Parametric Resonance Ionization Cooling and Reverse Emittance Exchange for Muon Colliders

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Introduction

6D cooling in HCC

Parameter	*Unit*	*equilibrium rms value*
Beam momentum, p	MeV/c	100
Synchrotron emittance, ε_s	μm	260
Relative momentum spread	%	2.5
Beam width due to $\Delta p / p$	mm	1.5
Bunch length	mm	11
Transverse emittances, $\varepsilon_+ / \varepsilon_-$	mm-mr	100/300
Beam widths, σ_1 / σ_2	mm	2.8/4.5

Transverse emittance is still be too large for the required luminosity

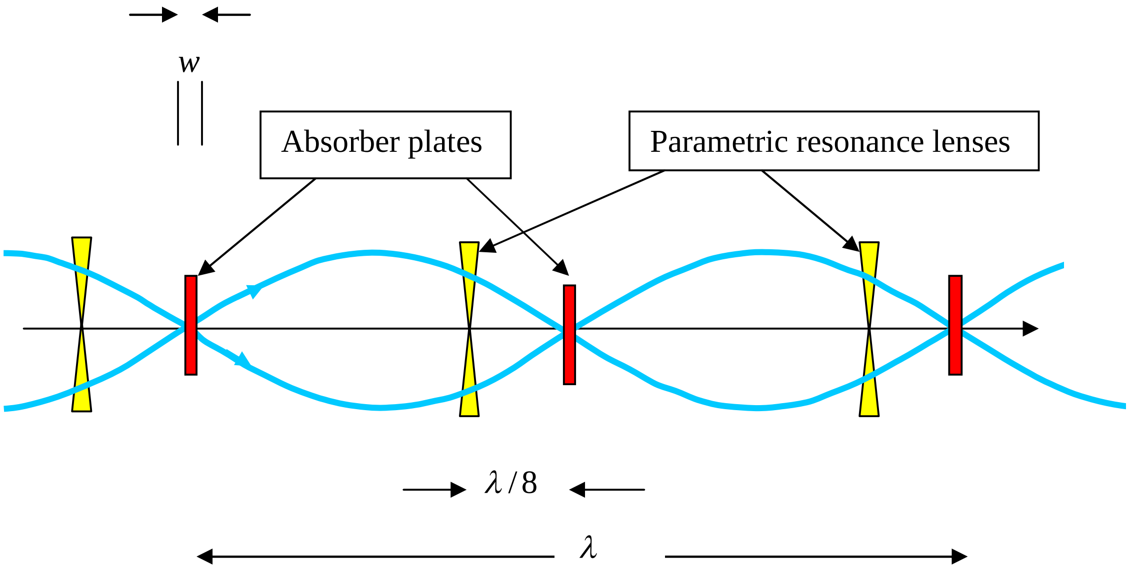
On the other hand, the longitudinal emittance appears exceedingly small to be useful in a collider (when preserved)

Basic principles of PIC

- **Assume initially the tune spread for a beam in a focusing channel to be smaller than the cooling decrement**
- **Weak lenses installed every half oscillation period drive a half-integer parametric resonance that creates a hyperbolic beam evolution at the absorber plates:**

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{n+1} = -\begin{pmatrix} k^{-1} & 0 \\ 0 & k \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix}_n \;;\; k = \exp(\Lambda_d \lambda / 2)$$

$$0 < \Lambda_d \lambda << 1$$



The lattice magnets and RF cavities to replace energy loss are not shown.

