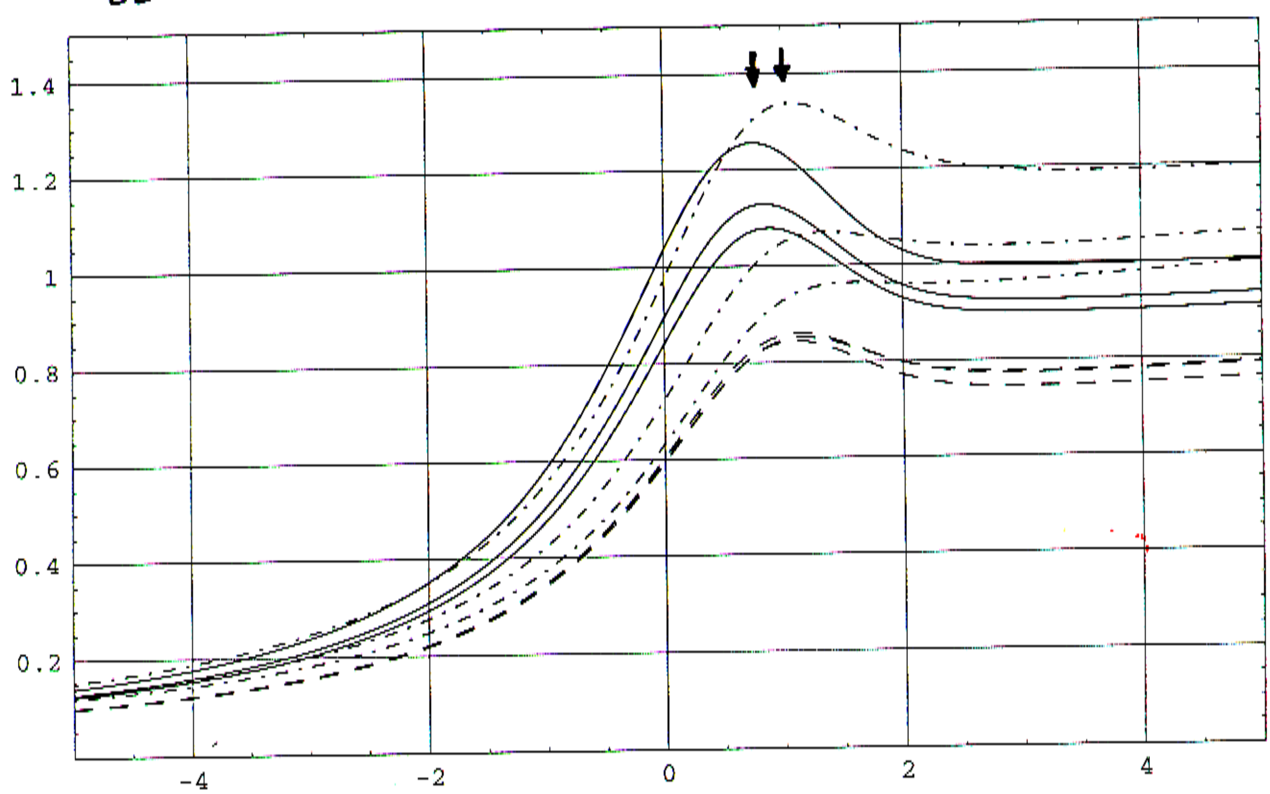
NLO - - - -

NNLO ———

$\mu_s = 25, 50, 75$ GeV

$\underline{m_{PS} = m_{PS}(\mu = 20)}$

$R_{t\bar{t}}$



$E = \sqrt{s} - 2\,m_{PS}$

- 1S peak is stable : shifts $\simeq$ 100-200 MeV (+)
- no stability in $R_{t\bar{t}}$ (−)

