

# Precise Predictions for MSSM Higgs boson production at $e^+e^-$ colliders

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1. Introduction
2. Higgs Production at  $e^+e^-$  colliders
3. Numerical results for  $e^+e^- \rightarrow hZ, hA$
4. Conclusions

# 1. Introduction:

## Higgs sector of the MSSM:

MSSM: Enlarged Higgs sector:

Two Higgs doublets

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix}$$
$$H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}$$

Higgs potential:

$$V = m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.})$$
$$+ \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2$$

Physical states:  $h^0, H^0, A^0, H^\pm$

Input parameters:

$$\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = -m_{12}^2 (\tan \beta + \cot \beta)$$

Diagonalization of tree-level mass matrix

$\Rightarrow m_h, m_H, \text{ mixing angle } \alpha$

Tree-level result for  $m_h, m_H$ :

$$m_{H,h}^2 = \frac{1}{2} \left[ M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta} \right]$$

$\Rightarrow m_h \leq M_Z$  at tree level

Large radiative corrections:

Yukawa couplings:  $\frac{e m_t}{2M_W s_w}, \frac{e m_t^2}{M_W s_w}, \dots$

$\Rightarrow$  Dominant one-loop corrections:  $G_\mu m_t^4 \ln \left( \frac{m_t^2}{m_{\tilde{t}_1} m_{\tilde{t}_2}} \right)$

$\tilde{t}$  sector:

$$M_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + DT_1 & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + DT_2 \end{pmatrix}$$

$X_t = A_t - \mu \cot \beta$ ; large mixing possible

Input parameters:  $M_{\tilde{t}_L}, M_{\tilde{t}_R}, X_t$

often used:  $M_{\text{SUSY}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$

$\Rightarrow$  Physical parameters:  $m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}$

At LC with  $\sqrt{s} = 500 - 1000$  GeV:

⇒ Need prediction for Higgs production cross sections and branching ratios at 1% level

Electroweak corrections at  $\text{LEP} \leftrightarrow \text{LC}$ :

At LEP1:

Z-boson resonance ⇒ description in terms of universal corrections ( $\Delta\alpha$ ,  $\Delta\rho$ , ...), effective couplings (propagator-type corrections)

⇒ Born-type structure

Genuine box contributions negligible  
(Partially still true at LEP2 energies)

At LC with  $\sqrt{s} = 500 - 1000$  GeV:

Propagator-type corrections no longer dominant

⇒ potentially large box corrections,  
Sudakov-logs, ...

Finite width effects:

→ need to include non-resonant contributions

→  $2 \rightarrow 4$ ,  $2 \rightarrow 6$ , ... processes

## Remaining theoretical uncertainties in MSSM Higgs calculations:

→ From unknown higher-order corrections

Present estimate:  $\Delta m_h^{\text{theo}} \approx 3 \text{ GeV}$

→ From uncertainties in input parameters

$m_t, \dots, M_A, \tan \beta, m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{g}}, \dots$

Present situation:

$$\Delta m_t \approx 5 \text{ GeV} \Rightarrow \Delta m_h^{\text{theo}} \approx 5 \text{ GeV}$$

At LC:

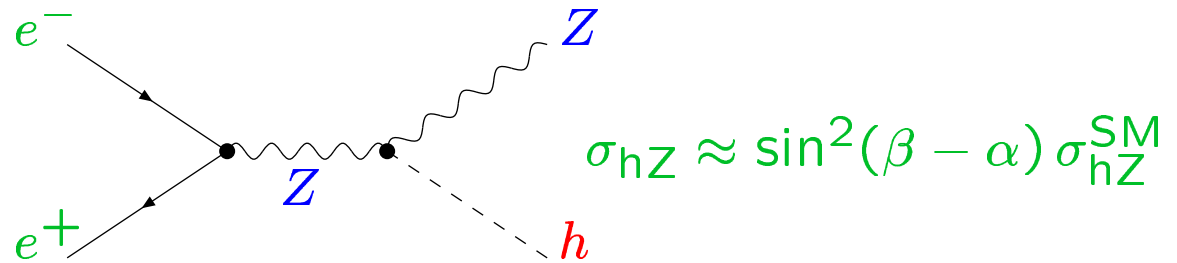
$$\Delta m_t = 0.2 \text{ GeV} \Rightarrow \Delta m_h^{\text{theo}} \approx 0.2 \text{ GeV}$$