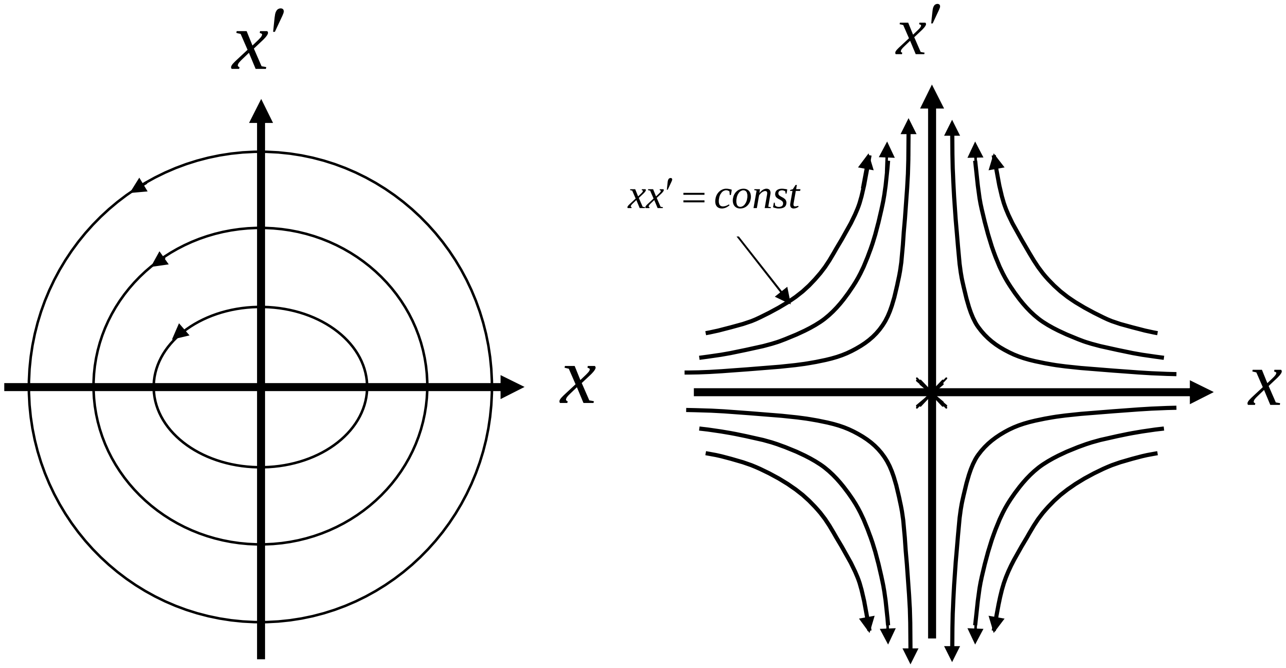


Fig. 1 Comparison of particle motion at periodic locations along the beam trajectory in transverse phase space for: LEFT ordinary oscillations and RIGHT hyperbolic motion induced by perturbations at a harmonic of the betatron frequency.

IC equations at Parametric Resonance

$$\theta^{2\,\prime} = \frac{\Lambda}{\beta^2}\left[-\left(1-2\beta^2\frac{\Lambda_d}{\Lambda}\right)\theta^2 + \frac{m_e}{2\gamma m_\mu}\left(Z+1+\frac{\gamma^2+1}{4\log}D'^2\right)\right]$$

$$\sigma^{2\,\prime} = \frac{\Lambda}{\beta^2}\left[\left(\frac{D}{2h}-2\beta^2\frac{\Lambda_d}{\Lambda}\right)\sigma^2 + \frac{m_e}{2\gamma m_\mu}\left(\frac{Z+1}{12}w^2+\frac{\gamma^2+1}{4\log}D^2\right)\right]$$

$$\varepsilon_z{}' = \frac{\Lambda}{\beta^2}\left[-\frac{D}{2h}+\frac{1}{\gamma^2}+\frac{m_e}{8\gamma m_\mu}\frac{\gamma^2+1}{\log}\left(\frac{p}{\Delta p}\right)^2\right]\varepsilon_z$$

Where

$$\Lambda = 2\frac{\gamma'_{abs}}{\gamma} = \frac{8\pi Z r_e r_\mu < n > \log}{\gamma\beta^2}; \qquad \log \approx 12$$

is the invariant 6D cooling decrement,

Λ_d **is the dynamical increment due to the parametric resonance, and** D/h **is a parameter of emittance exchange (dispersion/absorber height).**

The other terms are associated with scattering and energy straggling in the absorber.

