Applications of Schottky Spectrocopy at the Storage Ring ESR of GSI

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Cool05

Galena, IL

September 20, 2005

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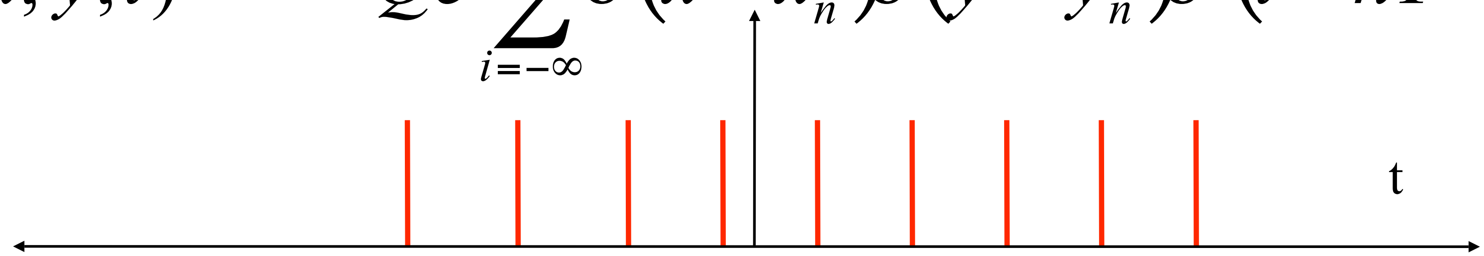
Single Particle Signal

$$j(x,y,t) = Qe\sum_{i=-\infty}^{\infty}\delta(x-x_n)\,\delta(y-y_n)\,\delta(t-nT-t_0)$$



convolution

$$U(t) = \frac{S(x,y,t)}{2} * j(x,y,t)$$

Spectrum depends very much on the x and y dependence of S!

Pick-up Response Models

- Longitudinal Pick-up: $S(s,y,\Omega) = S^L(\Omega)$
- Horizontal Pick-up: $S(s,y,\Omega) = xS_x^{'}(\Omega)$
- Vertical Pick-up: $S(s,y,\Omega) = yS_y^{'}(\Omega)$

Voltage Spectral Density

U(t): Voltage from Schottky probe with Fourier transform *U(Ω)*

First definition of voltage spectral density of a stationary process

$$\langle U(\Omega)U(\Omega')\rangle = C_U(\Omega)\delta(\Omega+\Omega')$$

or

Define $C_U(\Omega)$ as the Fourier transform of the autocorrelation function

$$R_U(\tau) = \langle U(t)U(t+\tau)\rangle$$

True expectation value can only be estimated from single measurements

Both definitions are equivalent (Wiener-Khinchine)

Longitudinal Schottky Spectral Density

Longitudinal harmonics

good for heavy ions

good for small rings

good for cool beams

$$C_U(m\omega) = \frac{(Z_L Q e)^2 \omega}{8\pi |m\eta|} \left| S^L(m\omega) \right|^2 \Psi\left(\frac{\delta p}{p}\right)$$

!

