New Ideas for 6 D Ionization Cooling

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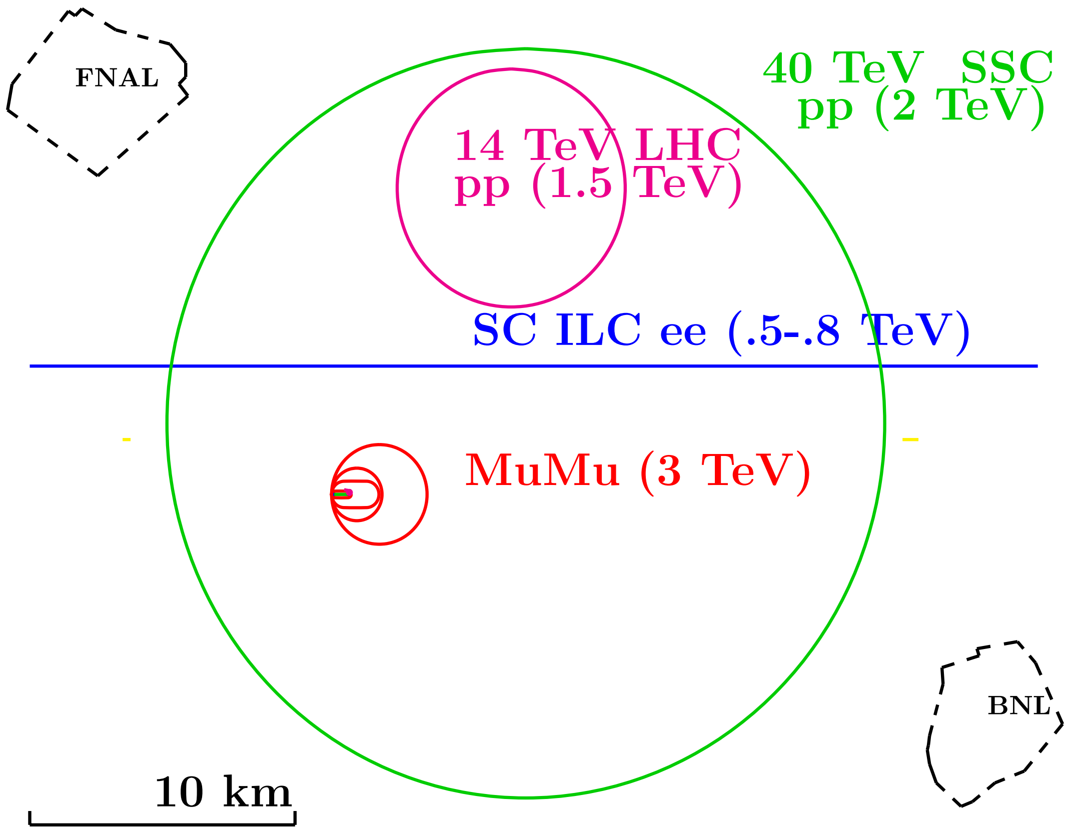
- Muon Collider requirements
- Transverse Ionization Cooling Theory
- How to get low beta
  - Solenoids
  - Focusing Lattices
  - Li Lenses
- Longitudinal Emittance Cooling
- Concept of complete system
- Conclusion

Why a Muon Collider



- Muons are point like, similar to electrons
- Can probe the same physics, and some more
- But have 40,000 less radiation
- So Muon Colliders can be much smaller than Linear Colliders

3 TeV Collider requirements

- **Luminosity limited independent of emittance:**

$$\mathcal{L} \propto n_{\rm turns}\ f_{\rm bunch}\ \frac{N_\mu^2}{\sigma_\perp^2} \qquad \Delta\nu \propto \frac{N_\mu}{\epsilon_\perp}$$

$$\mathcal{L} \propto B_{\rm ring}\ P_{\rm beam}\ \Delta\nu\ \frac{1}{\beta_\perp}$$

- **Higher $\mathcal{L}/P_{\rm beam}$ requires lower $\beta_\perp$ or correction of $\Delta\nu$**
- **But required emittance still small**

$$\epsilon_\perp \propto \frac{N_\mu}{\Delta\nu}$$

For the record

$$\mathcal{L} = n_{\rm turns}\ f_{\rm bunch}\ \frac{N_\mu^2}{2\pi\sigma_1\sigma_2}$$

$$\Delta\nu_i = \left(\frac{r_o}{2\pi}\right)\ \left(\frac{N_\mu}{\gamma}\right)\ \left(\frac{\beta_i}{\sigma_i(\sigma_1+\sigma_2)}\right) \qquad \text{see}^1$$

$$n_{\rm turns} = \int_o^\infty \left(e^{\frac{2\pi r n}{\gamma\tau_\mu}}\right)^2\ dn = \frac{\gamma\ \tau_\mu c}{4\pi r}$$

$$r = \frac{p}{<B>} = \frac{(m_\mu c^2)\ \gamma}{B\ c}$$

if $\sigma_1 = \sigma_2$ **then:**

$$\mathcal{L} = \left(\frac{\tau_\mu\ c^2}{4\ \pi\ (mc^2)^2\ r_o}\right)\ <B_{\rm ring}>\ P_{\rm beam}\ \frac{2\ \Delta\nu}{\beta_\perp}$$

$$\epsilon_\perp \propto \frac{N_\mu}{\Delta\nu}$$

For $\sigma_2 \ll \sigma_1$ **then**

$$\mathcal{L} = \left(\frac{\tau_\mu\ c^2}{4\ \pi\ (mc^2)^2\ r_o}\right)\ <B_{\rm ring}>\ P_{\rm beam}\ \frac{\Delta\nu_2}{\beta_2}$$