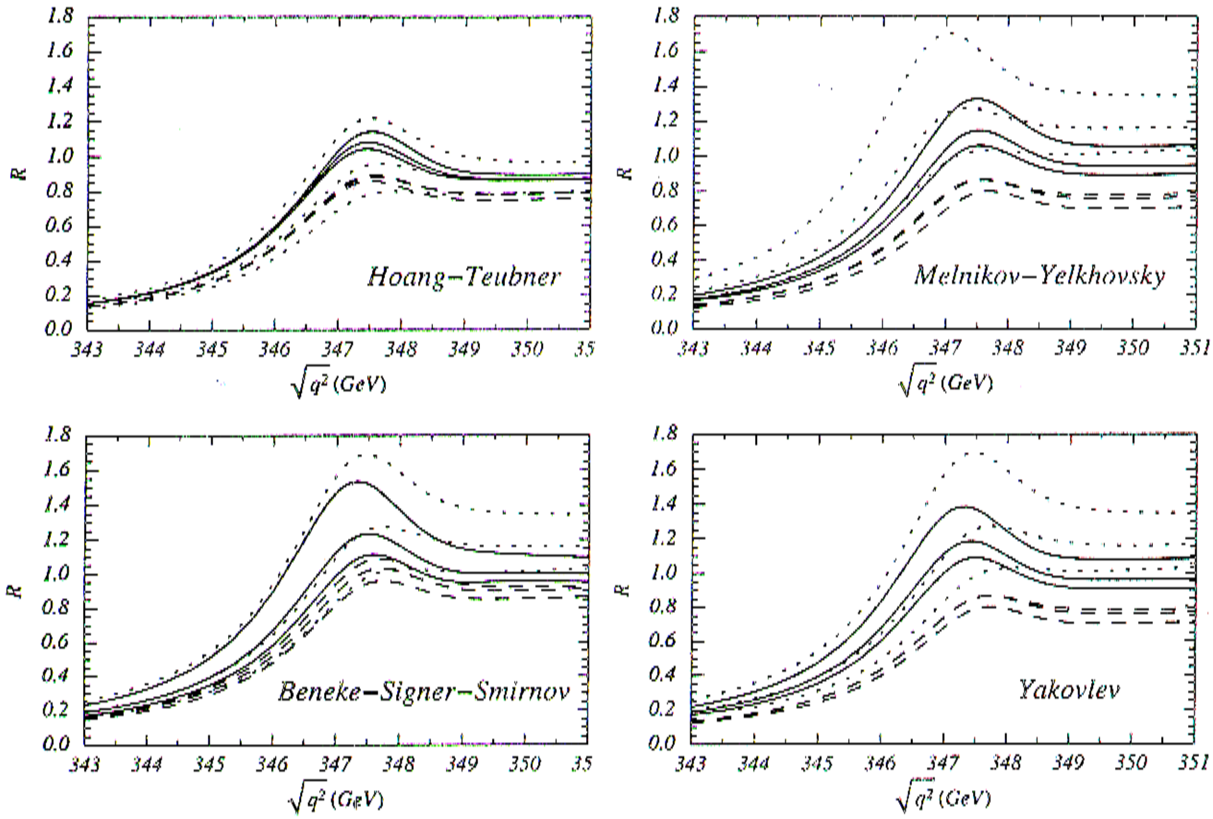


Figure 2: The total normalised photon-induced $t\bar{t}$ cross section R at the LC versus the c.m. energy in the threshold regime at LO (dotted curves), NLO (dashed) and NNLO (solid). Hoang–Teubner used the 1S mass scheme with $m_t^{1S} = 173.68$ GeV, Melnikov–Yelkhovsky the kinetic mass at 15 GeV with $m_{t,15\,\mathrm{GeV}}^{\mathrm{kin}} = 173.10$ GeV, and Beneke–Signer–Smirnov and Yakovlev the PS mass at 20 GeV with $m_{t,20\,\mathrm{GeV}}^{\mathrm{PS}} = 173.30$ GeV. The plots have been generated from results provided by the groups Hoang–Teubner (HT), Melnikov–Yelkhovsky (MY) and Beneke–Signer–Smirnov (BSS) and Yakovlev.

§ 4 Heavy Quark Mass Definitions

① quark masses are INPUT of SM

② although, it is widely accepted that quark mass is generated by HIGGS mechanism

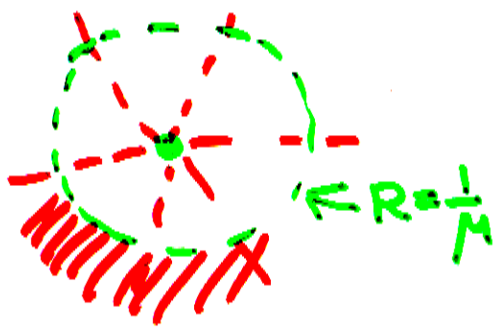
$$g_y h \bar{\Psi} \Psi \rightarrow \underbrace{\langle h \rangle g_y}_{\equiv m_Q} \bar{\Psi} \Psi$$

m_Q can not be calculated from S.M. (unfortunately)

③ m_Q has to be determined from comparison of theoretical prediction / exp. data

THRESHOLD MASSES

The Idea is general:
to take a pole mass and cut off
„gluon cloud" (or soft part, soft QCD)



$$m = m_{pole} - \delta m$$

$$\delta m = 4\pi\alpha_s C_F \int_0^{\mu} \frac{d\vec{k}}{(2\pi)} \frac{1}{2|\vec{k}|^2}$$

Zoo of definitions (differ in realization)

1) LS (Low scale) Shifman, Bigi, Vainshtein, Uraltsev ('94)

2) PS (potentially subtracted)
M. Beneke ('98)

3) 1S $\approx \frac{1}{2}$ perturb-ve constituent mass of 1S quarkonium state m_{1S} (Hoang, Teubner '99)

$1S = \frac{m_Q}{m_{\bar{Q}}}$ + nonpert. corr.

not really "running" mass.

4) $\overline{PS}$ → extension of PS, but includes 1. RECOIL. 2. RADIATIV.
Lo.Ye., Groote] 2000

* NO UNIQUE DEFINITION EXISTS!

- Because of the quark can not be observed as a free particle (electron), THE QUARK MASS is a PURE THEORETICAL DEFINITION/NOTION DEPENDS ON CONCEPT ADOPTED FOR DEFINITION.