Momentum distribution function

Transverse Schottky Spectral Density

betatron sidebands

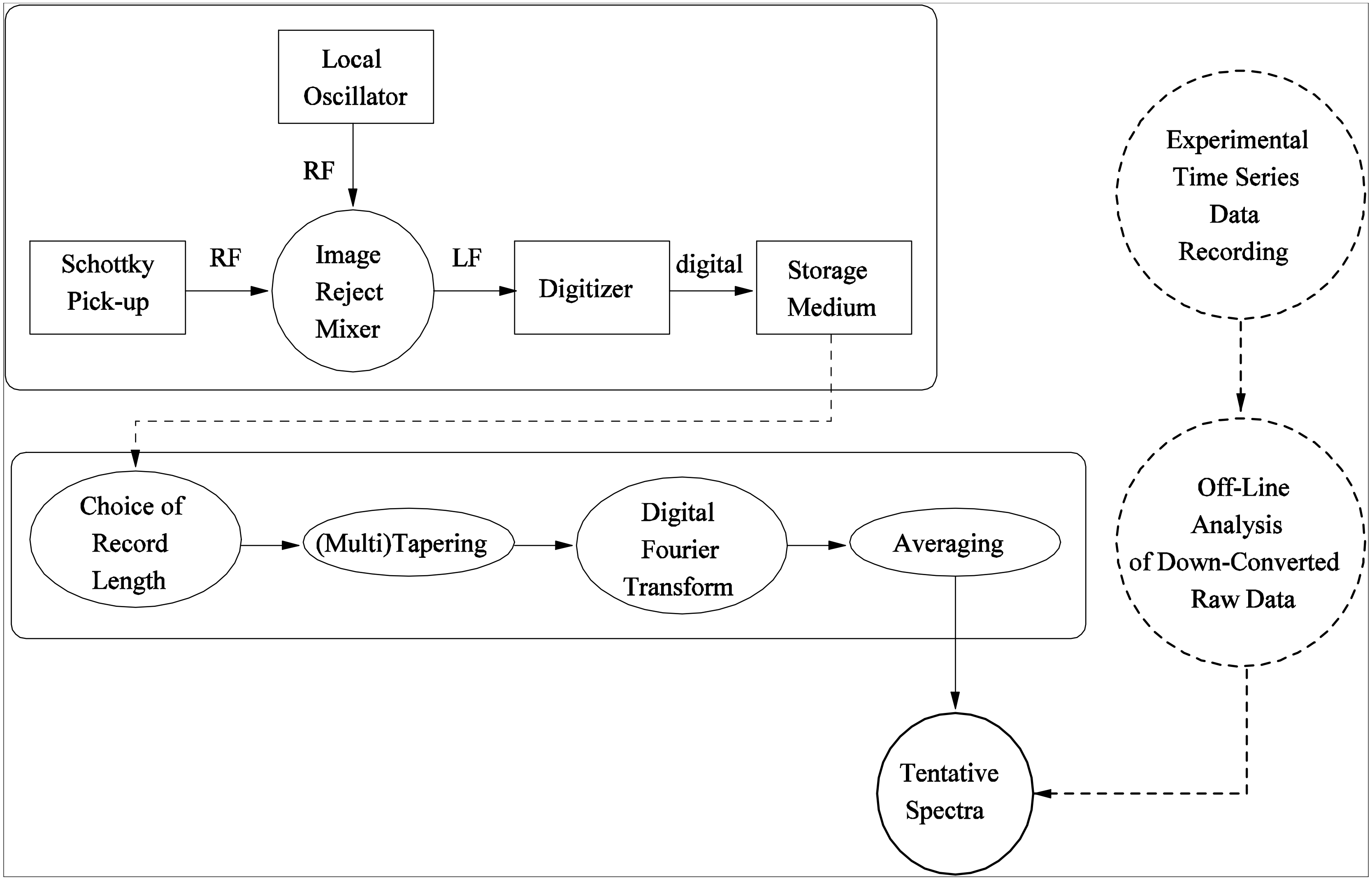
$$C_U\left(\left[m \pm Q_{x,y}\right]\omega\right) = \frac{\left(Z_L Q e\right)^2 \omega}{32\pi \left|m\eta\right|} \left\langle \beta_{x,y} E_{x,y} \right\rangle \left|S'_{x,y}\left(m\omega\right)\right|^2 \Psi\left(\frac{\delta p}{p}\right)$$

Average square of betatron amplitude at given $\dfrac{\delta p}{p}$

Power Considerations

- Total power in one longitudinal harmonic: $P \propto NQ^2$
- Total power in one betatron sideband: $P \propto NQ^2 \langle J \rangle$
- We are not dealing here with signal suppression, which is $\propto N(\delta p/p)^{-2}$