$$\Delta m_h^{\text{exp}} = 0.05 \text{ GeV}$$

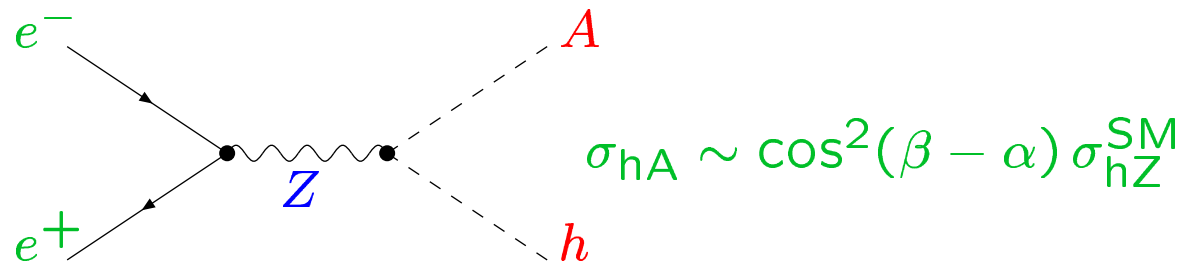
⇒ High-precision measurement of  $m_t$  important!

## 2. Higgs production at $e^+e^-$ colliders:

SM like:  $e^+e^- \rightarrow Zh$



Complementary:  $e^+e^- \rightarrow hA$



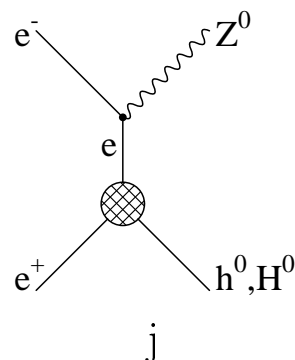
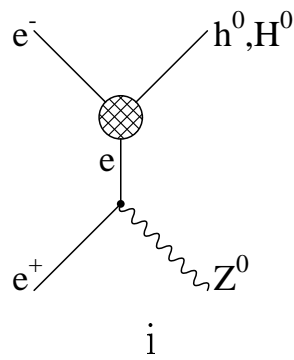
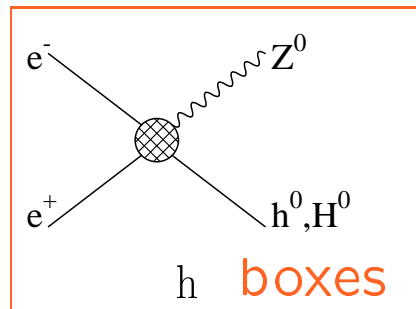
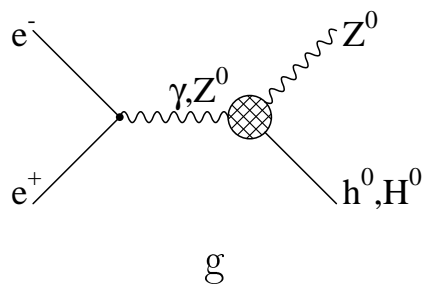
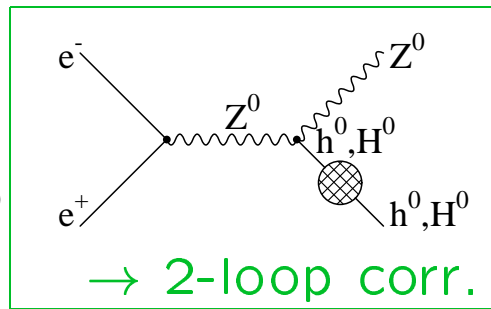
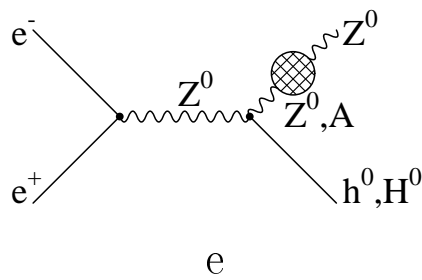
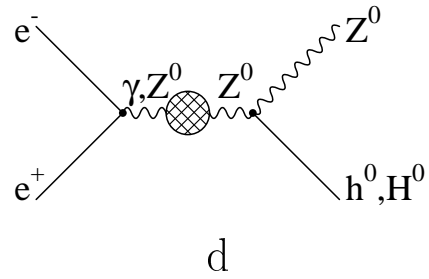
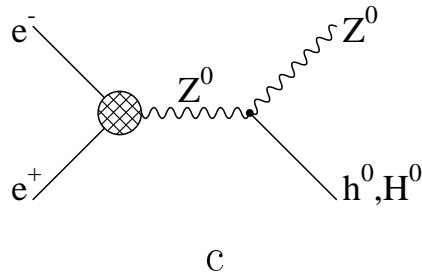
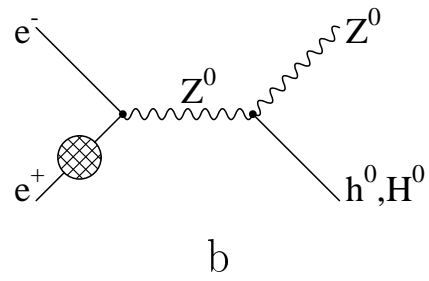
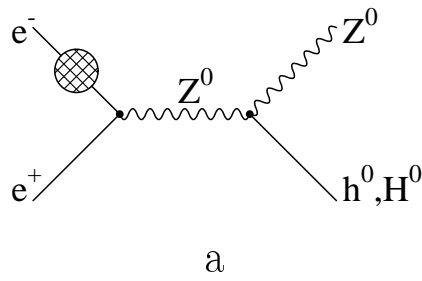
- Complete Feynman-diagrammatic one-loop result in on-shell scheme
  - box corrections included
  - full momentum dependence included