Popular definitions

1. Pole mass: $\dfrac{i}{\not p - m - \Sigma(\not p)}$

Solution of: $\underline{\not p - m - \Sigma(\not p) = 0}$

2. $\overline{MS}$ mass (minimally subtracted)

$\delta m_p =$  $+$  $+ \ldots - \left[\frac{1}{\varepsilon}\right]^n \alpha_s^k$

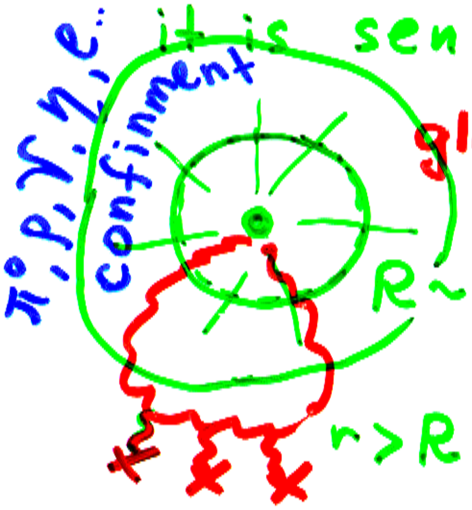
$$\boxed{m_{\overline{MS}} = m_p + \delta m}$$

m_{pole} & $m_{\overline{MS}}$ are NOT adequate masses for top quark threshold analysis!

<u>Why?</u>

1) m_{pole} has an ambiguity in QCD

i.e.: it is sensitive to non-perturbative QCD



$R \sim \frac{1}{\Lambda_{QCD}} \sim 1\,fm$

$r > R$ strongly non pert. regime

* in practice it gives LARGE QCD correction $\simeq \alpha_s^n \cdot \beta_0^n\, \underline{n!}$

renormalons

2) $\overline{MS}$ mass

is Eucledean mass → appropriate for VIRTUAL quark / $\delta m \simeq 7\,GeV$

• DESTROYS non-relativistic regime.

§ PS and $\overline{\text{PS}}$ masses

1) Beneke '98
2) O. Ya, Groote 2000

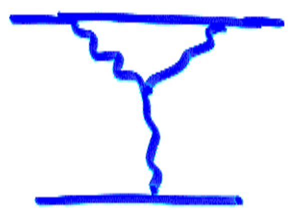
What is PS mass? Why do we need $\overline{\text{PS}}$?

* PS mass is invented to cancel renormalons in observables

$$\underbrace{m_{pole} - \delta m}_{= m_{PS}} \quad \text{with} \quad \delta m = \frac{1}{2}\int V(k) \frac{d\vec{k}}{(2\pi)^3}$$

* $\overline{\text{PS}}$ is a generalization of PS mass on $\frac{1}{m}$ (and higher) order

Renormalon at $\frac{1}{m}$:

$$V_{NA}(r) \simeq \int d\vec{k}$$  $$= \int d\vec{k}\, \frac{\pi^2 \alpha_s^2(k) C_A}{m\,|k|}$$

$$\simeq \frac{1}{m} \sum_n^{\infty} (\alpha_s \beta_0)^n \cdot n!$$

This divergent series, $\sim n!$, indicates a problem!

* $\overline{\text{PS}}$ mass is free from any renormalons at $\frac{1}{m}$ ....

(PS - causes a problem at $1/m$)

S.Eq. $\left[-\frac{\Delta}{m} + \underbrace{V(r) + 2m} - \sqrt{s}\right] G(r) = \delta(r)$

self energy

Σ

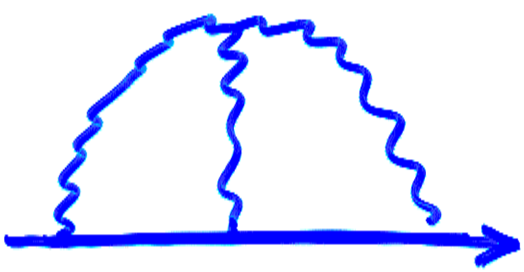
$= -\frac{1}{2}\int \frac{d\vec{k}}{(2\pi)^3}$ 

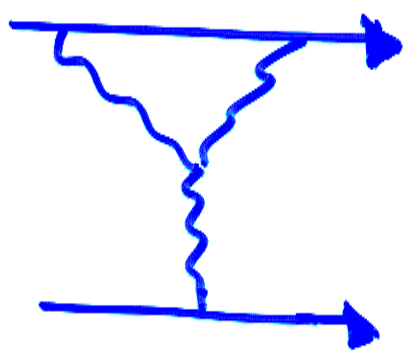
$V(k)$

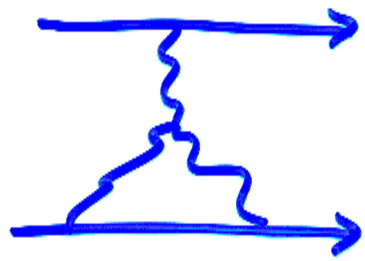
$$\Sigma = \frac{1}{2}\int V(k)\, \frac{d\vec{k}}{(2\pi)^3}$$

---

at $\frac{1}{m}$ order



$= -\frac{1}{2}\int$ 

$+$ 

We now impose the restriction $|\vec{k}_1| < \mu_f$ on the integrand to simplify it and obtain

$$\Sigma^{6b}_{\rm soft1} = \frac{g_s^4 C_F C_A}{4m} \int^{\mu_f} \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \frac{\vec{k}_1^2 \vec{k}_2^2 - (\vec{k}_1 \vec{k}_2)^2}{\vec{k}_1^2 (\vec{k}_2^2)^2 (\vec{k}_1 - \vec{k}_2)^2}. \tag{27}$$