PIC potential

Optimum conditions for PIC:

$$\frac{D}{h} = 2 - \frac{4}{3}\beta^2 \;; \qquad \frac{\Lambda_d}{\Lambda_\theta} = 1 - \frac{\beta^2}{3}$$

Equilibrium beam state:

$$\theta \approx \sqrt{\frac{3(Z+1)m_e}{2\gamma\beta^2 m_\mu}}\;; \quad \sigma \approx w\theta/2\sqrt{3}\;;$$

$$(\varepsilon_\perp)_{\min} \approx \frac{\sqrt{3}}{4}(Z+1)\frac{m_e}{m_\mu} w \equiv \varepsilon_{\perp 0}\;; \qquad w = \frac{\lambda}{2}\frac{< E'_{abs} >}{E'_{abs}}$$

$$< E'_a > = < E'_{acc} >$$

at conditions:

$$\theta << \frac{h}{w} 2\sqrt{3} << (\frac{Z+1}{\gamma^2+1}\log)^{1/2} / (1 - \frac{2}{3}\beta^2)$$

$$D' << 2\sqrt{\frac{Z+1}{\gamma^2+1}\log}$$

- **PIC results in reduction of $\varepsilon_\perp$ by a factor of** $\frac{\pi}{2\sqrt{3}} \frac{< E'_{acc} >}{E'_{abs}}$
- Z **-dependence of the equilibrium emittance:**

$$\varepsilon_{\perp 0} = \frac{\sqrt{3}}{16}\beta(1+\frac{1}{Z})\frac{\lambda/2\pi}{n r_e^2 \log}\gamma'_{acc}$$

Potential PIC effect

Parameter	**Unit**	**Initial**	**Final**
Beam momentum, p	MeV/c	100	100
Transverse beta function, $\lambda/2\pi$	cm	3	3
Distance between absorber plates, $\lambda/2$	cm	9.5	9.5
Plate thickness, w	mm	1.6	1.6
Intrinsic energy loss rate (Be)	MeV/m	600	600
Average energy loss	MeV/m	10	10
Accelerating RF field amplitude	MV generate /m	30	30
Transverse emittance	mm-mrad	600	24
Beam transverse size at plates, $\sigma_x = \sigma_y$	mm	4.0	0.17
Angle spread at plates, $\theta_x = \theta_y$	mrad	140	200
Phase cooling channel length	m	70	
Integrated energy loss	GeV	0.7	
Beam loss due to muon decay	%	10	
Number of particles/bunch*		10^{10}	
Space charge tune spread	%	.02	

***To overcome the space charge impact on tuning, one can implement a beam recombining scheme: generate a low charge/bunch beam and recombine the bunches after cooling and acceleration to sufficiently relativistic energy (under investigation)**

Reverse Emittance Exchange

The normalized longitudinal emittance after the basic and resonance cooling appears too small to be efficiently used (or maintained) in a collider. Therefore, reverse emittance exchange should be implemented after PIC to gain luminosity while preserving the achieved 6D emittance

The absorber based concept under study:

- Continue PR regime in order to maintain minimum beam size at plates
- Use the strongest reverse wedge: $h = \sigma / \xi ; \quad (\xi = 1/3)$
- Make dispersion at plates maximal: $D(\Delta p / p) = \sigma$
- Design the PR to equalize the σ and θ decrements:

$$\Lambda_d = -\frac{\Lambda}{4\beta^2}\left(\frac{\xi}{2}\frac{p}{\Delta p} - 1\right)(1-\chi) ; \qquad \chi \equiv \frac{\gamma}{4\xi}\frac{p}{\Delta p}\frac{m_e}{m_\mu \log} \quad (<1)$$

Then the relationship $\sigma^2 = \frac{w}{12}\theta^2$ is maintained.

- The relative momentum spread should also be maintained at a reasonable value implying bunch stretching (first stage) and then beam acceleration (second stage).

- The optimum maximum energy of REMEX is determined by energy straggling, which grows with energy.