[V. Driesen, W. Hollik, J. Rosiek '95]

- Dominant two-loop contributions  $\mathcal{O}(G_\mu \alpha_s m_t^4)$  incorporation via propagator corrections

[S.H., W. Hollik, J. Rosiek, G. Weiglein '00] → FF

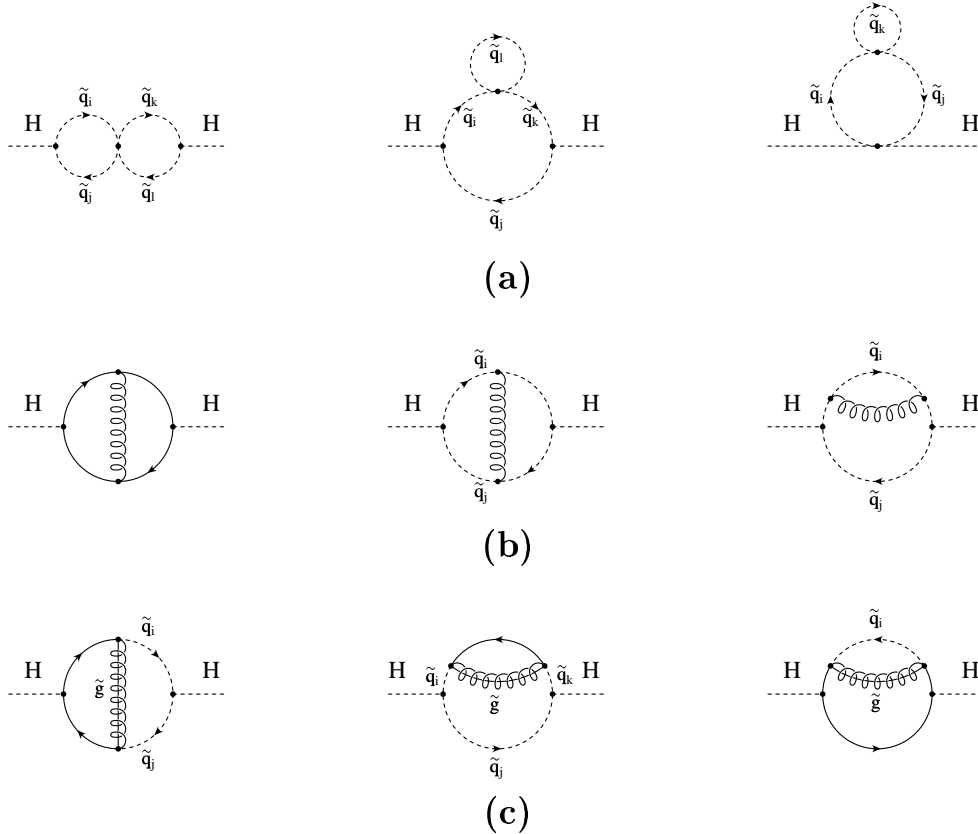
# Diagrams for $e^+e^- \rightarrow hZ$ :