which is 1/2 that for $\sigma_1 = \sigma_2$

[1] e.g. Accelerator physics, S.Y.Lee, p387

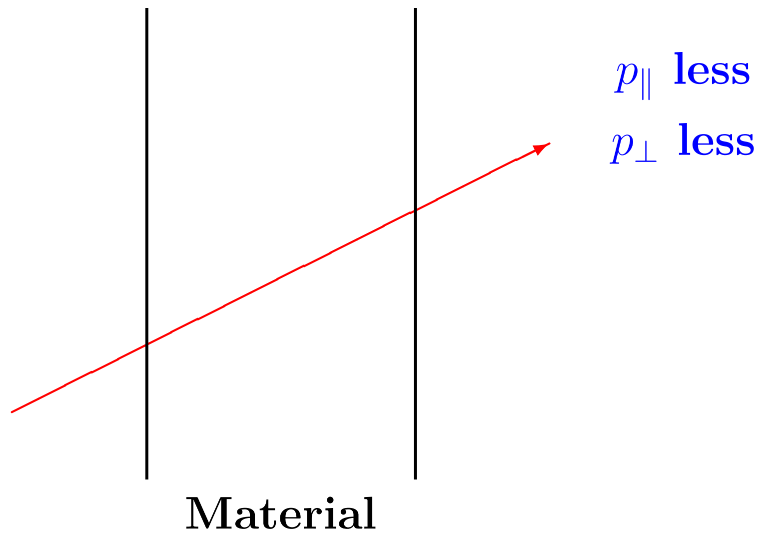
Snowmass 98 Assumed

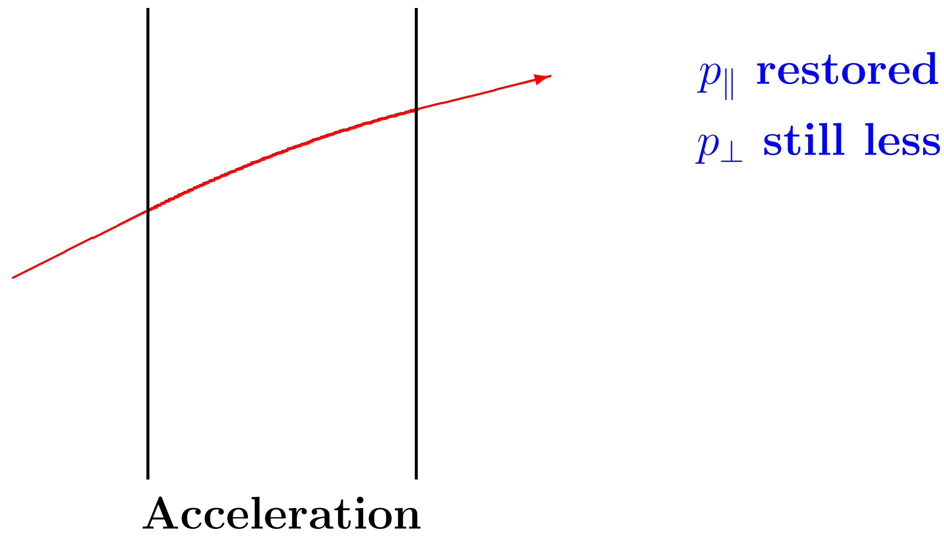
Average bending field	**T**	**5.2**
Tune Shift (from e rings)		**0.044**
Luminosity	$10^{33} cm^{-2}$	**70**

	$\mathbf{E}_{\mathrm{cm}}$ TeV	$\mathbf{N}_\mu$ 10^{12}	**f** Hz	$\mathbf{P}_\mu$ MW	$\beta_\perp = \sigma_z$ mm	**dp/p** %	$\mathbf{emit}_\perp$ π mm	$\mathbf{emit}_\parallel$ π mm	ϵ_6 $(\pi \text{ mm})^3$
Required	**3**	**2**	**30**	**28**	**3**	**0.16**	**.05**	**72**	**170**
Initial							**20**	**2000**	10^9
Factor							**400**	**30**	**6** 10^6

Required Cooling 6 D by 1/6,000,000

Transverse Cooling





Rate of Cooling without scattering

$$\frac{d\epsilon}{\epsilon_{x,y}} = \frac{dp}{p} \quad J_{x,y}$$

For the moment the "partition functions" Explanation later

$$J_{x,y} = 1$$

Minimum (Equilibrium) Emittance

$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{\beta_v\ J_{x,y}}\ C(mat,E)$$

$$J_{x,y} = 1 \qquad\qquad C(mat,E) \propto \frac{1}{L_R\ d\gamma/ds}$$

At minimum of dE/dx ($\approx$ 300 MeV/c)

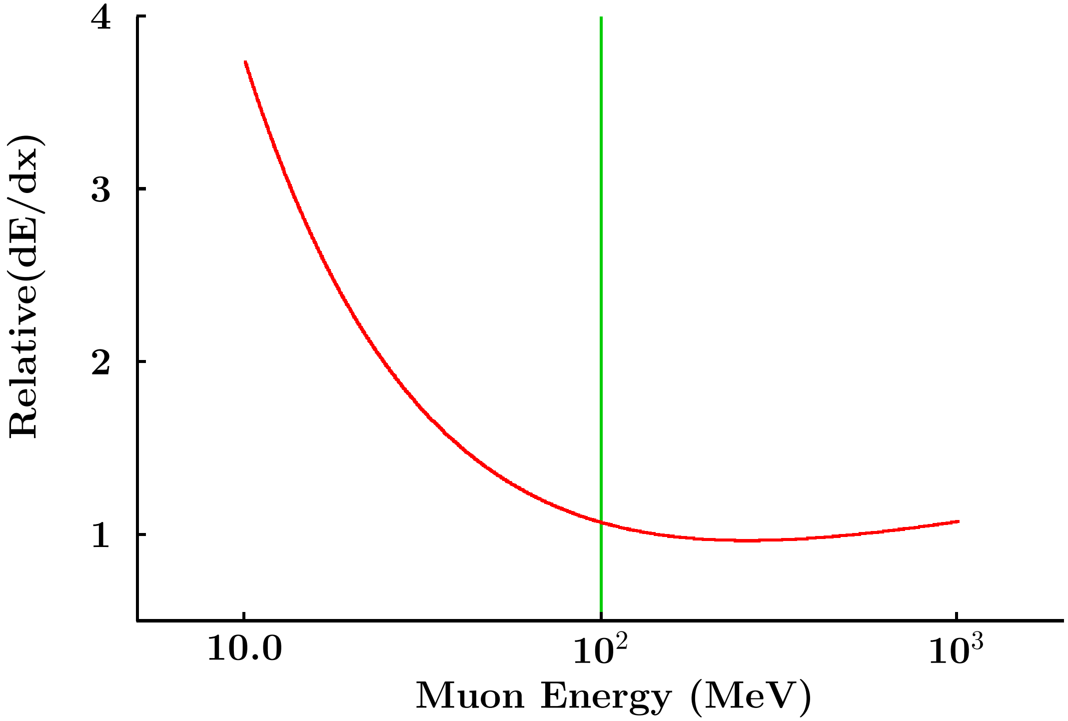
material	density kg/m^3	dE/dx MeV/m	L_R m	C_o %	A_o %
Liquid H_2	71	28.7	8.65	0.38	1.36
Li	530	87.5	1.55	0.69	1.31
Be	1850	295	0.353	0.89	1.28
C	2260	394	0.47	1.58	1.25
Al	2700	436	0.089	2.48	1.23

- **Hydrogen much the best material**
- **Coefficient A_o is for longitudinal cooling - explanation to come**

Choice of Momentum

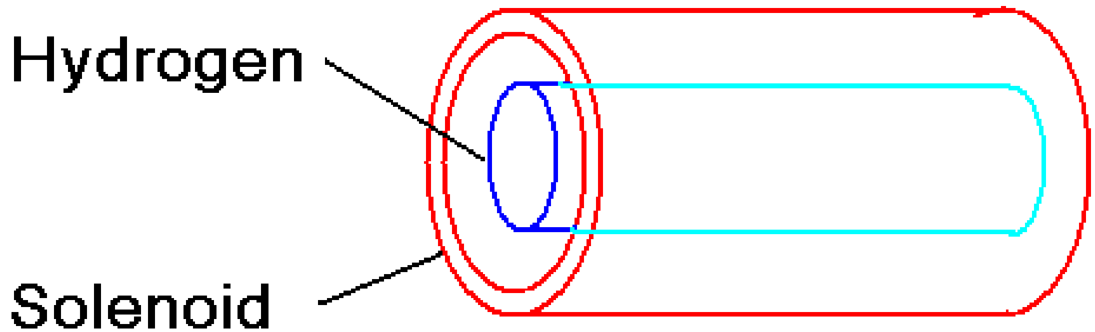
At lower Momentum,

- **Easier to get low betas**
- **dE/dx greater, so C is smaller**
- **But low E losses more than high E → increased dE/E**



