Detailed Studies of Electron Cooling Friction Force

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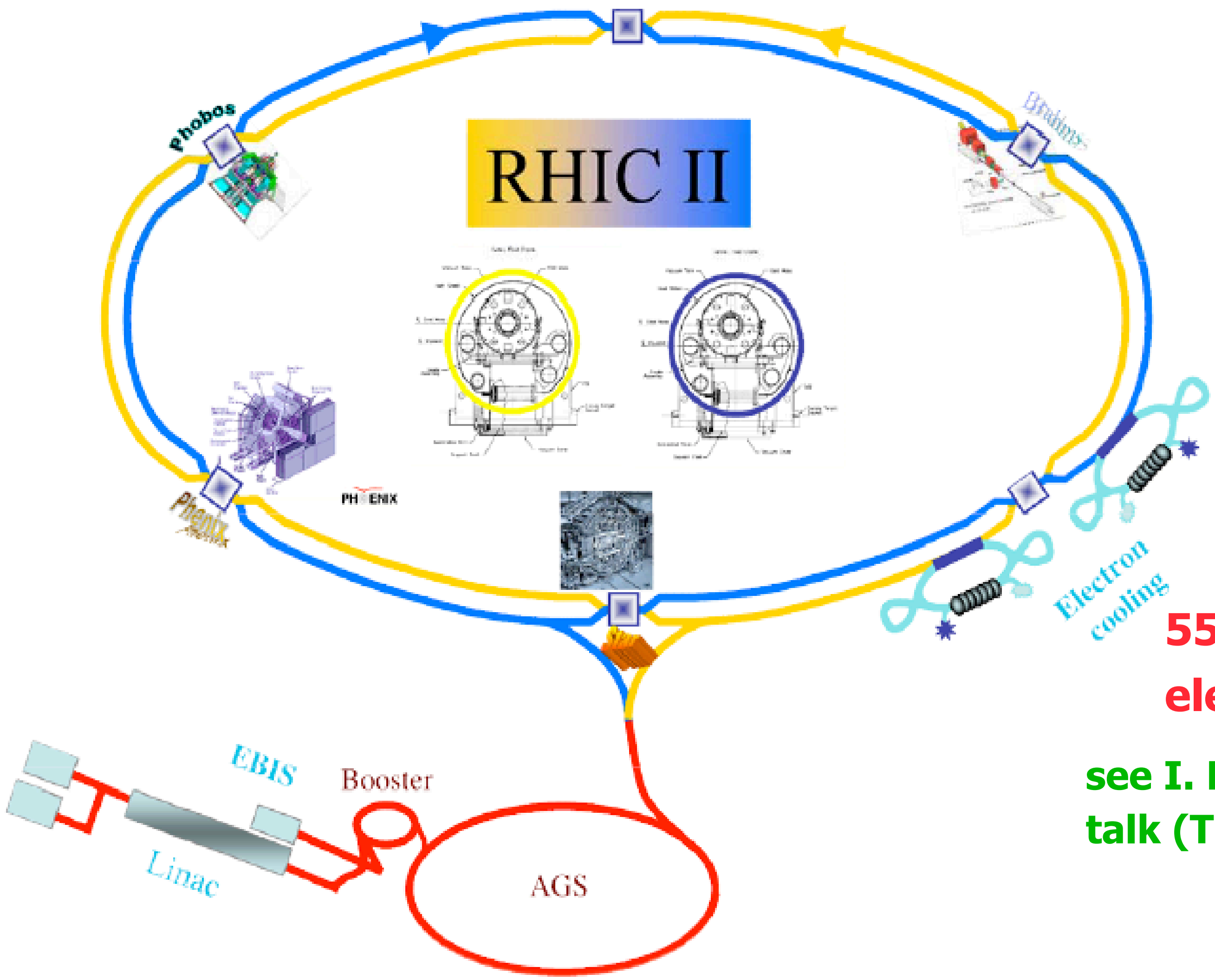
(COOL05, September 2005)



55 MeV electrons

see I. Ben-Zvi talk (Thursday)

Cooling theory/simulations and experimental benchmarking collaboration

BNL: A. Fedotov, I. Ben-Zvi, J. Kewisch, V. Litvinenko,

N. Malitsky, G. Parzen, G. Wang, others

JINR, Russia: I. Meshkov, A. Sidorin, A. Smirnov, G. Trubnikov

BINP, Russia: V. Parkhomchuk, A. Skrinsky, Yu. Eidelman, others

Tech-X, Colorado: D. Bruhwiler, D. Abell, R. Busby, J. Cary, others

FNAL: A. Burov

JLAB: Ya. Derbenev

INTAS "Advanced beam dynamics" collaboration: GSI (Darmstadt), ITEP (Russia), JINR (Russia), FZ (Julich), TEMF (Darmstadt), TSL (Uppsala), U. of Kiev (Ukraine)

Theory, Simulation, Experiments

I. Understanding ion beam dynamics in RHIC

I.1 Need for accurate description of heating mechanisms in RHIC

I.2 Accurate models of IBS

Dedicated IBS experiments and comparison with theory were done during 2004 run with Au and 2005 run with Cu ions.

V. ~~Theory and Numerical simulation~~ of cooling dynamics

~~II.1 Direct simulations of cooling force~~

Subject of this talk

II.2 Cooling dynamics codes/studies: cooling + IBS +

reported at PAC'05

IX. Experimental benchmarking of theory and simulations

reports on CELSIUS studies (Thursday)

ACCURATE benchmarking of the cooling force

At a minimum, we want to be sure that we are using the most appropriate and accurate cooling force formulas.

1. Infinite magnetic field appoximation (Derbenev-Skrinsky (D-S), Derbenev-Skrinsky-Meshkov (D-S-M)).

3. Empiric formula (V. Parkhomchuk (VP)) (any strength of the field) – can show very different cooling dynamics for some parameters. Also, has different numerical factors.

