Should we believe the Precision Electroweak Upper Bound on the Higgs Boson Mass?

M. E. Peskin
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It is well known that the precision electroweak measurements are in excellent agreement with the predictions of the Minimal Standard Model.

Within this model, the measurements are sensitive enough to distinguish between low and high values for the Higgs boson mass.

If the Higgs boson mass were known to be light, this would give tremendous encouragement to Run II Higgs searches and to the Linear Collider.

But, nobody believes in the Minimal Standard Model.

So, why should we take its implications seriously?
A reminder of the current situation:

A. Gurtu (Osaka ICHEP):

\[ m_h < 165 \text{ GeV} \quad 95\% \text{ CL} \]

(203 GeV w. BES-II data ?)
But, at the same time, we read ...

'a model with decoupling … widens the allowed range for the Higgs mass …'
- Casalbuoni et al., hep-ph/9805446

'large Higgs masses (up to 500 GeV) … can provide a good fit to precision data.'
- Rizzo, Wells, hep-ph/9906234

'we find that a Higgs mass up to 500 GeV is allowed.'
- Chivukula, Evans, Hoelbing, hep-ph/0002022

'the precision bound on the SM-like Higgs boson mass is shown to be significantly relaxed.'
- He, Polonsky, Su, hep-ph/0102144
In this talk, I would like to discuss

how does the Higgs boson affect the precision electroweak measurements?

how can the bound on the Higgs boson mass be avoided if we go beyond the Standard Model?

what is the price of relaxing this constraint?
Precision electroweak measurements mainly involve the weak interactions of light quarks and leptons.

The direct Higgs boson couplings to these particles is very small, so, the Higgs boson enters precision electroweak predictions mainly through vacuum polarization diagrams, e.g.,

![Diagram of vacuum polarization](image)

This is typical of new particles associated with electroweak symmetry breaking.

Probe for their effects through a general analysis of vacuum polarization effects ('oblique corrections').
To evaluate the effects of new particles with large mass $M$, expand vacuum polarization diagrams in powers of $\left( \frac{q^2}{M^2} \right)$.

\[ W \rightarrow 1,2 \quad Z \rightarrow 3 + s^2 Q \quad A \rightarrow Q \]

\[
\begin{align*}
1 & \quad 1 = \Pi_{11}(0) + q^2 \Pi_{11}'(0) + \ldots \\
3 & \quad 3 = \Pi_{33}(0) + q^2 \Pi_{33}'(0) + \ldots \\
3 & \quad Q = q^2 \Pi_{3Q}(0) + \ldots \\
Q & \quad Q = q^2 \Pi_{QQ}(0) + \ldots
\end{align*}
\]

Eliminate 3 coefficients in favor of $\alpha, G_F, m_Z$

What remains?

\[
\begin{align*}
\alpha S &= 4 e^2 \left( \Pi_{33}'(0) - \Pi_{3Q}'(0) \right) \\
\alpha T &= \frac{e^2}{s^2 c^2 m_Z^2} \left( \Pi_{11}(0) - \Pi_{33}(0) \right)
\end{align*}
\]

\[ s^2 = \sin^2 \theta_W \quad c^2 = \cos^2 \theta_W \]
A heavy Higgs contributes

\[ \Delta S = + \frac{1}{12 \pi} \log \frac{m_h^2}{m_Z^2} \]

\[ \Delta T = - \frac{3}{16 \pi c^2} \log \frac{m_h^2}{m_Z^2} \]

c.f. contribution of a 4th generation U,D

\[ \Delta S = \frac{1}{6 \pi} N_c \]

\[ \Delta T = \frac{1}{12 \pi s^2 c^2} \frac{(m_U - m_D)^2}{m_Z^2} \]

Define the zero of S, T to be:

\[ m_t = 174.3 \, \text{GeV} \quad \text{and} \quad m_h = 100 \, \text{GeV} \]
The most important constraints come from $Z$ asymmetries, $m_W$, $\Gamma_Z$

The recent LEP results on $m_W$ have significantly increased the pull to small $m_h$. 
What if we go beyond the MSM?