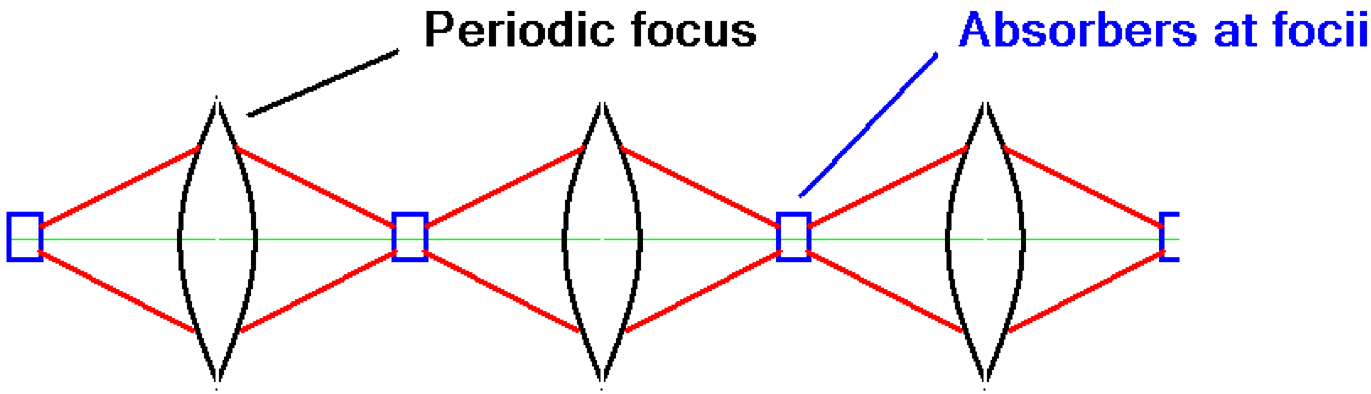
How to get low beta (strong focus) ?

- **Strong Solenoid**

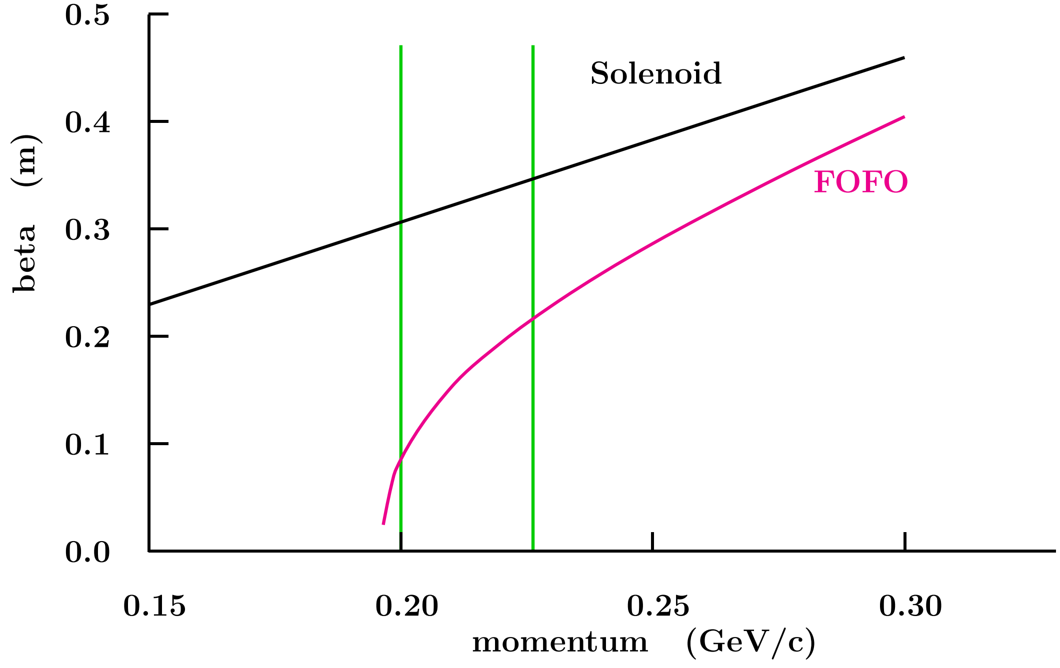


  - **HTS superconductors now allow,**
    for small bore,
    Maximum field $\approx$ 20 T

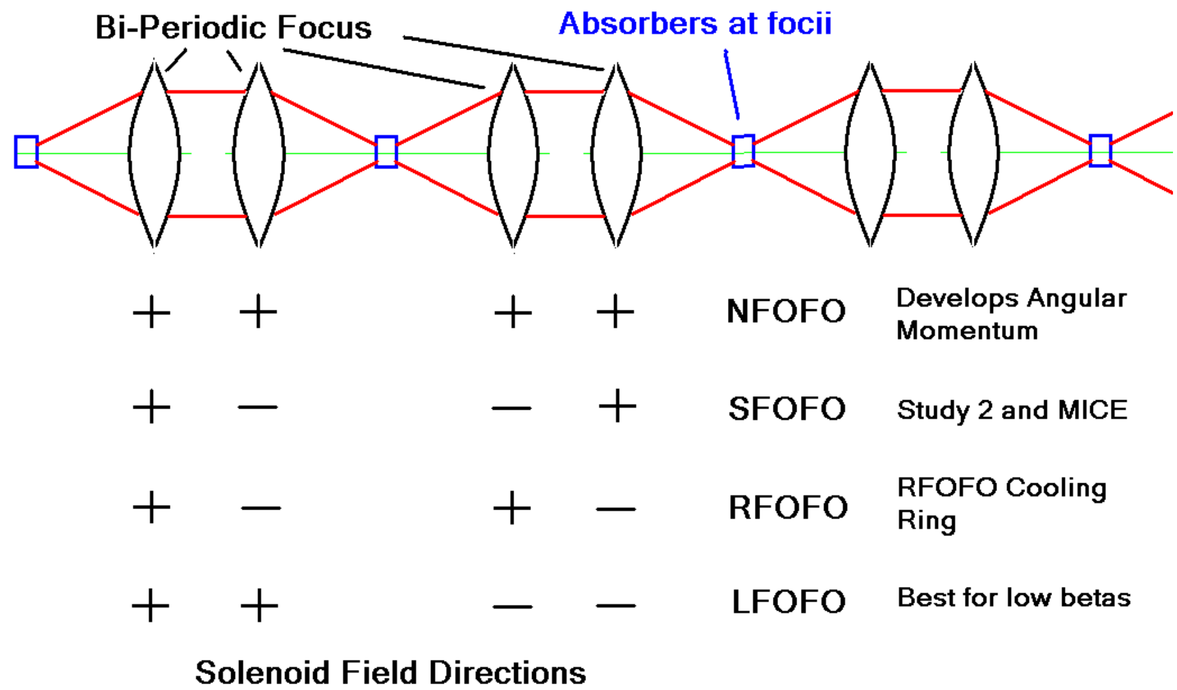
- **At Simple Multiple foci (Focus-Focus, or FOFO)**