Schottky Probe with Off-line Analysis



How to get to the ultimate frequency resolution

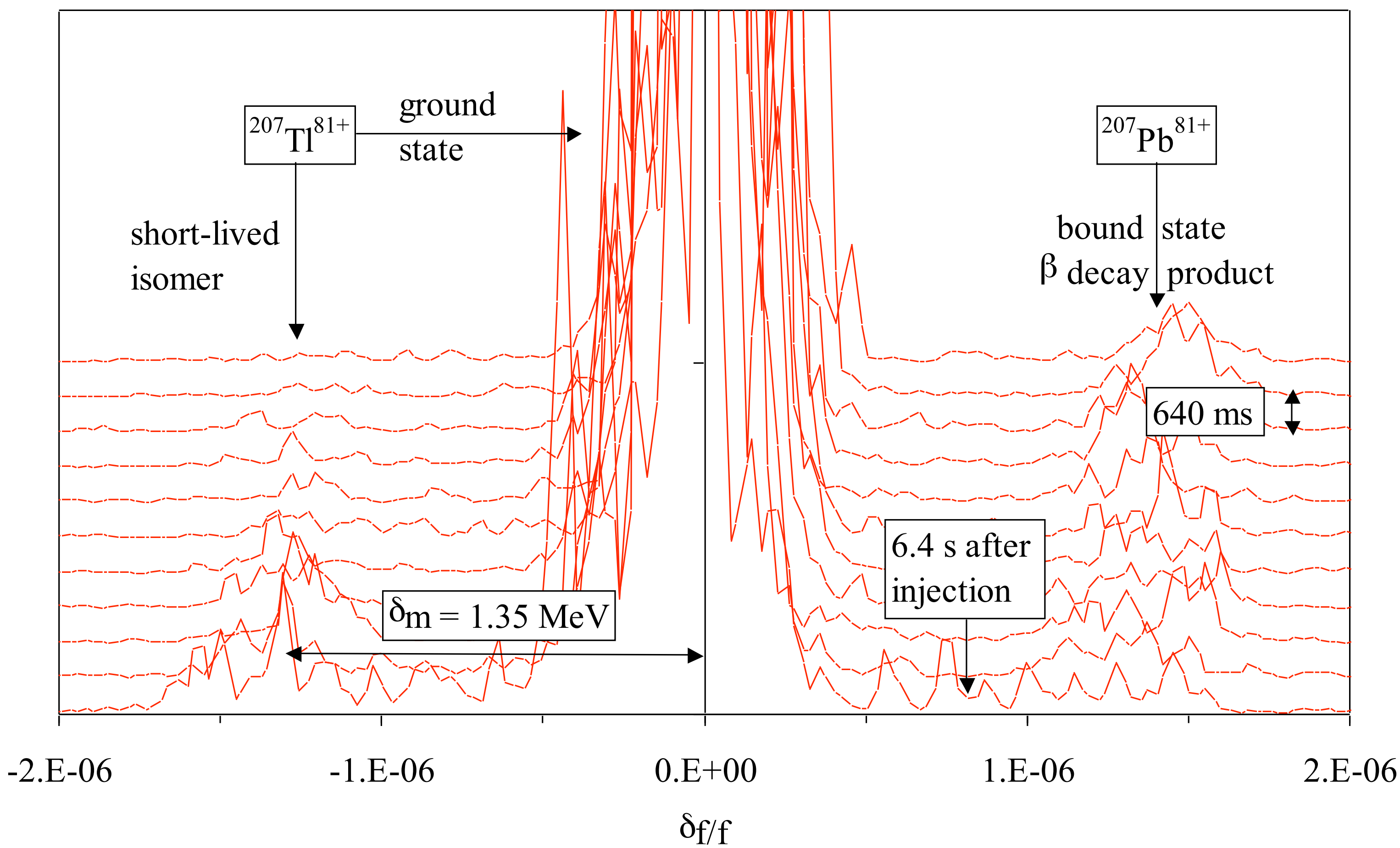
relative frequency difference in a multi-species beam: $$\frac{\delta f}{f} = \eta \frac{\delta(\beta\gamma)}{\beta\gamma} - \alpha_p \left[\frac{\delta(m/q)}{m/q} - \frac{\delta B}{B} \right]$$

<u>Properties of one-dimensional ordered beams in the ESR</u>

- Sudden drop of $\delta f / f$ as N decreases below ≈ 1000
- Typical fwhm frequency width $4 * 10^{-7}$