As a result, cooling dynamics is very different if one uses one formula or another. While an order of magnitude estimate was sufficient for typical low-energy coolers it becomes unacceptable for high-energy cooling in RHIC with very long cooling times.

We tried to understand these difference by

- Direct simulation of ion/electron collisions - testing of formulas.

  We should note that a lot of studies of similar questions were done by the Univ. of Erlangen (Germany) group and we plan to compare our findings with their results in the future.

- Need precise measurements of the force (well controlled experiments with known conditions/parameters) to provide data needed for benchmarking (see CELSIUS experiments – these proceedings

For electron cooling in RHIC (Au ions at γ=108) we studied two approaches:

2. Magnetized cooling – B=2-5T, q=20nC, ε_e=50 um
3. Non-magnetized cooling (with helical wigglers to control recombination) – B=20-50G, q=2-5nC, ε_e=2-3um

Recent studies of cooling dynamics for both approaches can be found, for example, in

A.V. Fedotov et al., 'Cooling dynamics studies and scenarios for RHIC cooler"; "Simulations of high-energy cooling", Proceedings of PAC05.

See also:

RHIC E-cooler Design Report http://www.agsrhichome.bnl.gov/eCool

Present status will be summarized in

Ilan Ben-Zvi talk – this conference (Thursday).

Codes used for Friction Force studies

We use:

1. VORPAL code – uses molecular dynamics techniques to explicitly resolve close binary collisions and thus capture friction and diffusion tensors with a bare minimum of physical assumptions.

   C. Nieter, J. Cary, J. Comp. Phys. 196, p. 448 (2004)

   D. Bruhwiler et al., AIP Conf. Proc. 773 (Bensheim, 2004), p. 394.

   see D. Bruhwiler talk for more details (this conference)

2. Numerical integration of analytic formulas over electron velocity distribution and comparison with simple asymptotic expressions using BETACOOL code

   The BETACOOL program, http://lepta.jinr.ru

   see A. Smirnov presentations for more details (this conference)

Non-magnetized friction force (B=0) – isotropic electron distribution

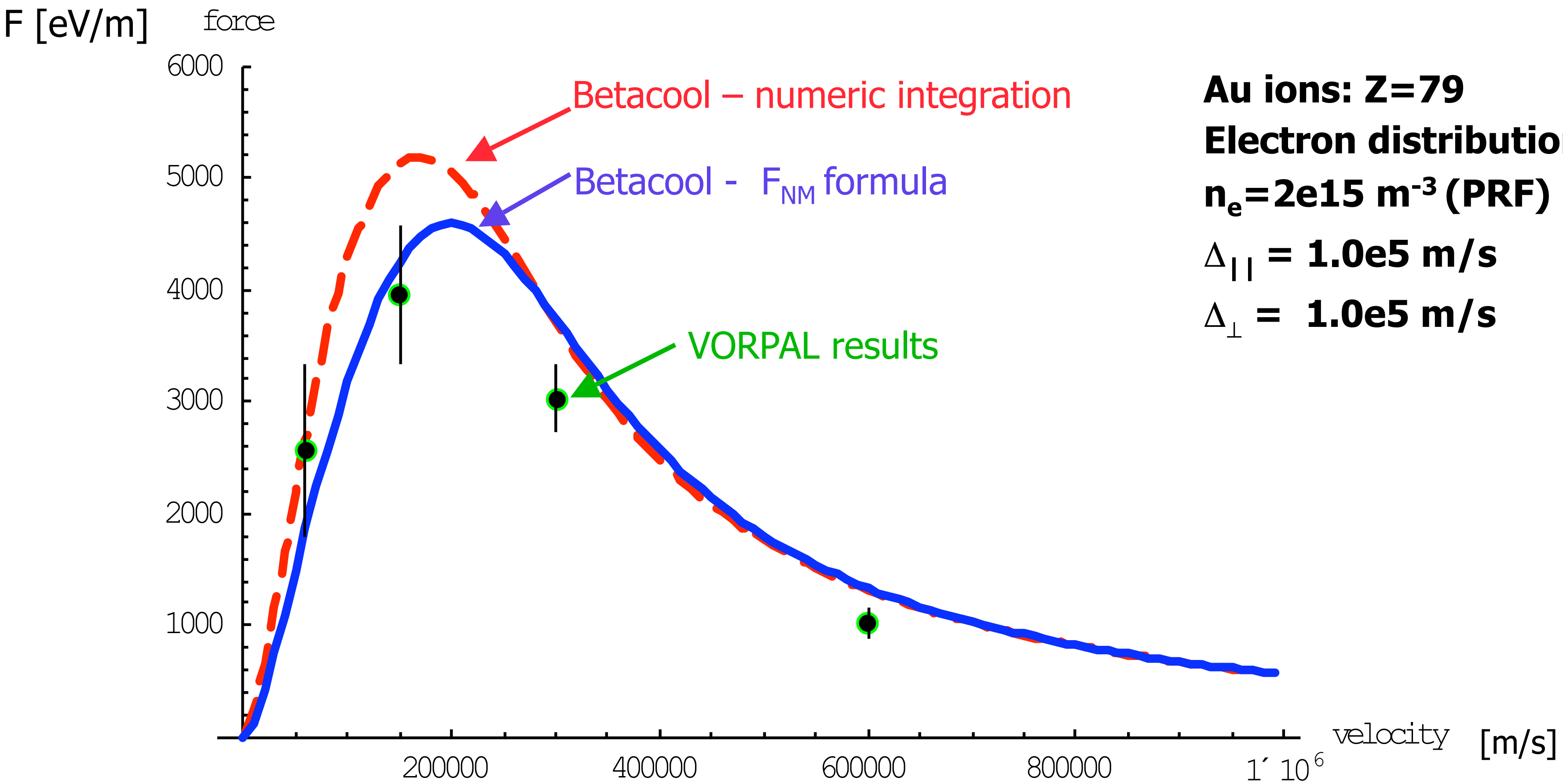
$$\vec{F} = -\frac{4\pi n_e e^4 Z^2 L}{m} \int \frac{\vec{v}_i - \vec{v}_e}{\left|\vec{v}_i - \vec{v}_e\right|^3} f(v_e) d^3 v_e$$