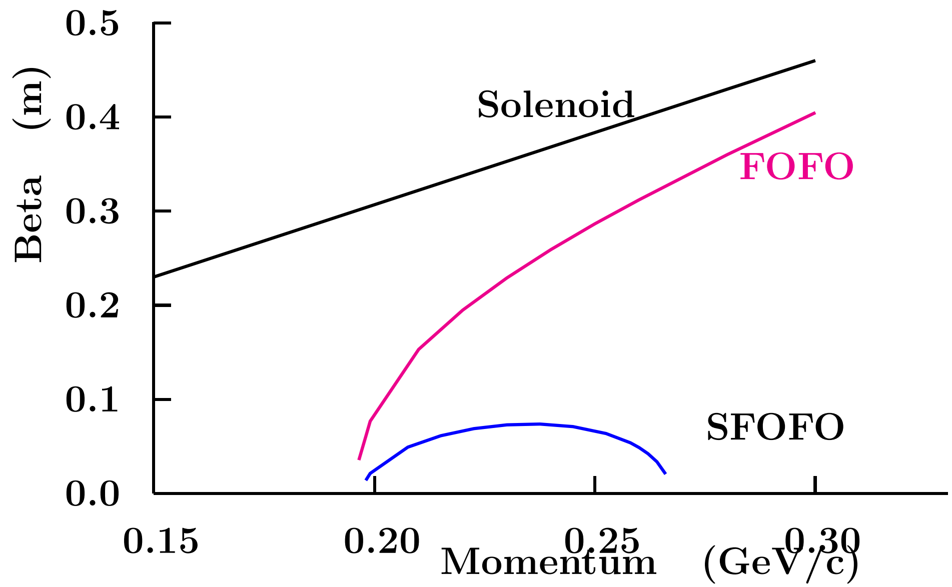


  - **Lower beta for same $\int B$**
  - **But for narrow dp/p**



• **Bi-Periodic Lattices (Andy Sessler)**





– Two resonaces

– Low beta between

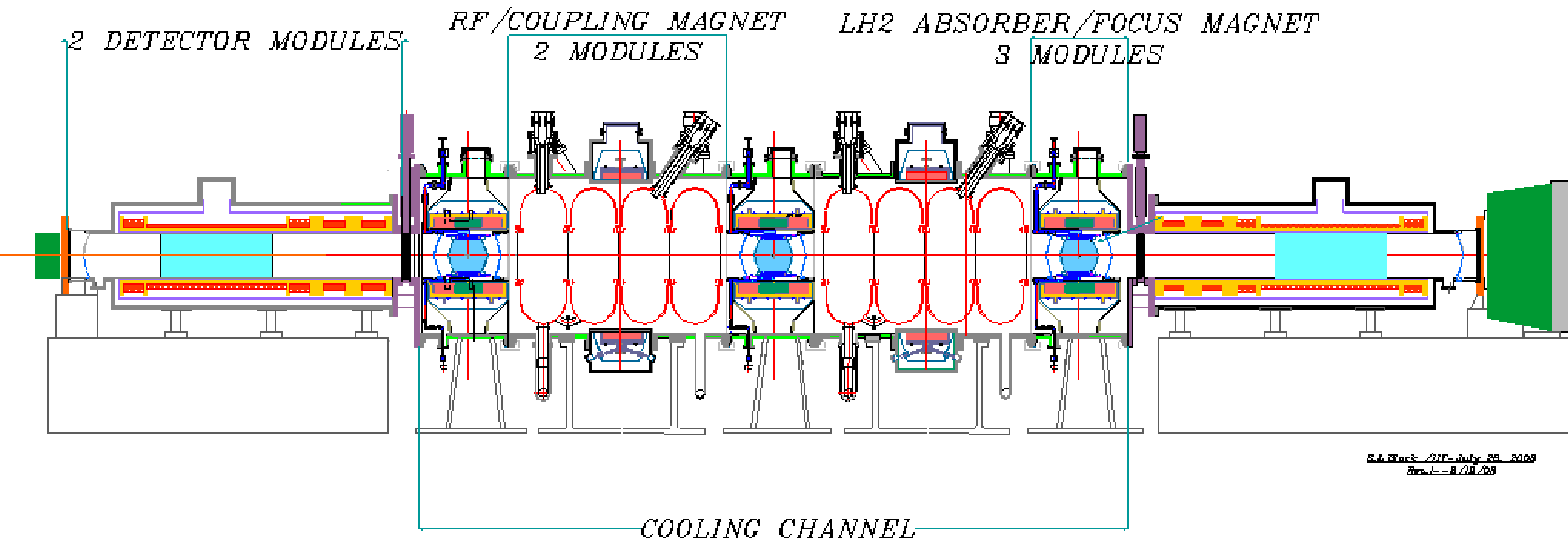
– Lower beta than FOFO

– Wider dp/p than FOFO but less than Solenoid

SFOFO used in Study 2 and Cooling Experiment

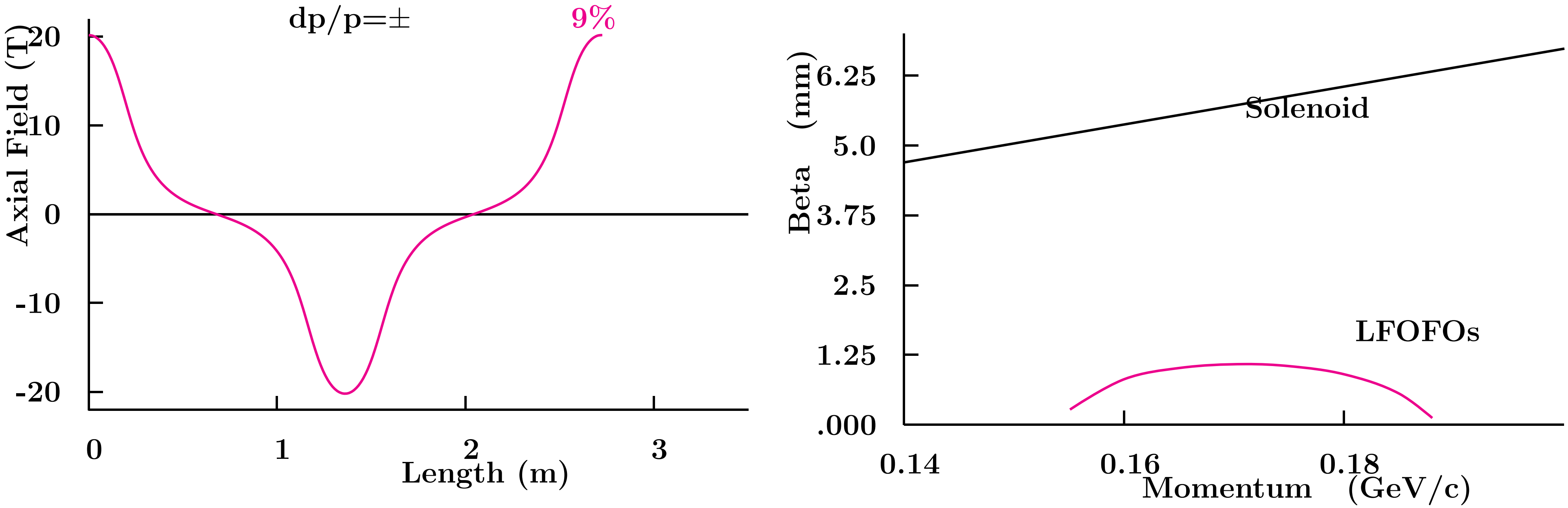
Muon Ionization Cooling Experiment MICE

- International Collaboration: (US, Europe, Japan)
- Proposal Approved at RAL
- Funding for phase I NSF (NSF, DOE, Europe, Japan)