Frequency resolution and record length: $\delta f = \frac{2.56}{t_{record}}$

$^{207}Tl^{81+}$ Decay Spectra with Isomer



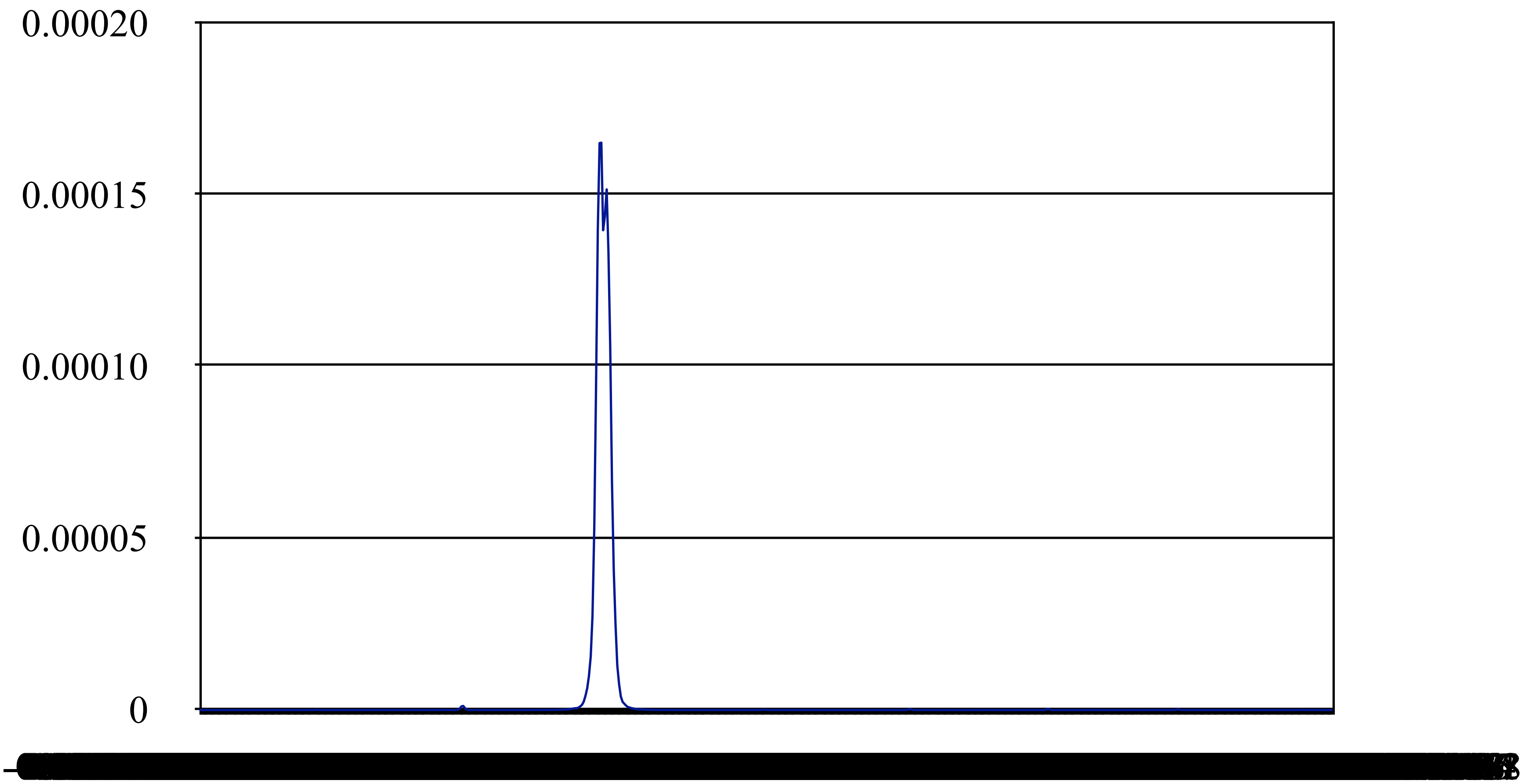
Schottky Mass Spectrometry: Resolution

Frequency separation: $$\frac{\delta f}{f} = -\alpha_p \frac{\delta (B\rho)}{B\rho}$$