For isotropic Maxwellian distribution f(v_e) (Chandrasekar 1942):

$$\vec{F}_{NM}(\vec{v}_i) = -\frac{\vec{v}_i}{v_i^3} \frac{4\pi n_e e^4 Z^2 L}{m} \varphi\left(\frac{v_i}{\Delta_e}\right)$$

$$\varphi(x) = \sqrt{\frac{2}{\pi}} \int_0^x e^{-y^2/2} dy - \sqrt{\frac{2}{\pi}} x e^{-x^2/2}$$

B=0, isotropic electron distribution for ion velocity along the longitudinal direction



Au ions: Z=79

Electron distribution:

n_e=2e15 m^{-3} (PRF)

$\Delta_{||}$ = 1.0e5 m/s

$\Delta_{\perp}$ = 1.0e5 m/s

B=0 – anisotropic electron velocity distribution (typical situation for electron coolers)

Numerical evaluation (BETACOOL):

$$F_{\perp} = -\frac{4\pi Z^2 e^4 n_e}{m \times Int} \int_0^{3\Delta_\perp} \int_{-3\Delta_\parallel}^{3\Delta_\parallel} \int_0^{\pi} \ln\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{(V_\perp - v_\perp \cos\varphi)\exp\left(-\frac{v_\perp^2}{2\Delta_\perp^2} - \frac{v_\parallel^2}{2\Delta_\parallel^2}\right)}{\left((V_\parallel - v_\parallel)^2 + (V_\perp - v_\perp \cos\varphi)^2 + v_\perp^2 \sin^2\varphi\right)^{3/2}} v_\perp d\varphi dv_\parallel dv_\perp$$

$$F_{\parallel} = -\frac{4\pi Z^2 e^4 n_e}{m \times Int} \int_0^{3\Delta_\perp} \int_{-3\Delta_\parallel}^{3\Delta_\parallel} \int_0^{\pi} \ln\left(\frac{\rho_{max}}{\rho_{min}}\right) \frac{(V_\parallel - v_\parallel)\exp\left(-\frac{v_\perp^2}{2\Delta_\perp^2} - \frac{v_\parallel^2}{2\Delta_\parallel^2}\right)}{\left((V_\parallel - v_\parallel)^2 + (V_\perp - v_\perp \cos\varphi)^2 + v_\perp^2 \sin^2\varphi\right)^{3/2}} v_\perp d\varphi dv_\parallel dv_\perp$$

Asymptotic formula – can significantly overestimate friction force, especially near the longitudinal rms velocity spread

($v_i << \Delta_{||}$)

$$F_\parallel = -\frac{4\pi Z^2 e^4 n_e L}{m} \frac{v_\parallel}{\Delta_\parallel \Delta_\perp^2}$$

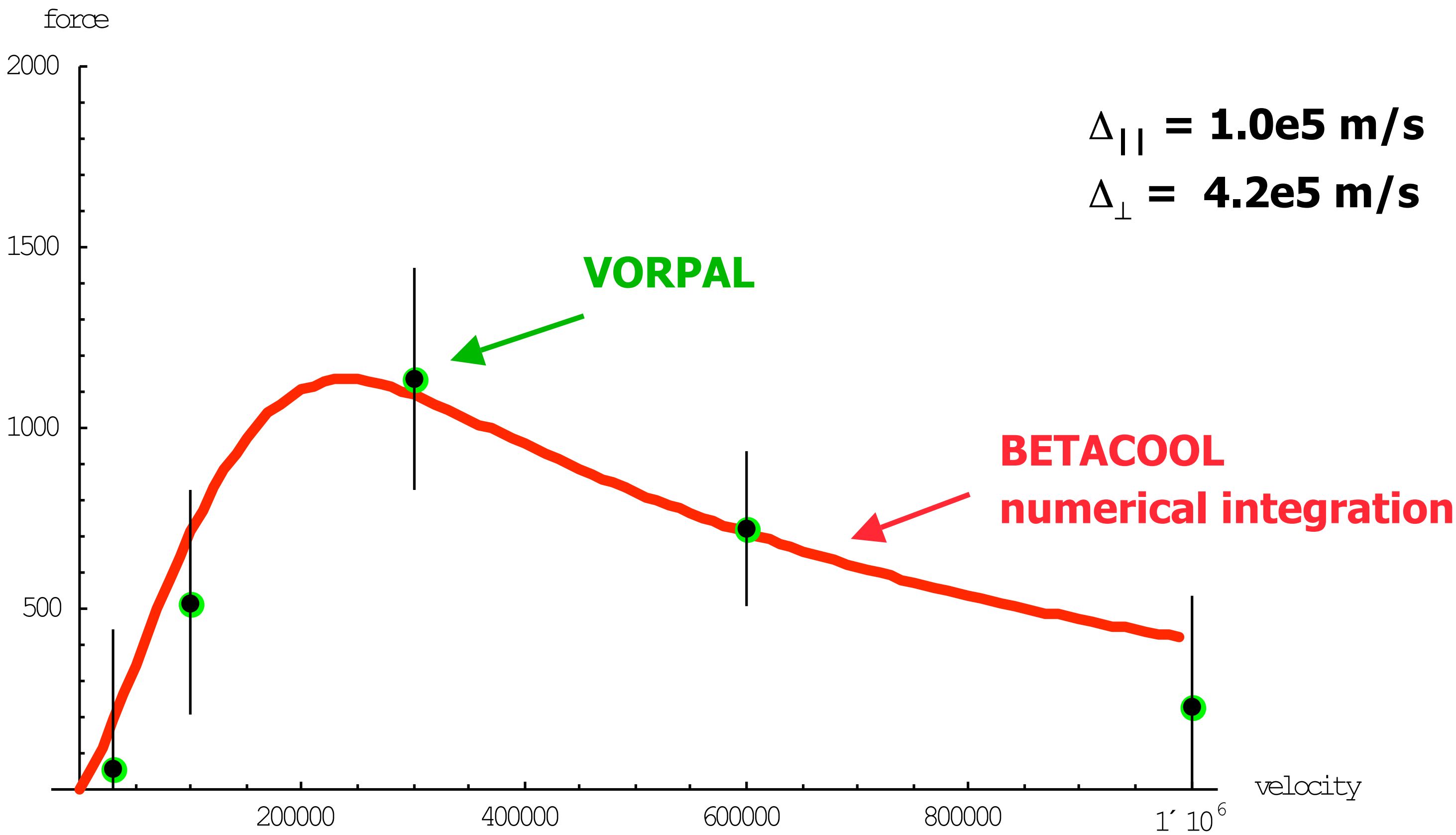
($\Delta_{||} << v_i << \Delta_\perp$)

