

# Toward Determination of $m_t$ at 50 MeV Accuracy

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## PLAN:

§ Motivation

§ New Results

§ Order Counting After Renormalon Cancellation

## § Motivation

At LCWS '99 (Sitges),

Significant theoretical progress reported:

- higher order calculations of threshold cross sections
- renormalon cancellation

Hoang, Teubner  
Melnikov, Kalinowski  
⋮

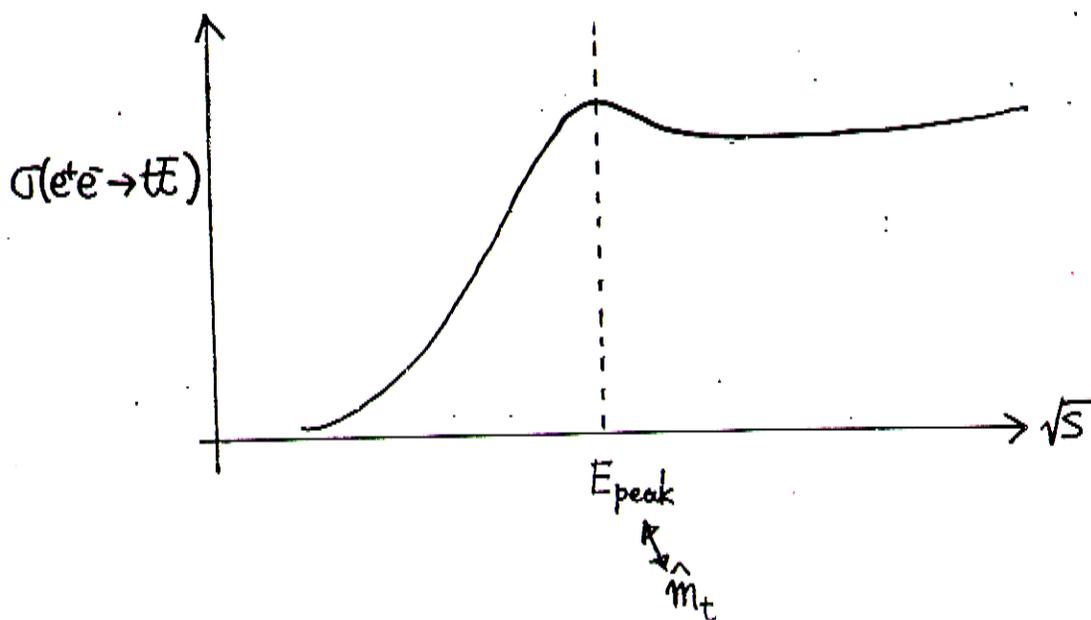
Hoang, Smith, Stegner, Wilkenbrock  
Beneke

⇒ Precise determination of  $\hat{m}_t \equiv m_{\overline{MS}}(m_{\overline{MS}})$

$$\delta \hat{m}_t \sim \begin{cases} 50 \text{ MeV} & \text{Exp. error (stat.)} \\ 150 - 200 \text{ MeV} & \text{Theor. uncertainty} \end{cases}$$