The integral over the space components of k_2 can be easily done by executing the angular integration followed by the radial integration. We obtain

$$\int \frac{d^3k_2}{(2\pi)^3} \frac{\vec{k}_1^2 \vec{k}_2^2 - (\vec{k}_1 \vec{k}_2)^2}{\vec{k}_1^2 (\vec{k}_2^2)^2 (\vec{k}_1 - \vec{k}_2)^2} = \frac{1}{16|\vec{k}_1|} \tag{28}$$

and therefore, finally

$$\Sigma^{6b}_{\rm soft1} = \frac{\alpha_s^2 C_F C_A}{16m} \mu_f^2. \tag{29}$$

Symmetry considerations show that $\Sigma^{6b}_{\rm soft2}$ gives exactly the same contribution. As mentioned before, there are no other non-abelian contributions, therefore we obtain

$$\Sigma^{NA}_{\rm soft} = \frac{\alpha_s^2 C_F C_A}{8m} \mu_f^2. \tag{30}$$

This result has been anticipated, too, to be minus one half of the non-abelian correction to the QCD Coulomb potential, which is known in the literature (see for example Refs. [39, 51]),

$$\Sigma^{NA}_{\rm soft} = -\frac{1}{2} \int^{\mu_f} \frac{d^3k}{(2\pi)^3} \left\{ -\frac{\pi^2 \alpha_s^2 C_F C_A}{m|\vec{k}|} \right\} = \frac{\alpha_s^2 C_F C_A}{8m} \mu_f^2. \tag{31}$$

This calculation concludes the considerations of the two-loop diagrams shown in Fig. 3.

3.4 Our final result

Summarizing all contribution up to NNLO accuracy, we obtain

$$m_{\overline{\rm PS}}(\mu_f) - m = -\frac{\alpha_s(\mu) C_F}{\pi} \mu_f \left\{ 1 + C_0' \frac{\mu_f}{m} + C_0'' \frac{\mu_f^2}{m^2} + \frac{\alpha_s(\mu)}{4\pi} \left(C_1 + C_1' \frac{\mu_f}{m} \right) + C_2 \left(\frac{\alpha_s(\mu)}{4\pi} \right)^2 \right\} \tag{32}$$

where m is the pole mass, μ is the renormalization scale, μ_f is the factorization scale, and

$$\begin{aligned}
C_0 &= 1, \qquad C_0' = 0, \qquad C_0'' = -\frac{1}{2}, \\
C_1 &= a_1 - 2\beta_0 \ln\left(\frac{\mu_f}{\mu}\right), \qquad C_1' = C_A \frac{\pi^2}{2}, \\
C_2 &= a_2 - 2(2a_1\beta_0 + \beta_1)\left(\ln\left(\frac{\mu_f}{\mu}\right) - 1\right) + 4\beta_0^2 \left(\ln^2\left(\frac{\mu_f}{\mu}\right) - 2\ln\left(\frac{\mu_f}{\mu}\right) + 2\right).
\end{aligned} \tag{33}$$

The constants a_1, a_2, β_0, and β_1 are given in Appendix A. The coefficients C_1 and C_2 have been derived in Ref. [29] by using known corrections to the QCD potential. In this work we have derived the coefficients C_0', C_0'', and C_1'. Note that our result can be represented in a condensed form as

$$m_{\overline{\rm PS}}(\mu_f) - m = -\frac{1}{2} \int^{\mu_f} \frac{d^3k}{(2\pi)^3} \left(V_C(|\vec{k}|) + V_R(|\vec{k}|) + V_{NA}(|\vec{k}|) \right) \tag{34}$$

where the first term V_C is the static Coulomb potential, V_R is the relativistic correction (which is related to Breit-Fermi potential but does not coincide with it), and V_{NA} is the non-abelian correction.

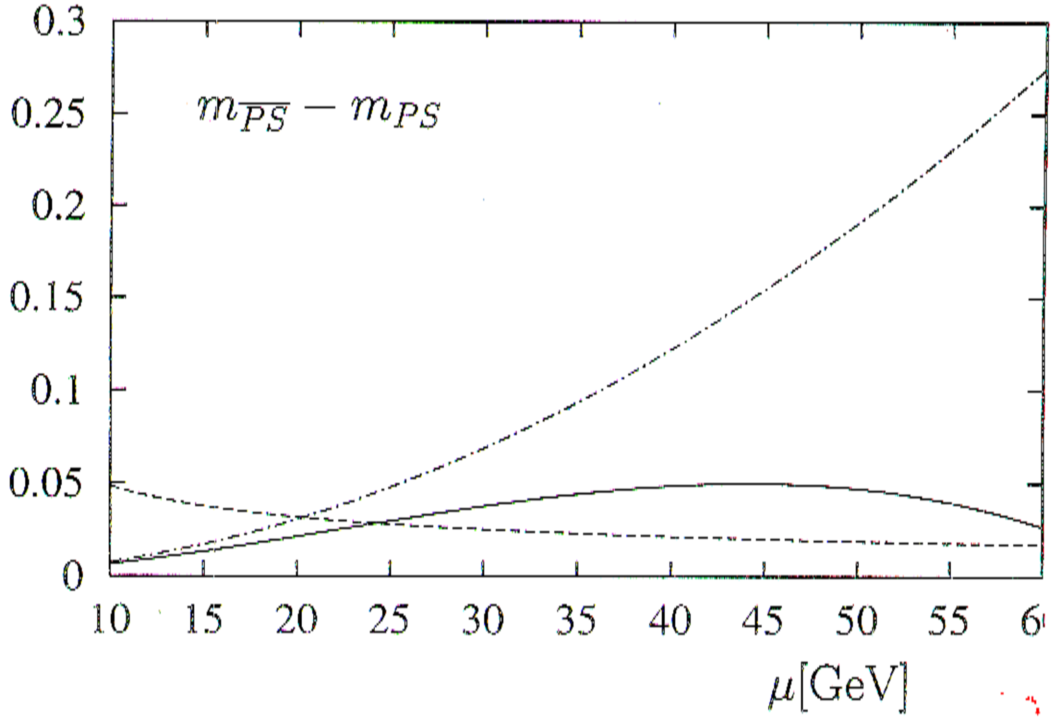
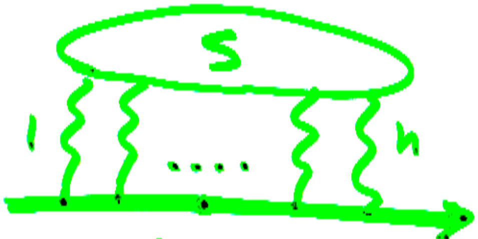