$$\vec{F}_\perp = -\frac{4\pi Z^2 e^4 n_e L}{m} \times \frac{v_{i,\perp}}{\Delta_\perp^3} \qquad F_\parallel = -\frac{4\pi Z^2 e^4 n_e L}{m} \frac{v_\parallel}{|v_\parallel| \Delta_\perp^2}$$

($v_i >> \Delta_\perp$)

$$\vec{F} = -\frac{4\pi Z^2 e^4 n_e L}{m} \times \frac{\vec{v}_i}{v^3}$$

B=0, anisotropic velocity distribution



Non-magnetized force - summary

For anisotropic velocity distribution:

1. VORPAL gives good agreement with numerical integrals.
2. Asymptotic formulas overestimate friction force by a significant factor for typical RHIC parameters.

We are presently using numerical integrals in our cooling dynamics studies for non-magnetized cooling.

Magnetized friction force
- approximation of strong magnetic field

Numerical integration using Derbenev-Skrinsky (D-S) expressions for the magnetized collisions (BETACOOL):

$$F_\perp(V_\perp, V_\parallel) = -\frac{2\pi Z^2 e^4 n_e L_M}{m} \int \frac{V_\perp \left(V_\perp^2 - 2(V_\parallel - v_e)^2 \right)}{\left(V_\perp^2 + (V_\parallel - v_e)^2 \right)^{5/2}} f(v_e) dv_e$$

$$F_\parallel(V_\perp, V_\parallel) = -\frac{2\pi Z^2 e^4 n_e}{m} \int \left(L_M \frac{3V_\perp^2 (V_\parallel - v_e)}{\left(V_\perp^2 + (V_\parallel - v_e)^2 \right)^{5/2}} + 2 \frac{V_\parallel - v_e}{\left(V_\perp^2 + (V_\parallel - v_e)^2 \right)^{3/2}} \right) f(v_e) dv_e$$

Asymptotic expressions for all three type of collisions (Derbenev-Skrinsky-Meshkov (D-S-M)):

$$F_\perp \approx -\frac{2\pi Z^2 e^4 n_e}{m} v_\perp \begin{cases} \frac{1}{v^3}\left(2L_F + \frac{v_\perp^2 - 2v_\parallel^2}{v^2} L_M \right), \{I\} \\ \frac{2}{\Delta_\perp^3}(L_F + N_{col} L_A) + \frac{v_\perp^2 - 2v_\parallel^2}{v^2} \frac{L_M}{v^3}, \{II\} \\ \frac{2}{\Delta_\perp^3}(L_F + N_{col} L_A) + \frac{L_M}{\Delta_\parallel^3}, \{III\} \end{cases}$$

$$F_\parallel \approx -\frac{2\pi Z^2 e^4 n_e}{m} v_\parallel \begin{cases} \frac{1}{v^3}\left(2L_F + \frac{3v_\perp^2}{v^2} L_M + 2 \right), \{I\} \\ \frac{2}{\Delta_\perp^2 v_\parallel}(L_F + N_{col} L_A) + \left(\frac{3v_\perp^2}{v^2} L_M + 2 \right) \frac{1}{v^3}, \{II_a\} \\ \frac{2}{\Delta_\perp^2 \Delta_\parallel}(L_F + N_{col} L_A) + \frac{L_M}{\Delta_\parallel^3}, \{II_b, III\} \end{cases}$$

Finite magnetic field

Empiric formula by V. Parkhomchuk (VP) (NIM, 2000):