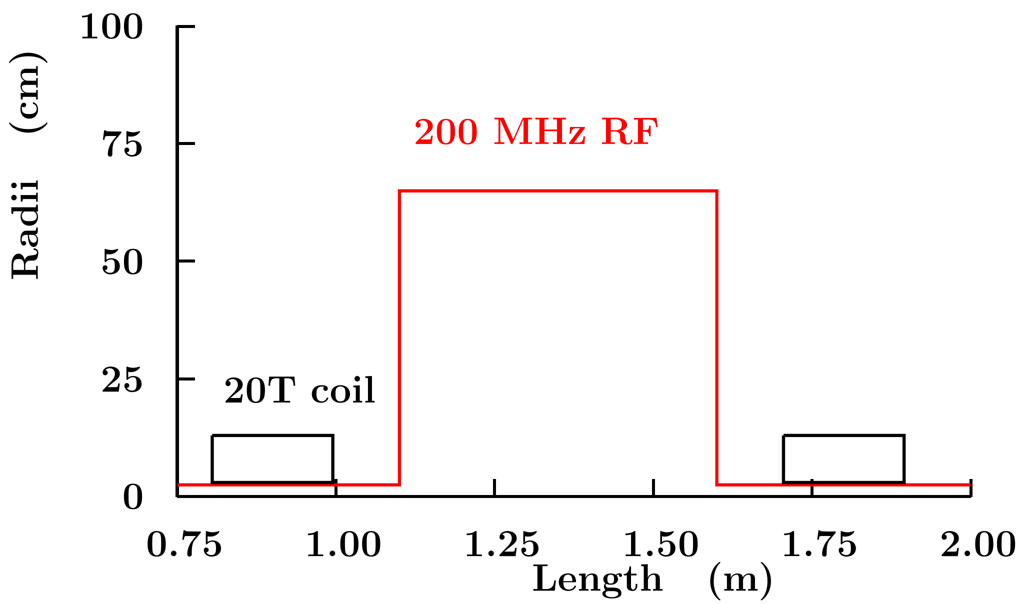
LFOFO (For Lowest betas)