Figure 11: The difference between $\overline{PS}$ and PS mass (in GeV) as a function of factorization scale μ (solid line). Only non-abelian part of the difference between $\overline{PS}$ and PS mass (dot-dashed line). The dependence of $m_{\overline{PS}} - m_{PS}$ as a function of the normalization scale (dashed line).

$\overline{PS}$ mass (O. Ya. S. Groote 2000)

$$m_{\overline{PS}} = m_{pole} - \delta m(\mu)$$

$\delta m =$ soft part of self energy $\equiv$

$\sum_n$ 

$p^2 = m^2$ $\quad \prod_n \frac{d^4 p_n}{(2\pi)^4}$

$\pi\delta(p_k^2 - m^2)$

$$\frac{1}{x+i0} = P\frac{1}{x} - i\pi\delta(x)$$

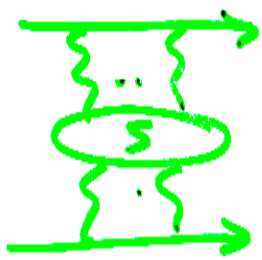
no IR contr.

$\delta m =$ 

$\frac{1}{2p_0^k} \frac{d\vec{p}_k}{(2\pi)^3} \ldots$

$$\delta m(\mu) = \frac{1}{2}\int \frac{d\vec{p}}{(2\pi)^3} V(\vec{p})$$

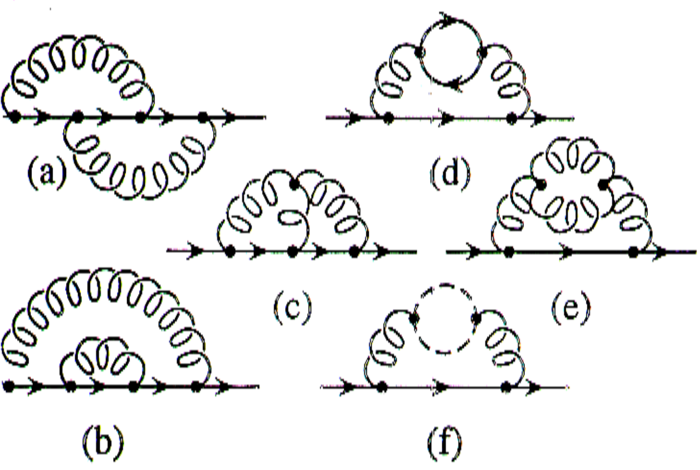
def.



coincides with
$Q\bar{Q}$ POTENTIAL

we have calculated:

1) recoil corr. 2) QCD[α_s^2] nonabelian and abelian ones.



e 3: Two-loop contributions to the quark self energy

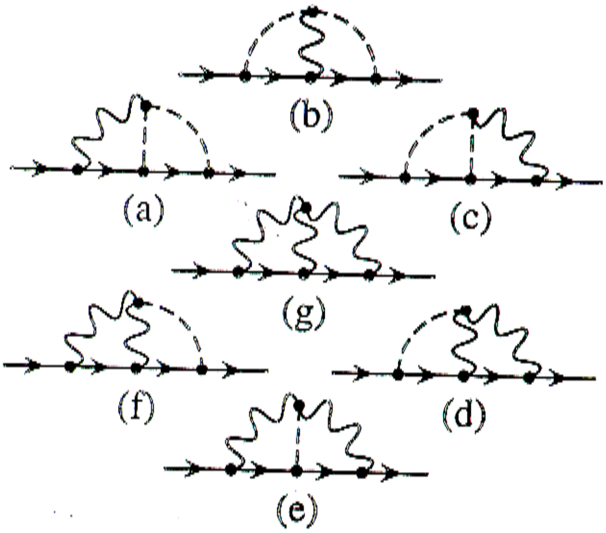


Figure 6: The non-abelian self energy diagram in Coulomb gauge; displayed are Coulomb gluons (wiggles) and transverse gluons (dashed lines)