Peralta, Martinez, M.  
Cinabro

Top Theory WGr



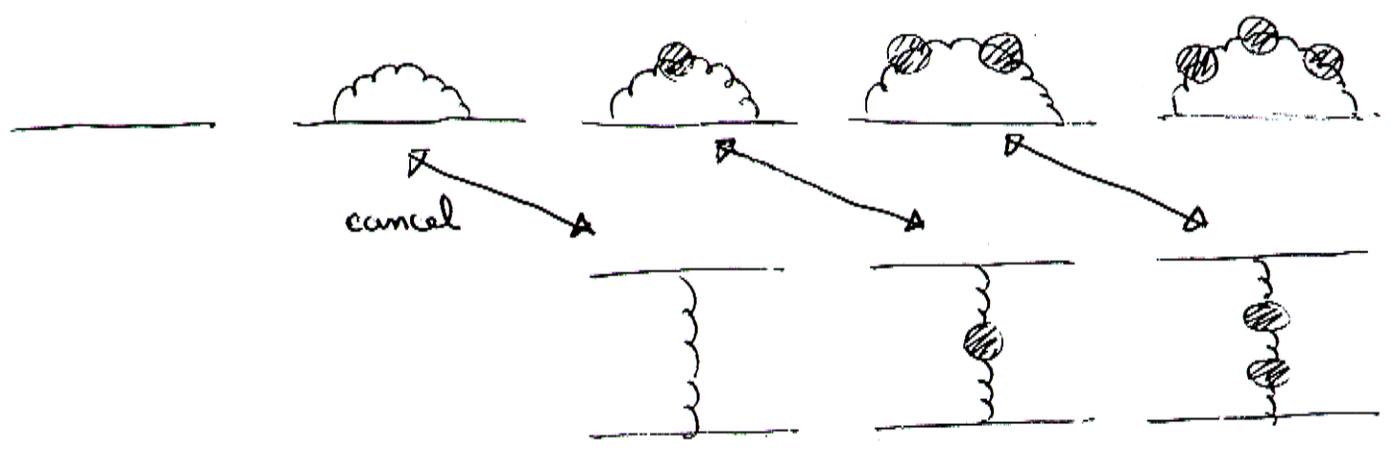
How to cancel renormalons in the quarkonium spectrum

Renormalon cancellation takes place between the terms of shifted orders:

$$2m_{\text{pole}} = 2m_{\overline{\text{MS}}} \left( 1 + \star \alpha_s + \star \alpha_s^2 + \star \alpha_s^3 + \dots \right)$$

$$E_{\text{binding}} = 2m_{\overline{\text{MS}}} \left( \dots + \star \alpha_s^2 + \star \alpha_s^3 + \dots \right)$$

Hoang, Ligeti, Manohar



$$\bullet - \left\langle G_F \frac{\alpha_s}{r} \right\rangle = -G_F \alpha_s \underbrace{\langle r^{-1} \rangle}_{\sim \alpha_s^{1/2}} \sim \alpha_s^{3/2} m$$

$$\bullet E_{\text{binding}} = 2m_{\overline{\text{MS}}} \sum_{n=2}^{\infty} \underbrace{P_n(\log \alpha_s)}_{\sim \alpha_s^{-1} \text{ for } n \gg 1} \alpha_s^n$$

"Toponium" 1S-state spectrum

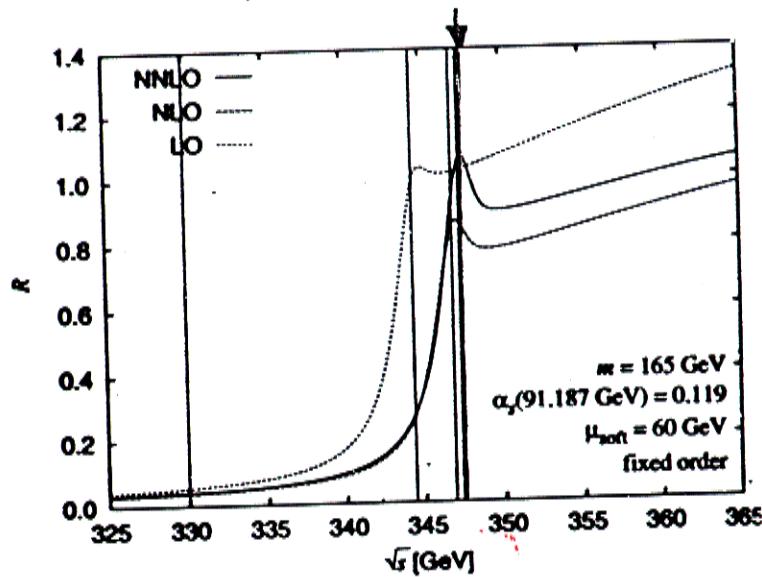
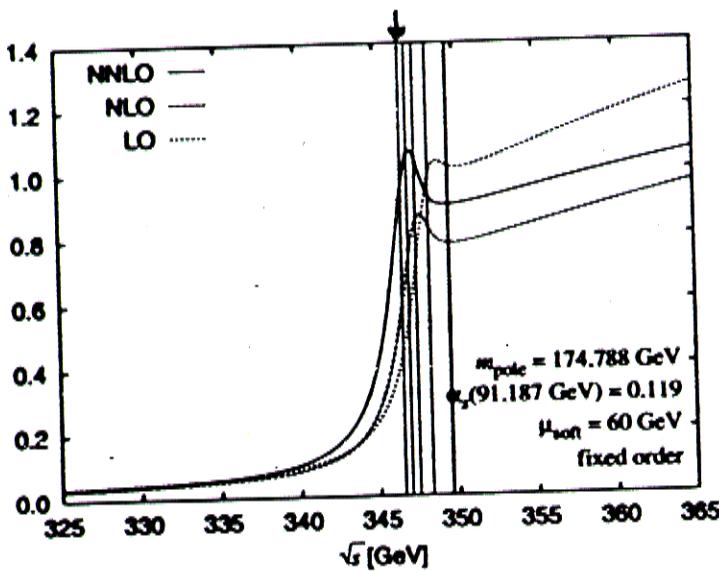
For  $\hat{m}_t = 165 \text{ GeV}$  ( $\Leftrightarrow m_{\text{pole}} = 174.79 \text{ GeV}$ )