## Leading two-loop corrections:

$\mathcal{O}(\alpha\alpha_s)$  contribution from  $t - \bar{t}$ -sector  
(Yukawa contribution,  $p^2 = 0$ )

[S.H., W. Hollik, G. Weiglein '98]



Complete result for

$$m_h, \quad \sigma(e^+e^- \rightarrow hZ, hA), \quad \text{BR}(h \rightarrow f\bar{f})$$

included in:

*FeynHiggs, FeynHiggsXS*

available from: [www.feynhiggs.de](http://www.feynhiggs.de)



Alternative approach:

## Renormalization group $\alpha_{\text{eff}}$ approximation:

Higgs mass matrix: (in  $\phi_1 - \phi_2$  basis)

incl. dominant higher-order corrections:

$$M_{\text{Higgs}}^2 = \begin{pmatrix} m_{\phi_1}^2 - \hat{\Sigma}_{\phi_1}(0) & m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) \\ m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) & m_{\phi_2}^2 - \hat{\Sigma}_{\phi_2}(0) \end{pmatrix}$$

$\Downarrow \leftarrow$  Diag. ,  $\alpha_{\text{eff}}$

$$\begin{pmatrix} M_H^2 & 0 \\ 0 & M_h^2 \end{pmatrix}$$

$\Rightarrow M_h, M_H$ , mixing angle  $\alpha_{\text{eff}}$  at higher order

$\alpha_{\text{eff}}$  contains dominant higher order corrections

$\rightarrow$  production  $\rightarrow \sin(\beta - \alpha_{\text{eff}}), \cos(\beta - \alpha_{\text{eff}})$

$\rightarrow$  decay

$\Rightarrow$  Deviations from other approaches:

- different accuracies for  $\hat{\Sigma}_s(0)$
- momentum dependence
- vertex, box corrections

### 3. Numerical results

#### Comparison:

FD (1-loop, 2-loop)  $\longleftrightarrow$  RG- $\alpha_{\text{eff}}$  (2-loop)

[S.H., W. Hollik, J. Rosiek, G. Weiglein '01]

#### FD:

- Fortran code including all FD corrections ( $\rightarrow$  *FeynHiggsXS*)  
[V. Driesen, S.H., J. Rosiek '95/'01]

#### RG- $\alpha_{\text{eff}}$ :

- Fortran code *subhpole*  
[M. Carena, M. Quiros, C. Wagner '95]
- $m_t(M_S)$  corrections included  
[M. Carena, H. Haber, S.H., W. Hollik, C. Wagner, G. Weiglein '00]
- On-shell  $\overline{\text{MS}}$  transition ( $M_{\text{SUSY}}, X_t$ ) included  
[M. Carena, H. Haber, S.H., W. Hollik, C. Wagner, G. Weiglein '00]
- $\rightarrow \alpha_{\text{eff}}$  approximation