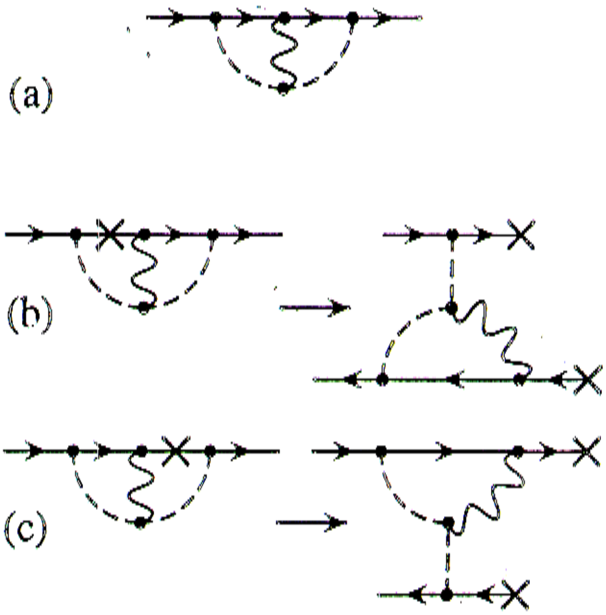


Figure 7: The soft part of the non-abelian diagram under consideration

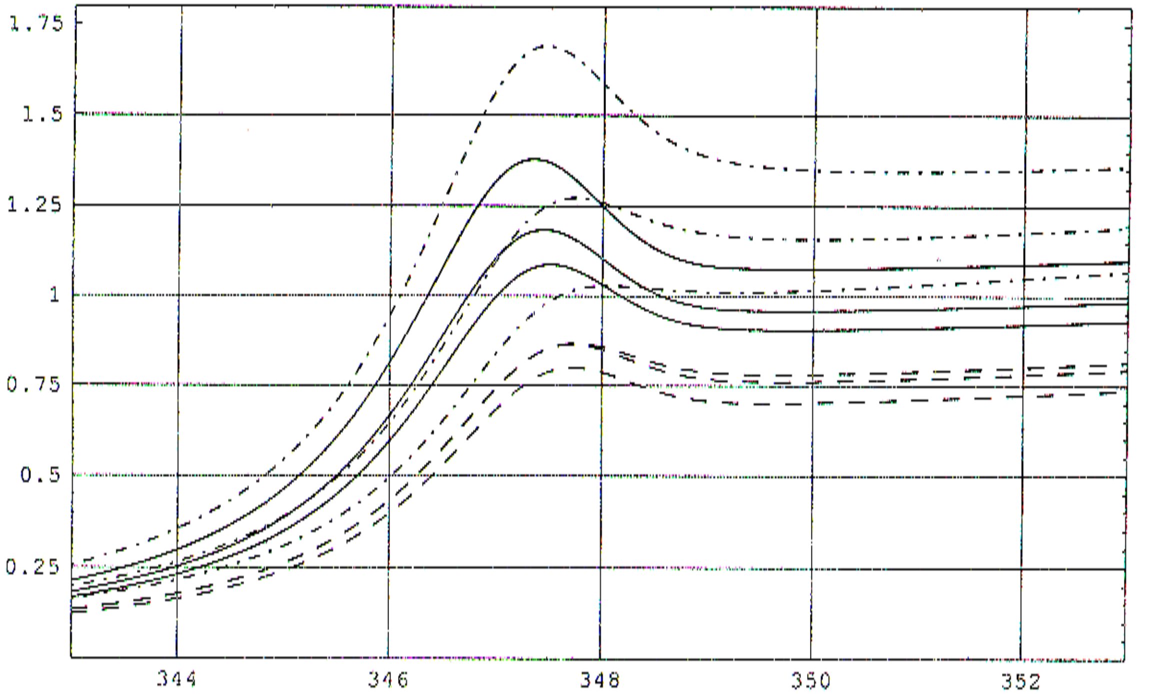
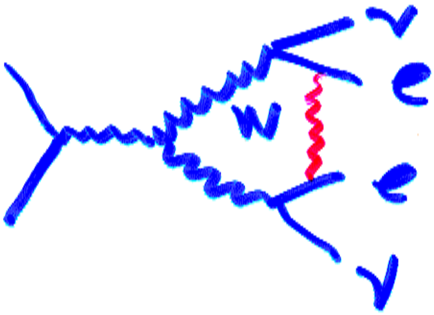


Figure 10: The scheme with $\overline{PS}$ mass. $R(e^+e^- \to t\bar{t})$ for the LO (dashed-dotted lines), NLO (dashed lines), NNLO (solid lines) approximation as a function of c.m. energy. In all cases we use $\overline{PS}$ mass m_{PS} =173.30 GeV,Γ_t =1.43 GeV, $\alpha_s(m_Z)$ =0.119 but different values of the soft scale $\mu_s = 15, 30, 60$ GeV .