New physics can contribute to S and T.

In principle, this can compensate the contribution to S, T from a heavy Higgs.

To do this, we need

\[ \Delta S \sim -0.1 \quad \text{or} \quad \Delta T \sim +0.3 \]

or some combination of these.
A general way to analyze this problem would be to add to the MSM the most general effective Lagrangian that might result from integrating out new massive particles.

This turns out to include operators

\[ L = \frac{g_1}{\Lambda^2} \text{tr}[U^+ W_{\mu\nu} U B^{\mu\nu}] + \frac{g_2}{\Lambda^2} (\text{tr}[U^+ D_{\mu} U \tau^3])^2 \]

that directly shift \( S \) and \( T \).

If \( g_1, g_2 \) are taken to be free parameters, the bounds on \( m_h \) relax almost completely.

Barbieri, Strumia
Bagger, Falk, Swartz
Kolda, Murayama
Chivukula, Hoelbing, Evans
Some weak constraints follow from restrictions on the size of the effective Lagrangian coefficients contributing to $S, T, Higgs$ self-interaction.
I feel that this argument is incomplete.

New particles that have significant effects on S and T may also lead to other experimental signatures. We should examine whether these are observable and interesting.

In the process, we should try to understand what kind of models lead to compensation of the heavy Higgs effect on precision electroweak measurements.

Jim Wells and I have systematically reviewed the literature on this question.

see: hep-ph/0101342
Wells and I found that all explicit models in the literature which allow a heavy Higgs use one of three well-defined mechanisms.

I will review these in a moment.

First, I would like to remind you of an important case in which this analysis is not necessary.
In a grand unified theory with a fundamental Higgs boson, the Higgs self-coupling $\lambda$ decreases in running from the GUT scale to the weak scale. 

This by itself implies an upper bound on the Higgs mass of about 200 GeV.

This argument applies, in particular, to supersymmetric theories with grand unification.

For this case, Espinosa and Quiros have done an exhaustive search of models and have concluded

$$m_h < 205 \text{ GeV}$$

In the MSSM, there is an even stronger result

$$m_h < m_Z + \text{(radiative correction)}$$

or    $$m_h < 130 \text{ GeV}$$
Now I will review three methods for compensating the $S$, $T$ contributions of a heavy Higgs boson:

Method A: Negative $S$

Method B: New Vectors

Method C: Positive $T$
Method A:  Negative $S$

It is not so easy to find weak interaction multiplets which, when integrated out, give negative $S$.

Extra generations, QCD-like technicolor give $\Delta S > 0$.

Elementary scalars typically give very small contributions to $S$.

But, ...
Dugan-Randall:

Consider SU(2) x SU(2) broken to SU(2) custodial.

Consider a scalar field in the representation \((j_L, j_R)\) of SU(2) x SU(2). This representation will break up into final SU(2) multiplets with isospin from \(|j_L - j_R|\) to \(|j_L + j_R|\). If the multiplet with the smallest isospin is the lightest, these scalars contribution \(\Delta S < 0\).

Gates-Terning:

Fermions with both Dirac and Majorana masses, in some regions of their parameter space, can contribute \(\Delta S < 0\).

In both cases, the formula is

\[
S = - \frac{C}{3\pi} \log \frac{m_1}{m_2}
\]

where the two masses are split by electroweak symmetry breaking. **A large effect requires a light particle with electroweak charge.**
Dugan-Randall model

\[ m = 100 \text{ GeV} \]

\[ (j_L, j_R) \]

\[ \Delta S \]

\[ m \text{ (GeV)} \]

\[ \Delta m = 100 \text{ GeV} \]
An example of the Gates-Terning mechanism in supersymmetry:

The open circles are models with $m(\chi^+) < 60$ GeV
Method B: New Vectors

Enlarge the gauge group by adding a Z'. Small mixing of Z' and Z will induce subtle changes in the precision observables.