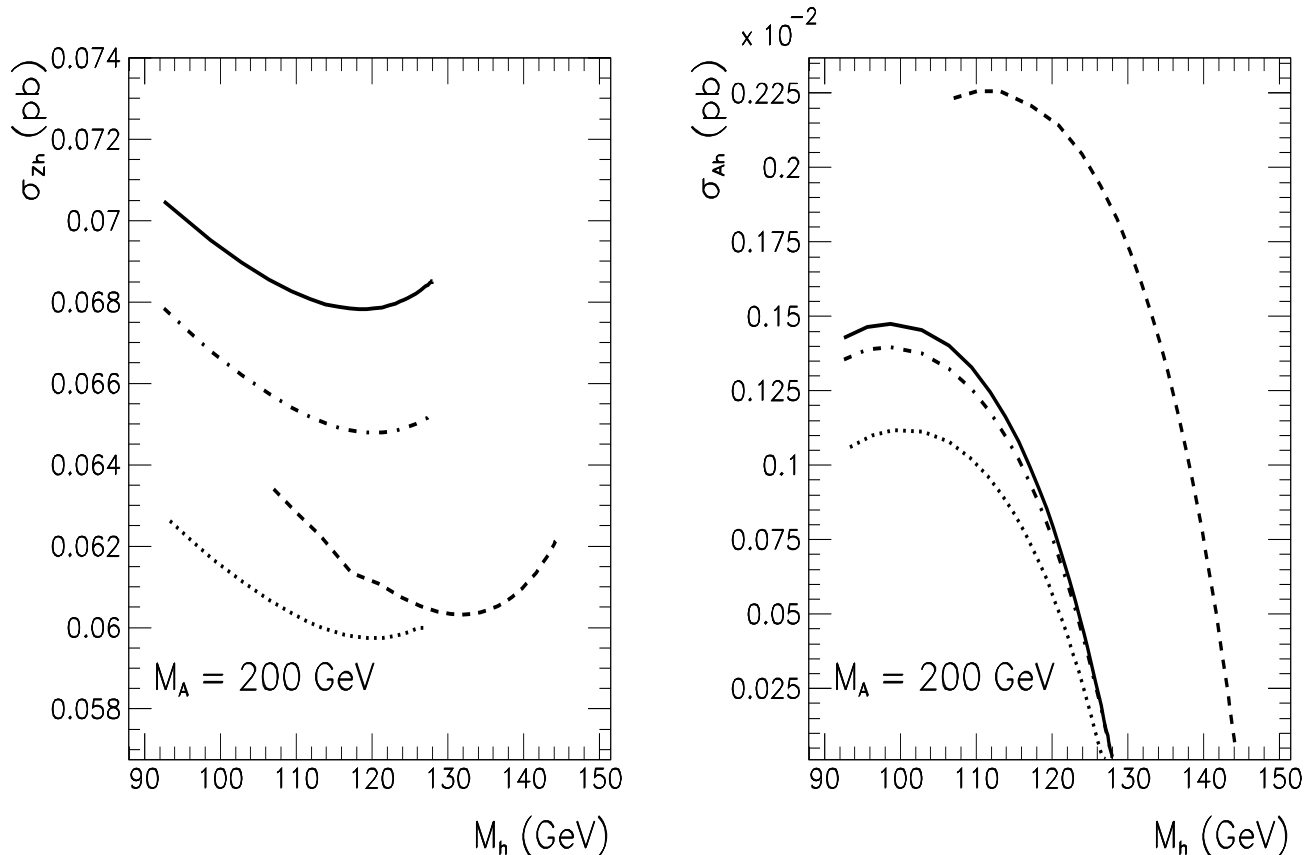
$e^+e^- \rightarrow hZ, hA, \sqrt{s} = 500 \text{ GeV}$

$M_{\tilde{q}} = 1 \text{ TeV}, M_{\tilde{l}} = 300 \text{ GeV}, M_2 = \mu = 200 \text{ GeV}$

$X_t = 2M_{\tilde{q}}, M_A = 200 \text{ GeV}$

solid: FD 2L (with box) , dot-dashed: FD 2L (no box)

dashed: FD 1L , dotted: RG 2L



Diff. FD 2L – RG- $\alpha_{\text{eff}}$  result:  $\sigma_{Zh} \approx 10\text{--}15\%$