Non-factorizable QCD

it was ignored (difficult) for a long time! For QED:

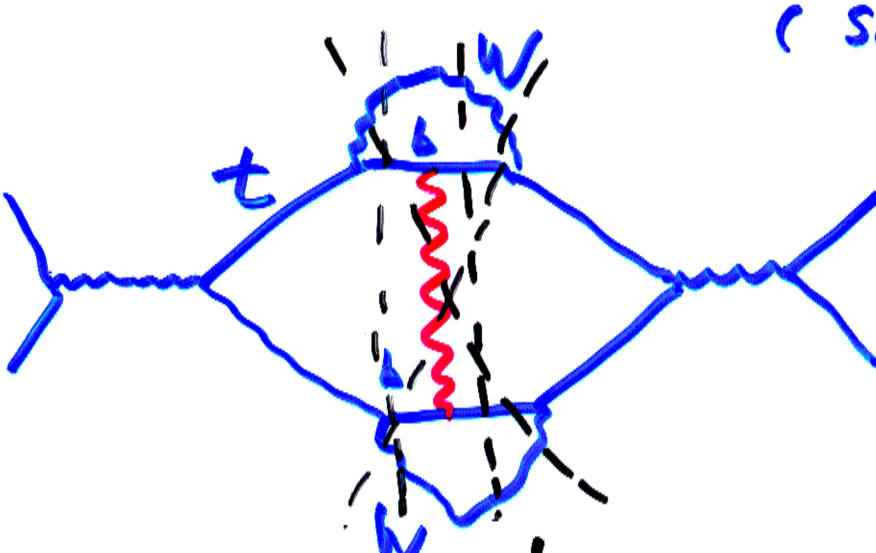


– ? till 1994–1996!

1-loop

cancellation in inclusive cross-section

(V. Fadin, Khoze '94) (O.Ya. Melnikov '94 [redacted])
(Sumino '94)



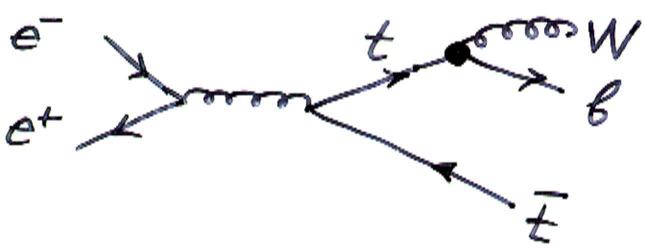
sum over cuts

$$= \int I_1 + I_2 + I_3 + I_4 = 0$$

$$= \alpha_s \cdot 0 + \alpha_s \frac{\Gamma}{M}$$

Non-factorizable Corrections

- consider $e^+e^- \to t\bar{t} \to Wb + \bar{t}$ with (t-unstable, $\bar{t}$-stable)



$$= \frac{A}{D} + B \qquad D = p^2 - m^2 + im\Gamma \to \mathcal{O}(\Gamma m) \to 0.$$

- additional coupling e (vertex $t \to Wb$) does not give suppression, is compensated by t-propagator

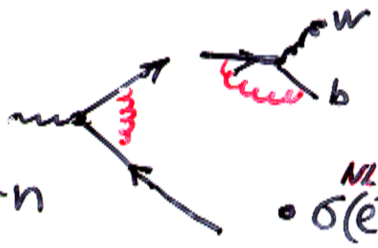
- Cross section

$$\frac{d\sigma}{dp^2} = \underbrace{\sigma(e^+e^- \to t\bar{t})}_{\text{production}} \underbrace{\frac{1}{\pi} \frac{m\Gamma}{(p^2-m^2)^2 + m^2\Gamma^2}} \underbrace{Br(t \to Wb)}_{\text{decay}}$$

- Breit-Wigner distribution $\to \delta(p^2 - m^2)$, $\Gamma \to 0$.
- p^2 - invariant mass of (Wb).

- the production and decay processes are FACTORIZED
- QCD corrections:

i) standard procedure



★ factorizable QCD corr-n

- $\sigma_{NLO}(e^+e^- \to t\bar{t})$ and
- $\Gamma_{NLO}(t \to Wb)$

ii) Non-fact. NEW

- PRODUCTION and DECAY STAGES ARE INVOLVED

- Consider in more detail:

Only <u>soft gluons</u> contribute, hard gluons with $\underline{\omega \gg \Gamma_t}$ shift t-prop. from resonance

$\Rightarrow$  $\sim \alpha_s \cdot \left(\frac{\Gamma_t}{\omega}\right) \cdot M_{BORN}$

$|\omega| \lesssim \Gamma_t$ ↯

- Khoze, Fadin
- Melnikov, Yakovlev '94

- <u>Result:</u>

$$\left(\frac{d\sigma}{dp^2}\right)^{NLO} = \left(\frac{d\sigma}{dp^2}\right)^{LO} \cdot \left[1 - \frac{(1-\beta)^2}{2\beta}\,\underline{\alpha_s C_F} \cdot \arctan\left(\frac{p^2-m^2}{m\Gamma}\right)\right]$$

$\Delta =$

1) <u>inclusive zero</u>

$$\int \left(\frac{d\sigma}{dp^2}\right)^{NLO} dp^2 = \sigma^{LO}$$

: • NO non-factorizable correction in $\underline{\sigma}$.