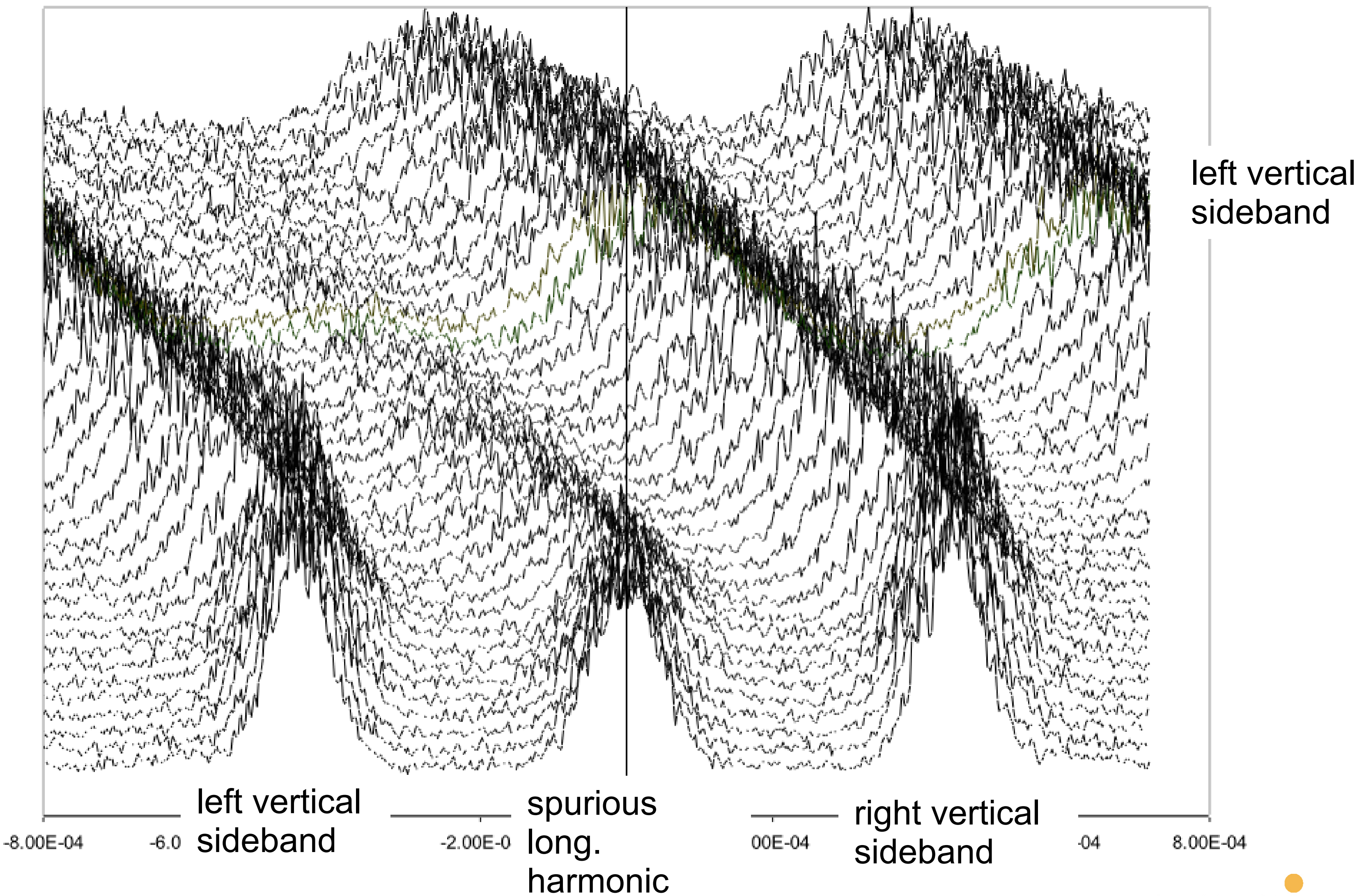
Available Resolution: $$\Delta f = \frac{2.56}{t_{record}}$$

Available Mass Resolution: $$\frac{\delta (m/q)}{m/q} \approx \frac{2.56}{|\alpha_p| f\, t_{record}}$$