$\sigma_{Ah} \approx 25\%$

Effect of 2L corrections:  $\sim 20\%$

Effects of box contributions: 5–10%

larger effects in differential cross sections

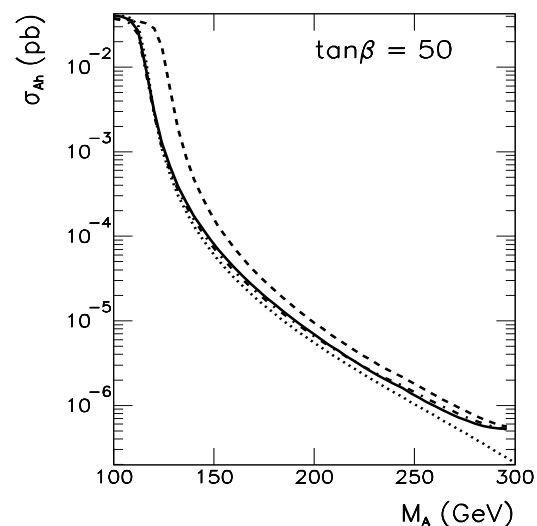
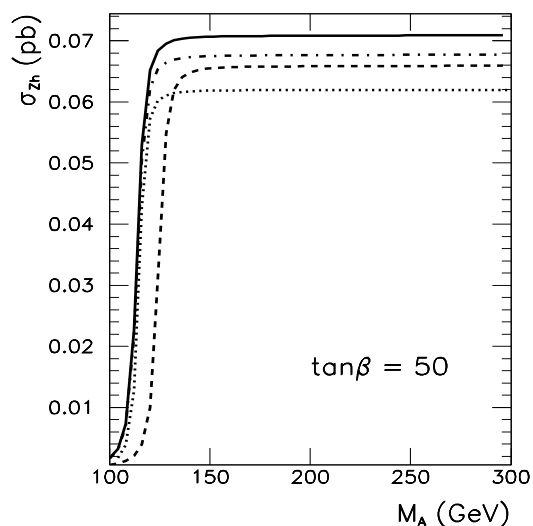
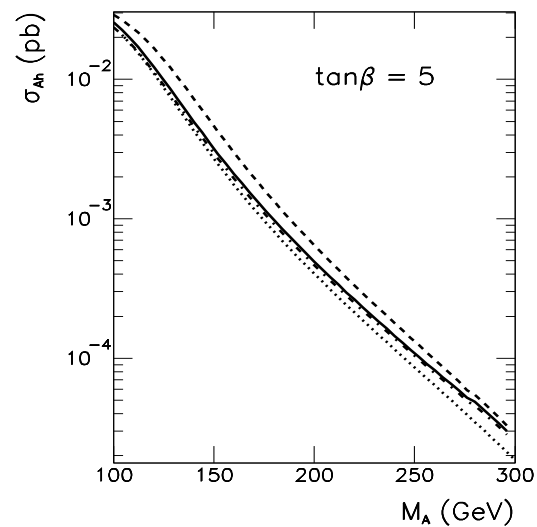
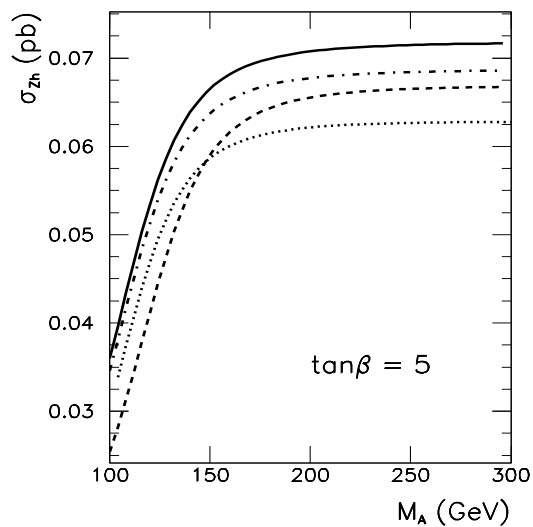
$e^+e^- \rightarrow hZ, hA, \sqrt{s} = 500 \text{ GeV}$

$M_{\tilde{q}} = 1 \text{ TeV}, M_{\tilde{l}} = 300 \text{ GeV}, M_2 = \mu = 200 \text{ GeV}$

$X_t = 0, \tan\beta = 5, 50:$

solid: FD 2L (with box) , dot-dashed: FD 2L (no box)

dashed: FD 1L , dotted: RG 2L



⇒ Sizable deviations between FD and RG- $\alpha_{\text{eff}}$  result for large  $M_A$