HTS (BSCCO) superconductors now allow field = 20 T

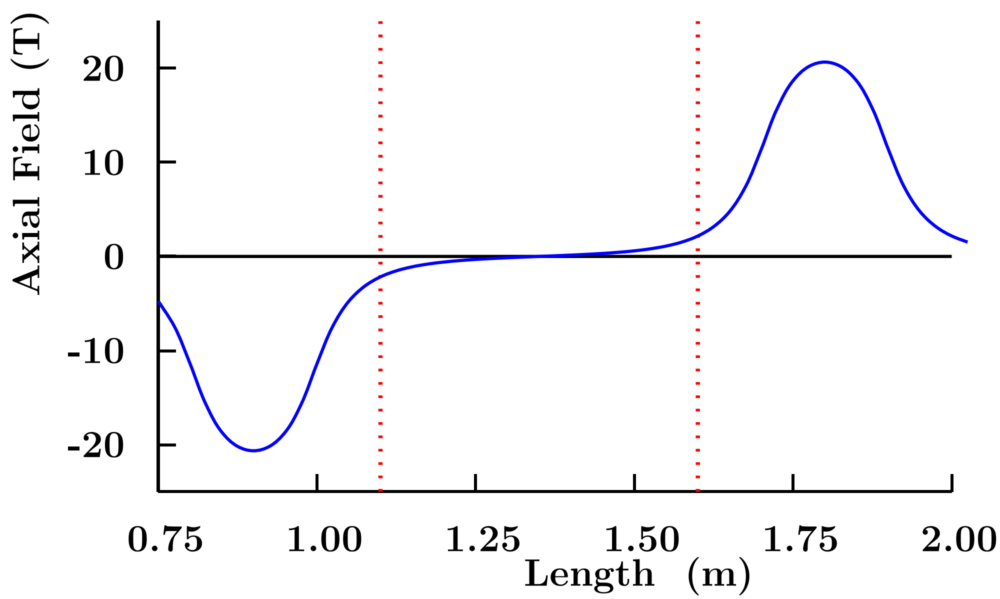


- For final cooling
- Short cells
- Beta below 0.7 cm at 70 MeV/c

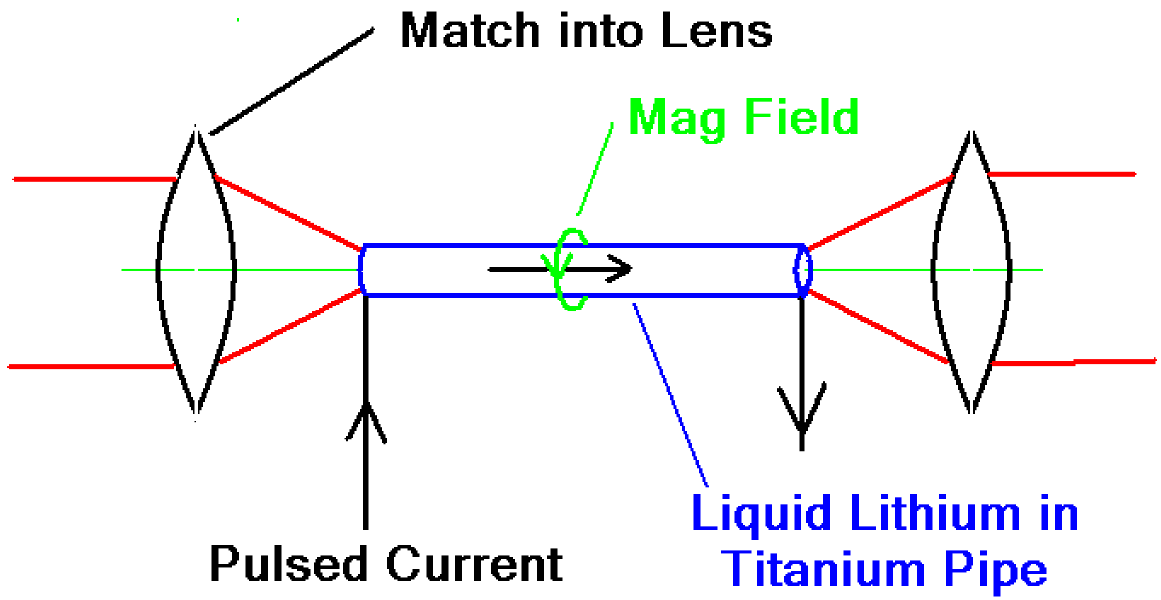
One Cell of 20T LFOFO Cooling Lattice



– Magnets are small
– RF is in low field



- **Lithium Lens**



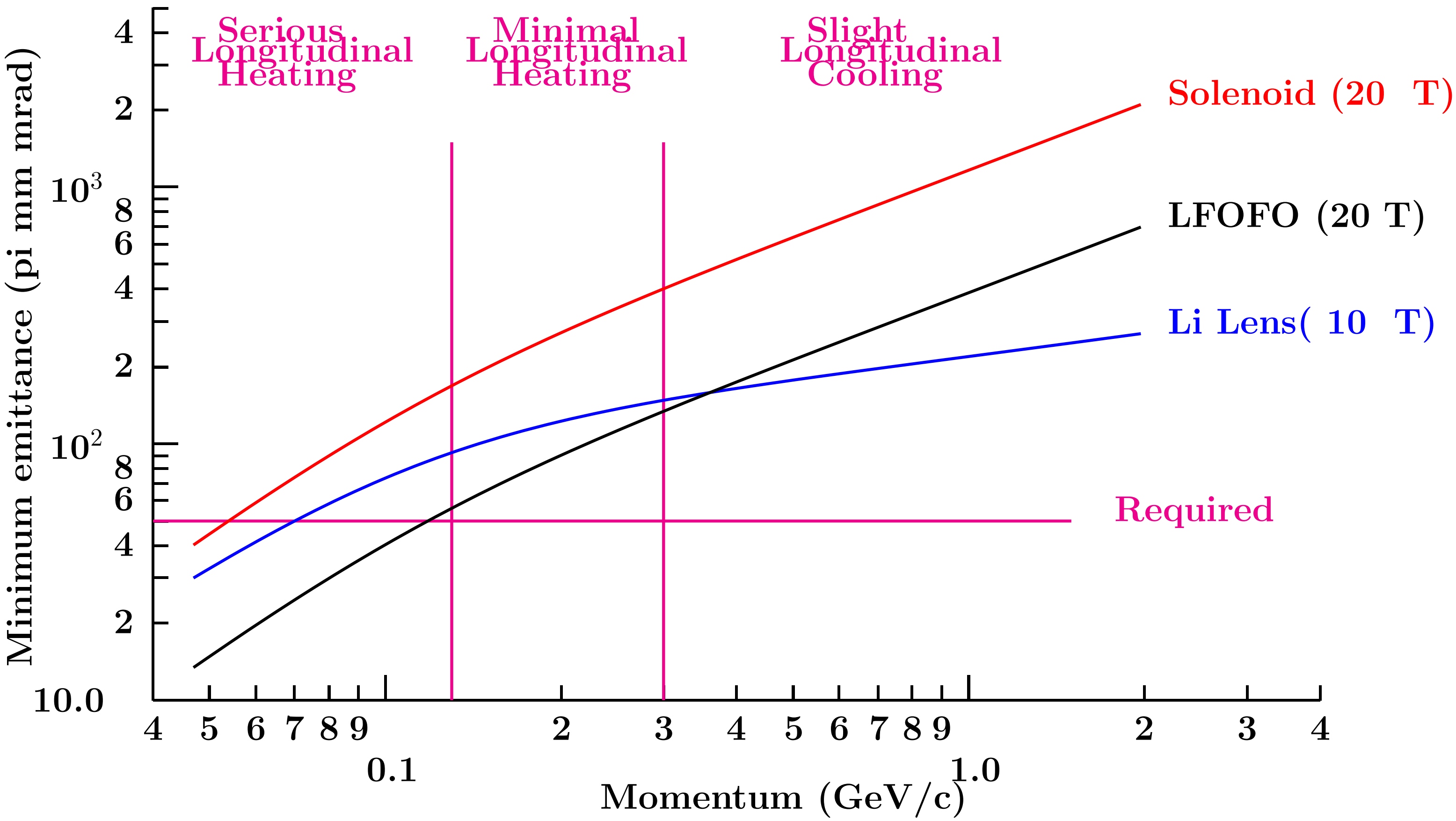
  - **For uniform i then perfect lens**

$$I \propto r^2 \qquad\qquad \text{Bending} \propto B \propto I/r \propto r$$

  - **Maximum current limited by breaking containment tube**
  - **Pressure $\propto$ Surface Field   Current lenses 10 T**

Equilibrium Transverse emittances vs Energy

- Both Li Lens and LFOFO achieve req. at low energies
- But longitudinal emittance will be rising fast



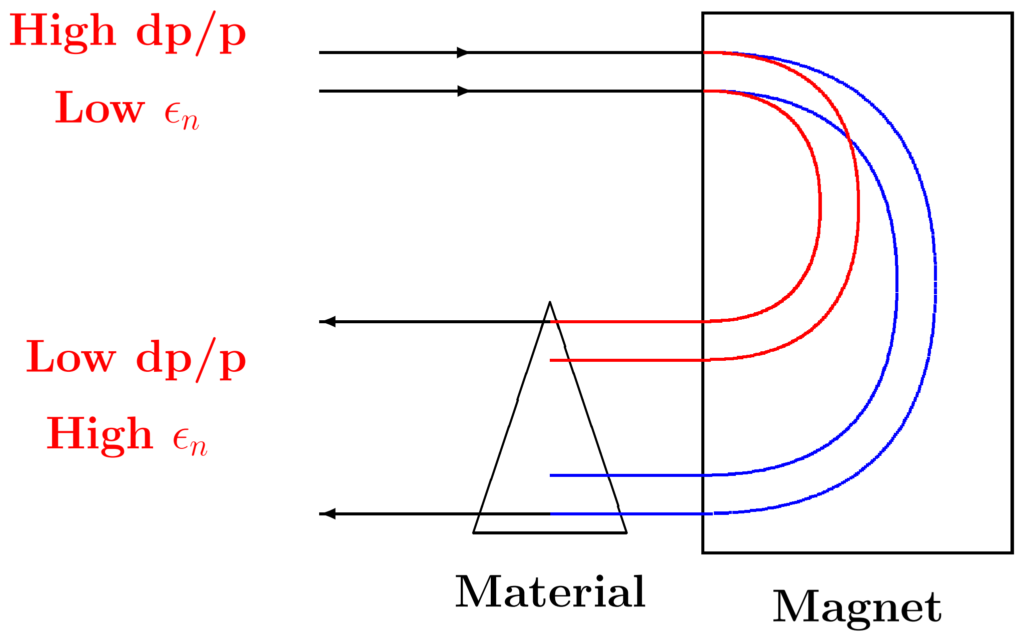
- Ok If we know how to reduce it first

Work in Progress

- **Final Li Lens successfully simulated**
- **Including matching in and out**
- **But engineering of lens not demonstrated**

- **20T LFOFO appears to achieve required emittance**
- **Momentum acceptance limited to $\pm$ 9 %**
- **Not yet simulated**