expansion parameter =  $\alpha_s^{(5)}(\hat{m}) = 0.1092$

$M_{1S} = 2 \times (174.79 - 0.46 - 0.39 - 0.28 - 0.19^*) \text{ GeV}$  : Pole-mass scheme

$= 2 \times (165.00 + 7.21 + 1.24 + 0.22 + 0.052^*) \text{ GeV}$  :  $\overline{\text{MS}}$  scheme

Kiyo + Y.S.



Nagano + Y.S.

The present perturbative evaluation of  $M_{15}(m_{\text{PS}}, \alpha_s)$  has a "genuine accuracy" at  $\mathcal{O}(\alpha_s^3 m)$ .

Formally  $\mathcal{O}(\alpha_s^n m)$  term

$$= \underbrace{\text{"Leading renormalon part"}}_{V?} + \text{"genuine } \mathcal{O}(\alpha_s^n m) \text{ part"}$$

$\Lambda_{\text{QCD}}$

In order to achieve a "genuine  $\mathcal{O}(\alpha_s^4 m)$  accuracy", it is sufficient to calculate further

- (I)  $\mathcal{O}(\alpha_s^4 m)$  relation between  $m_{\text{pole}}$  and  $m_{\text{PS}}$   
 $\Rightarrow$  replaced by large- $\beta_0$  approx.
- (II)  $E_{\text{bin}}$  at  $\mathcal{O}(\alpha_s^5 m)$  in the large- $\beta_0$  approx.  
 $\Rightarrow$  computed

i.e. The full  $\mathcal{O}(\alpha_s^5 m)$  corrections are not necessary!

# Renormalon cancellation

Chetyrkin, Steinhauser  
Melnikov, Ritbergen

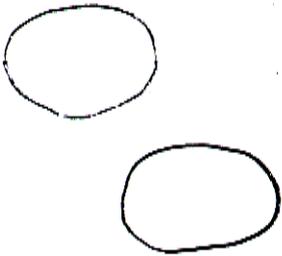
$$2m_{\text{pole}} = 2m_{\overline{\text{MS}}} (1 + \star \alpha_s + \star \alpha_s^2 + \star \alpha_s^3 + \star \alpha_s^4 + \star \alpha_s^5 + \dots)$$

$$E_{\text{bin}} = 2m_{\overline{\text{MS}}} ( \star \alpha_s^2 + \star \alpha_s^3 + \star \alpha_s^4 + \star \alpha_s^5 + \dots )$$

known

known

our calculation  
in large- $\beta_0$  approx.