- Angle scattering and energy straggling limit the achievable transverse emittance.

$$\varepsilon_z' = \Lambda_z \varepsilon_z; \qquad \Lambda_z \equiv \frac{\Lambda}{2\beta^2}\frac{p}{\Delta p}\frac{1+\chi}{\xi} >> \Lambda$$

$$\varepsilon_\perp' = -\frac{\Lambda_z}{2}\frac{1-\chi}{1+\chi}\varepsilon_\perp + \frac{\Lambda}{3}\frac{w\beta_0}{w_0\beta}\varepsilon_{\perp 0};$$

$$\frac{\varepsilon_\perp(z)}{\varepsilon_{\perp 0}} \Rightarrow \approx \sqrt{\frac{\varepsilon_{z0}}{\varepsilon_z}} + \frac{4\beta\beta_0}{3\xi(1-\chi)}\frac{w}{w_0}\frac{\Delta p}{p}$$

Potential REMEX effect

Parameter	**Unit**	**Initial**	**Final**
Momentum	MeV/c	100	2500
Bunch length	cm	.5	10
Momentum spread	%	3	3
Longitudinal emittance, norm	cm	1.5x10	7.5
Transverse emittance, norm	μm	25	2

Parametric Resonance Aberrational Demands

The challenge is to have all particles reach the minimum radial position at the same place in the channel (at the absorber positions)

Resonance coherence will be violated by the tune spreads

- **There are constant and alternating tune spreads**
  PR efficiency condition for the constant spreads:

$$\Delta\psi_{const} = \frac{\Delta f_{const}}{f^2} l_{cool} << 1$$

  PR efficiency condition for the alternating parts
  (period $2\pi\beta_{alt}$, at $\beta_{alt} << l_c$):

$$\Delta\psi_{alt} = \frac{\Delta f_{alt}}{f^2} \beta_{alt} << 1$$

- **Tuning for full PR efficiency:**

$$\Delta\psi_{tot} \equiv \sqrt{(\Delta\psi)^2{}_{const} + (\Delta\psi)^2{}_{alt}} << \frac{w}{2\sqrt{3} f}$$

- **PIC design includes compensation for aberrations**
- **Canonical relationships between non-linear tunes must be taken into account**
- **For non-compensated chromaticity, there are two important effects of the RF field to be taken into account:**
  1) **Synchrotron oscillation of energy suppresses the chromaticity detuning impact on PR, although not to a level of negligibility**
     ($\Delta\psi_{const} \rightarrow \Delta\psi_{alt} << \Delta\psi_{const}$)
  2) **The auto-synchronism between particle longitudinal motion and RF field creates an energy shift proportional to the transverse momentum squared (correlated energy)**
- **The non-linear detuning due to space charge will limit the number of muons in each bunch**

PIC in alternating solenoids without chromaticity compensation

For parametric resonance in a solenoidal channel:

- **Canonical angular momentum is conserved**
- **The ½ resonance is radial and driven by a small modulations of the solenoidal field strength**
- **At resonance, the transverse momentum grows and the minimum radial position decreases**

Expansion of focusing strength in solenoid (RF field is on; correlated energy is taken into account):

$$\frac{1}{f^2} \propto \left(\frac{B_z}{4p_z}\right)^2; \quad \left(\frac{p_0}{p_z}\right)^2 \approx 1 - 2\frac{\tilde{p}}{p_0} + \left(\frac{\tilde{p}}{\gamma_0 p_0}\right)^2 - \theta^2$$

$$B_z^{\,2}(z,\rho) \approx B_0^{\,2}(z) - \frac{1}{2} B_0 B_0'' \rho^2$$

- **The correlated energy transforms the positive angle aberration of the focusing strength to the negative one**
- **This renormalization offers a possibility to compensate for angle aberration by use of the fringe field of an alternating solenoid channel**

$$\Delta\psi_{const} \Rightarrow 0; \qquad (\Delta\psi)_{alt} = \frac{\tilde{p}}{p}(f_s / f)$$