Longitudinal Cooling: Emittance Exchange

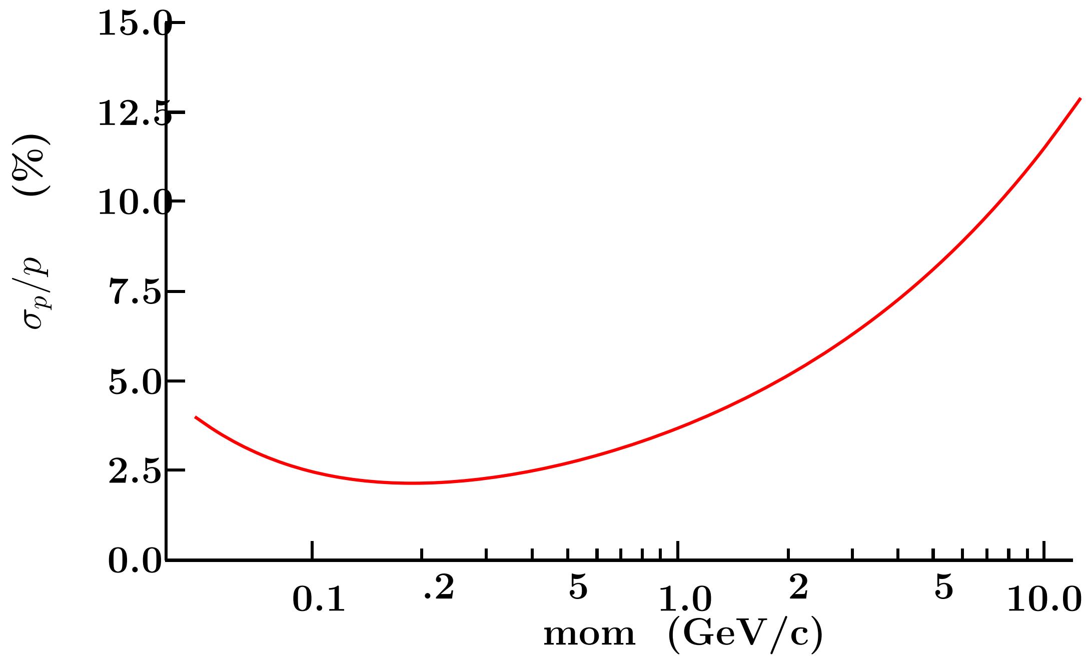


- dp/p (and Longitudinal emittance) reduced
- But σ_y (and transverse emittance) increased
- Transverse cooling from mean loss in absorber
  $\longrightarrow$ "Emittance Exchange"
- J's are modified, but Jx + Jy + Jz = constant

"Emittance Exchange" + Transverse Cooling = 6 D cooling

Minimum (equilibrium) dp/p

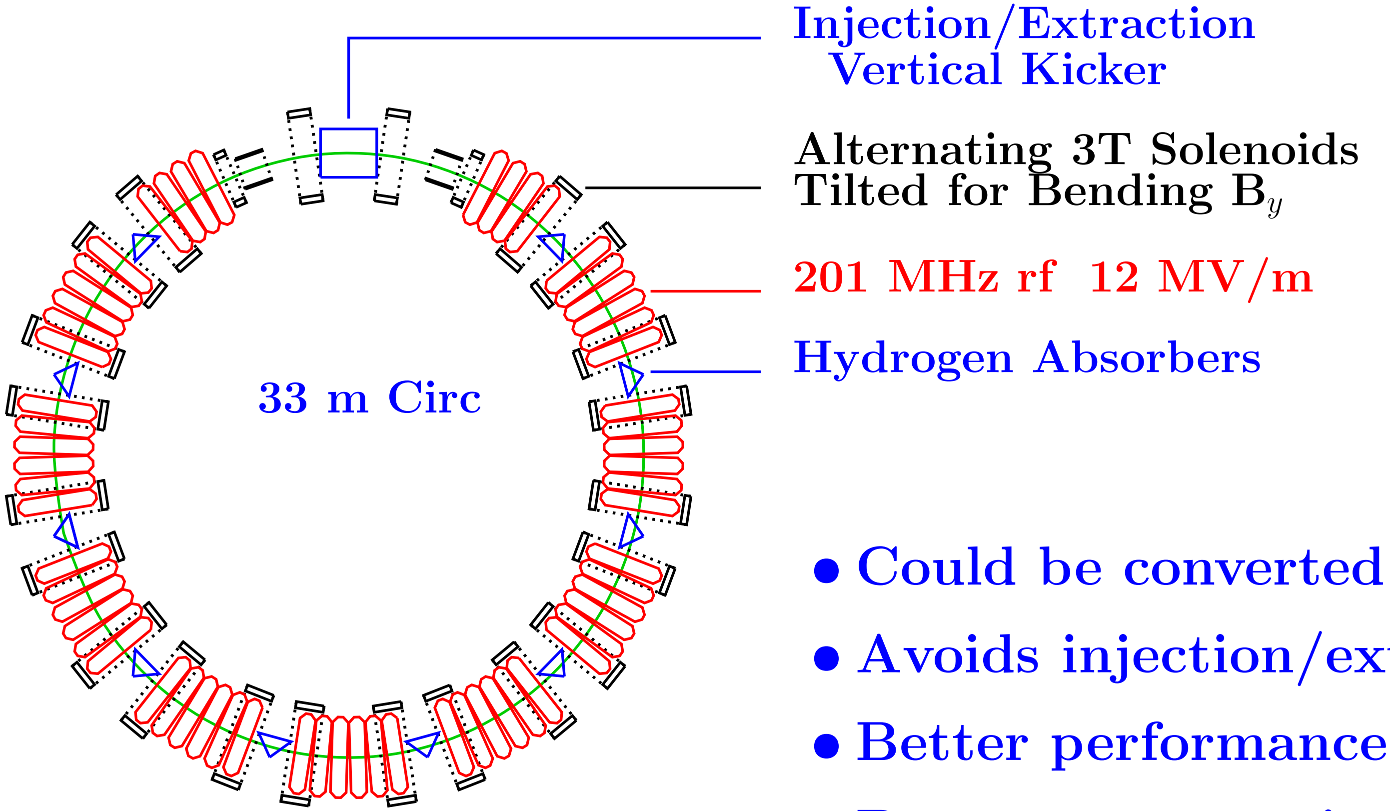
If equal cooling in 3 dimensions:



- **Minimum at 2.5% around 200 MeV/c**
- **Encouraging for the above LFOFO acceptances ≈ 9%**