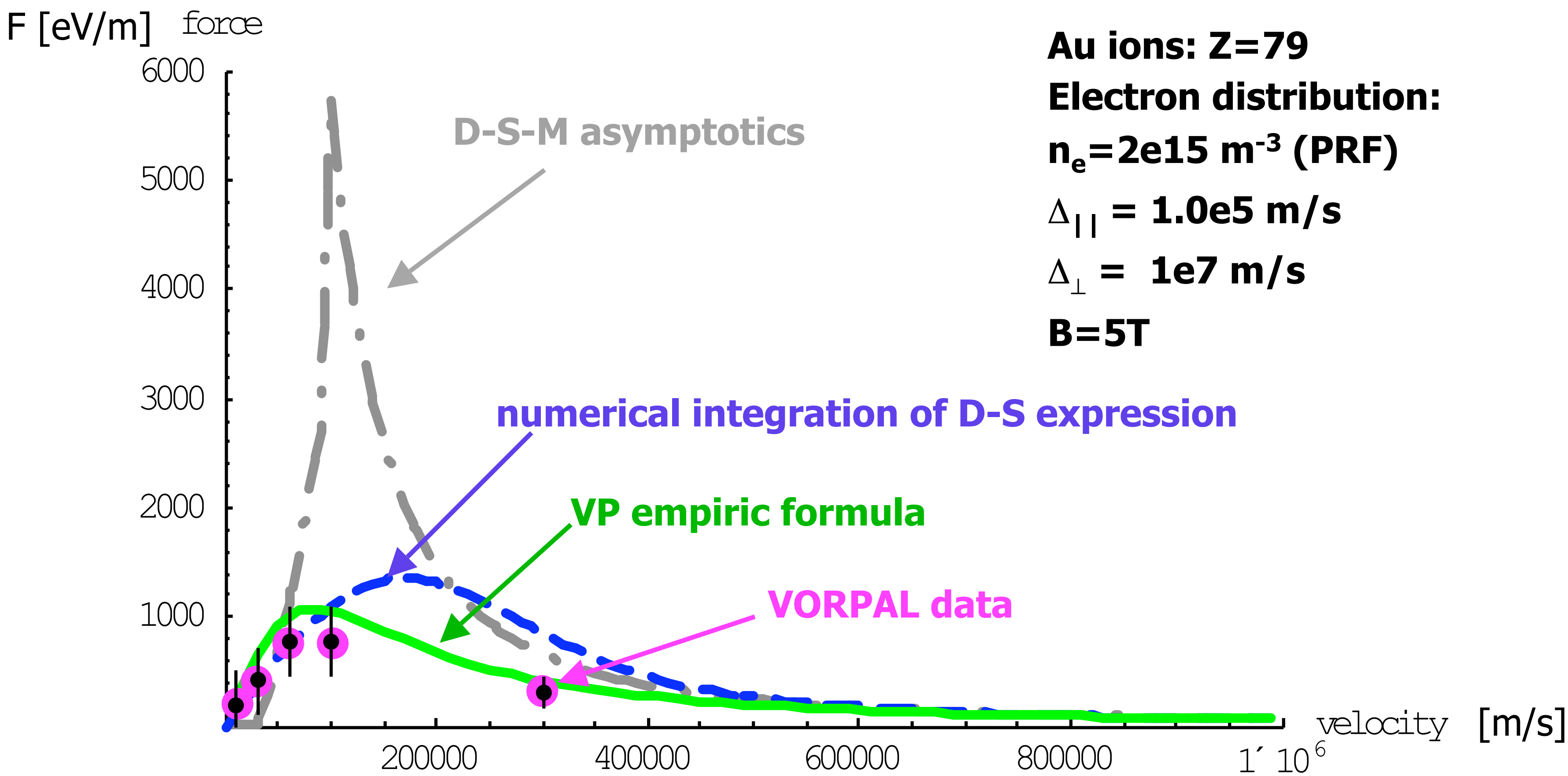
$$\vec{F} = -\vec{v}\frac{4Z^2e^4n_eL_P}{m}\frac{1}{\left(v^2+\Delta_{e,eff}^2\right)^{3/2}} \qquad L_P = \ln\left(\frac{\rho_{max}+\rho_{min}+\langle\rho_\perp\rangle}{\rho_{min}+\langle\rho_\perp\rangle}\right)$$

1. Similar to D-S asymptotics at low velocities v < $\Delta_{||}$

2. Very different at large velocities v >> $\Delta_{||}$ - both in numerical factor and dependence on angle with respect to the magnetic field direction.

Studies were done to explore magnetized friction force formulas in various regimes. Some of these studies are reported in the next slides, using parameters of the RHIC-II cooler based on the magnetized approach.

Friction force for ion velocity along magnetic field line $V_\perp = 0$



Au ions: Z=79

Electron distribution:

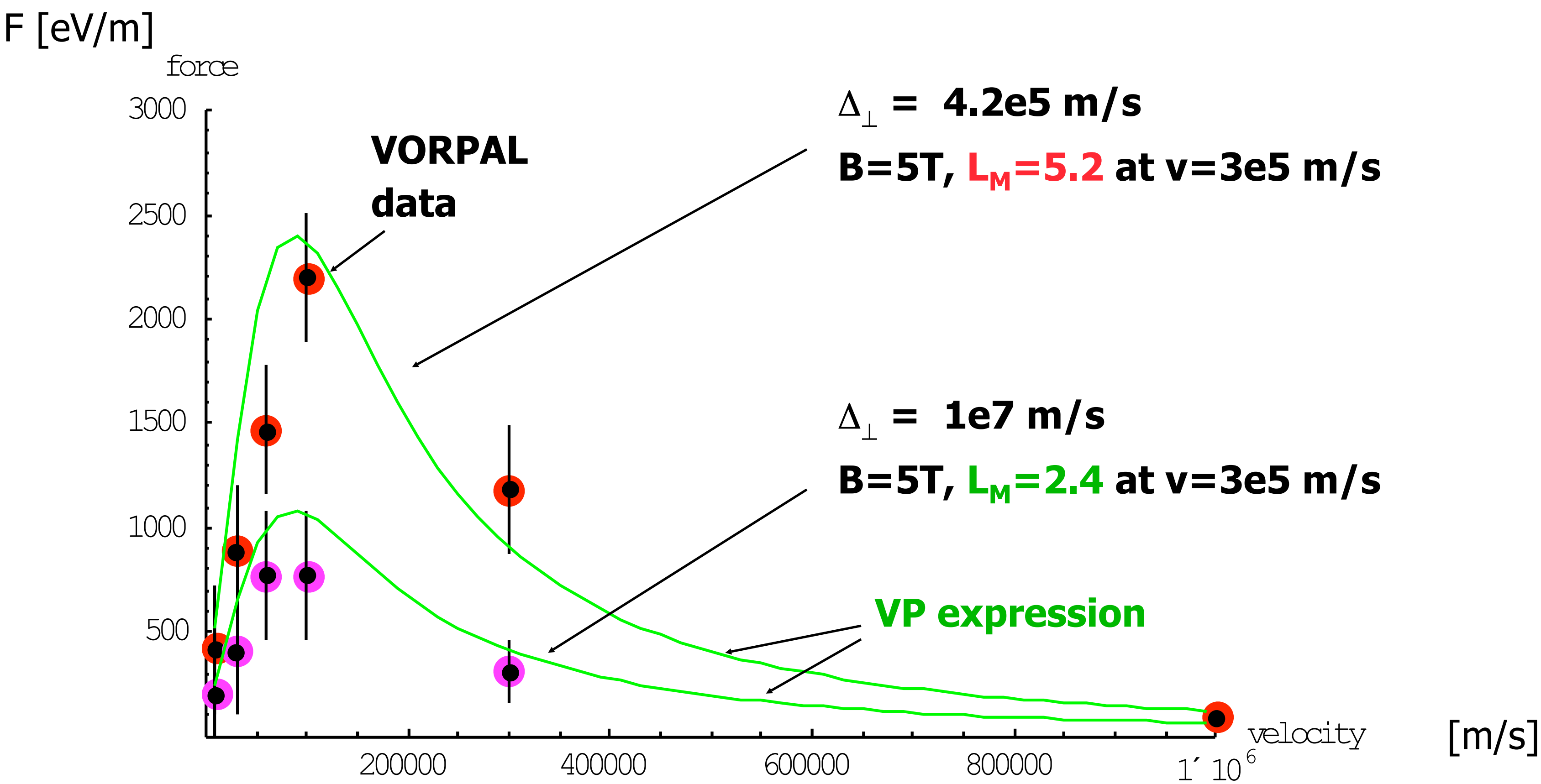
n_e=2e15 m^{-3} (PRF)

$\Delta_{||}$ = 1.0e5 m/s

$\Delta_\perp$ = 1e7 m/s

B=5T

Friction force for ion velocity along magnetic field line ($V_\perp$ =0) for two different degrees of magnetization



Angular dependence at large relative velocities

Strong magnetic field results in friction force dependence on the angle with respect to the direction of magnetic field.

Very different expressions for the transverse and longitudinal components of the friction force both of which now depend on both transverse and longitudinal velocity.

But how important is such "angular anisotropy" of the friction force for finite magnetization?

This question was already addressed by Parkhomchuk (NIM, 2000), using simulations with zero temperature electrons. Here we try to examine this question for finite temperatures of electron beam.

Angular dependence for longitudinal component of the friction force

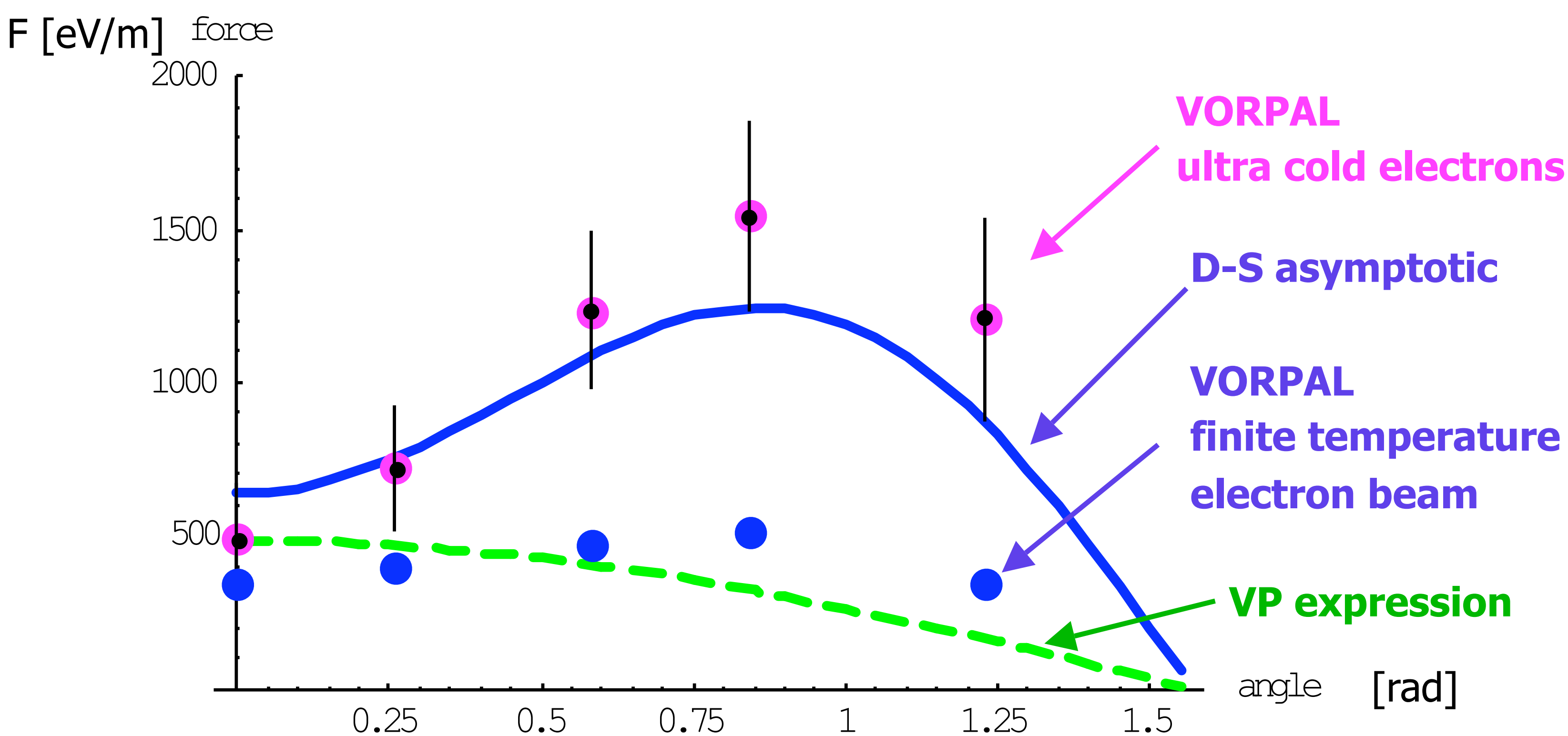
empiric formula by V. Parkhomchuk (VP)