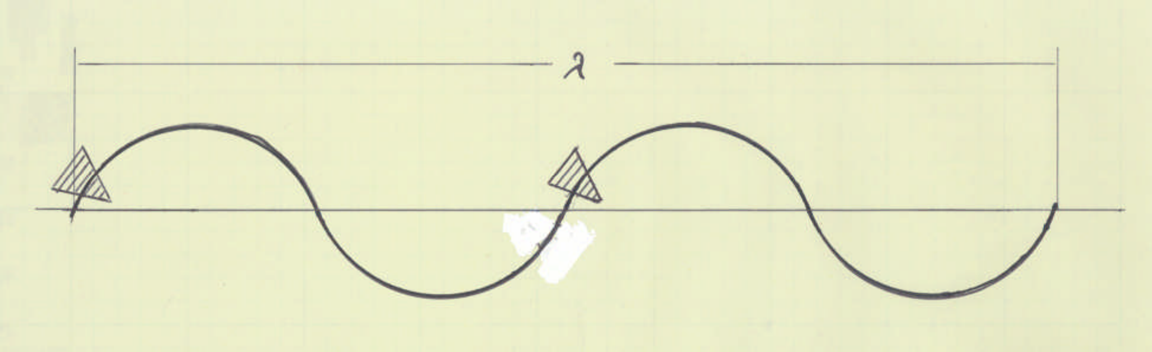
An important feature of the alternating solenoid is that the non-linear radial tune appears to be a function of only the radial Courant-Snyder invariant I_ρ but not a function of $\hat{M}$ (since there is no a certain helicity in average in such system).

Therefore, there is the only condition to fulfill : $f^2 \approx (Al)_{sol}$.

Achromatic channel for PIC

- **Compensation for chromaticity requires relatively large orbit dispersion – which is a constraint to PIC because of increase of energy straggling impact on transverse emittance**
- **A resolution of this constraint is: design a dispersion function that follows the beam envelope at PR**

Scheme: **Achromatic wiggler**



- **Field index $n = 1/2$ (symmetric focusing, $f \equiv (\lambda/2\pi) = R\sqrt{2}$)**
- **Betatron phase advance $\pi/2$ per bend segment (bend angle $\pi/\sqrt{2}$)**
- **Dispersion then oscillates with period equal to half of the betatron oscillation period**
- **Sextupole alternates in tact with the beam bend**
- **Orbit plane interchanges**

However, compensation for chromaticity leads to a revival of the angle aberration. This seems possible to compensate by superimposing octupole field in combination with solenoid one (both relatively week) /under study/

Achromatic channel for REMEX

First stage: REMEX accompanied by beam stretching

Focusing is similar to PIC. The scheme includes sections with RF resonators and optics for de-bunching.

Second stage: REMEX accompanied by acceleration

PIC, REMEX and MC Luminosity

$$L = \frac{N^2}{4\pi\sigma^2} f_r = E\frac{\langle J\Delta\nu\rangle}{e^3\beta^*} = J_0 E\frac{\langle q\Delta\nu\rangle}{e^3\beta^*}\,; \qquad \Delta\nu = \frac{N r_\mu}{4\pi\varepsilon_\perp}$$

- **A desirable bunch length is determined by a minimum beta-star that can be designed**
- $l_{\min} = (\varepsilon_z / \Delta\gamma_{adm}) \le \beta^*$**;** $\qquad \beta^* >> \Delta F$
- **Aberrations of focal parameter,** $\Delta F / F$ **, can be compensated to a level of** 10^{-5}
- **Then,** $\beta^* \approx 1mm$ **at 2.5 TeV seems possible to design**

Circumference	**KM**	**12**
Energy	**TeV**	**2.5**
IP focal parameter, F	**M**	**20**
Beta-star/bunch length	**mm**	**1/1**
Energy spread	**%**	**.1**
Emittance ε_z**, norm**	**cm**	**2. 5**
Emittance $\varepsilon_\perp$**, norm**	μ**m**	**3**
Beam size at ff magnets, σ_f	**mm**	**6**
Number of muons/bunch, N		$1.5 \cdot 10^{11}$
Beam-beam tune shift/IP, $\Delta\nu$		**.1**
Number of bunches		**10**
Number of revolutions, q		**1500**
Luminosity/IP	$10^{35} / cm^2 s$	**1**
Muon flux from linac	**1/s**	$1.5 \cdot 10^{12}$

Conclusions

- **There exists a good potential for raising the efficiency of the Ionization Cooling after basic 6D cooling for a Muon Collider by implementing Parametric Resonance Ionization Cooling and Reverse Emittance techniques**

- **Some concepts of beam focusing and tune spread compensation for best parametric resonance effect and emittance exchange have been proposed**

- **Compatibility of beam resonance focusing with effective emittance exchange in both directions is understood, though it suggests some absorber technology issues for study and design**

Thank you