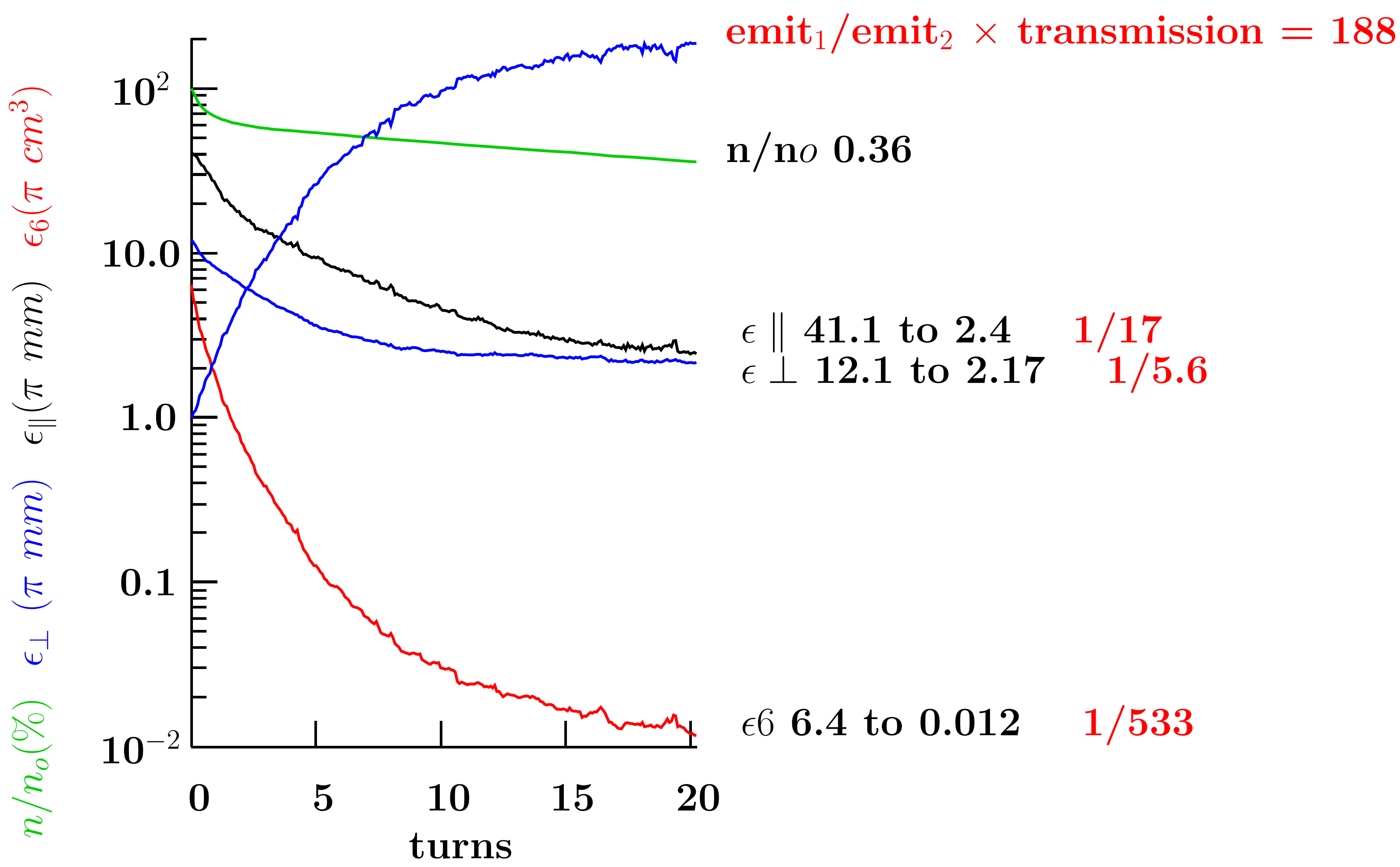
e.g. 6 D cooling in "RFOFO" Ring with Wedges

- Bending gives dispersion
- Wedge absorbers give cooling also in longitudinal
- Many turns in ring gives more cooling at lower cost
- But Injection/extraction Hard



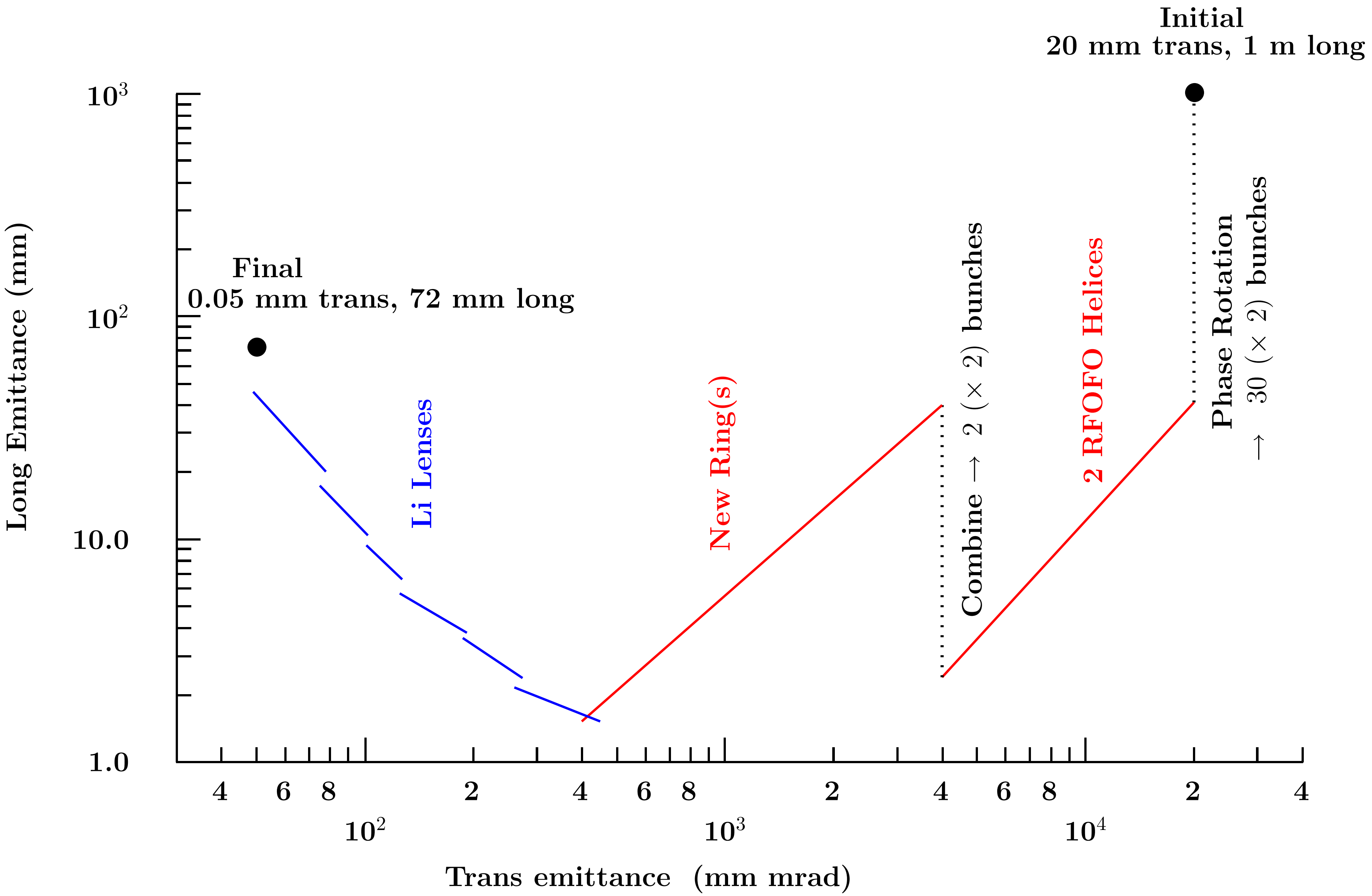
- Could be converted to Helix
- Avoids injection/extraction
- Better performance by tapering
- But more expensive

Performance of RFOFO Ring



- Final Long Emittance 2400 (pi mm mrad)
- Less that (7200 pi mm mrad) Required for Collider

Plausible Solution to Collider Requirements



Simulations Done

- **Initial Phase Rotation and Bunching**
- **Most of Initial RFOFO Helix**
- **All Li Lenses**
- **Match from last Lens (the hardest)**

Work needed on

- **Bunch Combiner**
- **Low beta cooling rings**
- **Matching in and out of all Li Lenses**

- **Version using LFOFO instead of Li Lenses**

Conclusion

1. RFOFO Rings/helices with wedges (emittance exchange)
   can lower longitudinal emittance below requirement

2. Li lenses and/or LFOFO Lattices can achieve trans. req.
   but only at low momenta where
   Long emittance is growing (reverse emittance exchange)

By using 1) first, and then 2),
all requirements appear attainable

- Much more Study is needed
- There are many other problems
- But there is reason to hope the problem is soluable