$$\mathbf{F}^{VP} = -\frac{1}{\pi}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\Lambda^{M}\frac{\mathbf{V}_{ion}}{\left(V_{ion}^2+V_{eff}^2\right)^{3/2}}$$

Derbenev-Skrinsky (D-S) asymptotic

$$F_{\parallel}^{DS} = -\frac{3}{2}\omega_{pe}^{2}\frac{(Ze)^2}{4\pi\varepsilon_0}\left[\Lambda^{A}(V_{ion})\left(\frac{V_{\perp}}{V_{ion}}\right)^2+\frac{2}{3}\right]\frac{V_{\parallel}}{V_{ion}^3}$$

Angular dependence for Vion=3e5 m/s (B=5T, for $\Delta_{ex,y}$=8e6 m/s, L_M=2.4)



Transverse component of friction force for high velocities V > Δ $_{e\|}$

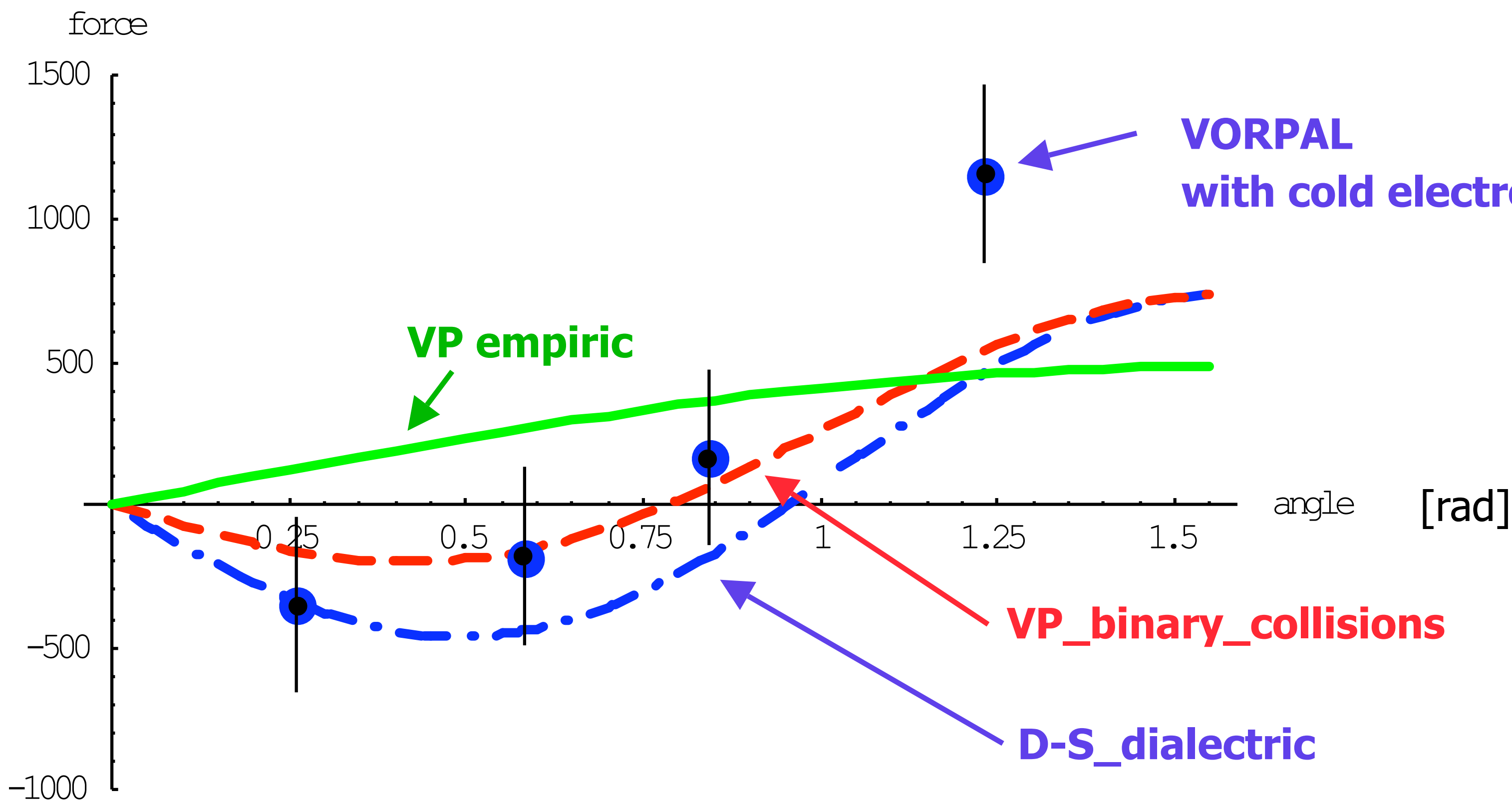
D-S$_d$ (1977)

$$F^{DS}_{\perp,dialectric} = -\frac{1}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \Lambda^A(V_{ion}) \frac{\left(V_\perp^2 - 2V_\|^2\right)}{V_{ion}^2} \frac{V_\perp}{V_{ion}^3}$$