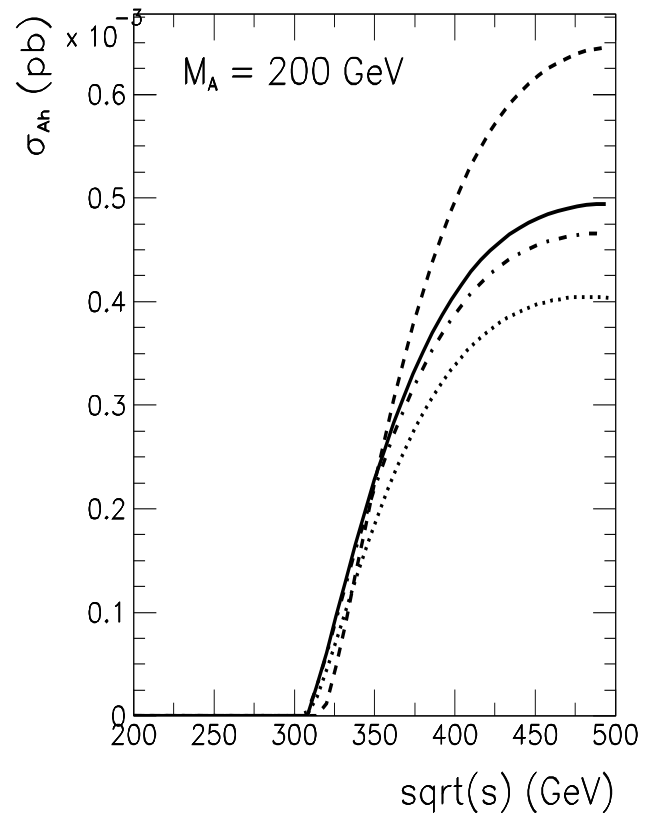
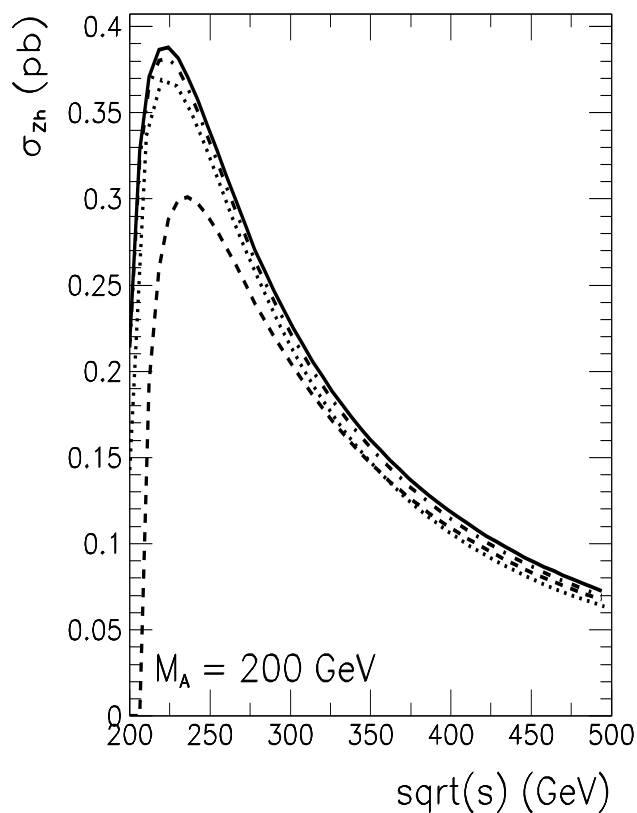
$e^+e^- \rightarrow hZ, hA$  as a function of  $\sqrt{s}$ :

$$M_{\tilde{q}} = 1 \text{ TeV}, M_{\tilde{l}} = 300 \text{ GeV}, M_2 = \mu = 200 \text{ GeV}$$

$$X_t = 0, M_A = 200 \text{ GeV}$$

solid: FD 2L (with box) , dot-dashed: FD 2L (no box)

dashed: FD 1L , dotted: RG 2L



$\Rightarrow$  Relative difference between FD and RG result grows with increasing  $\sqrt{s}$