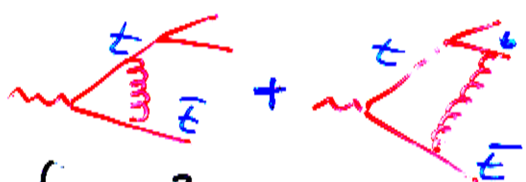
NF α_s – antisymmetric in $\Delta = \frac{p^2-m^2}{m}$ „OFF-SHELLNESS"

$$\int \frac{1}{\Delta^2+\Gamma^2} \cdot \arctan\left(\frac{\Delta}{\Gamma}\right) d\Delta$$

2) <u>High energy zero</u>

AT $s \gg m_t$ or $\beta \to 1$

$$\left(\frac{d\sigma}{dp^2}\right)^{NLO} = \left(\frac{d\sigma}{dp^2}\right)^{LO}$$

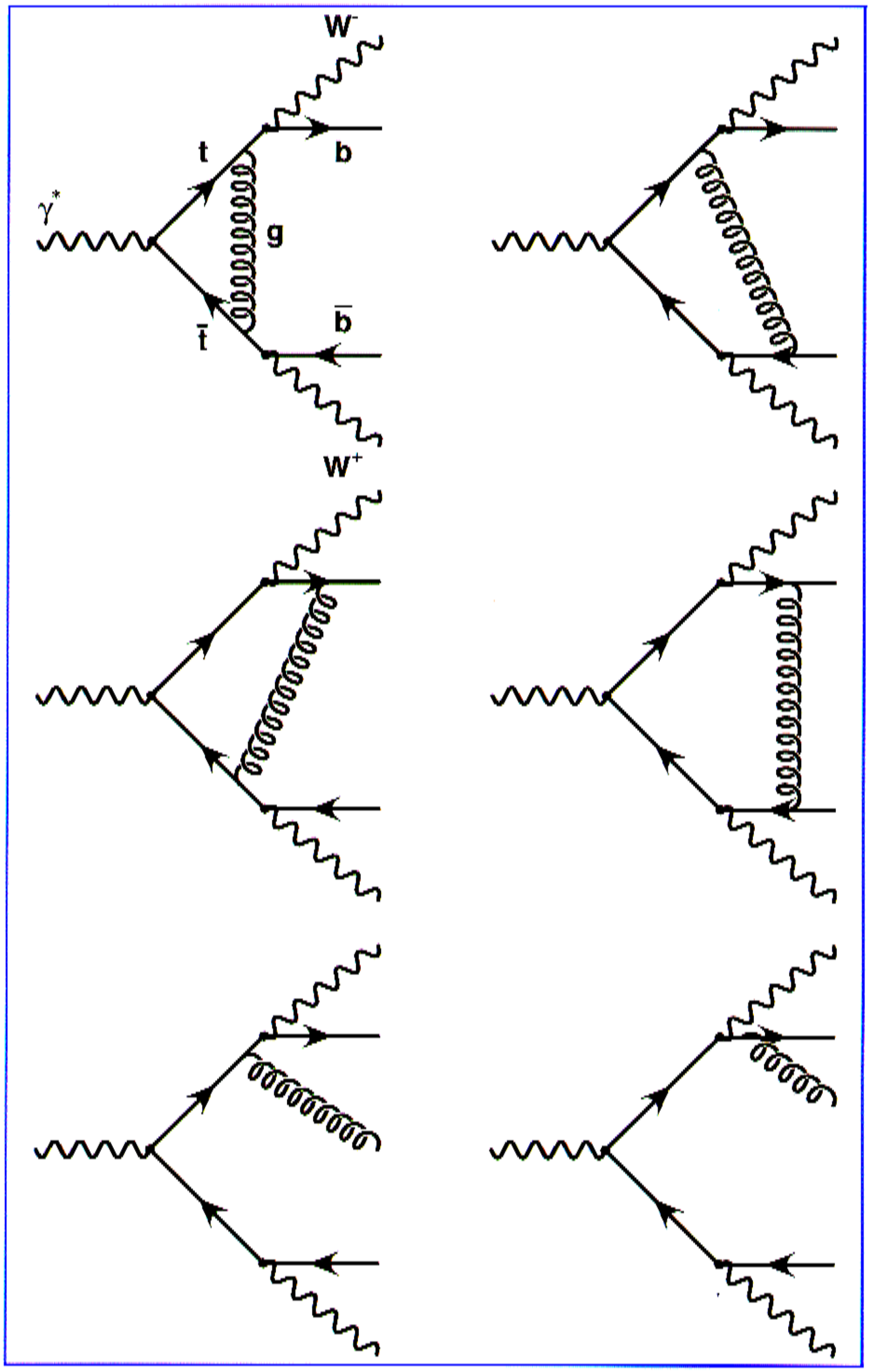


$$\frac{\alpha_s}{\beta}\left[1+\beta^2 \quad -2\beta\right]$$

$$2 \quad - 2$$

- NO Non-Fact. Corr-ns at $\beta \to 1$

Non-factorizable correction
to the reaction $e^+e^- \to t\bar{t} \to WbW\bar{b}$



Summary

- NNLO corrections have been calculated ⇒ large, but the origin is understood → soft QCD in the pole mass
- we suggested a new definition of mass of heavy quark: $m_{\overline{PS}}$ ↔ which is related to $m_{\overline{MS}}$ by α_s^n series which has well behavior (no $n!$ like for pole mass)
- extraction of $m_{\overline{PS}}$ with accuracy 0.1-0.2 GeV is possible! ⇒ $m_{\overline{MS}}$ as well!
- non-factorizable effects studied at one-loop only

★ they cancel in the inclusive σ

Prelimenary: it happens at all orders.