P$_{\text{binary collisions}}$ (1984)

$$F^{P}_{\perp,binary} = -\frac{1}{2}\omega_{pe}^2 \frac{(Ze)^2}{4\pi\varepsilon_0} \Lambda^A(V_{ion}) \frac{\left(V_\perp^2 - V_\|^2\right)}{V_{ion}^2} \frac{V_\perp}{V_{ion}^3}$$

Angular dependence for the transverse component of the friction force



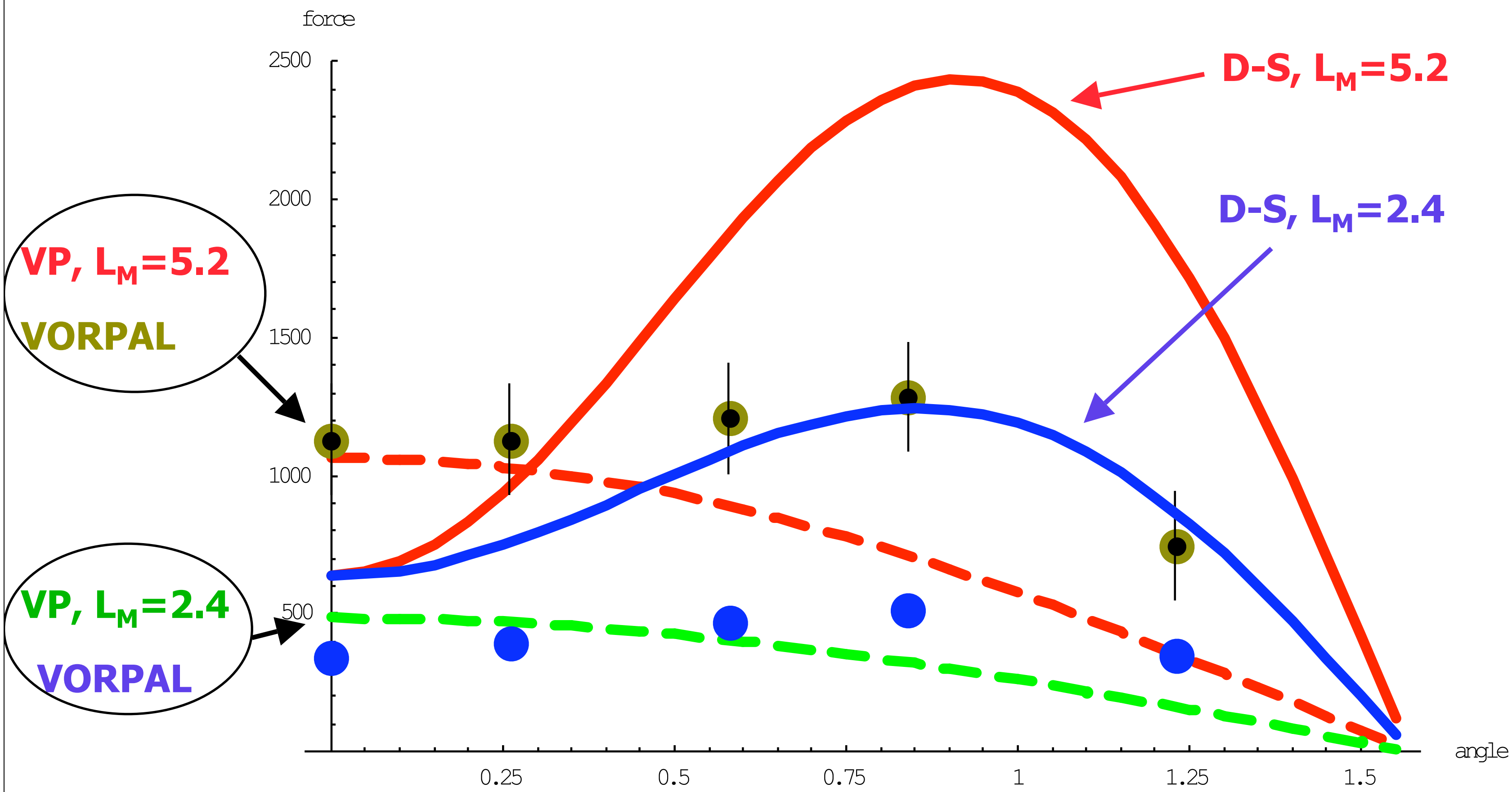
Longitudinal friction force at zero transverse angle (for large relative velocities)

We did studies for various strength of magnetic field - both for cold electron beam and finite temperature electron beam of different anisotropy.

Constant term in D-S expression does not help to correctly describe cooling at zero angle since it comes from collective plasma behavior and does not scale with the magnetized logarithm.

VORPAL shows scaling with magnetized logarithm at zero angle in agreement with VP expression (for the parameters of the RHIC cooler where maximum time of interaction is determined by the length of the cooling section).

Longitudinal friction force – scaling with magnetized logarithm for finite temperature electron beam



Conclusions on magnetized formulas

Using VORPAL code we are now able to explore fine effects in magnetized cooling.

We are exploring various available formulas and are trying to determine their accuracy in various regimes.

Based on this studies we should be able to simulate magnetized cooling with much better accuracy than an order of magnitude estimate – which is a critical task for the magnetized cooling estimates for future high-energy electron cooling such as HESR and RHIC-II.

Other important effects – such as an accurate treatment of magnetic field errors, are also being studied:

(see talk by D. Bruhwiler)

Acknowledgements

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