Code currently used for the LEP2 analysis: **HZHA**

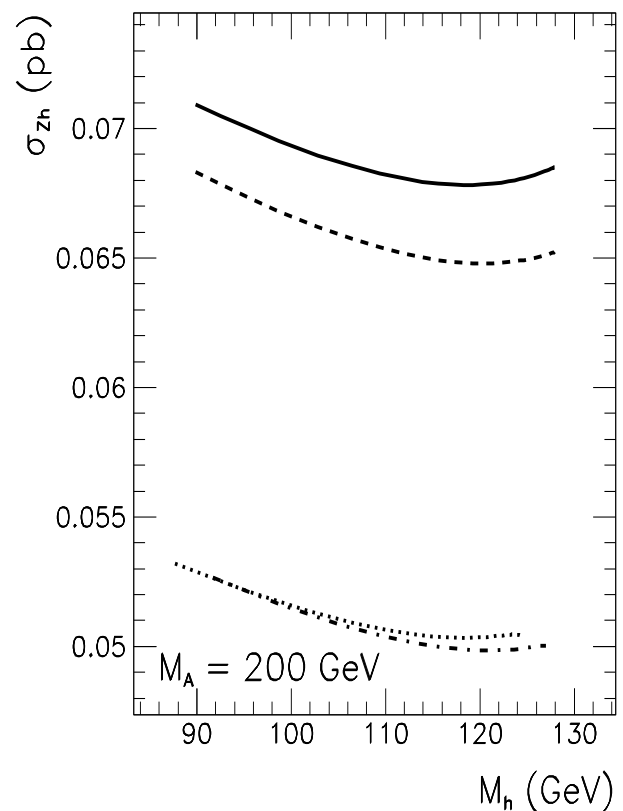
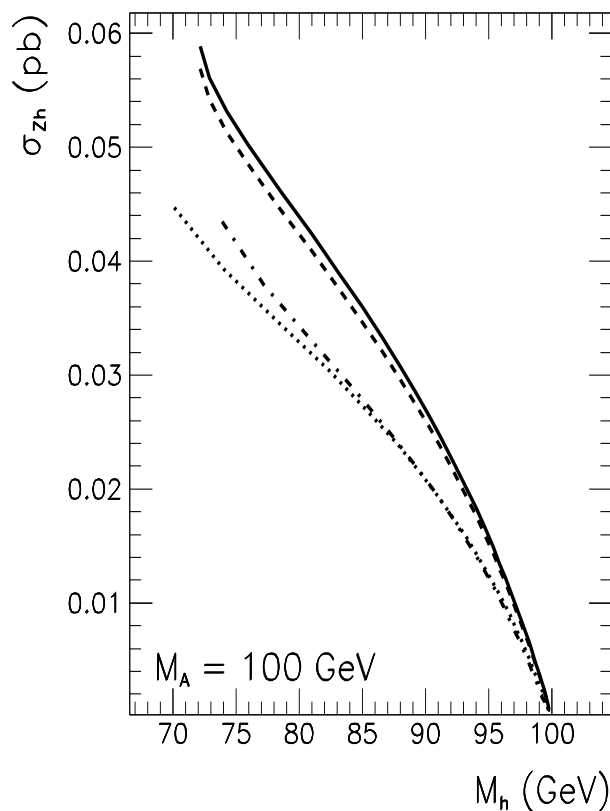
[*P. Janot '96-'00*]

$e^+e^- \rightarrow hZ$ , comparison with HZHA:  $M_{\tilde{q}} = 1 \text{ TeV}$ ,  
 $M_{\tilde{l}} = 300 \text{ GeV}$ ,  $M_2 = \mu = 200 \text{ GeV}$

$X_t = 2M_{\tilde{q}}$ ,  $M_A = 100, 200 \text{ GeV}$

solid: **FD 2L (with box)** , dashed: **FD 2L (no box)**

dotted: **HZHA (FeynHiggs)**, dot-dashed: **HZHA (subhpole)**



$\Rightarrow$  sizable deviations for large  $M_A$

## Outlook:

- Further comparison with HZHA  
→ full effect of FD calculation
- Implementation of *FeynHiggsXS*  
into existing codes ?
  - Pythia
  - HZHA
- leading two-loop (process specific) vertex corrections?  
⇒ needed for 1% accuracy

## 4. Conclusinos

- Calculation for Higgs production:

$$e^+e^- \rightarrow hZ, hA$$

Higgs propagator correction combined with complete one-loop on-shell result  
( $\rightarrow$  vertices, boxes)

$\Rightarrow$  *FeynHiggsXS*

- Comparison of FD and RG- $\alpha_{\text{eff}}$  :
  - $\sigma_{Zh} : @ \sqrt{s} = 500 \text{ GeV}$ 
    - box corrections up to 7%
    - two-loop corrections  $\sim 10\%$
    - FD larger than RG up to 15%
  - $\sigma_{Ah} : @ \sqrt{s} = 500 \text{ GeV}$ 
    - box contributions up to 10%
    - two-loop corrections  $\sim 20\%$
    - FD larger than RG up to 25%
- Improvement factor of 10 needed in theoretical prediction to match the experimental precision