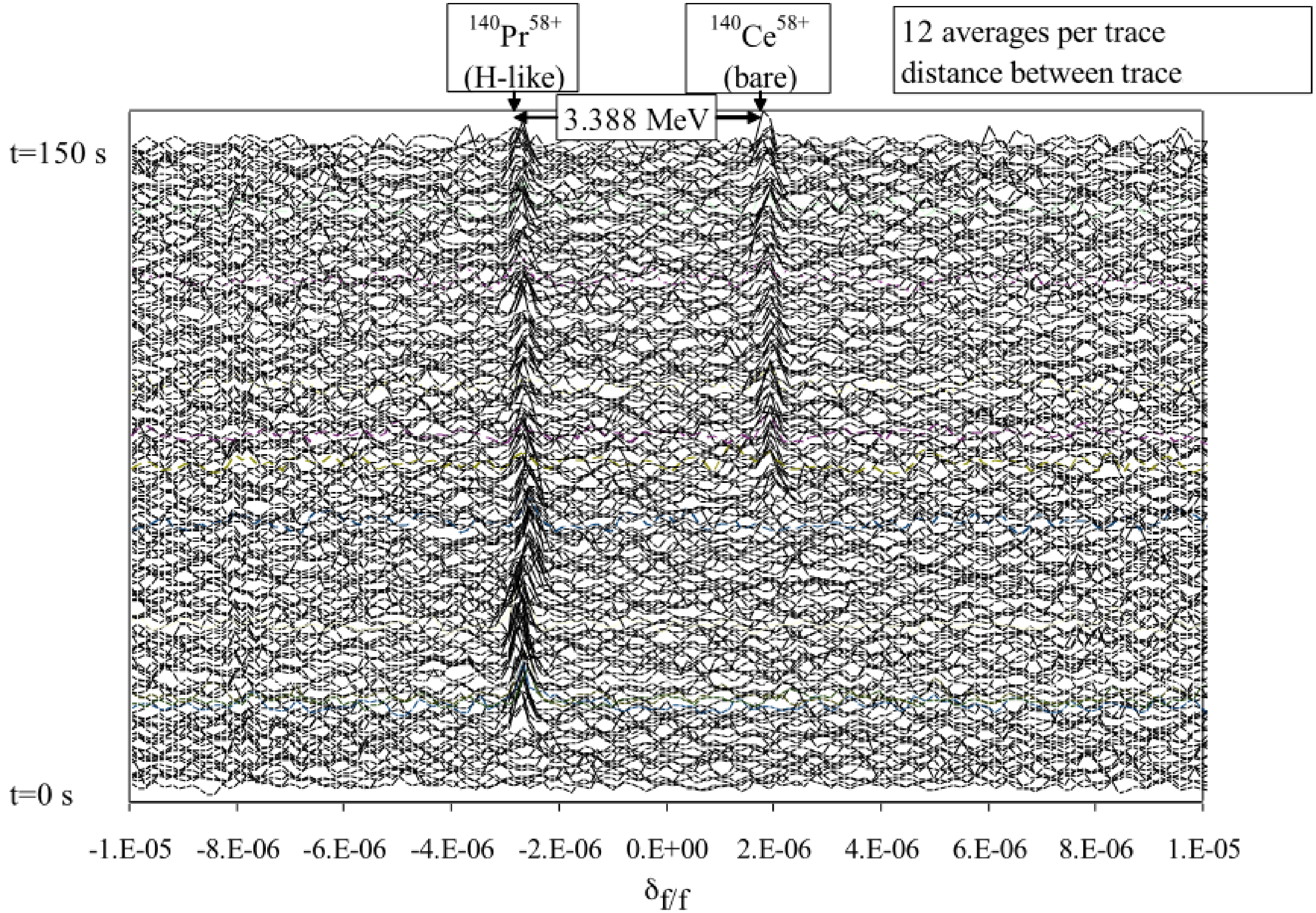
Typical Fragment Spectrum



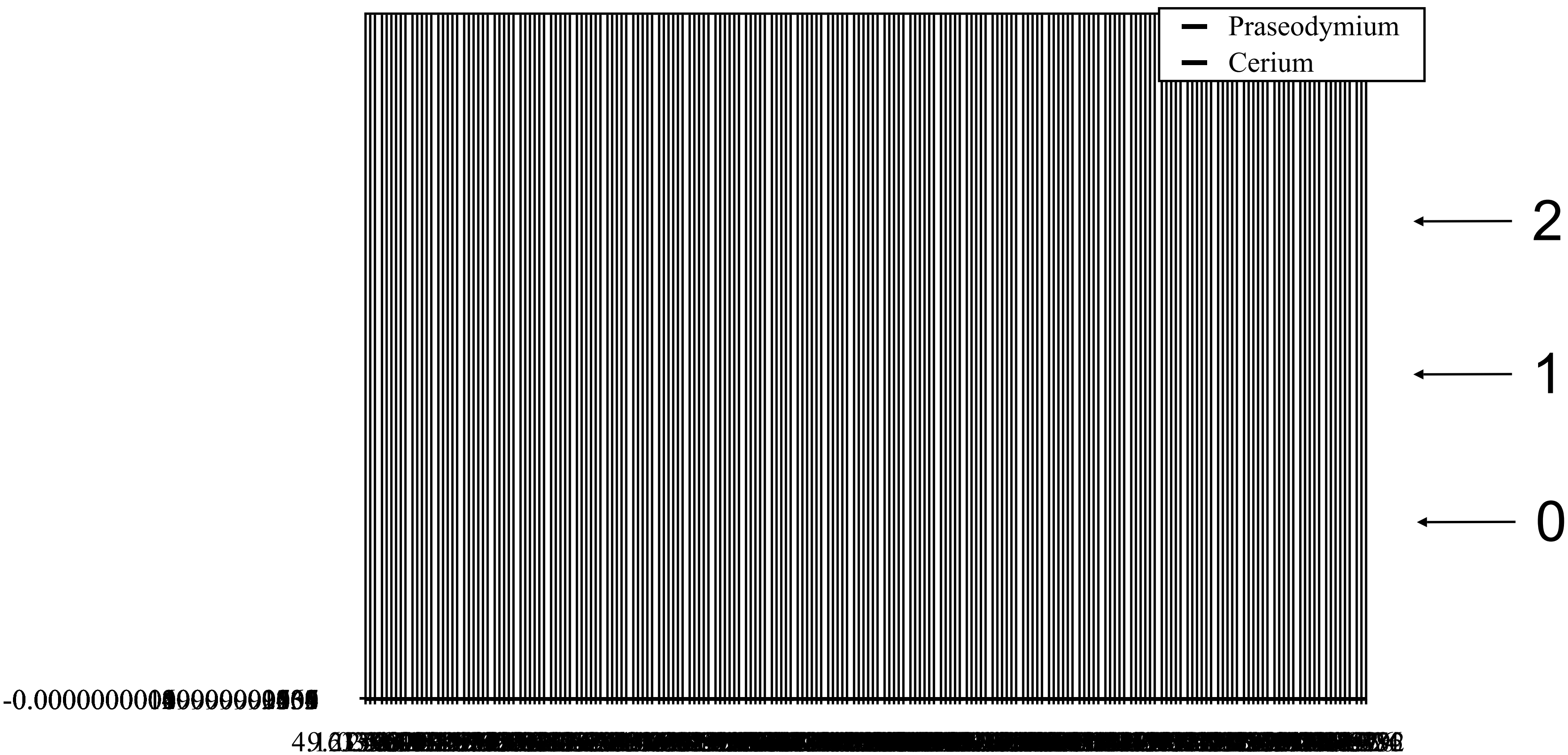
Vertical Schottky Spectra



Decay of one Ion out of Two



Particle Counting in Schottky Spectrum



Multitapering

A taper is a window in the time domain: $x_n \text{ ® } h_{nm}\, x_n$

Use orthogonal taper families: $\sum_n h_{nl}\, h_{nm} = \delta_{lm}$

yielding uncorrelated weighted spectral averages $S_j = \frac{1}{K} \sum_{k=0}^{K-1} S_{kj}$

with $$S_{kj}(f) = t_{record} \sum_{l=0}^{N-1} x_n\, h_{nk}\, \exp\left(\frac{2\pi\, i\, f\, l\, t_{record}}{N} \right)$$

DPSS Multitapers

Chose the appropriate taper family

Solution: Discrete Prolate Spheroidal Sequences

Prolate Speroidal Functions are solutions to

$$\int_{-F}^{F} \frac{\sin\left(N\pi\,(x-x')\right)}{\sin\left(\pi\,(x-x')\right)}\, h_k^F(x')\,dx' = \lambda_k\; h_k^F(x)$$

The DPSS are the discrete version of these and satisfy the energy concentration criterion