## Consistency checks

### • Toponium $1S$

$$M_{1S} = 2 \times (165.00 + 7.21 + 1.24 + 0.22 + 0.052^*) \text{ GeV}$$

$\downarrow$   
0.22  
0.31

### • Bottomonium $1S$

$$M_{1S} = 2 \times (4.20 + 0.36 + 0.13 + 0.040 + 0.0051^*) \text{ GeV}$$

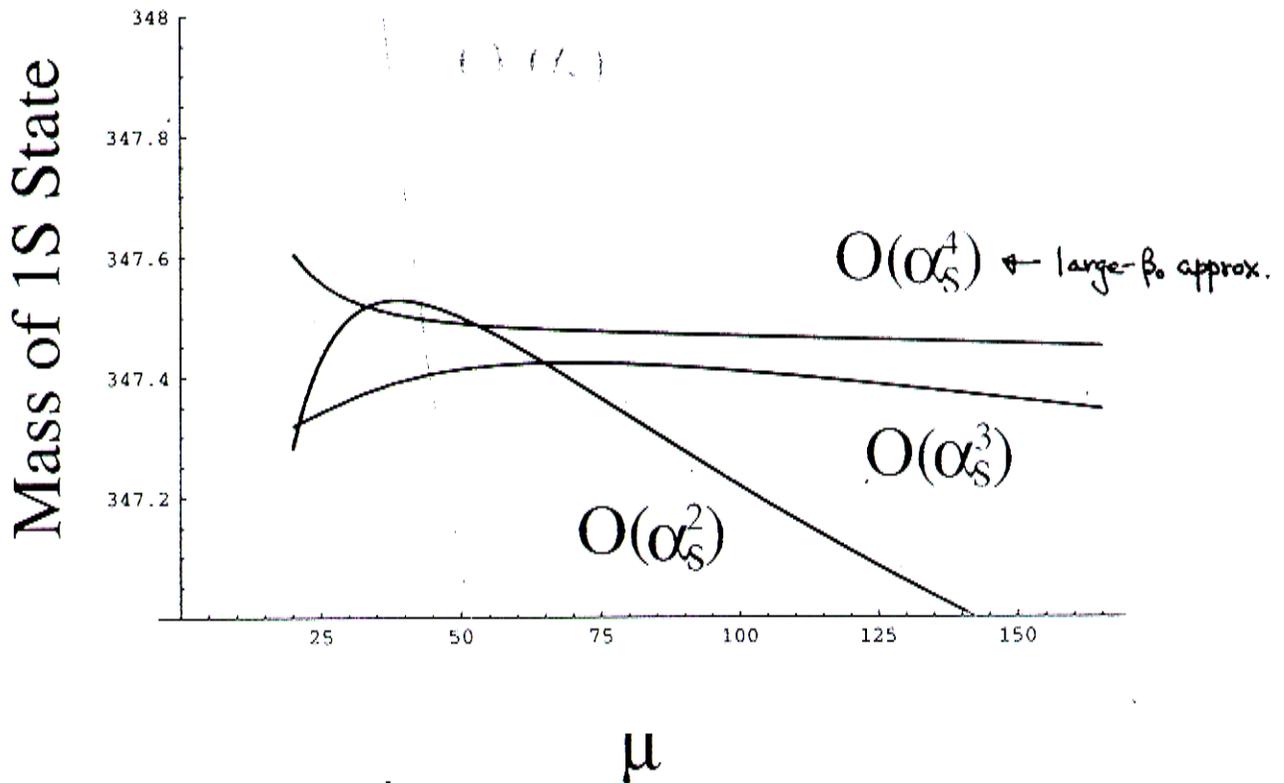
$\downarrow$   
0.043  
0.056

(1)  $\mathcal{O}(\alpha_s^4 m)$  term of  $E_{\text{bin}}$  is replaced by large- $\beta_0$  approx.

(2)  $\mathcal{O}(\alpha_s^3 m)$  term of  $m_{\text{pole}} \Leftrightarrow m_{\overline{\text{MS}}}$  relation is replaced by large- $\beta_0$  approx.

## Scale dependences

$$M_{1S} = M_{1S}(\alpha_s(\mu), \hat{m}_t) \quad ; \quad \hat{m}_t \equiv m_{\overline{MS}}(m_{\overline{MS}})$$



## Other necessary calculations to achieve 50 MeV accuracy

- 4-loop relation  $m_{\text{pole}} \leftrightarrow m_{\overline{MS}}$
- Final-state interaction corrections to  $\sigma(e^+e^- \rightarrow t\bar{t})$  at NNLO
- EW corrections (Non-resonant / 1-loop)