Rizzo, Casalbuoni et al.: In some cases, these effects can mimic $\Delta S < 0$.

To analyze such models, we take the following approach:

- compute the shifts in $A_e$, $m_W$, $\Gamma_Z$

- refit the data including these shifts and the effect of a 500 GeV Higgs

- compare to the MSM fit
Simplest example:

\[ Z' \text{ with no coupling to light } q, l \]

\[
m^2 = \begin{pmatrix} m_{Z0}^2 & \gamma m_Z^2 \\ \gamma m_Z^2 & M^2 \end{pmatrix}
\]

Mixing shifts the Z mass by the fraction

\[ \delta = \gamma^2 \frac{m_Z^2}{M^2} \]

with effects on the precision electroweak observables

fit for \( S, T \), including effects of such a \( Z' \) with \( \gamma = 1 \) and variable \( M \)
for each value of $M$, the fits tells us the effective displacement in $S, T$

the heavy Higgs effect is almost completely compensated for $M \sim 2$ TeV.
Repeat this analysis with other $Z'$ models, including general $E_6$ models with symmetry breaking by a Higgs $H_u$ or $H_d$.

Numbers denote the values of $M$, in TeV.

(KK) is the extra-dimension model of Rizzo and Wells.
So, adding a $Z'$ can significantly alter the precision electroweak fit.

In the model space, we can engineer the $\Delta S, \Delta T$ to compensate the heavy Higgs.

But, this works only for sufficiently small $M$.

I remind you that LHC and a 500 GeV LC are sensitive to a $Z'$ up to 4 TeV and to KK vectors above 6 TeV.

If this is the model of a heavy Higgs, we will have very interesting experiments to do at these machines.
Method C: Positive $T$

Though it is difficult to generate negative $S$ in many models of new physics, it is always straightforward to generate positive $T$.

Any small amount of weak isospin breaking will do this.

Looking at the contour of the current $S$, $T$ fit, it is possible to have a consistent solution with a heavy Higgs and positive $\Delta T$. 
examples:

Dobrescu-Hill `topcolor seesaw'

\[ T = \frac{3}{64\pi} \frac{g_{tc} v^2}{m_\chi^2} \left( 1 + 2 \frac{\lambda_t}{g_{tc}} \log \frac{m_\chi^2}{m_t^2} \right) \]

2-Higgs doublet model: Chankowski et al.

walking technicolor: Appelquist and Terning

4 generation models: He, Polonsky, Su
topcolor seesaw, $m_\chi = 5$ TeV, with $m_h = 500$ GeV

In technicolor models, typically $S > 0.12$; but still there is room.
It is possible to plot this effect in a much more suggestive way:

Collins, Grant, Georgi
Many realistic topcolor and technicolor models predict new and interesting physics visible at accessible energies:

new gauge bosons, light technirho, extra dimension signatures

But it is possible that we have a heavy Higgs and `no new physics'. What then?

In this case it will be crucial to do improved electroweak experiments at a higher level of precision:

Giga-Z (and WW threshold scan in $e^+e^-$):

\[
\begin{align*}
\sin^2 \theta_W & \quad \text{to} \quad 0.00002 \\
m_W & \quad \text{to} \quad 6 \ \text{MeV} \\
\Gamma_Z & \quad \text{to} \quad 0.4 \ \text{MeV}
\end{align*}
\]
Then we can distinguish new physics from the MSM at > 5\sigma .
so,

should we believe the precision electroweak upper bound on the Higgs boson mass?

I have shown a number of counterexamples,

but

each mechanism for avoiding the electroweak bound has its price.

Typically these mechanisms lead to interesting new physics at accessible energies.
I conclude:

We must take the precision electroweak constraint seriously as guidance for our current and future experimental program.